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LABELED POINT PATTERN MATCHING BY FUZZY RELAXATION

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Abstract—A method of matching labeled point patterns is described on the basis of a fuzzy relaxation. The method is applicable to labeled point patterns which differ by geometrical transformations, such as translation, rotation and scale change. In this method, the point pairs are considered to be the pattern primitives. The pattern primitives are geometrically transformed in the process of relaxation so as to minimize a measure of mismatch between the primitive pairs. The compatible primitive pairs between the two labeled point patterns are established after only a few iterations of relaxation and the corresponding points are obtained from the primitive pairs. As an example, the method is applied to the matching of constellations.

Matching   Registration   Point patterns   Fuzzy relaxation

1. INTRODUCTION

In pattern recognition and image processing, pattern matching is a fundamental technique in order to register or identify the objects of interest. Therefore, the methodologies concerning pattern matching have been much studied. In recent years, point pattern matching has attracted notice and is studied because of its potential usefulness to patterns which consist of a finite set of feature points obtained from an image by local observations and feature extractions. Some experiments in point pattern matching based on a brute-force approach and a relaxation approach are described in the literatures.[1,2] Ranade *et al.*[2] also describes the usefulness of point pattern matching and give experimental results showing that the relaxation approach is more tolerant of global distortions in the patterns. However, in these methods it is assumed that the patterns differ only by translation, so the matching process is quite sensitive to geometrical transformations, such as rotation and scale change.

In this paper, a method of matching two labeled point patterns is described on the basis of a fuzzy relaxation. The method is applicable to labeled point patterns which differ by the geometrical transformations mentioned above. One of the two labeled point patterns can be thought of as a template or model of some object of interest and the other can be thought of as a description of a world in which we are searching for an instance of this object, as mentioned in Kitchen and Rosenfeld[3] and Kitchen.[4] In this method, the point pairs are considered to be the pattern primitives. The pattern primitives are geometrically transformed in the process of relaxation so as to minimize a measure of mismatch between the primitive pairs. The compatible primitive pairs between the two point patterns are established after only a few iterations of relaxation and the corresponding points are obtained from the primitive pairs. It should be noted that by a geometrical transformation we mean a translation, a rotation or a scale change in this paper. In Section 2, a brief definition or notation of a labeled point pattern is given. The labeled point pattern matching algorithm is proposed in Section 3, and the method is applied to the matching of constellations in Section 4.

2. DEFINITION OF LABELED POINT PATTERN

A labeled point pattern M is a finite set of labeled points P_i. As mentioned later, the symbol M will be used to denote a template or model of some object of interest in this paper. A labeled point P_i is described with the coordinates (X_{p_i}, Y_{p_i}) and the label F_{p_i}. The coordinates show the location of the point on a template or picture in general. The label F_{p_i} corresponds to the properties which characterize the point or a local region around the point on a picture. Accordingly, a labeled point pattern M can be written as follows.

$$M = \{P_i\} = \{(X_{p_i}, Y_{p_i}, F_{p_i})\}, \quad i = 1, 2, \ldots, m(M). \tag{1}$$

In the problem of labeled point pattern matching discussed in this paper, one of the two labeled point patterns can be thought of as a model of some object of interest, so denoted with the symbol M, and the other can be thought of as a description of a world in which we are searching for an instance of this object. The world, denoted with the symbol W, can be written in the same manner as the model M, as follows.

$$W = \{Q_j\} = \{(X_{q_j}, Y_{q_j}, F_{q_j})\}, \quad j = 1, 2, \ldots, m(W). \tag{2}$$

In the following section, an idea of primitive elements (primitives for short) of the point patterns is introduced.

3. MATCHING METHODOLOGY

Though point pair displacements are considered in the literatures[1,2] in order to evaluate the local consistency of point pairings, the method is not directly applicable to the matching of geometrically transformed point patterns. Accordingly, the concept of point pairs is employed in this paper. Point pairs are regarded as the primitive elements (the primitives for short) of the point patterns and denoted with the symbol $\mathbf{P}_{ik}$ or (P_i, P_k). A measure of match between the two primitive pairs is defined to evaluate the local consistency of the primitive pairings. Namely, by considering four points (i.e. two primitive pairs) at a time, it becomes possible to apply the geometrical transformation to the point pattern in the process of relaxation in order to achieve an optimum matching.

Composition of primitives

Three typical means to compose the primitives of a model M can be proposed, as follows.

Type 1: Compose all of the possible pairs, as shown in Fig. 1(a).

Type 2: Compose pairs on condition that any point is not involved in plural pairs, as shown in Fig. 1(b). In a situation where the number of points is odd, only one point can be involved in two pairs.

Type 3: Pair a point P_i which has a distinguishing label with other remaining points P_k, as shown in Fig. 1(c).

In this way, a set of primitives of a model, denoted by the symbol Ω_m, is composed. The first means provides a complete set of primitives, but the number of primitives is so large that the following process (i.e. relaxation) becomes computationally costly. On the other hand, the second means provides the smallest number of primitives. The last one is quite practical because of the fact that every primitive has a point with a distinguishing label, therefore, in the process of relaxation, the compatibility of primitive pairs can be easily judged by comparing the labels of the primitives. This implies that computation cost can be considerably reduced.

The primitives of a description of a world should be composed referring to the set of primitives of the model of some object which we are searching for an instance of in the world, in order to guarantee the existence of the corresponding primitives when an instance of the object or model really exists in the world. This can be attained by the following steps.

Step 1: Take a pair of points of the world and denote as $\mathbf{Q}_{jl}$ or (Q_j, Q_l).

Step 2: Compare the labels of $\mathbf{Q}_{jl}$ (i.e. (F_{q_j}, F_{q_l})) with the labels of the model primitives (i.e. (F_{p_i}, F_{p_k})).

Step 3: Employ $\mathbf{Q}_{jl}$ as a primitive of the world when there exists a model primitive $\mathbf{P}_{ik}$ with 'similar' labels, or discard $\mathbf{Q}_{jl}$ otherwise.

Step 4: Repeat the above steps until all of the possible pairs of points of the world are examined.

In this way, a set of primitives of a world, denoted by the symbol Ω_w, is composed. The following paragraph is devoted to describing the method of measuring the goodness of primitive pairings.

Goodness of primitive pairings

Let us assume that a primitive $\mathbf{P}_{ik} \in \Omega_m$ corresponds to a primitive $\mathbf{Q}_{jl} \in \Omega_w$. Then for any $\mathbf{P}_{su} \in \Omega_m$, there should be a $\mathbf{Q}_{tv} \in \Omega_w$ which is in the similar position relative to $\mathbf{Q}_{jl}$ that the $\mathbf{P}_{su}$ has relative to $\mathbf{P}_{ik}$. To evaluate the virtual displacement of the two primitive pairs, i.e. $(\mathbf{P}_{ik}, \mathbf{P}_{su})$ and $(\mathbf{Q}_{jl}, \mathbf{Q}_{tv})$, a geometrical transformation $\Phi: (\mathbf{Q}_{jl}, \mathbf{Q}_{tv}) \rightarrow (\hat{\mathbf{Q}}_{jl}, \hat{\mathbf{Q}}_{tv})$ which minimizes the measure of mismatch (defined in equation (4)) between $(\mathbf{P}_{ik}, \mathbf{P}_{su})$ and $(\hat{\mathbf{Q}}_{jl}, \hat{\mathbf{Q}}_{tv})$ is found. The amount of mismatch is used to measure the goodness of the pairings, i.e. the measure of match defined in equation (11). The transformation Φ and the minimum of the amount of mismatch can be obtained as follows.

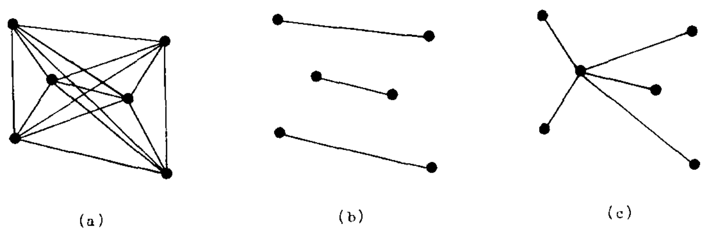
When we are considering the two primitive pairs (i.e. $(\mathbf{P}_{ik}, \mathbf{Q}_{jl})$ and $(\mathbf{P}_{su}, \mathbf{Q}_{tv})$), there are four possibilities of point pairings as follows.

Case 1: $(P_i, P_k)\,(P_s, P_u) \leftrightarrow (Q_j, Q_l)\,(Q_t, Q_v)$.

Case 2: $(P_i, P_k)\,(P_s, P_u) \leftrightarrow (Q_j, Q_l)\,(Q_v, Q_t)$.

Case 3: $(P_i, P_k)\,(P_s, P_u) \leftrightarrow (Q_l, Q_j)\,(Q_t, Q_v)$.

Case 4: $(P_i, P_k)\,(P_s, P_u) \leftrightarrow (Q_l, Q_j)\,(Q_v, Q_t)$.



(a) (b) (c)

Fig. 1. Three types of composing primitives: (a) type 1; (b) type 2; (c) type 3.

Case 1, for example, means that P_i corresponds to Q_j, P_k to Q_l, P_s to Q_t and P_u to Q_v, respectively. But in fact, some of the possibilities can be discarded by employing an appropriate function to evaluate the similarity of the labels of points, as well as a similarity tolerance T. The function and tolerance must be identical to those used in the process of composing primitives for a world. In the experiments shown in the next section, the distance function $|F_{p_i} - F_{q_j}|$ and $T = 0$ are used.

For each remaining case, ε^2 in equation (4) or (10) is calculated and then the minimum value is employed as the measure of mismatch between the two primitive pairs, i.e. $(\mathbf{P}_{ik}, \mathbf{Q}_{jl})$ and $(\mathbf{P}_{su}, \mathbf{Q}_{tv})$. Here we define the measure of mismatch* precisely. For the simplicity of notation, let us denote the considered point pairs as

$$(P_1, P_2)\,(P_3, P_4) \leftrightarrow (Q_1, Q_2)\,(Q_3, Q_4),$$

where the labels of the points are omitted for expediency, namely $P_i = (X_{p_i}, Y_{p_i})$ and $Q_j = (X_{q_j}, Y_{q_j})$. Furthermore, introducing the notation of complex numbers, the geometrical transformation and the measure of mismatch ε^2 can be formulated as

$$\hat{Q}^c_j = ZQ^c_j + W, \quad j = 1, 2, 3, 4, \tag{3}$$

$$\varepsilon^2 = \min_{Z,W}\left\{\sum_{j=1}^{4} |P^c_j - \hat{Q}^c_j|^2\right\} \tag{4}$$

where $Z = z_1 + iz_2$, $W = w_1 + iw_2$, $\hat{Q}^c_j = X_{\hat{q}_j} + iY_{\hat{q}_j}$, $Q^c_j = X_{q_j} + iY_{q_j}$ and $P^c_j = X_{p_j} + iY_{p_j}$ and $i = \sqrt{-1}$. The coefficients, i.e. z_1, z_2, w_1, w_2, of the transformation which minimize ε^2 are given by, with the vector notations,

$$\mathbf{X}_p = (X_{p_1}\,X_{p_2}\,X_{p_3}\,X_{p_4})^{\mathrm{T}}, \mathbf{Y}_p = (Y_{p_1}\,Y_{p_2}\,Y_{p_3}\,Y_{p_4})^{\mathrm{T}}, \mathbf{X}_q = (X_{q_1}\,X_{q_2}\,X_{q_3}\,X_{q_4})^{\mathrm{T}},$$

$$\mathbf{Y}_q = (Y_{q_1}\,Y_{q_2}\,Y_{q_3}\,Y_{q_4})^{\mathrm{T}} \text{ and } \mathbf{U} = (1\ 1\ 1\ 1)^{\mathrm{T}},$$

$$z_1 = (4\mathbf{X}^{\mathrm{T}}_p\mathbf{X}_q + 4\mathbf{Y}^{\mathrm{T}}_p\mathbf{Y}_q - (\mathbf{X}^{\mathrm{T}}_q\mathbf{U})(\mathbf{X}^{\mathrm{T}}_q\mathbf{U}) - (\mathbf{Y}^{\mathrm{T}}_q\mathbf{U})(\mathbf{Y}^{\mathrm{T}}_p\mathbf{U}))/a, \tag{5}$$

$$z_2 = (4\mathbf{Y}^{\mathrm{T}}_p\mathbf{X}_q - 4\mathbf{X}^{\mathrm{T}}_p\mathbf{Y}_q + (\mathbf{Y}^{\mathrm{T}}_q\mathbf{U})(\mathbf{X}^{\mathrm{T}}_p\mathbf{U}) - (\mathbf{X}^{\mathrm{T}}_q\mathbf{U})(\mathbf{Y}^{\mathrm{T}}_p\mathbf{U}))/a, \tag{6}$$

$$w_1 = ((\mathbf{X}^{\mathrm{T}}_q\mathbf{X}_q + \mathbf{Y}^{\mathrm{T}}_q\mathbf{Y}_q)(\mathbf{X}^{\mathrm{T}}_p\mathbf{U}) - (\mathbf{X}^{\mathrm{T}}_p\mathbf{X}_q + \mathbf{Y}^{\mathrm{T}}_p\mathbf{Y}_q) \times (\mathbf{X}^{\mathrm{T}}_q\mathbf{U}) + (\mathbf{Y}^{\mathrm{T}}_p\mathbf{X}_q - \mathbf{X}^{\mathrm{T}}_p\mathbf{Y}_q)(\mathbf{Y}^{\mathrm{T}}_q\mathbf{U}))/a, \tag{7}$$

$$w_2 = ((\mathbf{X}^{\mathrm{T}}_q\mathbf{X}_q + \mathbf{Y}^{\mathrm{T}}_q\mathbf{Y}_q)(\mathbf{Y}^{\mathrm{T}}_p\mathbf{U}) - (\mathbf{X}^{\mathrm{T}}_p\mathbf{X}_q + \mathbf{Y}^{\mathrm{T}}_p\mathbf{X}_q) \times (\mathbf{Y}^{\mathrm{T}}_q\mathbf{U}) - (\mathbf{Y}^{\mathrm{T}}_p\mathbf{X}_q - \mathbf{X}^{\mathrm{T}}_p\mathbf{Y}_q)(\mathbf{X}^{\mathrm{T}}_q\mathbf{U}))/a, \tag{8}$$

where

$$a = 4\mathbf{X}^{\mathrm{T}}_q\mathbf{X}_q + 4\mathbf{Y}^{\mathrm{T}}_q\mathbf{Y}_q - (\mathbf{X}^{\mathrm{T}}_q\mathbf{U})^2 - (\mathbf{Y}^{\mathrm{T}}_q\mathbf{U})^2. \tag{9}$$

From equation (4) we have

$$\varepsilon^2 = \sum_{i=1}^{4}\{(X_{p_i} - z_1X_{q_i} + z_2Y_{q_i} - w_1)^2 + (Y_{p_i} - z_1Y_{q_i} - z_2X_{q_i} - w_2)^2\}. \tag{10}$$

Finally, using ε^2, the measure of goodness of the primitive pairings, i.e. the measure of match, can be defined as follows.

$$G\{(\mathbf{P}_{ik}, \mathbf{Q}_{jl}), (\mathbf{P}_{su}, \mathbf{Q}_{tv})\} = \frac{1}{1 + \alpha\varepsilon^2}, \tag{11}$$

where α is an arbitrary constant.

Relaxation process

The fuzzy relaxation scheme to calculate the amount of total support, denoted as $C(\mathbf{P}_{ik}, \mathbf{Q}_{jl})$, for the primitive pairing $(\mathbf{P}_{ik}, \mathbf{Q}_{jl})$ can be formulated as follows.

(1) Set the initial values $C^0(\mathbf{P}_{ik}, \mathbf{Q}_{jl}) = 1.0$ for all $\mathbf{P}_{ik} \in \Omega_m$ and $\mathbf{Q}_{jl} \in \Omega_w$.

(2) Iterate the following process until the convergence criterion defined later is satisfied.

$$C^n(\mathbf{P}_{ik}, \mathbf{Q}_{jl}) = \min_{\substack{\mathbf{P}_{su}\in\Omega \\ \mathbf{P}_{su}\neq\mathbf{P}_{ik}}}\left[\max_{\substack{\mathbf{Q}_{tv}\in\Omega_w \\ \mathbf{Q}_{tv}\neq\mathbf{Q}_{jl}}}\{\min[\mathrm{G}\{(\mathbf{P}_{ik}, \mathbf{Q}_{jl}) \times (\mathbf{P}_{su}, \mathbf{Q}_{tv})\}, C^{n-1}(\mathbf{P}_{su}, \mathbf{Q}_{tv})]\}\right], \tag{12}$$

for $n = 1, 2, 3, \ldots$

(3) Convergence criterion:

$C^n(\mathbf{P}_{ik}, \mathbf{Q}_{jl}) = C^{n-1}(\mathbf{P}_{ik}, \mathbf{Q}_{jl})$, for all $\mathbf{P}_{ik} \in \Omega_m$ and $\mathbf{Q}_{jl} \in \Omega_w$.

This condition is necessarily satisfied.[4]

After the process converges, the primitive pairs which have the maximum value of $C^n(\mathbf{P}_{ik}, \mathbf{Q}_{jl})$ are taken. It is then straightforward to derive the corresponding point pairs from the primitive pairs.

4. EXPERIMENTS

The proposed matching methodology was applied to the matching of constellations. Figure 2 shows a description of a world, i.e. part of a star map, which was obtained by translating each star on a star map to the nearest point on a 60 × 60 grid. Figure 3 shows models

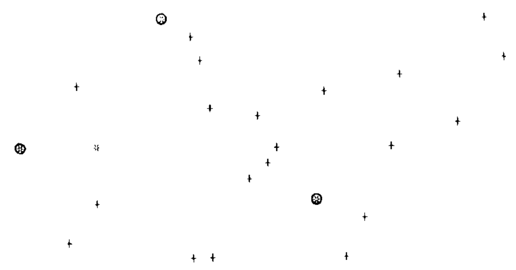


Fig. 2. A point pattern (a world) obtained from part of a star map. ⊛ Star of the first magnitude; * star of the second magnitude; + star of the third magnitude.

* The procedure of measuring the amount of mismatch employed here is similar to the method described in Davis.[5] A different way, based on the angle and relative distance between the primitives, has been proposed by Ogawa and Taniguchi.[6]

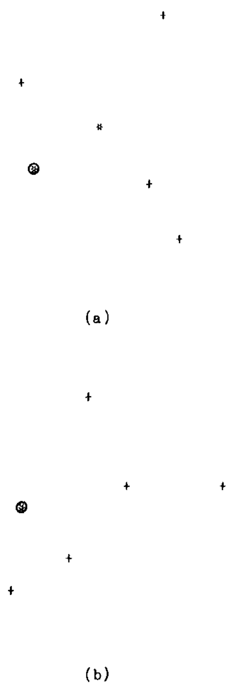


Fig. 3. Point patterns (models) obtained from constellations: (a) Cygnus; (b) Aquila.

of the constellations, i.e. Cygnus and Aquila, which were obtained in the same manner from a star map of different position, scale and orientation. This also introduced systematic distortions between the models and the world. The magnitude of a star was employed as the label of a star or point. In these experiments, the distance function $|F_{p_i} - F_{q_j}|$ was used to evaluate the dissimilarity of the labels and the tolerance $T = 0$ was used. The correct point pairings were obtained after four to six iterations for both Cygnus and Aquila, and also for each manner of composing primitives. From the viewpoint of computation cost, when the model has a point with a distinguishing label, the third means (type 3) of composing primitives becomes considerably practical. For example, Cygnus has a star with a distinguishing label, i.e. a star of the second magnitude, therefore, the matching operation between the primitives composed by this means (type 3) was quite fast. In fact, type 3 composed 5 primitives for the model and 23 primitives for the world, respectively. However the second means (type 2) composed 3 primitives for the model and 193 primitives for the world. On the contrary, in the situation where no point has a distinguishing label, the means of type 2 becomes useful. It is needless to say that the first means (type 1) requires much more computation cost compared with two other.

5. CONCLUSION

The labeled point pattern matching algorithm described in this paper is applicable to labeled point patterns which differ by geometrical transformations, such as translation, rotation and scale change. This was realized by employing the idea that the point pairs can be regarded as the pattern primitives of the labeled point patterns. From the viewpoint of computation cost, if the model has a point with a distinguishing label, the primitives should be composed in a manner such that the point which has the distinguishing label is paired with other remaining points. Then the computation cost can be greatly reduced. Finally, for the situation where some points of a world which really correspond to the points of a model are deleted, the formulation for the calculation of the amount of total support described in this paper is not necessarily adequate. Further studies on this problem and applications of this method are planned.

6. SUMMARY

Point pattern matching attracts notice because of its potential usefulness to patterns which consist of a finite set of feature points obtained by local observations and feature extractions from an image. In this paper, a labeled point pattern matching algorithm based on a fuzzy relaxation is described, which is applicable to point patterns which differ by geometrical transformations, such as translation, rotation and scale change. This was realized by regarding the point pairs as the pattern primitives of the labeled point patterns and then applying geometrical transformations to the primitives in the process of relaxation so as to minimize a measure of mismatch between the two primitive pairs. Three means of composing primitives are also described. From the viewpoint of computation cost, if the point pattern (model) has a point with a distinguishing label (properties), the primitives should be composed in a manner such that the point with the distinguishing label is paired with other remaining points. Then the computation cost is greatly reduced. Finally, as an example of applying the proposed method, experiments of matching constellations are described.

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