Multimodal Data Processing

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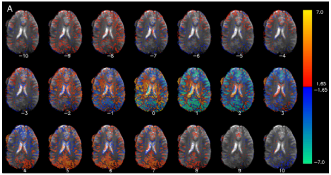
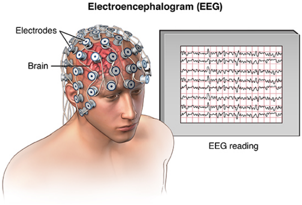
Overview

Multimodal datasets

With the increasing availability of data, many applications involve data drawn from more than one source (called *modalities*).



FMRI



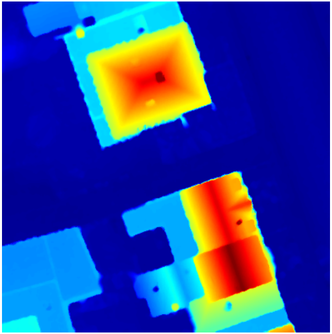
EEG

How to solve standard machine learning problems on multimodal data?

Example Multimodal Data

Remote sensing example: RGB + Elevation map of residential neighborhood in Belgium. Found in [Bampos-Taberner et al, 2016].

RGB Data



Lidar Data

Examples from the literature

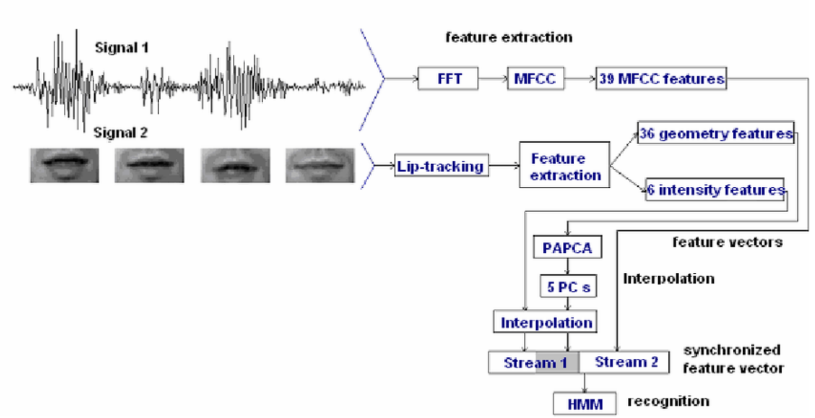
Exposure Fusion, from [Mertens et al, 2008].

(a) Exposure bracketed sequence

(b) Fused result

Examples from the literature

Audio-Visual speech recognition, from [Datcu et al, 2007].



Challenges in multimodality

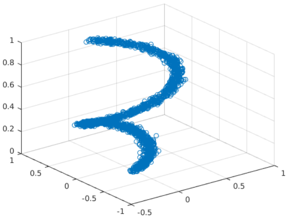
Most multimodal methods are developed specifically for one problem, BUT:

[Lahat et al, 2015]: "... a solution that is based on a sufficiently data-driven, model-free approach may turn out to be useful in very different domains."

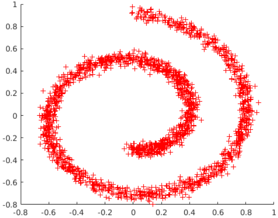
Manifold alignment

Attempt to address multimodality in general via manifold alignment.

For each modality, view the data as a manifold (have sets $X^1, X^2, \ldots, X^\ell$. $\ell =$ number of modalities).

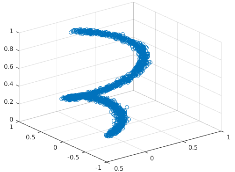


X^1

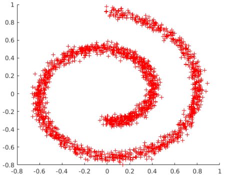


X^2

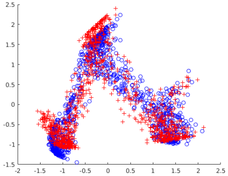
Create a *latent space* Y and maps $X^i \to Y$.



X^1



X^2



Images in latent space

Example from [Tuia et al, 2016]

Compare sets by using the latent space image.

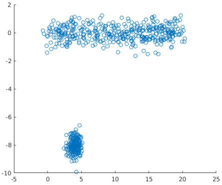
Manifold alignment: Methods from the literature

Some examples from the literature:

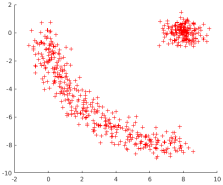
- [Yeh et al, 2014]: Canonical Correlation Analysis, linear or with nonlinear kernel (unsupervised)
- [Wang et al, 2013]: Graph-based methods (semi-supervised)
- [Tuia et al, 2016]: Similar to [Wang et al, 2013] with an added nonlinear kernel (semi-supervised)

Manifold alignment: Methods from the literature

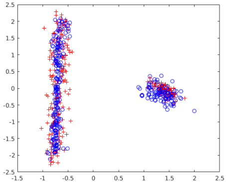
Common theme: Create the latent space by finding and correlating redundancies between sets.



X^1



X^2



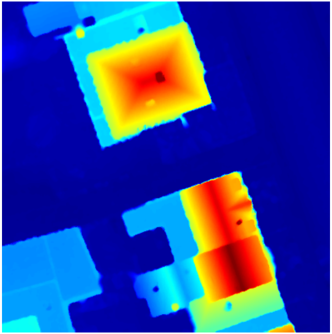
Images in latent space

Using code from [Tuia et al, 2016]

Manifold alignment: Our goal

Our idea: Can improve on these methods. Find and exploit the unique information that each modality brings.

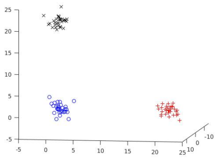
Distinguish road from grass



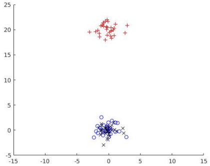
Distinguish roof from ground

Synthetic example: Data

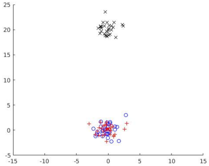
Synthetic example:
Ground truth = 3 point clouds in $\mathbb{R}^3$ (20 points per cloud).
Modality 1 = projection onto xy-plane.
Modality 2 = projection onto xz-plane.



Ground Truth



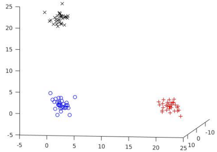
Modality 1



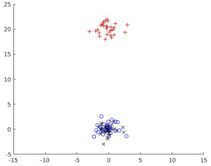
Modality 2

Synthetic example: Data

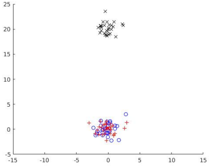
Assumption: Data is *co-registered*. *i*-th point from modality 1 corresponds to *i*-th point from modality 2.



Ground Truth



Modality 1



Modality 2

Synthetic Example: Result of CCA

Result of CCA algorithm from [Yeh et al, 2014] applied to the data:

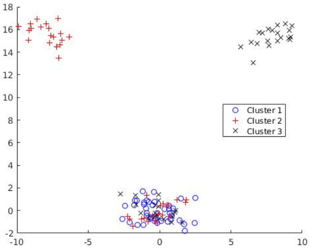


Image of clusters in latent space

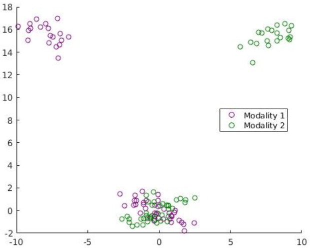
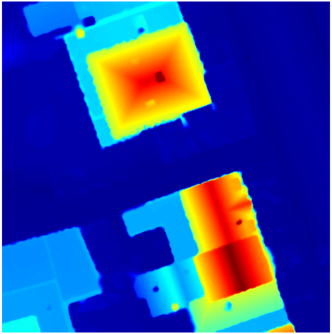


Image of data in latent space

Problem setup

We use co-registration assumption and Graph Laplacian theory for segmentation of multimodal datasets.

RGB Data



Lidar Data

Notation

From each modality, have a data set X^k. $\ell =$ number of modalities.

$N =$ number of observations.

$d_j =$ dimension of set X^k. (Can view $X^k \in \mathbb{R}^{N \times d_k}$).

From co-registration assumption: i-th point in X^{k_1} corresponds to i-th point in X^{k_2}. Create concatenated set $X = (X^1, X^2, \ldots, X^\ell) \subseteq \mathbb{R}^{N \times (d_1 + \cdots + d_\ell)}$.

$x_i =$ element i from X. $x_i^k =$ element i from X^k.

Weight Matrix: Background

For each pair $x_i, x_j \in X$, define a *weight* w_{ij} that measures the similarity between the points.

$\implies$ represent data as $N \times N$ weight matrix W.

Common similarity measure from the literature: RBF kernel

$$w_{ij} = \exp\left(-\left\|x_i - x_j\right\| / \sigma\right).$$

Need to adapt this to multimodal data.

Multimodal Weight Matrix

For each modality X^k, calculate the distance matrix E^k via

$$E_{ij}^k = \left\| x_i^k - x_j^k \right\|.$$

$\|\cdot\|$ chosen based on the details of the modality. (in our examples we use the 2-norm)

Scale each distance matrix by standard deviation

$$\bar{E}^k = \frac{E^k}{\text{std}\left(E^k\right)}.$$

Multimodal Weight Matrix

Define

$$w_{ij} = \exp\left(-\max\left(\bar{E}_{ij}^{1}, \ldots, \bar{E}_{ij}^{k}\right)/\sigma\right).$$

Heuristics:

- Standard deviation scaling allows us to directly compare $\bar{E}^{k_1}, \bar{E}^{k_2}$ with reasonable results.
- Because of the max, elements are similar under this measure only if they are similar in each modality.

Graph min cut

Using W, state the problem as graph-cut minimization.

Given a partition of X into subsets $A_1, A_2, \ldots, A_m$, we define the *normalized graph-cut*

$$\mathrm{Ncut}(A_1, \ldots, A_m) = \frac{1}{2}\sum_{k=1}^{m} \frac{W(A_k, A_k^c)}{vol(A_k)}.$$

$$W(A, B) = \sum_{i \in A, j \in B} w_{ij}.$$

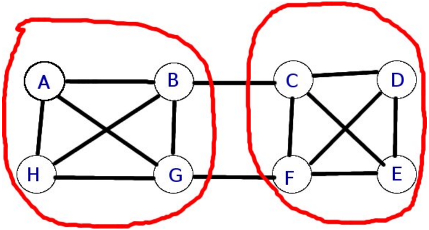
$$vol(A) = \sum_{i \in A, j \in \{1, \ldots, n\}} w_{ij}.$$

Graph min cut

$$\text{Ncut}(A_1, \ldots, A_m) = \frac{1}{2} \sum_{k=1}^{m} \frac{W(A_k, A_k^c)}{vol(A_k)}.$$

Minimize graph cut $\implies$ segment set. Compare the edges cut as a fraction of total edges.

Solving exactly is $O\left(|X|^{m^2}\right)$.



Example graph cut. $m = 2$

Graph Laplacian

Let $D = N \times N$ diagonal matrix, with

$$d_{ii} = \sum_{j=1}^{n} w_{ij}.$$

Graph Laplacian

$$L = D - W.$$

Graph Laplacian

From $A_1, \ldots, A_m$, get $H = N \times m$ indicator matrix.

$$H_{ij} = \begin{cases} \frac{1}{\sqrt{vol(A_j)}} & \text{if } x_i \in A_j \\ 0 & \text{else} \end{cases}$$

Columns of $H \iff$ classes. Rows of $H \iff$ data points.

$$\begin{aligned} \mathrm{Ncut}(A_1, \ldots, A_m) &= \frac{1}{2} \sum_{i=1}^{m} \frac{W(A_i, A_i^c)}{vol(A_i)} \\ &= \mathrm{Tr}\left(H^T L H\right). \end{aligned}$$

Relaxed graph min cut

Optimal graph cut is

$$\operatorname{argmin}_{H \text{ an indicator matrix}} \operatorname{Tr}\left(H^T L H\right).$$

This is an $O\left(|X|^{m^2}\right)$ problem. Instead we solve the relaxed problem:

$$\operatorname{argmin}_{H \in \mathbb{R}^{N \times m},\ H^T H = I} \operatorname{Tr}\left(H^T L H\right).$$

Solution: Columns of H = eigenvectors of L with smallest eigenvalues.

Relaxed graph min cut

In relaxed problem,

$$\begin{aligned} \text{columns of } H &\iff \text{features} \\ \text{rows of } H &\iff \text{data points.} \end{aligned}$$

Can use features for a variety of applications.

Our code: K-means on feature vectors $\rightarrow$ classification (this is called Spectral Clustering).

Nyström Extension

As $|X|$ becomes large, computing the $|X| \times |X|$ weight matrix W becomes prohibitive.

Instead choose $A \subseteq X$ *landmark nodes* with $|A| \ll |X|$. Up to permutation, we have

$$W = \begin{pmatrix} W_{A,A} & W_{A,A^c} \\ W_{A^c,A} & W_{A^c,A^c} \end{pmatrix}.$$

Nyström Extension

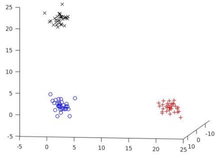
Nyström: Approximate Graph Laplacian eigenvectors using only $W_{A,A}$, $W_{A^c,A}$.

$$W \approx \begin{pmatrix} W_{A,A} \\ W_{A^c,A} \end{pmatrix} W_{AA}^{-1} \begin{pmatrix} W_{A,A} & W_{A,A^c} \end{pmatrix}.$$

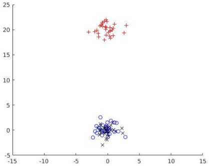
Compute and store matrices of size at most $|X| \times |A|$.

Synthetic example: Data

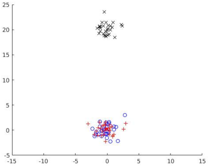
Synthetic example:
Ground truth $= 3$ point clouds in $\mathbb{R}^3$ (20 points per cloud).
Modality 1 $=$ projection onto xy-plane.
Modality 2 $=$ projection onto xz-plane.



Ground Truth



Modality 1



Modality 2

Synthetic Example: Result of Our Method

Result of our multimodal graph-based algorithm applied to the data:

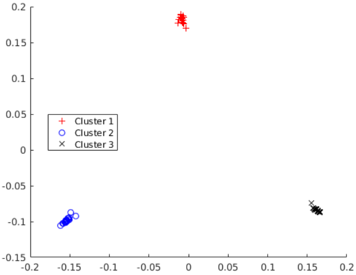
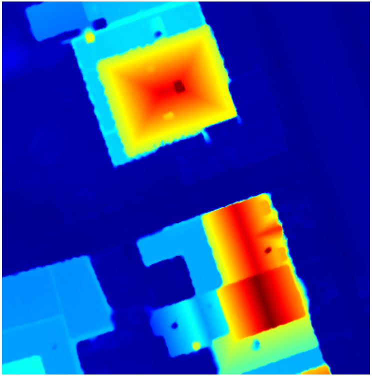


Image of clusters in latent space

Data

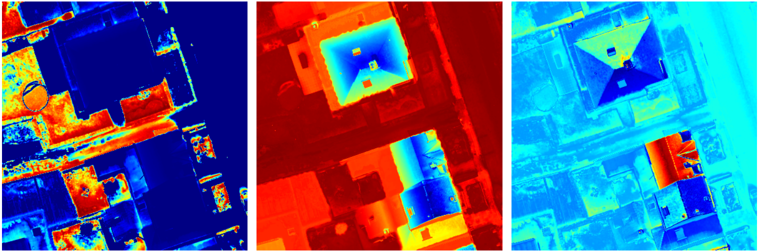
Our algorithm applied to [Bampos-Taberner et al, 2016] dataset.

RBG Modality



Lidar Modality

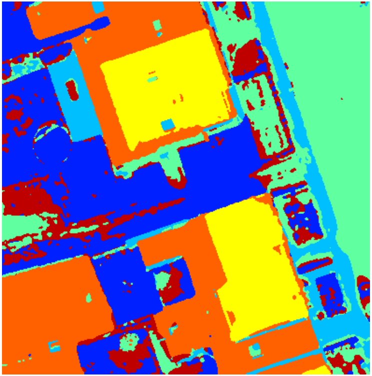
Results



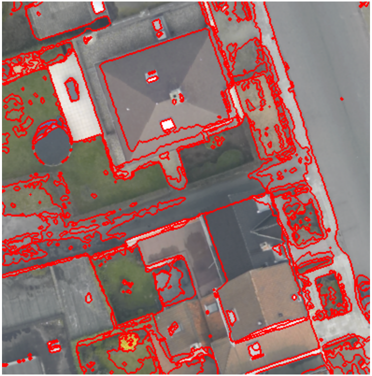
Example eigenvectors of Graph Laplacian

Results

Spectral Clustering result (unsupervised). $m = 6$ classes.



Classes

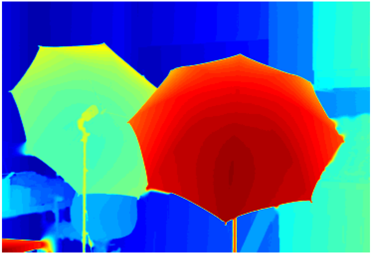


Regions on original image

Data

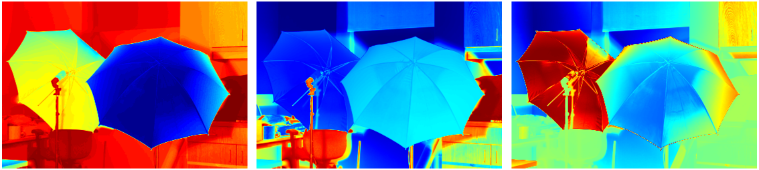
Our algorithm applied to [Scharstein et al. 2014] dataset.

RBG Modality



Lidar Modality

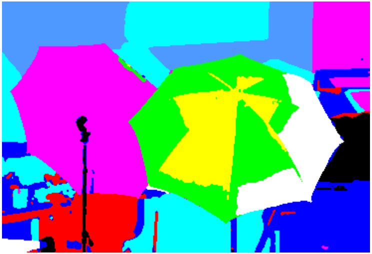
Results



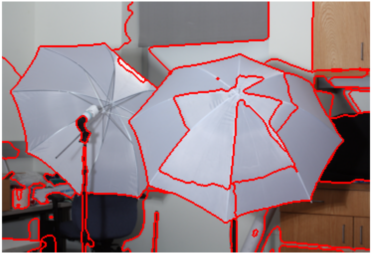
Example eigenvectors of Graph Laplacian

Results

Spectral Clustering result (unsupervised). $m = 8$ classes.



Classes



Regions on original image

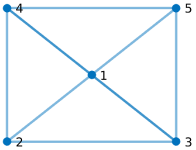
Future Work

Goal: Remove or weaken the coregistration assumption.

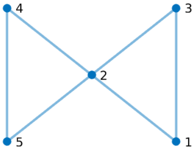
Current idea: Graph matching.

View each dataset as a (weighted) graph. Try to match nodes with similar structure.

Graph Matching Example



Graph 1



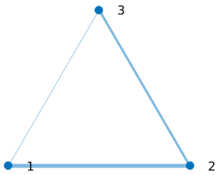
Graph 2

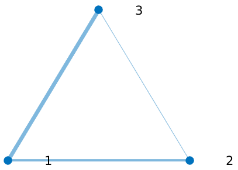
A reasonable matching would send $1 \to 2$. Difficult to match other nodes due to symmetry.

Problem Setup

Two weighted graphs, G_1, G_2, with weights W_1, W_2. Assume $|G_1| = |G_2| = N$

Search for an isomorphism of graphs $G_1 \to G_2$ that preserves the weights.





Best isomorphism is $1 \to 3$, $2 \to 1$, $3 \to 2$.

Problem Setup

Isomorphism $G_1 \to G_2$ corresponds to a permutation on nodes. Have P the corresponding permutation matrix. Want to minimize

$$\left\| P W_1 P^T - W_2 \right\|_F^2 .$$

Exact solution is too expensive. Can solve using Graph Laplacian trick from [Umetama 1988, Knossow et al. 2009].

Relaxation

Relax problem to

$$Q^* = \operatorname{argmin}_{QQ^T=I} \left\| QW_1Q^T - W_2 \right\|_F^2 .$$

Let L_1, L_2 the Graph Laplcians corresponding to W_1, W_2

U_1, U_2 the corresponding matrices of eigenvectors.

Then $Q^* = U_1 S U_2^T$.

S is a diagonal matrix with entries of ± 1 to account for sign ambiguity in eigenvectors.

Heuristics

Recall from Graph Laplacian

$$\begin{aligned} \text{columns of } U_i &\iff \text{features} \\ \text{rows of } U_i &\iff \text{data points.} \end{aligned}$$

Match rows of U_1 to rows of U_2 by considering $U_1 U_2^T$.

Matching Algorithm

Q^*_{ij} gives the similarity between node i from G_1 and node j from G_2.

Choose a matching $p : \{1, 2, \ldots, N\} \to \{1, 2, \ldots, N\}$ by maximizing

$$\sum_{i=1}^{N} Q^*_{i,p(i)}.$$

Hungarian algorithm finds this in $O(N^3)$.

Example Calculation

Say we have $N = 6$ and calculated:

$$Q^* = \begin{pmatrix} -0.1629 & -0.1711 & -0.1703 & 0.3426 & 0.3717 & -0.2100 \\ -0.1647 & -0.1662 & -0.1677 & 0.2966 & 0.3192 & -0.1172 \\ -0.1660 & -0.1653 & -0.1657 & -0.1477 & -0.1861 & 0.8308 \\ -0.4579 & 0.6860 & 0.2665 & -0.1787 & -0.1480 & -0.1678 \\ 0.4939 & -0.1039 & 0.1196 & -0.6689 & 0.3080 & -0.1486 \\ 0.4577 & -0.0795 & 0.1176 & 0.3561 & -0.6647 & -0.1872 \end{pmatrix}$$

Then

$$P^* = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

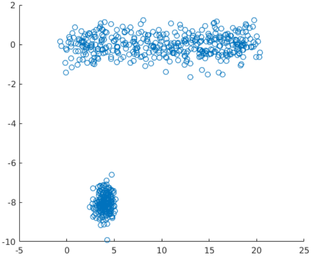
Benefits of Graph Matching

A precise number representing similarity between nodes gives us many options.

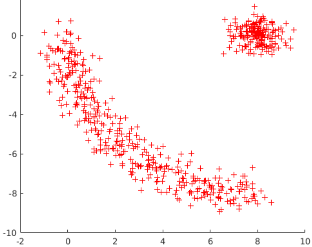
1. Thresholding
2. Many-to-many matching
3. Hierarchical matching
4. etc.

Example Matching

Recall from earlier.



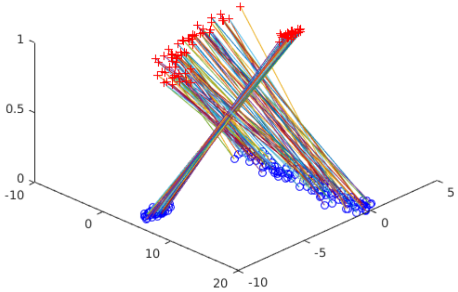
X^1



X^2

Synthetic Dataset

Example Matching



Result of our code

References I

Tom Mertens and J. Kautz and Frank Van Reeth
Exposure Fusion: A Simple and Practical Alternative to High Dynamic Range Photography
Computer Graphics Forum 28(1):161 - 171

D. Datcu and Z. Yang and L. Rothkrantz (2007)
Multimodal workbench for automatic surveillance applications
2007 IEEE Conference on Computer Vision and Pattern Recognition 1-2

Lahat, Dana and Adalı, Tülay and Jutten, Christian (2015)
Multimodal Data Fusion: An Overview of Methods, Challenges and Prospects
Proceedings of the IEEE 103(9), 1449-1477

Yi-Ren Yes and Chun-Hao Huang and Yu-Chiang Frank Wang
Heterogeneous Domain Adaptation and Classification by Exploiting the Correlation Subspace
IEEE Transactions on Image Processing 23(5), 2009-2018

References II

- Wang, Chang and Mahadevan, Sridhar (2013)
  Manifold Alignment Preserving Global Geometry
  Proceedings of the Twenty-Third International Joint Conference on Artificial Intelligence IJCAI '13, 1743-1749

- Tuia, Devis AND Camps-Valls, Gustau (2016)
  Kernal Manifold Alignment for Domain Adaptation
  PLOS ONE 11, 1-25

- M. Campos-Taberner and A. Romero-Soriano and C. Gatta and G. Camps-Valls and A. Lagrange and B. Le Saux and A. Beaupre and A. Boulch and A. Chan-Hon-Tong and S. Herbin and H. Randrianarivo and M. Ferecatu and M. Shimoni and G. Moser and D. Tuia (2015)
  Processing of Extremely High-Resolution LiDAR and RGB Data: Outcome of the 2015 IEEE GRSS Data Fusion Contest #8211;Part A: 2-D Contest
  IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing 9(12), 5547-5559

References III

Daniel Scharstein and Heiko Hirschmüller and York Kitajima and Greg Krathwohl and Nera Nešić and Xi Wang and Porter Westling (2014)
High-resolution stereo datasets with subpixel-accurate ground truth
Proceedings of the 36th German Conference on Pattern Recognition 31-42

Umeyama, S. (1988)
An Eigendecomposition Approach to Weighted Graph Matching Problems
IEEE Trans. Pattern Anal. Mach. Intell., 10(5), 695-703

Knossow, David and Sharma, Avinash and Mateus, Diana and Horaud, Radu" (2009)
Inexact Matching of Large and Sparse Graphs Using Laplacian Eigenvectors
Graph-Based Representations in Pattern Recognition: 7th IAPR-TC-15 International Workshop, 2009. Proceedings