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June 13th, 2017

### Overview

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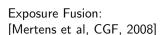
References

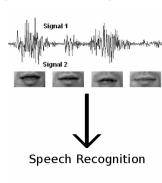
### Multimodal datasets

With the increasing availability of data, many applications involve data drawn from more than one source (called *modalities*).









Speech Recognition:
[Datcu et al, IEEE CVPR 2007]

# Challenges in multimodality

Most multimodal methods are developed specifically for one problem, BUT:

[Lahat et al, IEEE 2015]: "... a solution that is based on a sufficiently data-driven, model-free approach may turn out to be useful in very different domains."

### Our Previous Work

First attempt: Multimodal segmentation via graph methods.

Assumes datasets are coregistered (share a common indexing).

Focus on RGB/Lidar image segmentation.

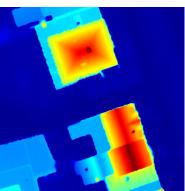
Accepted to ICIP, Sept 2017.

References

### Example Data

From IEEE Data Fusion Contest, 2015 [Bampos-Taberner et al, IEEE J-STARS 2016].





**RGB** Modality

Lidar Modality

#### Notation

From each modality, have a data set  $X^k$ .  $\ell =$  number of modalities.

N = number of observations.

 $d_j = \text{dimension of set } X^k$ . (Can view  $X^k \in \mathbb{R}^{N \times d_k}$ ).

From co-registration assumption: i-th point in  $X^{k_1}$  corresponds to i-th point in  $X^{k_2}$ .

Create concatenated set  $X = (X^1, X^2, \dots, X^{\ell}) \subseteq \mathbb{R}^{N \times (d_1 + \dots + d_{\ell})}$ .

 $x_i = \text{element } i \text{ from } X. \ x_i^k = \text{element } i \text{ from } X^k.$ 

## Graph Representation: Background

When using a single modality:

For each pair  $x_i, x_j \in X$ , define a weight  $w_{ij}$  that measures the similarity between the points.

 $\implies$  represent data as  $N \times N$  weight matrix W.

Common similarity measure from the literature: RBF kernel

$$w_{ij} = \exp\left(-\left\|x_i - x_j\right\|^2 / \sigma\right).$$

Need to adapt this to multimodal data.

## Multimodal Weight Matrix

For each modality  $X^k$ , calculate the distance matrix  $E^k$  via

$$E_{ij}^k = \left\| x_i^k - x_j^k \right\|.$$

 $\|\cdot\|$  chosen based on the details of the modality.

(in our examples  $\|\cdot\|$  is the 2-norm)

Scale each matrix by standard deviation (nondimensionalization)

$$\bar{E}^k = \frac{E^k}{\operatorname{std}(E^k)}.$$

## Multimodal Weight Matrix

#### Define

$$w_{ij} = \exp\left(-\max\left(\bar{E}_{ij}^1, \dots, \bar{E}_{ij}^k\right)/\sigma\right).$$

#### Heuristics:

- Standard deviation scaling allows us to directly compare  $\bar{E}^{k_1}$ ,  $\bar{E}^{k_2}$  with reasonable results.
- Because of the max, elements are similar under this measure only if they are similar in each modality.

### Graph min-cut

Introduction

Use weights W to segment X into  $A_1, \ldots, A_k$ . We want to

- group nodes with high similarity (weight) together
- ensure each set is a reasonable size

Use the Normalized graph-cut

$$\operatorname{Ncut}(A_1, \dots, A_k) = \frac{1}{2} \sum_{j=1}^k \frac{W(A_j, A_j^c)}{\operatorname{vol}(A_j)}.$$
 $W(A, B) = \sum_{i \in A, j \in B} w_{ij}.$ 
 $\operatorname{vol}(A) = \sum_{i \in A, j \in B} w_{ij}.$ 

Exact min-cut solution is computationally infeasible.

## Graph Laplacian

Introduction

A well-known relaxation for graph min-cut problem: eigenvectors of the (normalized) graph Laplacian L.

$$L = I - D^{-1/2}WD^{-1/2},$$

where  $D = N \times N$  diagonal matrix (degree matrix), with

$$d_{ii}=\sum_{j=1}^n w_{ij}.$$

#### Feature extraction

Result of [vonLuxburg, Stat Comput 2007]

eigenvectors of  $L \iff$  features extracted from data

Can use eigenvectors for a variety of applications.

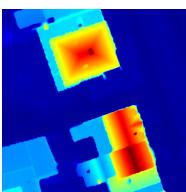
Our results: Use the eigenvectors to segment the data.

- K-means (unsupervised)
- Graph MBO (semisupervised) [Z. Meng et al, IPOL 2017]

#### Data

Our algorithm applied to DFC 2015 [Bampos-Taberner et al, IEEE J-STARS 2016].



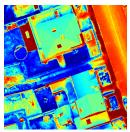


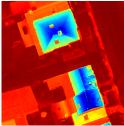
**RGB** Modality

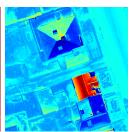
Lidar Modality

DFC2015 data

### Results



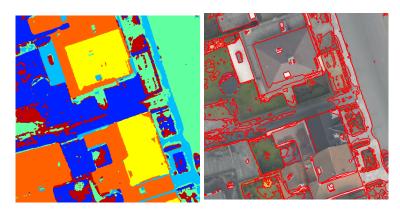




Example eigenvectors of graph Laplacian

#### Results

K-means result (unsupervised). m = 6 classes.

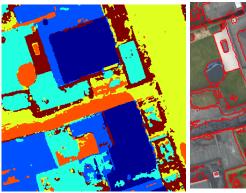


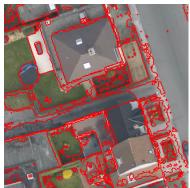
Classes

Regions on original image

### Results

### MBO (7% supervised). m = 6 classes.





Classes

Regions on original image

#### Data

Our algorithm applied to [Scharstein et al., GCPR 2014] dataset.

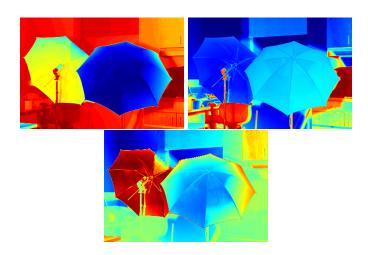


**RGB** Modality

Lidar Modality

Umbrella data

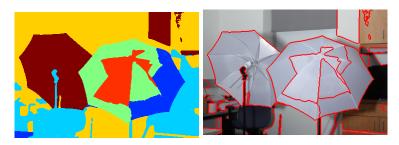
### Results



Example eigenvectors of graph Laplacian

#### Results

Spectral Clustering result (unsupervised). m = 6 classes.

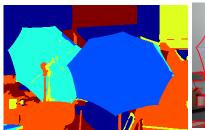


Classes

Regions on original image

### Results

MBO result (5% supervised). m = 6 classes.







Regions on original image

## **Graph Matching**

Goal: Remove or weaken the coregistration assumption.

Current idea: Graph matching.

View each dataset as a (weighted) graph.

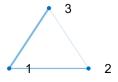
Try to match nodes with similar structure.

## Problem Setup

Two weighted graphs,  $G_1$ ,  $G_2$ , with weight matrices  $W_1$ ,  $W_2$ .

For now, 
$$|G_1| = |G_2| = N$$
.

Search for a graph isomorphism  $\textit{G}_1 \rightarrow \textit{G}_2$  preserving edge weights.





Best isomorphism is  $1 \rightarrow 2$ ,  $2 \rightarrow 3$ ,  $3 \rightarrow 1$ .

## Problem Setup

Isomorphism  $G_1 \to G_2$  corresponds to a permutation on nodes. Have P the corresponding permutation matrix. Want to minimize

$$\left\|W_1 - PW_2P^T\right\|_F^2.$$

Exact solution is too expensive. Can solve using graph Laplacian trick from

[Umeyama, IEEE TPAMI 1988, Knossow et al., GbRPR 2009].

#### Relaxation

Relax problem to

$$Q^* = \operatorname{argmin}_{QQ^T = I} \left\| W_1 - QW_2Q^T \right\|_F^2.$$

Let  $L_1, L_2$  the graph Laplacians corresponding to  $W_1, W_2$ 

 $U_1$ ,  $U_2$  the corresponding matrices of eigenvectors.

Then 
$$Q^* = U_1 S U_2^T$$
.

S is a diagonal matrix with entries of  $\pm 1$  to account for sign ambiguity in eigenvectors.

#### Heuristics

Recall from graph Laplacian

column of  $U_i \iff$  feature extracted from data row of  $U_i \iff$  image of data point in new feature space.

Match rows of  $U_1$  to rows of  $U_2$  by considering  $U_1U_2^T$ .

## Matching Algorithm

 $Q_{ii}^*$  gives the similarity between node i of  $G_1$  and node j of  $G_2$ .

Choose a permutation  $p:\{1,2,\ldots,N\} o \{1,2,\ldots,N\}$  via

$$\operatorname{argmax}_{\operatorname{permutations}\,p} \sum_{i=1}^{N} Q_{i,p(i)}^{*}.$$

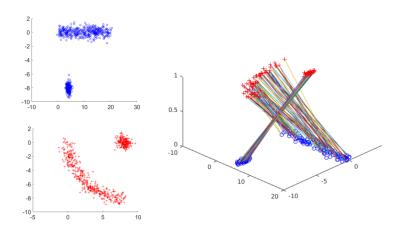
Hungarian algorithm finds this in  $O(N^3)$  [Munkres, SIAM 1957].

## Benefits of Graph Matching

#### Benefits of Graph Matching

- Invariant under conformal maps.
  - scaling, shifts, rotations, etc.
  - robust to continuous deformation.
- A precise number representing similarity between nodes gives us many options.
  - Thresholding
  - Hierarchical matching
- **3** Easy extension to the case  $|G_1| \neq |G_2|$ .

## Example Matching



Graph Match on Synthetic Data

## Change Detection

One possible application: Change detection.

Given images X and Y of the same scene, compare coregistration against results of graph matching. Use this to pick out large changes between X, Y.

From graph matching, get a permutation

$$\rho: \{1, \ldots, n\} \to \{1, \ldots, n\}.$$

Compare  $x_i$  to  $x_{\rho(i)}$ , and  $y_i$  to  $y_{\rho(i)}$ .

# Change Detection Example



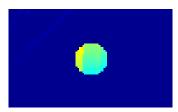
Image X



Naive difference ||X - Y||



Image Y



$$||x_i-x_{\rho(i)}||$$

#### References I



Tom Mertens and J. Kautz and Frank Van Reeth

Exposure Fusion: A Simple and Practical Alternative to High Dynamic Range Photography

Computer Graphics Forum 28(1):161 - 171



D. Datcu and Z. Yang and L. Rothkrantz (2007)

Multimodal workbench for automatic surveillance applications
2007 IEEE Conference on Computer Vision and Pattern Recognition 1-2

1

Lahat, Dana and Adalı, Tülay and Jutten, Christian (2015)

 $\label{eq:multimodal} \mbox{Multimodal Data Fusion: An Overview of Methods, Challenges and Prospects}$ 

Proceedings of the IEEE 103(9), 1449-1477



Yi-Ren Yes and Chun-Hao Huang and Yu-Chiang Frank Wang Heterogeneous Domain Adaptation and Classification by Exploiting the Correlation Subspace

IEEE Transactions on Image Processing 23(5), 2009-2018

### References II



Wang, Chang and Mahadevan, Sridhar (2013)

Manifold Alignment Preserving Global Geometry

Proceedings of the Twenty-Third International Joint Conference on Artificial Intelligence IJCAI '13, 1743-1749



Tuia, Devis AND Camps-Valls, Gustau (2016) Kernal Manifold Alignment for Domain Adaptation PLOS ONE 11, 1-25



M. Campos-Taberner and A. Romero-Soriano and C. Gatta and G. Camps-Valls and A. Lagrange and B. Le Saux and A. Beaupre and A. Boulch and A. Chan-Hon-Tong and S. Herbin and H. Randrianarivo and M. Ferecatu and M. Shimoni and G. Moser and D. Tuia (2015)

Processing of Extremely High-Resolution LiDAR and RGB Data: Outcome of the 2015 IEEE GRSS Data Fusion Contest #8211;Part A: 2-D Contest

IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing 9(12), 5547-5559

### References III



lyer, Geoffrey and Chanussot, Jocelyn and Bertozzi, Andrea (2017).

A Graph-Based Approach for Feature Extraction and Segmentation of Multimodal Images

Preprint



Daniel Scharstein and Heiko Hirschmüller and York Kitajima and Greg Krathwohl and Nera Nešić and Xi Wang and Porter Westling (2014) High-resolution stereo datasets with subpixel-accurate ground truth *Proceedings of the 36th German Conference on Pattern Recognition* 31-42



Umeyama, S. (1988)

An Eigendecomposition Approach to Weighted Graph Matching Problems *IEEE Trans. Pattern Anal. Mach. Intell.*, 10(5), 695-703



Knossow, David and Sharma, Avinash and Mateus, Diana and Horaud, Radu (2009)

Inexact Matching of Large and Sparse Graphs Using Laplacian Eigenvectors

Graph-Based Representations in Pattern Recognition: 7th IAPR-TC-15 International Workshop, 2009. Proceedings



vonLuxburg, Ulrike (2007)

A tutorial on spectral clustering Statistics and Computing, 17(4), 395-416



Ekaterina Merkurjev and Tijana Kostic and Andrea L Bertozzi (2013) An MBO scheme on graphs for classification and image processing SIAM Journal on Imaging Sciences, 6(4), 1903-1930

### References V



Charless Fowlkes and Serge Belongie and Fan Chung and Jitendra Malik (2004)

 ${\sf Spectral\ Grouping\ Using\ the\ Nystrom\ Method}$ 

IEEE Transactions on Pattern Analysis and Machine Intelligence, 26(2)



J. Munkres (1957)

Algorithms for the Assignment and Transportation Problems Journal of the Society for Industrial and Applied Mathematics, 5(1):3238



Z. Meng, E. Merkurjev, A. Koniges, and A. L. Bertozz (2017) Hyperspectral Image Classification Using Graph Clustering Methods IPOL 2017, preprint