A Survey of Point Pattern Matching Techniques and a New Approach to Point Pattern Recognition

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Abstract

This paper summarises recent work in the area of point pattern matching and introduces a new approach to the more general problem of point pattern recognition. The new technique developed by the authors is invariant to rotational, scaling and translational transformations, and is optionally reflection invariant. The techniques presented are compared, and specific applications are described.

1 Introduction

Point pattern analysis plays an important role in picture pattern recognition. It is often advantageous to represent sets of local features (e.g. corners) in an image by their coordinates. The coordinates can be used to identify patterns in the image, rather than matching pictures as arrays using techniques such as template matching.

The use of point pattern matching (PPM) is most efficient relative to array processing techniques in situations where the size of the image is large and the number of feature points is low. The speed of point pattern techniques in such situations, their flexibility, and their invariance to affine transformations make them the optimal solution in many applications.

The importance of PPM is indicated by the large amount of recent research in this field. There are many new techniques using widely varying methodologies, each with different strengths according to the applications that inspired their development. This paper concentrates on techniques that have a traditional pattern recognition approach. Examples of the application of neural networks to point pattern problems can be found in [1] and [2].

2 Point patterns

A point pattern is a non-empty, finite set of points on a two dimensional plane. In point pattern P each point p_i is described by the coordinates (x_i, y_i). The number of points in P is written as $|P|$.

In some applications, additional characteristics of the point or its neighbourhood are added to the description in the form of a label f. In this case we have a labeled point pattern, which is defined as

$$P = \{p_i\} = \{x_i, y_i, f_i\}, \quad i = 1, 2, \ldots, n. \tag{1}$$

where $n = |P|$.

The neighbourhood of a point is normally defined in terms of neighbouring points. Ahuja [3] discusses the notion of a neighbourhood *region* that is the area enclosed by the point's Voronoi polygon.

3 Techniques

The techniques are presented in three main categories: *clustering*, *interpoint distance algorithms* and *relaxation methods*. Techniques that do not fall into these categories are described separately, followed by a description of a new approach proposed by the authors.

To simplify the descriptions, one pattern will be referred to as the scene, Q, and the other as a prototype, P. In practice Q normally describes acquired data and P describes one of many known objects. It may be necessary to classify Q as one of the prototypes, or to detect an instance of P within Q.

All of the techniques that are described here tolerate random perturbations of points in P and Q, and other noise, to some degree. Refer to the individual articles referenced for a more in depth discussion of noise sensitivity for each technique.

3.1 Clustering

These techniques assume that the two point patterns differ by one or more transformations. These transformation parameters are calculated for all combinations of point pairs from both patterns. The strongest clusters in the parameter space then represent the most likely transformation parameters for the best match, from which the parameter set that matches the patterns best is chosen.

Figure 1: Rotation/scale transformation parameters for two point patterns. (a) point pattern P. (b) P rotated 50° and scaled by a factor of 2. (c) quantised rotation/scale parameter space.

Figure 1 shows a quantised parameter space for two point patterns that differ only by scale and rotation. In figure 1(c) there are two strong clusters, one of which represents the rotation angle and scale factor that map the points in figure 1(a) onto the points in figure 1(b).

Kahl *et al.* [4] proposed a technique that defines a quantised parameter space, T, for all the possible translations between P and Q. The vector difference is calculated between every combination of point pairs from P and Q. If the magnitude of this difference is less than a threshold t for (p_i, p_j) and (q_h, q_k) then the merit of the translation in T that maps p_i on to q_h is incremented. Clusters of translation parameters in T are then detected.

Goshtasby *et al.* [5] describe a matching procedure that determines the rotation, scaling and translation *(R,S,T)* parameters that match the largest number of points in two patterns.

A merit value is assigned to the *(R,S,T)* transformation for each combination of point pairs from P and Q, according to the number of other points that match within a given distance threshold, D, under this transformation. The best *(R,S,T)* transformation is then used to determine how many points match within D. The optimal *(R,S,T)* transformation is calculated by using these matched points and minimising the sum of squared errors.

Stockman *et al.* [6] describe an object recognition technique that uses clustering for the registration of image features to models. A least-squares estimation approach to determining the optimum *(R,S,T)* parameters that will match two point patterns is described by Umeyama [7].

3.2 Interpoint distance algorithms

Interpoint distances are used as the basis of comparison in these techniques. Lavine *et al.* [8] describe three different representations for the interpoint distance matrix (IDM), which are used to determine a similarity measure between two point patterns.

The first two representations are sorted vectors of interpoint distances. The *sorted interpoint distance vector* (SIDV) is computationally expensive and gives rise to problems of geometric probability. The *sorted nearest neighbour vector* (SNNV) is more efficient by only including distances between a point and its nearest neighbour.

The third representation is the *minimal spanning tree* (MST) which represents the cheapest way of connecting the nodes in a graph representation of the point pattern. The points define the nodes in the graph and the interpoint distances define the graph edges. Figure 2 shows the graph representation of a point pattern.

Wong *et al.* [9] also construct a tree from the graph representation of a point pattern. The tree represents every possible matching of p_i to q_i from root to leaf. They define a cost function for a matching of size n as the sum of the differences between all corresponding interpoint distances in P and Q. A match is found by searching the tree for the minimum cost mapping with the maximum number of points that match within a given tolerance.

3.3 Relaxation methods

Relaxation techniques iteratively assign values to mutually constrained objects in such a way as to ensure that the values remain consistent. A solution is found when these values converge. In PPM the objects are point- or primitive mappings, and the constraints are the match criteria.

Ranade *et al.* [10] propose relaxation for a translation invariant technique that is more tolerant of global distortions. Each point mapping, (p_i, q_j), is assigned a merit score according to how well all other points (p_h, q_k) match when p_i is mapped onto q_j. Subsequent iterations use the merit scores for (p_h, q_k) and the distance between p_h and q_k to assign a new merit score to the mapping (p_i, q_j). The algorithm converges as the merit scores of mappings (p_i, q_j) that correspond under the actual translation difference of P and Q remain high, while those of other mappings become low.

Ogawa [11] describes a fuzzy relaxation technique for labeled point patterns. A set of primitive pairs, (p_i, p_j), that each have a point with a distinguishing label, is derived for the model P. Primitives for the scene, Q, are derived with reference to the primitives of P. Iterations of the relaxation process geometrically transform compatible primitives in P and Q so as to minimise mismatch between the other primitive pairs.

3.4 Other techniques

Hong *et al.* [12] define a canonical form for point patterns under affine transformation, and use this representation as the basis for comparison.

Griffin *et al.* [13] describe a technique that uses the smallest enclosing circles of P and Q to determine a scaling factor. The translation offsets T_x and T_y are determined by comparing the centroids of the normalised patterns. A bipartite graph of feasible mappings is constructed, and the final mapping from P onto Q is determined from the maximum cardinality match of this graph. The bipartite graph for two point patterns is illustrated in figure 3.

Ogawa [14] describes a technique for matching labeled point patterns. A subset of the points in the model, P, are chosen, according to attention points and the similarity of labels in P and the scene, Q. This subset of points is partitioned into a set of triangles using the Delaunay triangulation.

For each of these triangles, there are candidates for a match in the Delaunay triangulation of Q. These triangle mappings are used to determine the affine transformation from P onto Q, and to provide the mapping candidates from Q for each point in P. The largest set of mutually consistent point mappings is determined from the largest maximal clique in the consistency graph of the candidate point mappings.

3.5 A new approach

In [15] the authors introduced a new technique that compares the triangulations of P and Q in order to discard points that do not belong to matching patterns. The triangulations of both point sets are represented in a quantised parameter space, the Triangle Accumulator Array (TAA), for comparison. The triangles are represented by their shape (interior angles) in the TAA, which is consequently invariant to rotation, scaling and translation of P and Q. Depending on the specific triangle shape representation, the technique can also be made reflection invariant.

Figure 4: Result using the TAA to match two point patterns differing by rotation, scale and translation. Matching points are solid

Since the TAA of matching patterns will be identical, any triangle that is not represented in the TAA of both P and Q can be discarded. Consistent point patterns are detected in the now incomplete TAA using a sequential relaxation algorithm, and the process is iterated with these new patterns. This continues until two patterns have identical TAA's, or all of the points in P or Q have been discarded. The former case indicates a possible match. This match is then verified by calculating the transformation parameters and applying them to the patterns.

4 Comparisons

Table 1 is a feature comparison of the abovementioned techniques. We define three levels of capability for PPM techniques:

Matching: These techniques find a one-to-one mapping between the points of two point patterns P and Q, where $|P| = |Q| = n$. They are normally used to classify a point pattern as one of many prototypes.

Deletion Tolerant Matching: These techniques also find a global match between two point patterns, but tolerate, to some degree, missing or additional points in either of the patterns.

Recognition: These techniques will detect a match when the matching points are an arbitrarily small subset of the original patterns. They will detect instances of a small prototype within a large point pattern scene.

Performance is compared for the case of a *matching* problem, in order to allow comparison of all the techniques. However, it must be noted that this biases the results against the techniques that were de-

Figure 2: Graph representation of a point pattern. (a) point pattern (b) graph representation

Figure 3: Bipartite graphs. (a) point pattern P. (b) point pattern Q. (c) bipartite graph of feasible mappings of P onto Q. (d) Maximum cardinality match.

Table 1: Comparison of point pattern matching techniques

Technique	*Author*	*Capability*	*Invariance*	*Performance*
Clustering	Kahl *et al.* [4]	Deletion tolerant matching	Translation	$O(n^4)$
(R,S,T) Clustering	Goshtasby *et al.* [5]	Deletion tolerant matching	Rotation, scale, translation	$O(n^3)$
Sorted interpoint distance vector	Lavine *et al.* [8]	Deletion tolerant matching	Rotation, translation	$O(n^3)$
Sorted nearest neighbour vector	Lavine *et al.* [8]	Deletion tolerant matching	Rotation, translation	$O(n^2 \log n)$
Minimal spanning tree	Lavine *et al.* [8]	Deletion tolerant matching	Rotation, translation	$O(n^2 \log n)$
Minimal cost search	Wong *et al.* [9]	Deletion tolerant matching	Rotation, translation	$O(n^2 \log n)$
Relaxation	Ranade *et al.* [10]	Deletion tolerant matching	Translation	$O(n^4)$
Fuzzy relaxation	Ogawa [11]	Deletion tolerant matching	Rotation, scale, translation	-
Canonical forms	Hong *et al.* [12]	Matching	Rotation, scale, translation, stretching	$O(n)$
Centroid bounding	Griffin *et al.* [13]	Matching	Rotation, scale, translation	$O(n^{7/2})$*
Delaunay triangulation and maximal cliques	Ogawa [14]	Recognition	Rotation, scale, translation	-
Triangle accumulator array	Cox *et al.* [15]	Recognition	Rotation, scale, translation, reflection**	$O(n^3)$

* worst case
** optional

signed to be deletion tolerant or solve the more general problem of recognition.

5 Applications

Point pattern techniques are used in many computer vision applications where local features can be extracted from an image and used for pattern recognition. Specific areas where PPM has been applied are cartography, satellite image analysis, automatic object recognition and inspection, and astronomy.

The application of clustering techniques to aerial photographs is described by Stockman *et al.* [6]. Automatically and manually detected features were used to register aerial photographs to maps, for automatic map revision. The detection of aircraft in aerial images was investigated for the purpose of aircraft guidance.

A similar application is the registration of different satellite images of the same area. Goshtasby *et al.* [5] applied a clustering technique to the matching of feature point sets that were extracted automatically from satellite image data.

There are a wide variety of computer vision applications in the automatic on-line analysis of production. Point pattern techniques are particularly suited to the detection and inspection of parts on a production line. Stockman *et al.* [6] and Griffin *et al.* [13] apply PPM techniques to the recognition parts in images.

In astronomy the matching of star constellations is reduced to a PPM problem once the stars have been identified in a stellar image. Ogawa [11, 14] has applied PPM to the matching of constellations.

The authors are involved in the development of an automatic photoelectric telescope that is the combined effort of the University of Cape Town and the South African Astronomical Observatory [16]. This telescope will use a computer vision system and PPM to acquire and track target stars [15]. This is an inexpensive and efficient alternative to using a telescope with a very high pointing accuracy.

6 Conclusion

There are many different approaches to point pattern matching. Those discussed include, among others, clustering techniques, relaxation methods and interpoint distance algorithms.

All of the techniques discussed are invariant to at least a simple affine transformation such as translation, and many are invariant to rotation, scaling and translation.

Three levels of capability have been defined for point pattern matching techniques: *matching*, *deletion tolerant matching* and *recognition*. For a matching problem the performance of the techniques is in the range: $O(n)$ to $O(n^4)$.

Areas of application for point pattern matching include cartography, satellite image analysis, automatic object recognition and inspection, and astronomy.

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