Graph-Based Multimodal Data Processing

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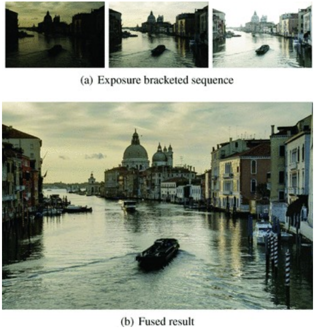
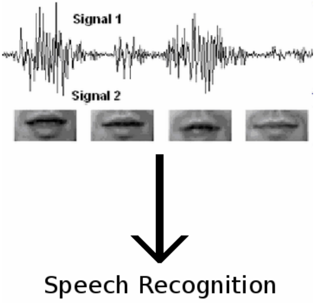
Overview

Multimodal Datasets

With the increasing availability of data, many applications involve data drawn from more than one source (called *modalities*).



Exposure Fusion:
[Mertens et al, CGF, 2008]

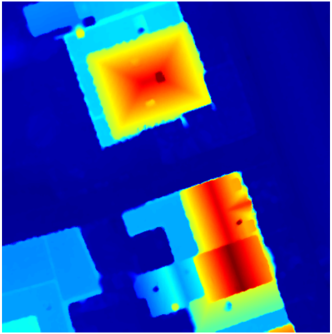


Speech Recognition:
[Datcu et al, IEEE CVPR 2007]

Example Multimodal Data

Remote sensing example: RGB + Elevation map.
From 2015 IEEE Data Fusion Contest.
[Bampos-Taberner et al, IEEE J-STARS 2016]

RGB Data



Lidar Data

Challenges in Multimodality

Most multimodal methods are developed specifically for one problem, BUT:

[Lahat et al, IEEE 2015]: "... a solution that is based on a sufficiently data-driven, model-free approach may turn out to be useful in very different domains."

Our approach:
Represent data via a graph, compare graph representations.

Graph Representation: Background

For a dataset X with N elements

For each pair $x_i, x_j \in X$, define a *weight* $w_{ij} \geq 0$ that measures the similarity between the points.

$\implies$ represent data as $N \times N$ weight matrix W.

Common similarity measure from the literature: RBF kernel

$$w_{ij} = \exp\left(-\left\|x_i - x_j\right\|^2 / \sigma\right).$$

Get a representation of X as a weighted graph (V, E).

Graph Laplacian

Perform analysis via the *Graph Laplacian* [Chung, AMS, 1997]

$$L = D - W.$$

$D =$ diagonal degree matrix, $d_{ii} = \sum_{j=1}^{N} w_{ij}$.

View L as an operator on the space of functions $f : V \to \mathbb{R}$.
Related to the standard differential Laplacian $\Delta = \nabla^2$.

A manifold $\mathcal{M}$ can be discretized and represented by a graph.

Under mild assumptions and some renormalization,
L converges to Δ as mesh size $\to 0$.

Graph Laplacian vs Differential Laplacian

Ex: A 1-D line graph. L is related to the discrete Laplacian.

$$L = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 2 & -1 & 0 & 0 & \cdots \\ \cdots & -1 & 2 & -1 & 0 & \cdots \\ \cdots & 0 & -1 & 2 & -1 & \cdots \\ \cdots & 0 & 0 & -1 & 2 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

1-D discrete Laplacian with stepsize h:

$$\Delta f(x) \approx \frac{-f(x-h) + 2 \cdot f(x) - f(x+h)}{h^2}.$$

Laplacian Eigenfunctions and Fourier Modes

Eigenfunctions of differential Laplacian give modes of vibration.
Corresponding eigenvalues describe the energy of the state.

Ex: On the interval $[0, 1]$, eigenfunctions are solutions to

$$\Delta f = \frac{d^2 f}{dx^2} = -\lambda f.$$

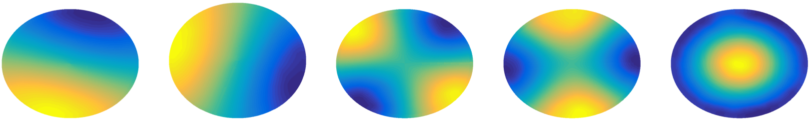
These are exactly the normal Fourier modes

$$\{\sin(k\pi x), \cos(\ell\pi x) \,:\, k \geq 1, \ell \geq 0\}.$$

Eigenvectors of Graph Laplacian give a discrete version.
Allow us to study symmetries and structure of the graph.

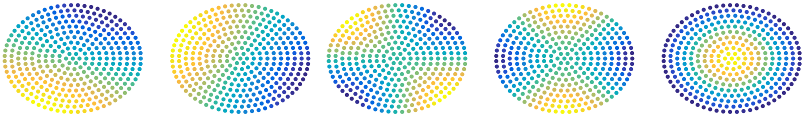
Laplacian Eigenfunctions and Fourier Modes

Example: A disc in $\mathbb{R}^2$, and a graph representation.



v_2 v_3 v_4 v_5 v_6

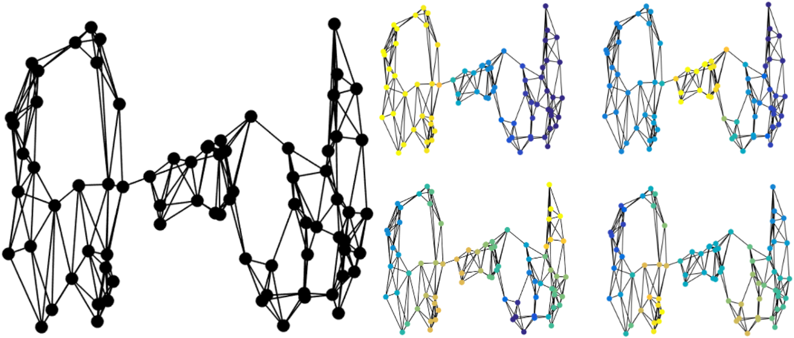
Differential Laplacian Eigenfunctions



v_2 v_3 v_4 v_5 v_6

Graph Laplacian Eigenvectors

Laplacian Eigenvectors and Fourier Modes



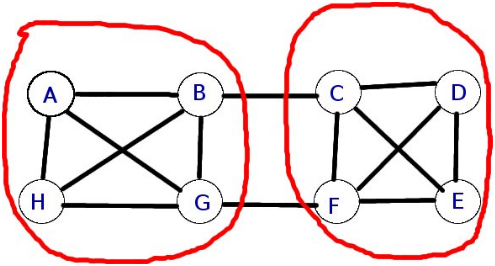
Synthetic Graph and Example Eigenvectors

Graph Min-Cut Problem

Graph Laplacian can also be understood via graph-cuts.
Partition V into k sets $A_1, \ldots, A_k$ minimizing graph-cut energy.

$$\text{Cut}(A_1, \ldots, A_k) = \frac{1}{2} \sum_{j=1}^{k} W(A_j, A_j^c).$$

$$W(A, B) = \sum_{i \in A, j \in B} w_{ij}.$$



Graph Laplacian and Min-Cut

Notation: $u = |V| \times k$ indicator matrix for $A_1, \dots, A_k$.

$$u_{ij} = \begin{cases} 1 & \text{if node } i \text{ is in set } A_j \\ 0 & \text{else.} \end{cases}$$

Then

$$Cut(A_1, \dots, A_k) = \operatorname{Tr}\left(u^T L u\right).$$

So we rephrase our min-cut problem as:

$$\operatorname{argmin}_{u \text{ an indicator matrix}} \operatorname{Tr}\left(u^T L u\right).$$

Laplacian Eigenvectors and Min-Cut

Exact min-cut solution is computationally infeasible.

One popular relaxation: find

$$\mathrm{argmin}_{u^T u = I} \mathrm{Tr}\left(u^T L u\right).$$

Solution: columns of u are eigenvectors of L corresponding to smallest eigenvalues.

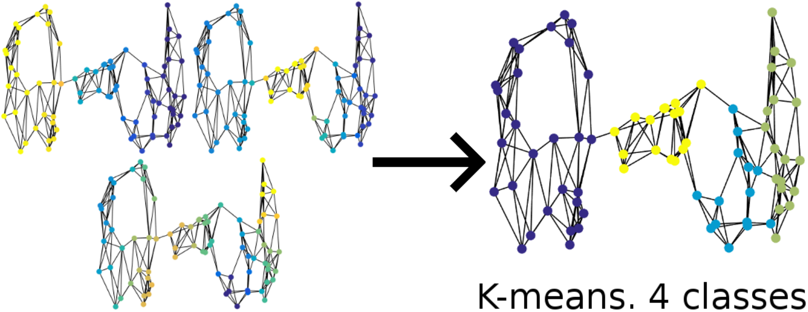
Heuristic [vonLuxburg, Stat Comput 2007]:

$$\text{eigenvectors of } L \iff \text{features extracted from data}$$

Laplacian Eigenvectors and Min-Cut

Can use eigenvectors for a variety of applications.

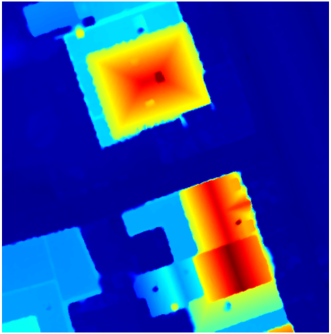
Ex: K-means on eigenvectors $\rightarrow$ segmentation:
(called Spectral Clustering [vonLuxburg, Stat Comput 2007])



Problem Setup

Perform segmentation on *coregistered*, multimodal datasets.
[Iyer et al., ICIP 2017]

RGB Data



Lidar Data

Example Multimodal Data

Problem Setup

From each modality, have a data set X^k. $N = \left|X^1\right| = \cdots = \left|X^k\right|$.

From co-registration assumption:
i-th point in X^{k_1} corresponds to i-th point in X^{k_2}.

x_i^k = element i from X^k.

For each modality X^k, calculate the distance matrix E^k via

$$E_{ij}^k = \left\| x_i^k - x_j^k \right\| .$$

$\|\cdot\|$ chosen based on the details of the modality.
(often $\|\cdot\|$ is the L^2-norm)

Multimodal Weight Matrix

Scale each matrix by standard deviation (nondimensionalization)

$$\bar{E}^k = \frac{E^k}{\operatorname{std}\left(E^k\right)}.$$

Define

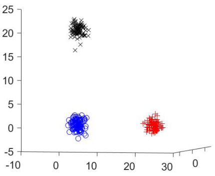
$$w_{ij} = \exp\left(-\max\left(\bar{E}^1_{ij}, \ldots, \bar{E}^k_{ij}\right)/\sigma\right).$$

Using this weight matrix and Graph Laplacian theory, perform *Spectral Clustering* and *Semisupervised Graph MBO* [Merkurjev, SIIMS 2013] to segment our datasets.

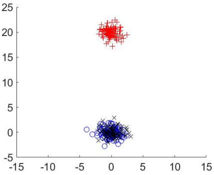
Synthetic Example: Data

Why use max?

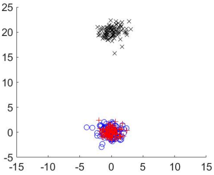
Ground truth = 3 point clouds in $\mathbb{R}^3$ (100 points per cloud).
Modality 1 = projection onto xy-plane.
Modality 2 = projection onto xz-plane.
Co-registration assumption: index is input to algorithm.



Underlying Data



Modality 1

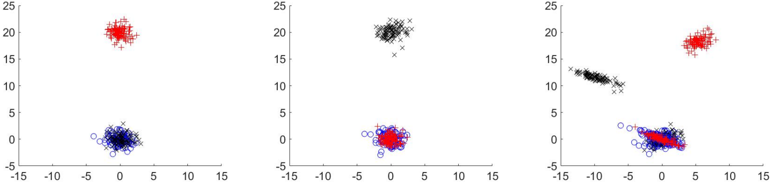


Modality 2

Synthetic Example: Result of CCA

Result of CCA algorithm from [Yeh et al, IEEE TIP 2014]:

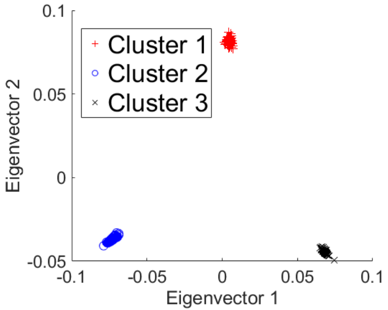
Both modalities are mapped to a common *latent space*.
(classification can then be done in this space.)



Modality 1 Modality 2 Image in latent space

Synthetic Example: Result of Our Method

Result of our method:
First 2 Laplacian Eigenvectors, plotted in $\mathbb{R}^2$.



2 eigenvectors of graph Laplacian

Nyström Extension

As $|X|$ becomes large, computing the $|X| \times |X|$ weight matrix W becomes prohibitive.

Instead choose $A \subseteq X$ *landmark nodes* with $|A| \ll |X|$. Up to permutation, we have

$$W = \begin{pmatrix} W_{A,A} & W_{A,A^c} \\ W_{A^c,A} & W_{A^c,A^c} \end{pmatrix}.$$

[Fowlkes et al., IEEE TPAMI 2004]:
Approximate graph Laplacian eigenvectors using only $W_{A,A}$, $W_{A^c,A}$.

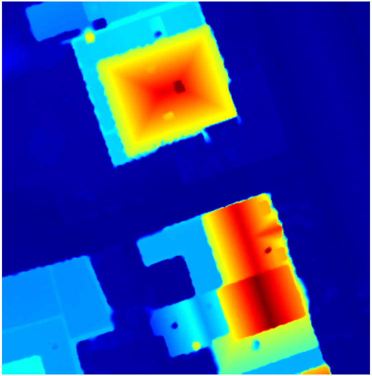
$$W \approx \begin{pmatrix} W_{A,A} \\ W_{A^c,A} \end{pmatrix} W_{AA}^{-1} \begin{pmatrix} W_{A,A} & W_{A,A^c} \end{pmatrix}.$$

Compute and store matrices of size at most $|X| \times |A|$.

DFC2015 Data

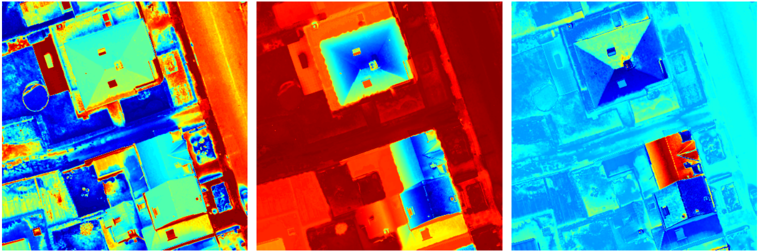
Our algorithm applied to DFC 2015
[Bampos-Taberner et al, IEEE J-STARS 2016].

RGB Modality



Lidar Modality

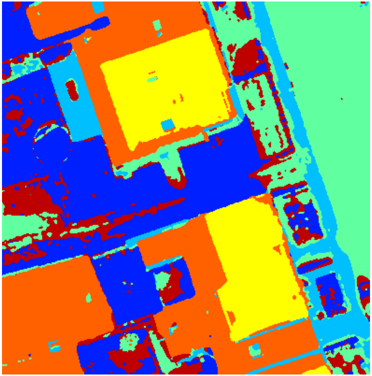
Results



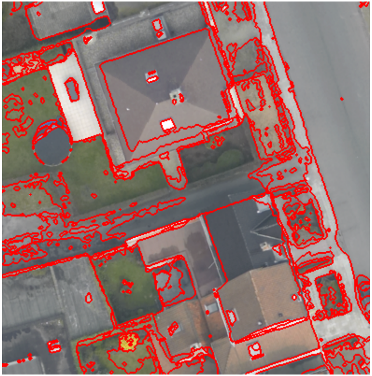
Example eigenvectors of graph Laplacian

Results

Spectral clustering result (unsupervised). $m = 6$ classes.



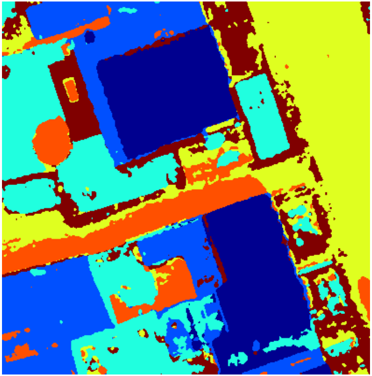
Classes



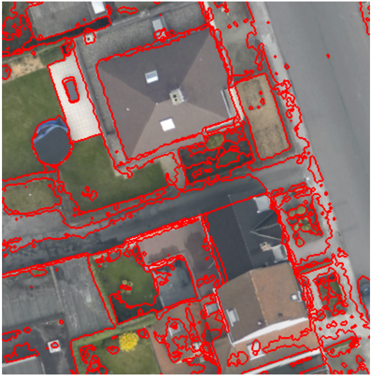
Regions on original image

Results

Graph MBO (7% supervised). $m = 6$ classes.



Classes

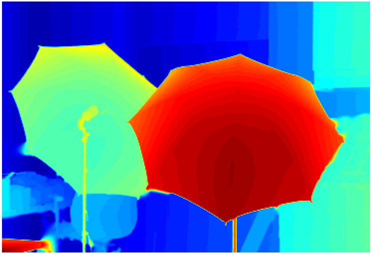


Regions on original image

Umbrella Data

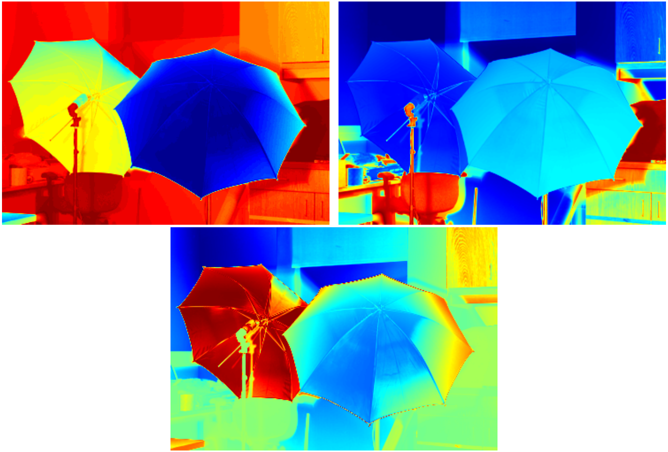
Our algorithm applied to [Scharstein et al., GCPR 2014] dataset.

RGB Modality



Lidar Modality

Results

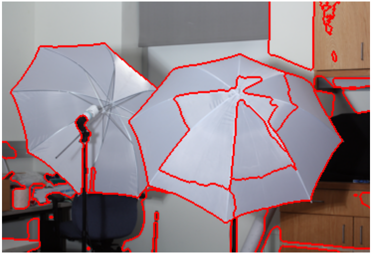


Example eigenvectors of graph Laplacian

Results

Spectral Clustering result (unsupervised). $m = 6$ classes.

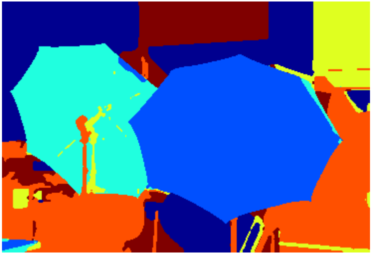
Classes



Regions on original image

Results

Graph MBO result (5% supervised). $m = 6$ classes.



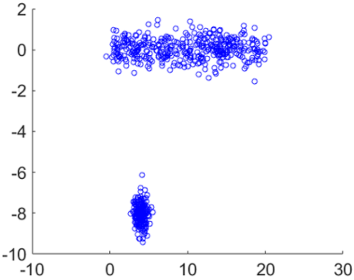
Classes

Regions on original image

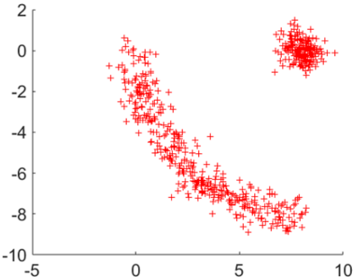
Graph Matching

Goal: Remove or weaken the coregistration assumption.
Create our own registration using topological qualities of data.

View each dataset as a (weighted) graph.
Try to match nodes with similar structure.



$\approx$

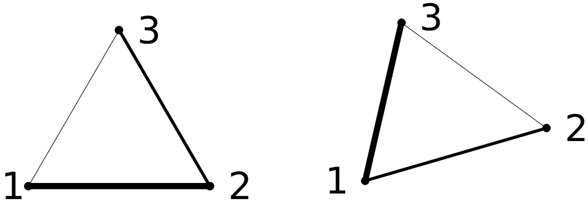


Problem Setup

Two weighted graphs, G_1, G_2, with weight matrices W_1, W_2.

For convenience, assume $|G_1| = |G_2| = N$.

Search for a graph isomorphism $G_1 \to G_2$ preserving edge weights.



Best isomorphism is $1 \to 3$, $2 \to 1$, $3 \to 2$.

Problem Setup

Isomorphism $G_1 \to G_2$ corresponds to a permutation on nodes. Have P the corresponding permutation matrix. Want to find

$$\text{argmin}_{P \text{ a Permutation matrix}} \left\| W_1 - P W_2 P^T \right\|_F^2 .$$

Exact solution is too expensive. Approximate via Graph Laplacian. [Umeyama, IEEE TPAMI 1988, Knossow et al., GbRPR 2009].

Relaxation

Relax problem to

$$Q^* = \operatorname{argmin}_{QQ^T=I} \left\| W_1 - QW_2Q^T \right\|_F^2 .$$

Let L_1, L_2 the graph Laplacians corresponding to W_1, W_2

U_1, U_2 the corresponding matrices of eigenvectors.

Then $Q^* = U_1 S U_2^T$.

S is diagonal matrix with entries ± 1 to handle sign ambiguity.

Heuristics

Recall from Laplacian Theory

$$\begin{aligned} \text{column of } U_i &\iff \text{feature extracted from data} \\ \text{row of } U_i &\iff \text{image of data point in new feature space.} \end{aligned}$$

Match rows of U_1 to rows of U_2 by considering $U_1 U_2^T$.

Currently, S is determined in a semisupervised manner, align eigenvectors using a few ($< 1\%$) known matches.

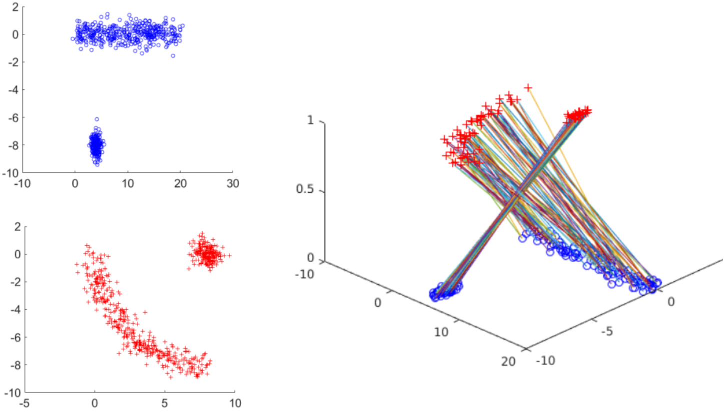
Matching Algorithm

Q^*_{ij} gives the similarity between node i of G_1 and node j of G_2.

Many choices for how to complete the matching:

- Match 1-to-many via maximum similarity.
- Match 1-to-1 via *Hungarian Algorithm*.
- Match many-to-many via a threshold on Q^*.
- **Hierarchical Matching:**
  Match some landmark nodes, extend to a full match.
  Similar to Nystöm, avoid creating the full $N \times N$ graph.

Example Matching

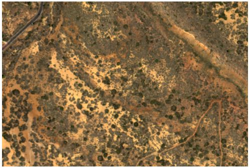


Graph Match on Synthetic Data

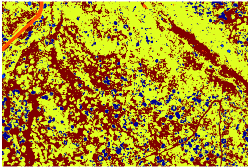
Knowledge Transfer

Knowledge transfer via Graph Matching.

- 2 hyperspectral images. 68 bands. (ARTEMISAT-2 Project)
- Area 1 is segmented into 6 classes. (Given with data)
- Match Area 1 to Area 2. Transfer classification via match.



Area 1 (RGB bands)

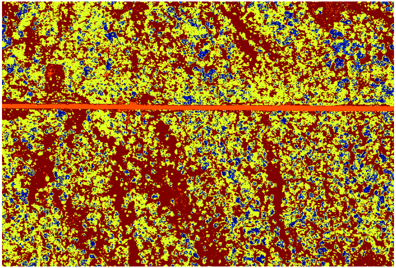


Area 1 Classification

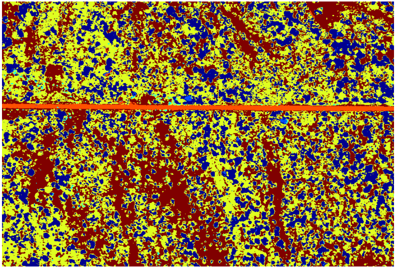
Area 2 (RGB bands)

Knowledge Transfer

Correctly classify 72% of pixels.



Transfer to Area 2



Area 2 Ground Truth

Change Detection

Change detection via Graph Matching:

Given *coregistered* data X and Y, compare the match via registration against the match via graphs.

From graph matching, get a permutation

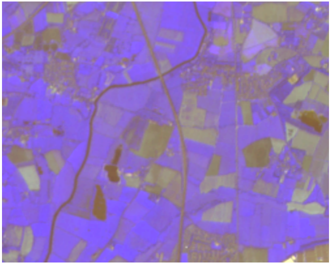
$$\rho : \{1, \ldots, n\} \to \{1, \ldots, n\}.$$

Look at $\left\| x_i - x_{\rho^{-1}(i)} \right\|$ and $\left\| y_i - y_{\rho(i)} \right\|$.

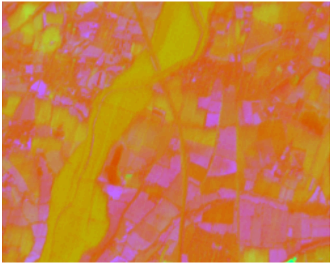
Change Detection Example

Flood data: Data Fusion Contest 2010
[Longbotham et al, IEEE,2012].

Satellite images before and after flood.
3 bands, near-infrared (displayed in false color).

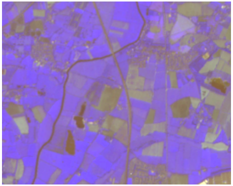


Before Flood

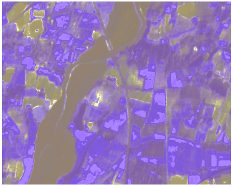


After Flood

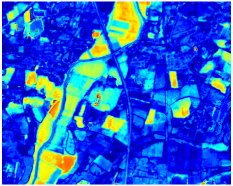
Change Detection Example



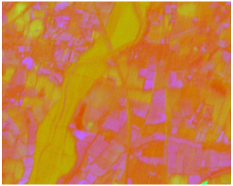
Data X



Approx Y by X

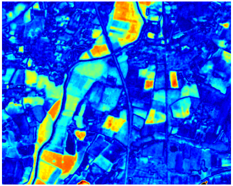


$\|X - \text{perm}(X)\|$



Data Y

Approx X by Y

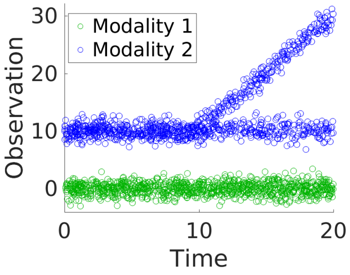


$\|Y - \text{perm}(Y)\|$

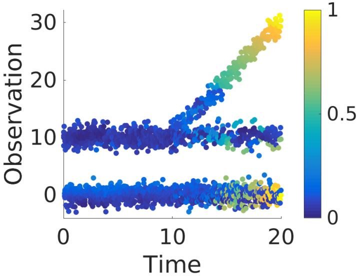
Change Detection

Change detection to find unique information in different modalities.

Synthetic Example: 1-D observations over time.



Input Data

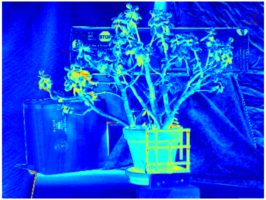


Norm Comparison (scaled)

Change detection to find unique information in different modalities.

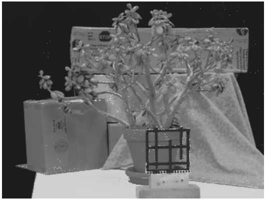
Data X

Approx Y by X



$\|X - \mathrm{perm}(X)\|$

Data Y



Approx X by Y

$\|Y - \mathrm{perm}(Y)\|$

References I

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Hungarian Algorithm

Say we have $N = 6$ and calculated:

$$Q^* = \begin{pmatrix} -0.1629 & -0.1711 & -0.1703 & 0.3426 & 0.3717 & -0.2100 \\ -0.1647 & -0.1662 & -0.1677 & 0.2966 & 0.3192 & -0.1172 \\ -0.1660 & -0.1653 & -0.1657 & -0.1477 & -0.1861 & 0.8308 \\ -0.4579 & 0.6860 & 0.2665 & -0.1787 & -0.1480 & -0.1678 \\ 0.4939 & -0.1039 & 0.1196 & -0.6689 & 0.3080 & -0.1486 \\ 0.4577 & -0.0795 & 0.1176 & 0.3561 & -0.6647 & -0.1872 \end{pmatrix}$$

Then

$$P^* = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Graph Matching Example

Graph 1 Graph 2

Any reasonable matching sends $1 \to 2$.

Other nodes can be matched in many ways (symmetry).

Change Detection Example 2

Synthetic Example: Graph representation is useful!

Continuous transform ($\mathbb{R}^3 \to \mathbb{R}^3$) applied to most pixels,
One extra artifact added.
Simulate data captured from different sources.

Original Picture

Altered Picture

Norm Comparison

Change Detection Example

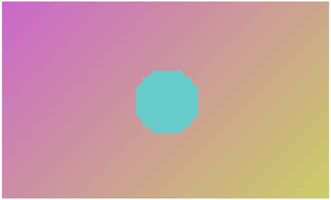


Image X

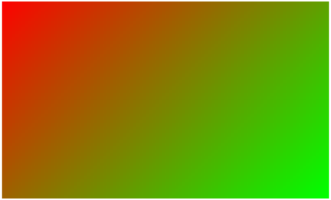
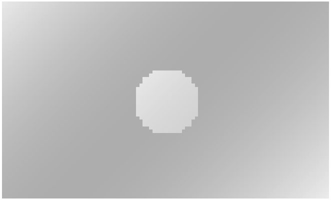
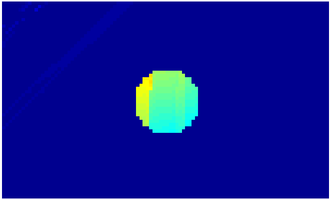


Image Y



Naive difference $\|X - Y\|$



$\left\| x_i - x_{\rho(i)} \right\|$