A Graph-Based Approach for Feature Extraction and Segmentation of Multimodal Images

Geoffrey Iyer, University of California, Los Angeles, Department of Mathematics
Jocelyn Chanussot, Univ. Grenoble Alpes, CNRS, GIPSA-lab, F-38000 Grenoble, France
and Andrea L. Bertozzi, University of California, Los Angeles, Department of Mathematics

***Abstract*—In the past few years, graph-based methods have proven to be a useful tool in a wide variety of energy minimization problems [1]. In this paper, we propose a graph-based algorithm for feature extraction and segmentation of multimodal images. By defining a notion of similarity that integrates information from each modality, we merge the different sources at the data level. The graph Laplacian then allows us to perform feature extraction and segmentation on the fused dataset. We apply this method in a practical example, namely the segmentation of optical and lidar images. The results obtained confirm the potential of the proposed method.**

I. Introduction

With the increasing availability of data we often come upon multiple datasets, derived from different sensors, that describe the same object or phenomenon. We call the sensors *modalities*, and because each modality represents some new degrees of freedom, it is generally desirable to use more modalities rather than fewer. For example, in the area of speech recognition, researchers have found that integrating the audio data with a video of the speaker results in a much more accurate classification [2], [3]. Similarly, in medicine, the authors of [4] and [5] fuse the results of two different types of brain imaging to create a final image with better resolution than either of the originals. In this paper we also focus on multimodal images, but rather than seeking to merge our images, we instead perform feature extraction, with applications toward segmentation.

Image fusion is roughly broken into two categories depending the structure of the fusion algorithm [6]. The first of these is feature-level fusion, where each data source is collected and processed independently, then some final algorithm combines the results. These methods are generally easier to implement, as the processing step simplifies the data and allows for a more straightforward fusion process. The other category, which our method falls under, is fusion at the data level (also called pixel-level fusion when dealing with images). Here the raw data is processed as a whole, rather than as individual images creating an intermediate feature space that is informed by each dataset. This object is then used for further analysis. These terms are quite nebulous, as it is difficult to precisely define the difference between data and features, but the concept of fusion at different levels is important to consider. We believe that to optimally make use of data, the fusion step should occur as close to the data level as possible. In the general case, there should be significant information to be gained from the interactions between the different datasets, and this can be lost by prematurely processing the data.

A major issue in data fusion is the difficulty of reconciling data from different modalities that at first glance may appear highly heterogeneous. Because of the wide variety of sensors used to acquire data, fusion methods are often tailor-made for specific problems, and are not useful in general [7]. In this paper we work towards solving this problem through graph-based methods. The major advantage of using graphs lies in the ability to compare information from disparate modalities without much need for pre-processing. The only requirements are the ability to measure similarity between points in the same dataset, as well as a co-registration between the different sets (so the i-th point in one set corresponds to the i-th point in another). This situation often occurs in image processing problems, where the sets may be images of the same scene obtained from different sensors (as is the case in our experimental data), or taken at different times.

Our method extracts features from the dataset by finding eigenvectors of the graph Laplacian, then uses standard data-segmentation algorithms on these features to obtain a final classification. In section III we give the general theory behind our method, and in IV we show the results of the method applied to several optical/LIDAR datasets.

II. Related Work

One very simple algorithm for multimodal image fusion is to simply take a weighted average of the

different modes. Unfortunately, this method is often too naive to produce meaningful results. In many cases there are various objects and regions that occur in multiple images but with opposite contrast, which would cancel out in an averaged image. However, this basic idea is still worth consideration, so long as the blending step is treated with more care. In [8] the authors use structural patch decomposition to perform roughly the same task, but with much better results, and in [9] the authors address the same problem with probabilistic methods. In each of these cases, the end product is an image that contains the most relevant features from each modality. Classical segmentation algorithms can then be performed on this fused image to create the desired results.

Another common way to fuse images is to transform each modality with some processing algorithm, then merge the data in the new feature space. In [10] the authors follow this methodology, using a multiresolution (MR) transformation to process information in each modality. The benefit of this algorithm is that the transformation is fully invertible, meaning that once the data has been synthesized in the feature space, the inverse transformation can be applied to recover the fused image. In [11], [12] the authors follow the same overall strategy, using Independent Component Analysis (ICA) as the initial processing algorithm.

Each of the above methods first fuses the different modalities (into either a new image, or into a new set of features), then uses this fused data to create a final segmentation. But another valid method is to instead segment each modality first, then combine the different classifications into a final result. Both [13] and [14] create a hierarchical segmentation of each modality (a chain of segmentations ranging from very coarse to very fine), then blend these segmentations using some decision algorithm. A related field of study is segmentation combination. Given multiple segmentations of the same image (possibly obtained from different modalities), the goal is to obtain a consensus segmentation by somehow fusing the different inputs. In [15] the authors accomplish this through general ensemble clustering methods, and in [16] this is done by using probabilistic methods and random walks.

In regard to spectral graph theory, these methods have been very successfully applied to data clustering problems and image segmentation [17]–[19]. Graph-cut algorithms are quite flexible. All that is required is a well-chosen affinity function to describe the similarity between different graph nodes. Graph cuts can even be used to minimize a wide variety of energy functions [1], allowing for the use of unsupervised [20], [21] or semi-supervised methods [22]. The standard theory behind this is described in [23], with a tutorial on spectral clustering given in [24].

III. The Method

In this section we explain the theory behind our algorithm. First, in section III-A we explain the graph framework used in the later segmentation steps, including the method for processing the different modalities to create objects which can be directly compared. We then exhibit two segmentation methods that we apply to the graph object. The first, *spectral clustering* III-B, is an unsupervised method that can be used to quickly obtain a reasonable set of "proof-of-concept" results. The second, *graph MBO* III-C, is a semisupervised method that more carefully handles the energy minimization to obtain a stronger final result.

A. Graph Representation

For notation, we label the sets, $X^1, X^2, \dots, X^k$. Let

$$n = \left|X^1\right| = \cdots = \left|X^k\right| \tag{1}$$

and let d_i be the dimension of the data X^i. Have

$$X = (X^1, X^2, \dots, X^k) \subset \mathbb{R}^{n\times(d_1+\cdots+d_k)}$$

be the concatenated dataset. We represent X using an undirected graph $G = (V, E)$. The nodes $v_i \in V$ of the graph correspond to elements of X, and we give each edge e_{ij} a *weight* $w_{ij} \geq 0$ representing the similarity between nodes v_i, v_j, where large weights correspond to similar nodes, and small weights to dissimilar nodes. This gives rise to a *similarity matrix* (also called the *weight matrix*)

$$W = (w_{ij})_{i,j=1}^{n}.$$

Since G is undirected, we require that $w_{ij} = w_{ji}$, which implies that W is a symmetric matrix. There are many different notions of "similarity" in the literature, and each has its own merits. In many applications, one defines

$$w_{ij} = \exp\left(-\left\|v_i - v_j\right\|/\sigma\right),$$

where σ is a scaling parameter. In this work we adapt this definition to apply to our multimodal dataset, as is explained below.

1) Multimodal Edge Weights: To calculate the weight matrix W, we first scale our sets $X^1, \dots, X^k$ to make distances in each set comparable. Let $X = (X^1, \dots, X^k) \subset \mathbb{R}^{n\times(d_1+\cdots+d_k)}$ be the concatenated dataset. Then for $\ell = 1, \dots, k$ define the scaling factor

$$\lambda_\ell = \text{stdev}\left(\left\|x_i^\ell - x_j^\ell\right\| \;;\; 1 \leq i, j \leq n\right) \tag{2}$$

For a graph node $x \in X$, we define

$$\|x\| = \max\left(\frac{\|x^1\|}{\lambda_1}, \cdots, \frac{\|x^k\|}{\lambda_k}\right). \tag{3}$$

Then define the weight matrix W by

$$W = (w_{ij})_{1 \leq i,j \leq n} \tag{4}$$

with $w_{ij} = \exp\left(-\|x_i - x_j\|\right)$.

Note that the $\|\cdot\|$ defined above is a norm on the concatenated dataset X. We specifically choose to use the maximum of the individual measurements to emphasize the unique information that each dataset brings. With this norm, two data points x_i, x_j are considered similar only when they are similar in each dataset.

2) The Graph Laplacian: Once we have created the weights, we define the *normalized graph Laplacian*. For each node $v_i \in V$, define the *degree* of the node

$$d_i = \sum_j w_{ij}. \tag{5}$$

Intuitively, the degree represents the strength of a node. Let D be the diagonal matrix with d_i as the i-th diagonal entry. We then define the *normalized graph Laplacian*

$$L_{sym} = I - D^{-1/2} W D^{-1/2}. \tag{6}$$

For a thorough explanation of the properties of the graph Laplacian, see [23]. In this paper, we will use the connection between the graph Laplacian and the graph min-cut problem, as explained below.

B. Spectral Clustering

To implement our first segmentation method, *spectral clustering*, we rephrase the data clustering problem as a graph-cut-minimization problem of the similarity matrix W. A more detailed survey of the theory can be found in [24]. Here we state only the results necessary to implement our algorithm.

Given a partition of V into subsets $A_1, A_2, \ldots, A_m$, we define the *graph N-cut*

$$\text{NCut}(A_1, \ldots, A_m) = \frac{1}{2} \sum_{i=1}^{m} \frac{W(A_i, A_i^c)}{vol(A_i)}. \tag{7}$$

Where

$$W(A, B) = \sum_{i \in A, j \in B} w_{ij}, \tag{8}$$

$$vol(A_i) = \sum_{i \in A, j \in A} w_{ij} = W(A, A), \tag{9}$$

and the $\frac{1}{2}$ is added to account for double-counting each edge. Heuristically, minimizing the N-cut serves to minimize the connection between distinct A_i, A_j, while still ensuring that each set is of a reasonable size. Without the $vol(A_i)$ term, the optimal solution often contains one large set and $m-1$ small sets.

Solving the graph min-cut problem is equivalent to finding an $n \times m$ indicator matrix u, where

$$u_{ij} = \begin{cases} 1 & \text{if } x_i \in A_j \\ 0 & \text{else} \end{cases}. \tag{10}$$

Here the columns of u correspond to the m different classes. Each row of u will contain a single 1, which represents the class given to that data point. It has been shown in [25] that explicitly solving this problem is an $O(|V|^{m^2})$ process. As this is infeasible in most cases, we instead introduce an approximation of the graph min-cut problem that we will solve using the graph Laplacian. Our main tool here is the following fact (proven in [24]).

Fact III.1. For a given graph-cut $A_1, \ldots, A_m$, define u as above, then

$$\text{NCut}(A_1, \ldots, A_m) = \text{Tr}\left(u^T L_{sym} u\right). \tag{11}$$

As explained above, we it is infeasible to find the u that minimizes the N-Cut. Instead we relax the problem to allow u to be an arbitrary orthogonal matrix. That is, we find

$$\text{argmin}_{u \in \mathbb{R}^{n \times m}} \text{Tr}\left(u^T L_{sym} u\right) \quad \text{where } u^T u = I. \tag{12}$$

As L_{sym} is symmetric and u is orthogonal, this problem is solved by choosing u to be the matrix containing the m eigenvectors of L_{sym} corresponding to the m smallest eigenvalues. Using the eigenvectors u we define a map $X \to \mathbb{R}^m$. For each graph node $x_i \in X$ we get a vector $y_i \in \mathbb{R}^m$ given by the i-th row of u. These y_i give the solution to the relaxed min-cut problem, and as such can be thought of as features extracted from the original dataset X.

To obtain a solution to the original min-cut problem, we then implement some classification algorithm on the y_i. Specifically, for spectral clustering we use k-means on the eigenvectors u to create a final classification into m classes.. Although k-means is unlikely to give an optimal classification, it is quite easy to implement, and the final results are strong enough to give a proof-of-concept.

Note that the eigenvectors u found above are useful for many more purposes than just spectral clustering. In IV we display some eigenvectors, and show they can be used to recognize objects in images. Furthermore, in III-C, we will use these same eigenvectors as part of our MBO algorithm.

C. Semisupervised Graph MBO

In this section we describe how to use eigenvectors of the graph Laplacian to segment data in a semisupervised setting. By "semisupervised", we mean that the final classification of a small amount of data points (roughly

5% of all data) is used as an input to our algorithm. Following the example set in [22], [26], [27], we formulate the problem as a minimization of the Ginzburg-Landau functional.

For the definition of the energy function, we use an $n \times m$ assignment matrix u (similar to the u from III-B). For intermediate steps of the algorithm, we require that

$$u_{ij} \geq 0 \quad \forall i, j \tag{13}$$

$$\sum_{j=1}^{m} u_{ij} = 1. \tag{14}$$

Heuristically, the value u_{ij} represents the probability that element x_i will be classified into class j. The final output of the algorithm will be a matrix u where each value is either 0 or 1. For notational convenience we let u_i represent the i-th row of u. With this notation, we define our energy function

$$\begin{aligned} E(u) = \epsilon \cdot \mathrm{Tr}\left(u^T L_{sym} u\right) \\ + \frac{1}{\epsilon} \sum_i W(u_i) \\ + \sum_i \frac{\mu}{2} \lambda(x_i) \left\| u_i - \hat{u}_i \right\|_{L_2}^2 . \end{aligned} \tag{15}$$

The first term of (15) is Dirichlet Energy, similar to III-B. The second term is the multiwell potential

$$W(u_i) = \prod_{k=1}^{m} \frac{1}{4} \left\| u_i - e_k \right\|_{L_1}^2 , \tag{16}$$

where e_k is the k-th standard basis vector. These two terms together produce an approximation of the classical real Ginzburg-Landau functional, and it has been shown in [28] that they converge to the (graph) total-variation norm as $\epsilon \to 0$. The last term includes the fidelity, where $\hat{u}$ represents the semisupervised input,

$$\lambda(x_i) = \begin{cases} 1 & \text{if } x_i \text{ is part of fidelity input} \\ 0 & \text{else} \end{cases} , \tag{17}$$

and μ is a tuning parameter.

The gradient descent update associated to this energy is given by

$$\frac{\partial u}{\partial t} = -\epsilon L_{sym} u - \frac{1}{\epsilon} W'(u) - \mu \lambda(x)(u - \hat{u}). \tag{18}$$

Similar to [22], [26], [29], we propose to minimize this via an MBO algorithm. If u^n represents the n-th iterate, then to calculate u^{n+1} we first diffuse

$$\frac{u^{n+\frac{1}{2}} - u^n}{dt} = -L_{sym} u^n - \mu \lambda(x)(u^n - \hat{u}). \tag{19}$$

Then threshold each row

$$u_i^{n+1} = e_r \quad \text{where } r = \mathrm{argmax}_j u_{ij}^{n+\frac{1}{2}}. \tag{20}$$

This method effectively splits our energy into two parts and minimizes each alternatively. The diffusion step (19) handles the semisupervised Dirichlet Energy (terms 1 and 3 in (15)), and the thresholding minimizes the potential function W (term 2 in (15)). The stopping criterion for this algorithm is based on the difference between two consecutive iterates u^n, u^{n+1}. In section IV, we stop the algorithm when u^n and u^{n+1} agree on 99.99% of data points.

The diffusion calculation can be done very efficiently by using the eigendecomposition of L_{sym} (the feature vectors described in III-B). If we write

$$L_{sym} = X \Lambda X^T \tag{21}$$

and change coordinates

$$u^n = X a^n \tag{22}$$

$$\mu \lambda(x)(u^n - \hat{u}) = X d^n \tag{23}$$

then the diffusion step reduces to solving for coefficients

$$a_k^{n+1} = (1 - dt \cdot \lambda_k) \cdot a_k^n - dt \cdot d_k^n. \tag{24}$$

where λ_k is the k-th eigenvalue of L_{sym}, in ascending order.

In practice, only a small number of leading eigenvectors and eigenvalues need to be calculated in order to achieve good accuracy. Therefore, in the eigendecomposition 21, we choose a number of eigenvectors to use, and truncate X to a rectangular matrix. This significantly improves the speed of the algorithm. Furthermore, in section III-D, we discuss how to approximate the leading eigenvectors of L_{sym} without calculating the full $n \times n$ matrix.

D. Nyström Extension

Calculating the full graph Laplacian is computationally intensive, as the matrix contains n^2 entries. Instead we use Nyström's extension to find approximate eigenvalues and eigenvectors with a heavily reduced computation time. See [21], [22], [30] for a more complete discussion of this method.

Let X denote the set of nodes of the complete weighted graph. We choose a subset $A \subset X$ of "landmark nodes", and have B its complement. Up to a permutation of nodes, we can write the weight matrix as

$$W = \begin{pmatrix} W_{AA} & W_{AB} \\ W_{BA} & W_{BB} \end{pmatrix}, \tag{25}$$

where the matrix $W_{AB} = W_{BA}^T$ consists of weights between nodes in A and nodes in B, W_{AA} consists of weights between pairs of nodes in A, and W_{BB} consists

of weights between pairs of nodes in B. Nyström's extension approximates W as

$$W \approx \begin{pmatrix} W_{AA} \\ W_{BA} \end{pmatrix} W_{AA}^{-1} \begin{pmatrix} W_{AA} & W_{AB} \end{pmatrix}. \quad (26)$$

where the error of approximation is determined by how well the rows of of W_{AB} span the rows of W_{BB}. As W is positive semidefinite, we can write it as a matrix transpose times itself, $W = V^T V$. In [31], the authors show that the Nyström extension estimates the unknown part of V (corresponding to W_{BB}) by orthogonally projecting it into the known part (corresponding to W_{AA}, W_{AB}). This approximation is extremely useful, as we can use it to avoid calculating W_{BB} entirely. It is in fact possible to find $|A|$ approximate eigenvectors of W using only the matrices W_{AA}, W_{AB}. This results in a significant reduction in computation time, as we compute and store matrices of size at most $|A| \times |X|$, rather than $|X| \times |X|$.

In practice, the details of choosing A will not significantly affect the final performance of the algorithm. Although it is possible to choose specific "landmark nodes", in most applications (including ours) the elements of A are selected at random from the full set X. Assuming the X is not overly patterned, then it is almost guaranteed that W_{AA}, W_{AB} will be full rank. Furthermore, the amount of landmark nodes m can be chosen to be quite small without noticeably affecting performance. This makes Nyström's extension especially useful in application, as very little work is required to tune the parameters. In Section IV we use $m = 100$, and choosing a larger set A does not give a significant change in the error of approximation.

IV. Experiment

A. Data Fusion Contest 2015 Images

We test our algorithm on an optical/LIDAR dataset from the 2015 IEEE Data Fusion Contest [32] (abbreviated as DFC), shown in figure 1a, 1b. The data consists of an RGB image and an elevation map of a residential neighborhood in Belgium. We choose this particular scene because of the large amount of non-redundancy between the two images. The lidar data is effective at differentiating the roofs of the buildings from the adjacent streets, and the optical data is useful for segmenting the many different objects at ground-level.

In figure 2a, 2b, 2c we show three example eigenvectors of the graph Laplacian. As explained in III-B, these vector can be thought of as feature of our dataset, and looking at them will give us a rough idea of the final segmentation. Notice how in 2a the dark-grey asphalt is distinct from both the nearby grass (which is at the same elevation), and the roofs of the buildings (which are a similar color). This shows at the feature level that our algorithm is successfully using both the optical and the lidar data when determining what pixels can be considered similar. Based on this example vector, the classification algorithm then separates those regions in the final results. One can note the similarities between each of the example eigenvectors and our classifications 2d, 2f.

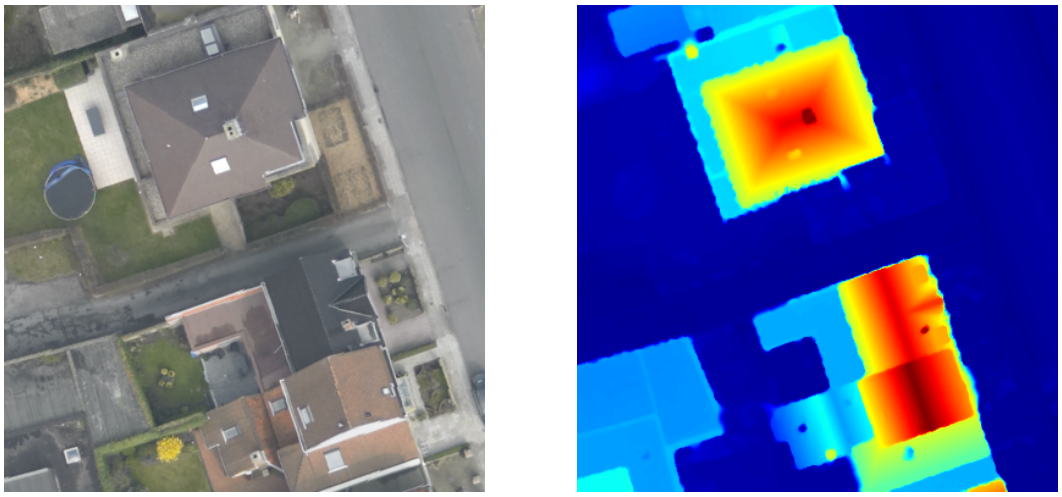
For this image, we choose to segment our data into 6 classes. As the data does not come with any ground truth attached, the number 6 was chosen based purely on personal opinion. The classes given in the semisupervised term (fig 2e) are roughly: tall buildings, mid-level buildings, asphalt (bright), asphalt (dark), white tiles, and grass. The exact choice of fidelity pixels was made by either manually choosing locations, or by characteristics of the data (ex: the 1% of pixels at highest elevation). Most importantly, these classes can all be separated using either color or lidar (or both).

As should be expected, the spectral clustering method (fig 2d) does select the same 6 classes that we have manually identified. As this algorithm is unsupervised, there is no way of encoding our human preference into the method. Therefore, the choice of exactly how to divide the different groups of pixels is made in accordance with just the graph min cut energy. In the end, this algorithm can still pick out the major features of our dataset, but the specific decisions of exactly which classes to combine and which to separate does not agree with our human intuition. By instead using a semisupervised algorithm such as graph MBO (fig 2f), we can input a small amount of information (in this case, 7% of total pixels) in order to align the energy minimization with our human expectations. Therefore, the final result aligns quite well with out initial expectations.

When choosing the exact parameters for the algorithm, there are several factors to consider. First, the diffusion step should be stable, which occurs when dt and μ are relatively small. Second, the final result should agree with the semisupervised input (fig 2e), which occurs when μ is relatively large. Third, the runtime should not be too long, which occurs when dt is relatively large. Balancing these different goals required multiple trial runs of the code. In this particular example we use parameters $dt = 0.1, \mu = 10^4$.

B. Umbrella Data

In fig 3 we show the results of our method applied to another optical/lidar set (found in [33]), which we will refer to as the umbrella data. Similar to the DFC set, the umbrella data serves as a good example because it cannot be easily analyzed using one modality alone. The umbrellas and the background walls are nearly the same shade of white, and can only be distinguished in the lidar data. Meanwhile, the different pieces of the background all lie at nearly the same depth, and can only



(a) DFC optical data

(b) DFC lidar data

Fig. 1: DFC Input Data

be separated by color. As was the case with the DFC data, the final classifications 3f, 3h can be understood by looking at the individual feature vectors. In figure 3c, we see very clearly the difference the major features of the dataset: the front umbrella, the back umbrella, and the background wall. Figures 3d, 3e show more of the small details of the data, separating the many different background objects.

As was the case with the DFC data, we chose to segment this image into 6 classes based primarily on personal opinion. The classes represented in the semisupervised input are: the front umbrella, the back umbrella, the wooden cabinet in the corner, and various different colors of background objects (fig 3g). Similar to the results from the DFC dataset, we can find many major features in the spectral clustering result (fig 3f), but the exact details of the classification do not match our expectations. In particular, the foremost umbrella of this set is overclassified, which in turn forces the algorithm to combine the background objects into a small number of classes. In the graph MBO result (fig 3h), we give include the class of 5% of pixels as part of our input, and as such the classification fits the original data much more closely.

C. Jade Plant Data

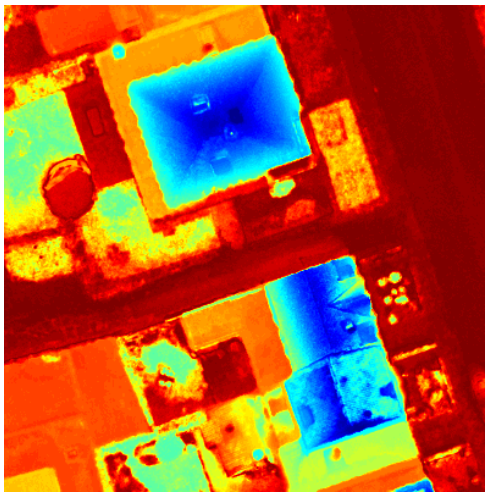
Found in the same paper as the umbrella data [33], we test our method against another optical/lidar scene of a jade plant. In figure 4 we once again display our classification result along with several example eigenvectors. As before, we can see in each eigenvector some pieces of the final classification.

V. Conclusions

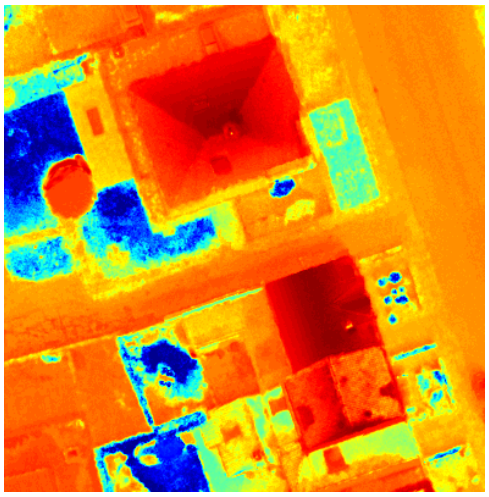
In conclusion, graph-based methods provide a straightforward and flexible method of combining information from multiple datasets. By considering the similarity between points in each individual dataset, we reduce the information from each modality into something more directly comparable. This in turn gives us a model that is more data-driven, using the information obtained from each modality without needing to know the details about the source from which the data was captured. Therefore the same algorithm could be applied in many different scenarios, with different types of data.

Once we have calculated and compared the different weight matrices, we can then create the graph Laplacian of the data and extract features in the form of eigenvectors. These features can then be used as part of many different data-segmentation algorithms. For this paper, we use k-means on the eigenvectors as a simple proof-of-concept, and graph MBO as a more in-depth approach. The main computational bottleneck is in calculation of the eigenvectors. Once we have these, there are many different viable classifications in the literature.

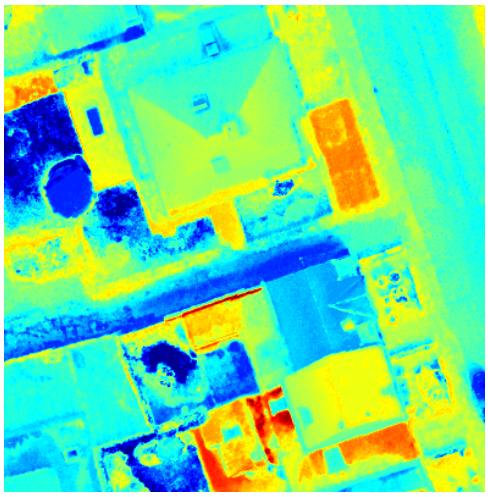
A future area of interest is to further generalize the method by removing or weakening the co-registration assumption. This segmentation algorithm only considers cases where the two images are of the same underlying scene, where pixels correspond exactly between images. But it would be interesting, for example, to process two images taken from different angles. In image processing problems, co-registration is usually a reasonable assumption. However, removing this assumption would allow this algorithm to be applied to data fusion problems across a huge number of fields.



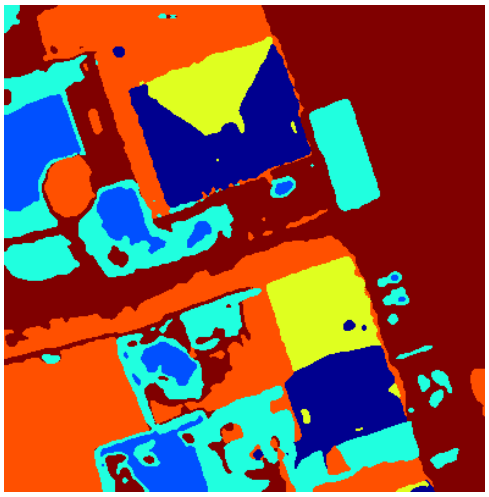
(a) Example eigenvector 1



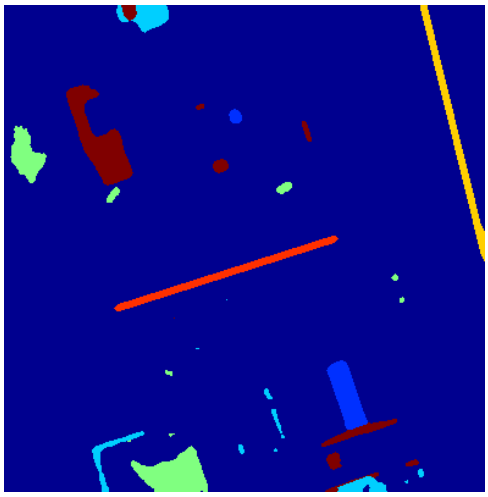
(b) Example eigenvector 2



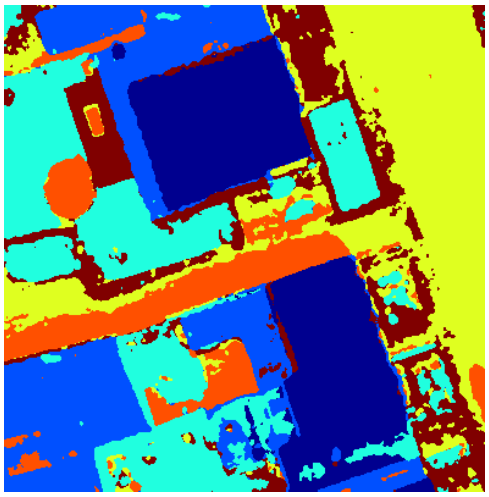
(c) Example eigenvector 3



(d) Spectral clustering segmentation



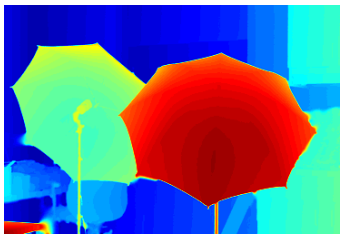
(e) Semisupervised Input



(f) MBO segmentation

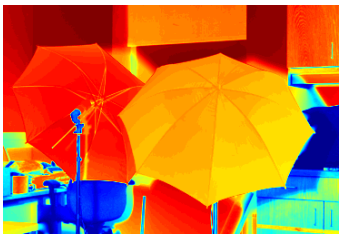
Fig. 2: DFC features and segmentations

(a) Umbrella optical data

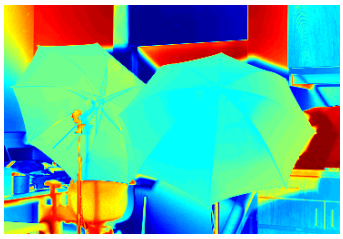


(b) Umbrella lidar data

(c) Example eigenvector 1

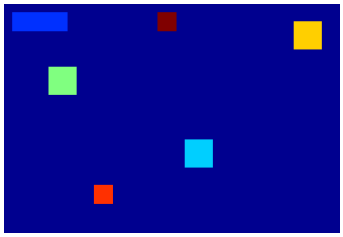


(d) Example eigenvector 2

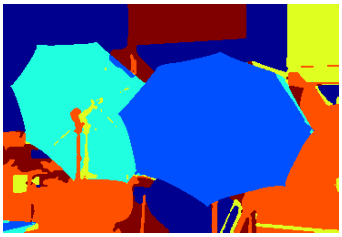


(e) Example eigenvector 3

(f) Spectral clustering segmentation



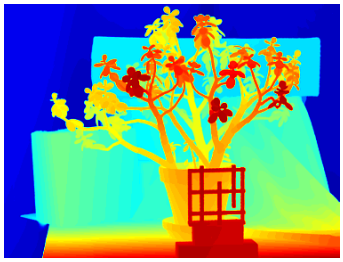
(g) Semisupervised Input



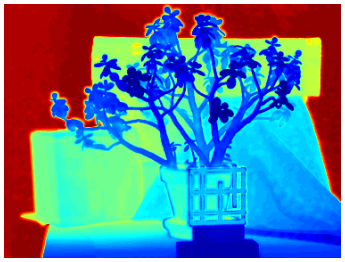
(h) MBO segmentation

Fig. 3: Umbrella data results

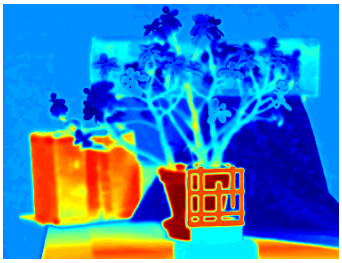
(a) Jade plant optical data



(b) Jade plant lidar data

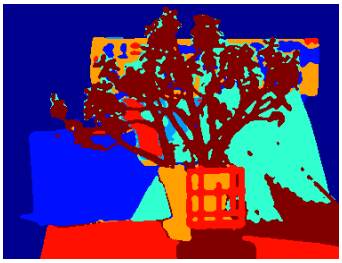


(c) Example eigenvector 1



(d) Example eigenvector 2

(e) Spectral clustering segmentation



(f) MBO segmentation

Fig. 4: Jade plant data results

VI. Acknowledgments

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