Research Project

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1 Introduction

With the increasing availibility of data we often come upon multiple datasets, derived from different sources, that describe the same object or phenomenon. We call the different sources *modalities*, and because each modality represents some new information, it is generally desirable to use more modalities rather than fewer. For example, in the area of speech recognition, researchers have found that integrating the audio data with a video of the speaker results in a much more accurate classification [1]. Similary, in medicine, the authors of [2] fuse the results of two different types of brain imaging to create a final image with better resolution than either of the originals. However, a correctly processing a multimodal dataset is not a simple task. Even the naive method of analyzing each modality separately still requires clever thinking when deciding how to combine the results, and this is rarely the optimal way to handle the data. Our goal is to create a general algorithm for feature extraction and data segmentation that can be applied to any multimodal dataset.

In the current state of the project, we consider the case where each dataset contains the same number of elements, and these elements are co-registered (so the i-th point in one set corresponds to the i-th point in another). This is often the case in image processing problems, where the sets may be images of the same scene obtained from different sources (as is the case in our experimental data), or taken at a different times. For notation, we label the sets, $X^1, X^2, \ldots, X^k$, with dimensions $d_1, d_2, \ldots, d_k$, and let $X = (X^1, X^2, \ldots, X^k) \subset \mathbb{R}^{n\times(d_1+\cdots+d_k)}$ be the concatenated dataset. Our method extracts features from the dataset by finding eigenvectors of the graph Laplacian, then uses standard data-segmentation algorithms on these features to obtain a final classification. In section 2 we give the general theory behind our method, and in 3 we show the results of the method applied to an optical/LIDAR dataset. Finally, in section 4 we discuss the extensions of the project that we hope to complete in France.

2 The Method

2.1 The Graph Min-Cut Problem

We represent our dataset X as a *similarity matrix*. That is, for each two data points x_i, x_j, we define a *weight* w_{ij} representing the similarity between the points. A large weight corresponds to very similar nodes, and a small weight to dissimilar nodes. There are many different choices of the weights w_{ij} in the literature, and each has its own merits. In many applications, one defines

$$w_{ij} = \exp\left(-\left\|v_i - v_j\right\|/\sigma\right),$$

where σ is a scaling parameter. In this work we adapt this definition to apply to our multimodal dataset. First we scale our sets $X^1, \ldots, X^k$ to make distances in each set comparable, then we define.

$$w_{ij} = \exp\left(-\max\left(\left\|x_i^1 - x_j^1\right\|, \ldots, \left\|x_i^k - x_j^k\right\|\right)\right).$$

Choosing to use the maximum of the individual values allows us to take advantage of the unique information in each dataset, as two points are considered similar here when they are similar in every modality.

Moving forward, we aim to find a classification that groups pairs with high weight together, while also separating pairs with low weight. This goal is formalized as the Ratio Cut problem. Given a partition of X into subsets $A_1, A_2, \ldots, A_m$, we define the *ratio graph-cut*

$$\mathrm{RatioCut}(A_1, \ldots, A_m) = \sum_{i=1}^{m} \frac{W(A_i, A_i^c)}{|A_i|},$$

where $W(A,B) = \sum_{i \in A, j \in B} w_{ij}$. Here the $W(A_i, A_i^c)$ term penalizes partitions that separate elements with a large weight between them, while the $|A_i|$ term ensures that each segment of the final partition is of a reasonable size (without the $|A_i|$ term, the optimal solution often contains one large set and $m-1$ small sets). It has been shown in [3] that explicitly solving this problem is an $O(|V|^{m^2})$ process. As this is infeasible in most cases, we instead introduce the graph Laplacian to handle an approximation of the minimization problem.

2.2 Graph Laplacian and Clustering

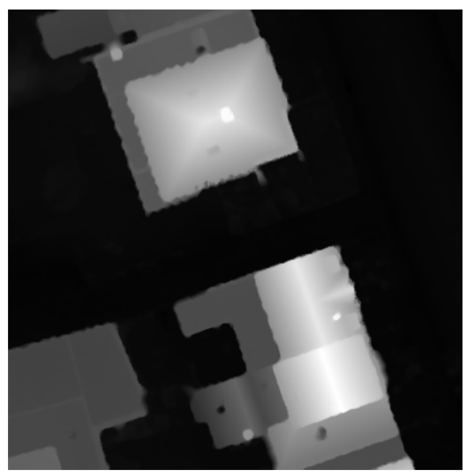
In [4] it is shown that the Ratio Cut problem can be approximately solved using the eigenvectors of the *graph Laplacian*, $L = D - W$. Here D is a diagonal matrix with entries $d_{ii} = \sum_j w_{ij}$. Each eigenvector represents a feature of the data, and if we let H be a matrix where the columns are eigenvectors, then the ith row of H represents the features of the data point x_i. We then get an approximate solution to the original min-cut problem by using any data clustering algorithm on these rows. In section 3 we use k-means to segment the row vectors, resulting in a well-known algorithm called *spectral clustering*.

Calculating the full graph Laplacian is computationally intensive, as the matrix contains n^2 entries. Instead we use Nyström's extension to find approximate eigenvalues and eigenvectors with a heavily reduced computation time. Essentially, this consists of choosing a small number $m \ll n$ of columns of the weight matrix, and using some clever linear algebra to approximately solve the eigenproblem using only these columns. This results in a significant reduction in computation time, as we compute and store matrices of size at most $m \times n$, rather than $n \times n$. See [5], [6] for a more complete discussion of this method. In practice, m can be chosen to be quite small without creating significant error. In Section 3 we use $m = n^{\frac{1}{4}}$.

3 Experiment

We test our algorithm on an Optical/Lidar dataset from the 2015 IEEE Data Fusion Contest. The data consists of an RBG image and an elevation map of a residential neighborhood in Belgium. Each picture contains roughly 160,000 pixels.

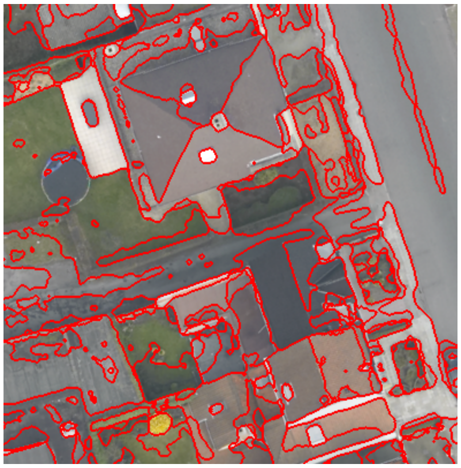
(a) Optical image



(b) Lidar image

Figure 1: Original Data

In figure 2 we show the results of spectral clustering applied to each dataset individually, and the final results of our algorithm are pictured in figure 3. These images show the importance of the multimodal approach, as the optical-only method is unable to differentiate the dark-grey roof from the adjacent street, and the lidar-only method cannot separate the white sidewalk from the green grass. In figure 3a we show an example eigenvector of the graph Laplacian. As explained in 2.2, this eigenvector represents one feature



(a) Optical-only classification

(b) Lidar-only classification

Figure 2: Single-set classifications

(a) Example eigenvector of our method

(b) Our clustering result

Figure 3: Experimental results

extracted from our dataset. Notice how the dark-grey street is highlighted, while both the light-grey sidewalk (which is at the same elevation) and the nearby roof (which is the same color) are blacked out. This difference in the feature-vector leads to the corresponding separation in the final clustering result in figure 3b.

4 Future Work

Our current algorithm allows us to perform feature extraction and data segmentation on multimodal sets, as desired, but the assumption that the datasets are *co-registered* is quite restrictive. In section 3 our two images are of the same underlying scene, where pixels correspond exactly between images. We could not, for example, process two images taken from different angles. Our goal for the future is to remove this restriction, so that our final algorithm can be applied to any dataset.

References

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