Triangulation and Genetic Algorithms for Fingerprint Matching

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1 Introduction

The use of fingerprints for identification is one of the most popular and promising biometric identity-verification methods, especially for daily personal authentication. In the later half of the seminar *Pattern Recognition*, we have reviewed literature concerning fingerprint verification in a few different aspects.

Compared to other biometric data, to authenticate by fingerprint may be the most cost effective choice. Although Pankanti et al.[3] have shown that the persistence and individuality of fingerprints are not as reliable as commonly believed, the low cost of capturing and storing fingerprint images makes it possible to reduce the problem by using multi-modal methods.

Many researches have been conducted to facilitate the performance of fingerprint verification system by combining multiple enrolled samples in either the feature level [4], [8] or the decision level [9]. Each methods proposed has its own advantages, and usually the better the performance is, the more costly the computation will be. Yang and Zhou claimed in their most recently published article [9] that a higher accuracy can be reached by performing multiple matching (i.e., decision fusion in their definition), and this is also the most costly method. However, all articles reviewed in the seminar are based on minutiae-based matching algorithms, which are usually not discussed in detail in the research articles.

This report primarily digests two articles about fingerprint matching algorithms in the list of references. Parziable and Niel [10] proposed a **minutiae triangulation** method based on Delaunay triangulation. In a later article, Tan and Bhanu [6] combined **genetic algorithm (GA)** and the triangulation for fingerprint matching. The minutiae triangulation is summarized in section 2, and in section 3 the GA approach is described. Finally, in section 4, some other non-minutiae-based matching algorithms are briefly discussed, as well as some possible research ideas.

2 Fingerprint Matching Using Minutiae Triangulation

2.1 Minutiae and fingerprint matching

A fingerprint image can be seen as the lines flow in various patterns, which are called ridges. And Minutiae are features of a fingerprint, such as bifurcations (a ridge splitting into two) and ridge endings.

Some commonly used features (minutiae) in fingerprints are:

- **ridge endings** - a ridge that ends abruptly
- **ridge bifurcation** - a single ridge that divides into two ridges
- **short ridges**, island or independent ridge - a ridge that commences, travels a short distance and then ends
- **ridge enclosures** - a single ridge that bifurcates and reunites shortly afterward to continue as a single ridge
- **spur** - a bifurcation with a short ridge branching off a longer ridge
- **crossover or bridge** - a short ridge that runs between two parallel ridges

In order to perform the minutiae-based fingerprint matching, an image of a fingerprint must be first analyzed and converted into a set of minutiae, $M = \{m_1, m_2, \ldots\}$. Each minutia, $m_i = (x, y, \theta, \{p_i\})$, is a vector containing its position, orientation, and other information about this minutia.

The main goal of matching algorithms is to compare two sets of minutiae, Q (query fingerprint) the and T (template fingerprint), to determine if they match each other. Because fingerprint images are usually noisy and deformed, the matching process will first finds an optimal geometric transformation between Q and T, then examines if the mismatch is smaller than a certain threshold.

This procedure can be mathematically expressed as following:

$Q = \{m_1^Q, m_2^Q, \ldots, m_M^Q\} \subset R^2$ and $T = \{m_1^T, m_2^T, \ldots, m_M^T\} \subset R^2$ are the sets of 2-D points extracted from the query and template image, respectively. A similarity transformation, $Tr_{s,\theta,t_x,t_y} : Q' \to T'$ that maps each point of a subset $Q' \subseteq Q$ with each point of a subset $T' \subseteq T$ has to be found. Then, $\forall m_j^{Q'}$, $\exists$ a unique $m_i^{T'}$ such that,

$$Tr \begin{pmatrix} x_j^{Q'} \\ y_j^{Q'} \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + s \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_j^{T'} \\ y_j^{T'} \end{pmatrix} \quad (1)$$

where s is a scale factor, θ is a rotation angle and t_x and t_y are the translations along x and y axes, respectively. In [10] (and most other studies), each minutia is defined as $m_i = (x, y, \theta)$, i.e. its position and orientation. And by setting thresholds, d_0 and θ_0, for Euclidean distance (ED) and angular difference (AD), respectively, two minutiae match if and only if:

$$ED(m_j^{Q'}, m_i^{T'}) = \left| Tr \begin{pmatrix} x_j^{Q'} \\ y_j^{Q'} \end{pmatrix} - \begin{pmatrix} x_j^{T'} \\ y_j^{T'} \end{pmatrix} \right| \le d_0 \quad (2)$$

$$AD(m_j^{Q'}, m_i^{T'}) = min(|\theta_j^{Q'} - \theta_i^{T'}|, 2\pi - |\theta_j^{Q'} - \theta_i^{T'}|) \quad (3)$$

$$AD(m_j, m_i) = min(|\theta_j - \theta_i|, \pi - |\theta_j - \theta_i|) \tag{3}$$

2.2 Delaunay triangulation

In 1934, Boris Delaunay [2] proposed a triangularization for a set P of points in a plane, which is commonly used in mathematics and computational geometry. Delaunay triangulation $DT(P)$ is a triangulation of P such that no point in P is inside the circumcircle of any triangle in $DT(P)$.

In the general n-dimensional case, it may be stated as : for a set P of points in the (n-dimensional) Euclidean space, the Delaunay triangulation is the triangulation

$DT(P)$ of P such that no point in P is inside the circum-hypersphere of any simplex in $DT(P)$.

It is known that the Delaunay triangulation exists and is unique for P, if P is a set of points in general position; that is, no three points are on the same line and no four are on the same circle, for a two dimensional set of points, or no $n+1$ points are on the same hyperplane and no $n+2$ points are on the same hypersphere, for an n-dimensional set of points.

Some algorithms have been shown for finding the Delaunay triangulation of a set of points in n-dimensional Euclidean space. Generalizations are possible to metrics other than Euclidean. However in these cases the Delaunay triangulation is not guaranteed to exist or be unique.

2.3 Minutiae triangulation

Based on the observation that, even if deformations, rotations and translations are applied to a fingerprint image, every minutia keeps always the same neighbor structure, a triangulation of minutiae may show some advantages in matching minutiae pairs.

In order to construct the triangulation, following quantities from each minutia pair are extracted:

- the distance L between the two minutiae;
- the angle α between the orientations of the two minutiae (angular difference between τ_i and τ_j);
- the angles β_1 and β_2 between the orientation of each minutia and the segment connecting them.

With these information, each minutia is used as a triangle vertex to establish the Delaunay triangulation. And then a match between the query and the template may be defined as:

$$D_L = \frac{|L^Q - L^T|}{min(L^Q, L^T)} < Th_L \tag{4}$$

$$D_\alpha = |\alpha^Q - \alpha^T| < Th_\alpha \tag{5}$$

$$D_\beta = \left| \begin{pmatrix} \beta_1^Q \\ \beta_2^Q \end{pmatrix} - \begin{pmatrix} \beta_1^T \\ \beta_2^T \end{pmatrix} \right| < Th_\beta \tag{6}$$

where Th_L, Th_α and Th_β are fixed thresholds where Th_L, Th_α and Th_β are fixed thresholds.

During the matching process, equations 5 and 6 are applied only if equation 4 is satisfied. As a result, the number of checks is largely reduced due to this triangulation structure.

The experimental results of [10] showed that minutiae triangulation method outperforms a orientation-based minutiae representation proposed by Tico and Kousmanen [7]. However, this comparison is based on some hand-derived minutiae sets, rather than automatic extraction.