MATHEMATICAL MODEL OF CABLE WINDING/UNWINDING SYSTEM

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ABSTRACT

The general form of mathematical model of cable winding/unwinding system is defined for several different constructions. The novelty of this mathematical model is detection and mathematical formulation of influence of new dynamic variables: winding/unwinding radius and cable length on dynamic response of cable winding/unwinding system. The validity of the obtained theoretical contribution has been illustrated through one case study by using a newly developed software package CWUSOFT which was generated in MATLAB. Theoretical and simulation results are confirmed through the experimental analysis of one novel construction of the cable winding/unwinding system.

Keywords: Winding/unwinding, Winch, Kinematics, Dynamics.

1. INTRODUCTION

Various complex mechatronic systems have cable (ropes as well) winding/unwinding systems as main sub-systems. There are a lot of these systems in different areas of science and engineering. Some of these systems are: measuring mechanism, machines in textile industry, cable logging systems in civil engineering and forestry, cranes, systems in shipping industry, CPR (Cable-suspended Parallel Robot) and other complex cable driven systems.

Phenomenon of cable winding/unwinding (hereinafter referred to as CWU) on winch is present in different applications, so in this paper the importance of its analysis will be emphasized. Some of papers which have inspired the research presented in this paper will be mentioned in the following paragraphs.

Authors of [1] have written a review paper on application of cable logging systems. Goals of [1] are far-reaching because authors define the instruction set for users that use those systems. These instructions help the users to solve several problems such are protection of workers, soil and forests. The author of [2] gives a detailed image about the history of CWU systems. The author points the fact that these systems were used in different areas for several millennia. One type of systems which use sub-systems for CWU are cranes and these systems are thoroughly analysed in [3].

In [4-6], authors present theoretical and experimental contribution to analysis and synthesis of kinematics and dynamics of winding/unwinding process of thread from a balloon. Fluctuating tensions in a perfectly flexible string unwinding from a stationary package were considered in [4]. Also, dependence of unwinding tensions on air resistance, unwinding speed, angle of winding on the package etc. was examined. Based on results from [4], in [5] over-end unwinding of yarn from a stationary helically wound cylindrical package was considered. An improved theory for the variation of yarn tension during high speed over-end unwinding from cylindrical yarn packages based upon the theory of bent and twisted elastic rods was presented in [6].

Authors of [7] presented a dynamic simulation for wire cable on the tower crane. They have considered not only the contact with the winch drum, but also the characteristics of the hydraulic system using SINDYS. Specifically, the following points have been demonstrated: (1) The contact between the winch drum and wire cable can be modelled by using the variable-length truss elements and the contact spring. (2) The dynamic behaviour of wire cable that occurs at hydraulic winch stopping is affected by the dynamic characteristics of the

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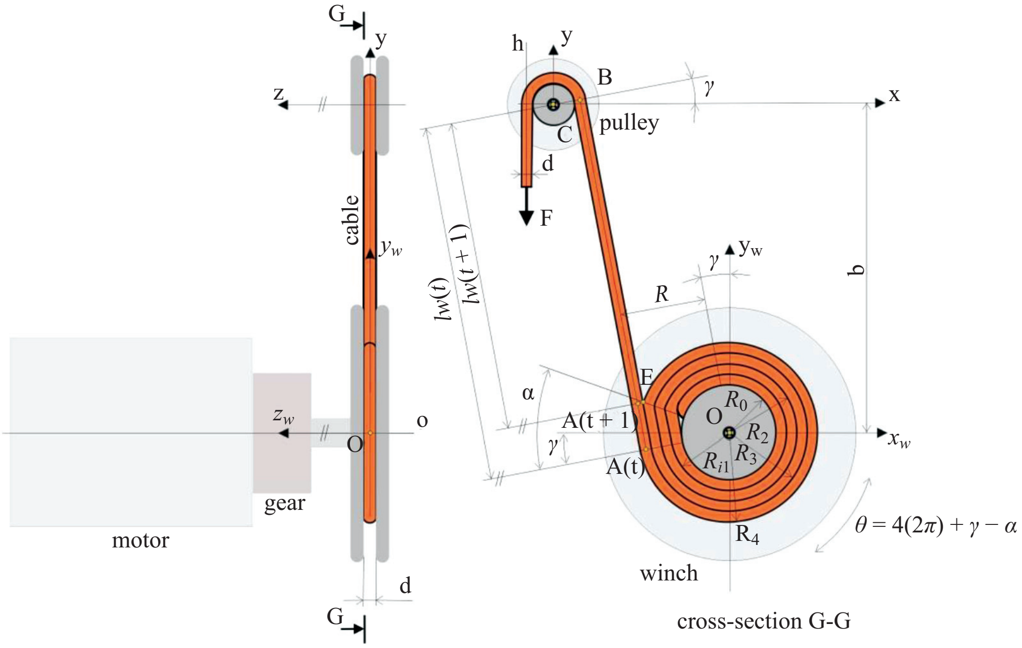


Fig. 1 Standard system for single – row radial multi-layered CWU process.

hydraulic system. (3) A slow-stopping hydraulic winch system has been proposed, and the system can prevent disordered winding even if the winch is rapidly operated.

In [8], authors presented the mathematical model of a pipelay spread. In the model, elasto-plastic deflections of the pipe, its large deformations and contact problems were considered. The modification of the rigid finite element method was used to discretize the pipe.

Wire-guided control technologies are widely used to increase the targeting accuracy of advanced military weapons through the use of unwinding dispensers to guarantee that unwinding occurs without any problems, such as tangling or cutting. In [9], the transient behaviour of cables unwinding from inner-winding cylindrical spool dispensers was investigated.

Authors of [10] dealt with design, analysis and synthesis of the cable suspended parallel robotic system (CPR system). They have used a well-known winding/unwinding sub-system presented in [11].

Based on analysis and synthesis of different types of CWU systems, in this paper a general form of mathematical model of these systems is presented. Mathematical model of CWU system indicates the impact of all the dynamic parameters of this system on its dynamic response. Theoretical contribution will be confirmed through experimental analysis of one chosen type of CWU system.

For the verification of the defined mathematical model of the system, the novel program package CWUSOFT was used.

After the Introduction, in Section 2, different types of CWU systems are presented. After that, the general form of mathematical model of CWU system is formulated in Section 3. By using the new mathematical model, the program package CWUSOFT is presented in Section 4. This program package is used to perform simulation of the motion of one chosen CWU system's load in Section 5. In Section 6, experimental confirmation of theoretical contributions is presented. In the last part of the paper the conclusions and observations are presented.

2. DIFFERENT TYPES OF CABLE WINDING/UNWINDING SYSTEMS

In this part of paper, several types of CWU systems will be presented. Their constructive differences will be indicated. The CWU system consists of: motor, gear, winch, pulley and cable that is wound/unwound on the winch.

2.1 Standard System for Single – Row Radial Multi-Layered CWU Process

This construction of CWU system has a circular shape of the winch and it is shown in Fig. 1. Detailed description of this system is presented in [12]. It was shown that this solution of the winch has adverse effects on the system's dynamic response and it causes instability and oscillations of the system. This structural instability of the winch from Fig. 1 has inspired the authors of this paper to design a new form of the winch for performing the smooth process of CWU. This novel solution of CWU system for smooth winding/unwinding in two variants will be presented in the next sub-section. The new constructive solution of the system intended for performing smooth CWU process is presented in Fig. 2.

2.2 Novel System for Smooth Single – Row Radial Multi-Lyered CWU Process

Figure 2 presents two new constructive solutions of

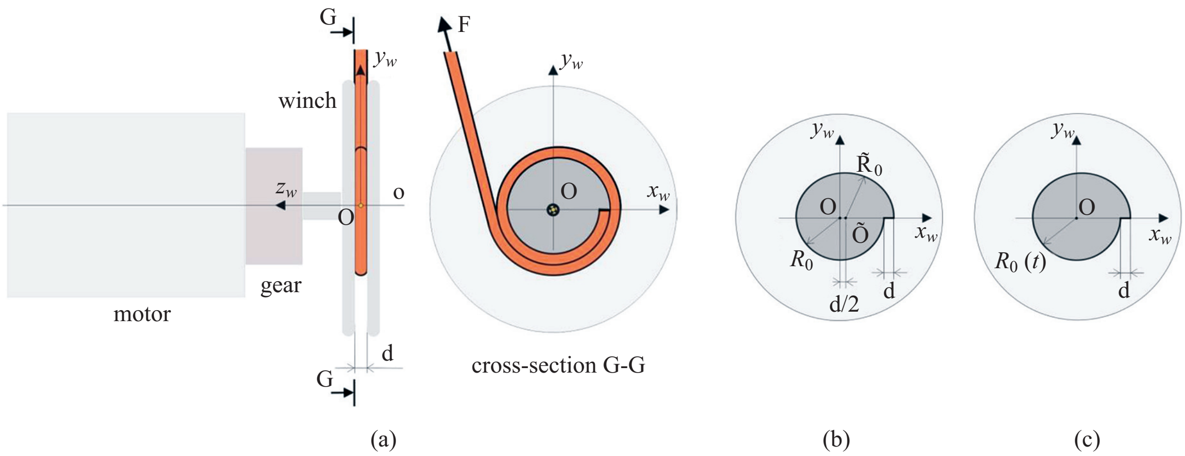


Fig. 2 The new winch for performing a smooth CWU process: (a) the winding/unwinding system, (b) the two – cylinder winch, (c) the spiral winch.

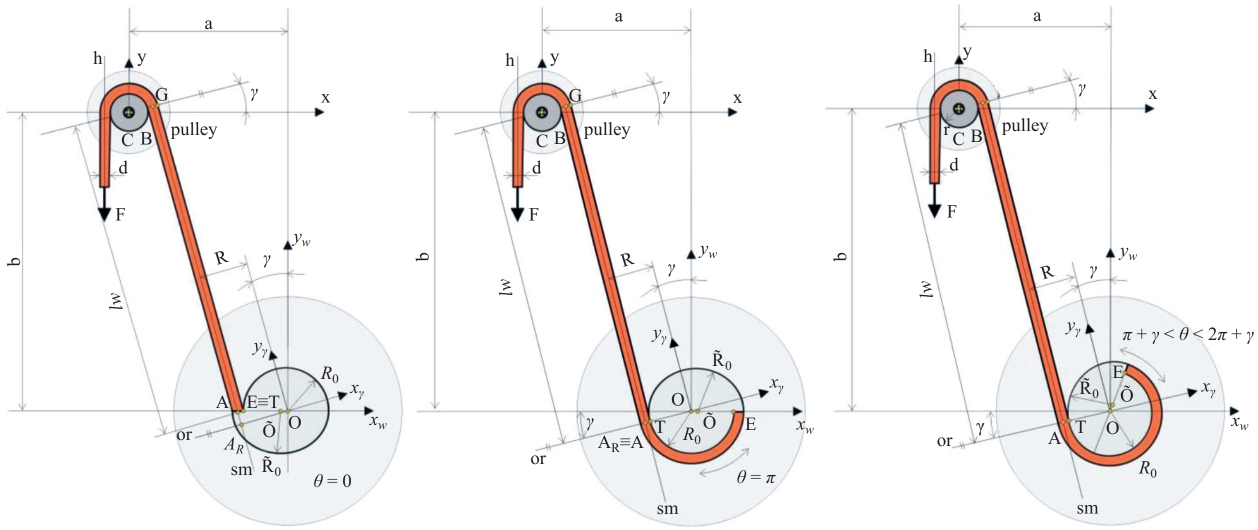


Fig. 3 The positions of the smooth CWU system for: (a) $\theta = 0$, (b) $\gamma < \theta < \pi + \gamma$, (c) $\pi + \gamma < \theta < 2\pi + \gamma$.

the winch which can be used to avoid the constructively generated unstable and oscillatory behaviour of the system from Fig. 1:

1. The first constructive solution consists of two semicylindrical bodies of different radii and it is presented in Fig. 2(b). Because of the characteristics of this winch, it has been named the two – cylinder winch.

2. The second constructive solution has a spiral shape and this winch is shown in Fig. 2(c). It has been named the spiral winch.

By using either of the two new constructive solutions of systems from Fig. 2: the two – cylinder winch (Fig. 2(b)) or the spiral winch (Fig. 2(c)), a smooth process of CWU on the winch is achieved. For further discussion, only the constructive solution from Fig. 2(b) will be used.

Figure 3 presents three currently selected positions during the process of the CWU on the winch. Angle θ is measured as deflection between the line $\overline{OE}$ and negative part of the x_w axis, in positive mathematical direction, around point O. The process presented in Fig. 3 is named as: single – row radial multi-layered smooth CWU process on the winch or abbreviated smooth process of CWU on the winch. Dynamic variables which characterize the process shown in Fig. 3 are the following: winding/unwinding radius R, length $lw = \overline{AB}$, and angle γ.

The change of these variables during the CWU process is smooth and nonlinear. Three phases of this process are to be observed:

a) in Fig. 3(a) the starting position of the CWU system is presented. In this case angle θ is $\theta = 0$.
b) in Fig. 3(b) the position of the smooth CWU process for the following values of angle θ is presented: $\gamma < \theta < \pi + \gamma$. In this period, dynamic variables R, $lw = \overline{AB}$, and γ of this process are constant, so this area has been named the *con* area,
c) in Fig. 3(c) the position of the smooth CWU process for the following values of angle θ is shown: $\pi + \gamma < \theta < 2\pi + \gamma$. In this period dynamic vari-

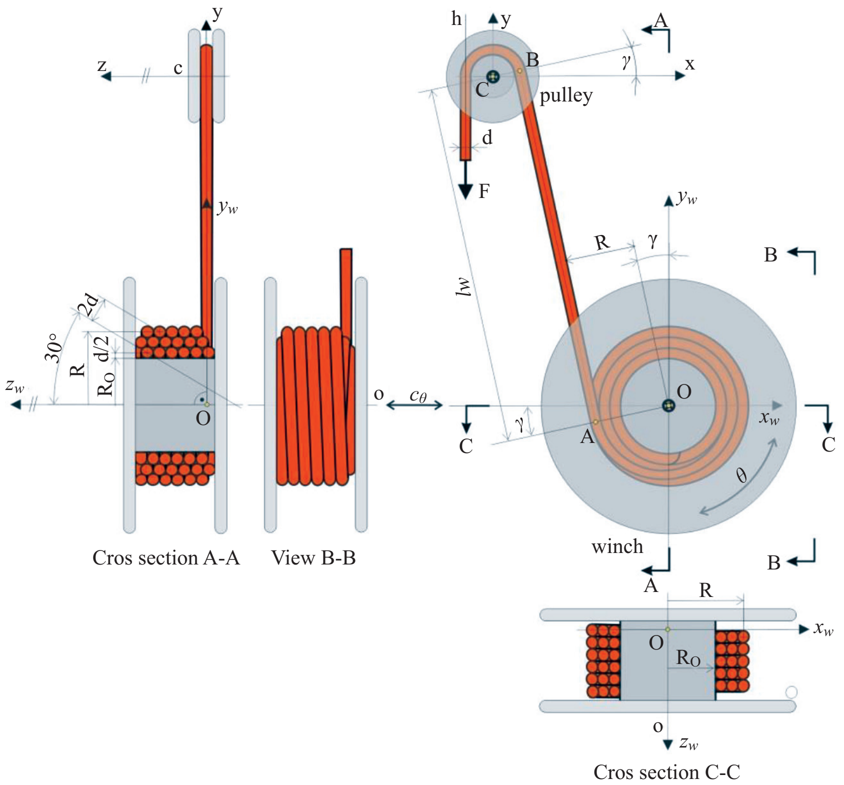


Fig. 4 The system for multi-row radial and axial CWU process.

ables R, $lw = \overline{AB}$ and γ of this process change their values, so this area has been named the *smvar* area. Changes of the characteristic variables are smooth and nonlinear, which is substantially different in comparison with the CWU process presented in Fig. 1.

During the process of the smooth CWU on the winch, areas *con* and *smvar* alternate cyclically. In the next sub-section, CWU system for multi-row radial and axial CWU process will be presented.

2.3 CWU System for Multi-Row Radial and Axial CWU Process

This system is shown in Fig. 4. This type of CWU system is characterized with one motor and two gears. One gear rotates the winch, while the other gear moves it translatory based on its rotation. It can be seen that this system has one DOF. The motor which drives the winch has a rotary motion around the winch's axis. This motion is labelled as θ. For successful and controllable winding/unwinding of the cable on the winch, this winch must have a translatory motion along its z_w axis as well. This motion is labelled as c_θ. In [13], authors have defined the relation between motions c_θ and θ with Eqs. (13) and (14) and Figs. 5 and 6. It can be seen that these two motions have linear and ideal relation [13]. Many researchers, i.e. designers of CPR systems assume application of CWU systems which keep relation between these two motions linear. In case of any error in construction or disturbance during such system's application, idealized mathematical relation from [13] does not apply. In that case, relation between these two motions is defined with another mathematical formulation which will not be analysed in this paper.

In order to facilitate understanding of this system's functionality, the first winding/unwinding layer will be described first. This example is shown in Fig. 5.

a) The first layer of winding

Figure 5 presents only the first layer of CWU process from Fig. 4. For example, authors of [11] only use cable winding/unwinding on the first layer, without the possibility and need for using the second layer, for implementation of their CWU system. The cable is only axially furled around the winch. Angular motion θ and translator motion c_θ must be coordinated for ideal cable furling to be achieved. Unlike examples from Figs. 1-3, this phase of CWU implies constant characteristics *R, lw* and γ, so their first derivatives are zero. Radius of winding/unwinding is $R = R_0 + \dfrac{d}{2}$. Next sub-section will present the CWU process of the second and third layer of system from Fig. 4.

b) The second and third layer of CWU process

Figure 6 shows the second layer of CWU process, while Fig. 4 presents the third layer of CWU process. Both figures relate to the same system, but because of easier understanding two different situations are examined. Like it was stated before, this type of the system implies coordinated angular θ and translator c_θ motion. Unlike the CWU phase on the first layer from Fig. 5, at the moment when the cable starts winding/unwinding on the second/third layer, characteristic variables *R, lw* and γ become changeable, so their first derivatives are not

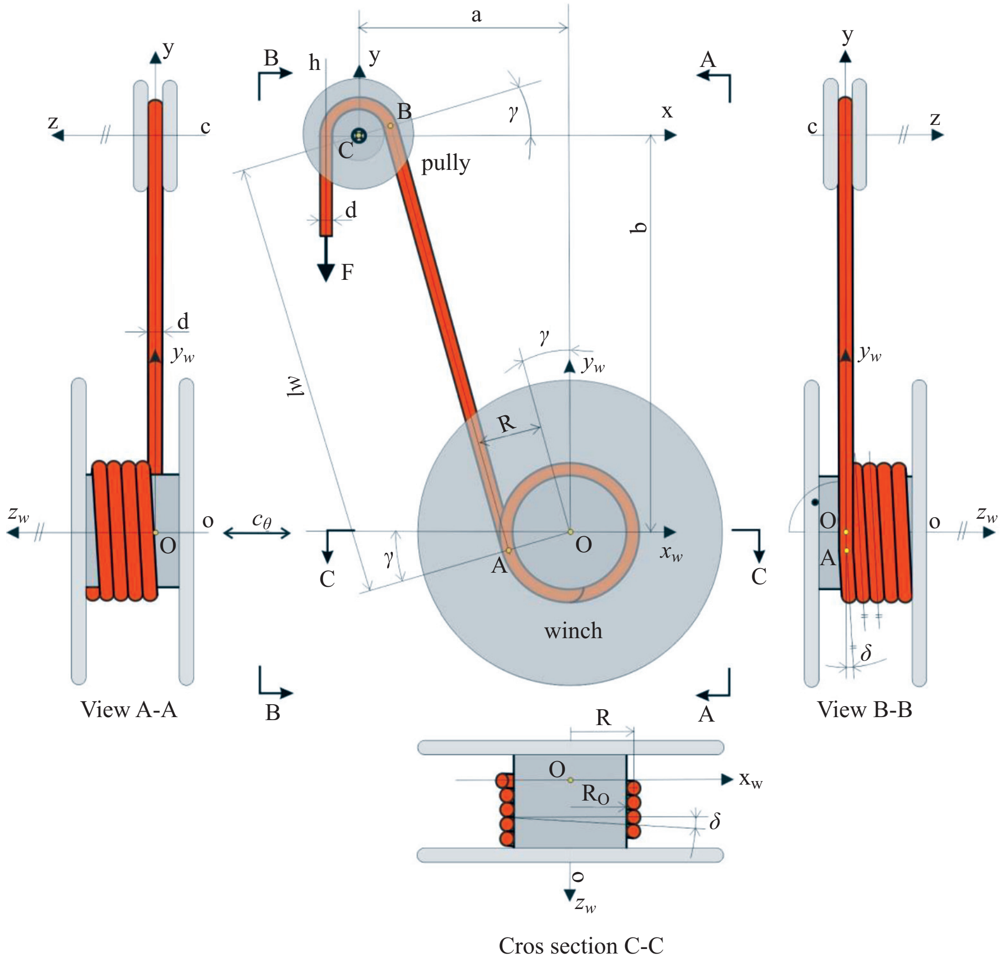


Fig. 5 The system for multi-row radial and axial CWU process – one layer.

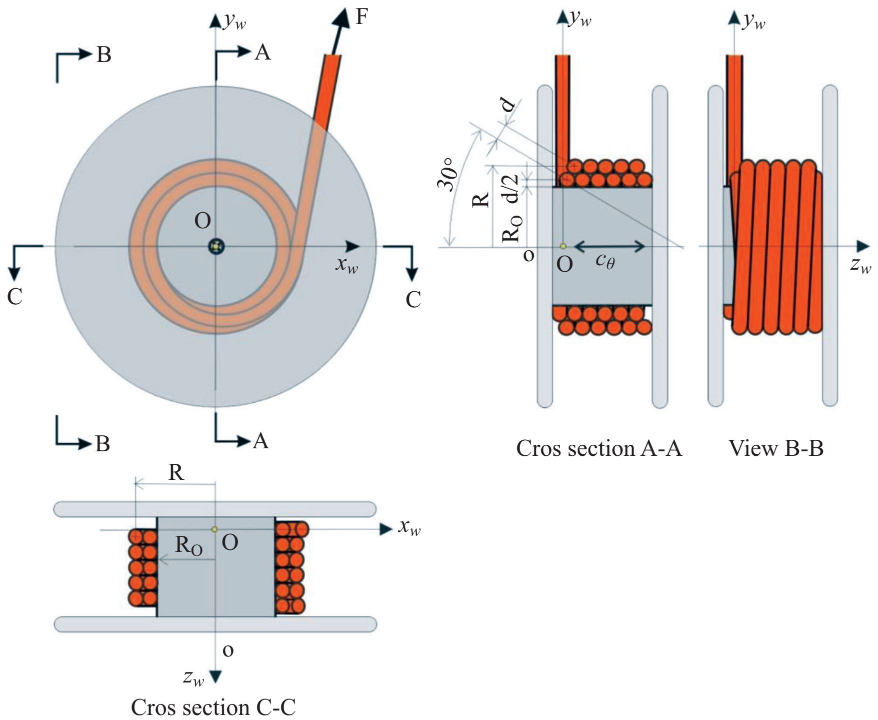


Fig. 6 The system for multi-row radial and axial CWU process – two layers.

zero. From Fig. 6 one can see that the winding/unwinding radius has a value of $R = R_0 + \frac{d}{2} + \frac{2d}{2}$ in the C-C cross-section, while this value is $R = R_0 + \frac{d}{2} + \frac{2d}{2} \cdot \cos(30°)$ when system traverses the next 90°, see the cross-section A-A. During the winding on the second layer, radius R continually increases and decreases each 90°.

In the case of CWU process on the third layer from Fig. 4, one can see that radius R has a value of $R = R_0 + \frac{d}{2} + \frac{4d}{2}$ in the cross-section C-C, while after system traverses next 90°, the radius is $R = R_0 + \frac{d}{2} + \frac{4d}{2} \cdot \cos(30°)$. This can be seen in the cross-section A-A from Fig. 4. During the winding on the third layer, radius R continually increases and decreases each 90°. This change of radius R causes the change of length lw and angle y.

Following rule applies for CWU systems from Figs. 1-6: coordinate system $x_w - y_w$ belongs to the plane defined by coordinate system x-y. This condition is constructively secured for systems from Figs. 1-3. For CWU systems shown in Figs. 4-6, this is secured with coordinated translational motion of the winch c_θ along z_w axis in dependence of angular rotation θ. This condition is especially noticeable in cross-section A-A shown in Figs. 4 and 5.

In the next Section, the general form of mathematical model of CWU systems from this Section will be presented.

3. THE GENERAL FORM OF MATHEMATICAL MODEL OF CWU SYSTEM

By observing the CWU systems from Figs. 1-6, it can be seen that they are different. Regardless of this fact, they are connected with same forms of kinematic and dynamic models. This will be presented in following sub-sections in detail. CWU system's cable is connected to a load via one pulley. Load's weight is labelled as m. Pulley's rotation axis is positioned at the coordinate system x-y. Whole system is presented in x-y plane. Coordinate system $x_w - y_w$ is in the same plane as well.

3.1 The Kinematic Model of CWU System

It has been presumed that the position of the load, defined in the Cartesian coordinate system, is labelled as $p = [x\ y]^T$. These coordinates are named external coordinates. The winding/unwinding angle of the winch θ is defined around axis z_w which contains the point O. It is also presumed that winding/unwinding angle of the winch is defined as internal coordinate.

In the observed examples, motion direction of the load is in direction of the y axis. It is presumed that load's x coordinate has a constant value. The load's weight m is affected by gravitational acceleration and this defines the force F.

In general, it can be said that CWU systems from Figs. 1-6 are characterized with changeable dynamic variables: radius of winding/unwinding R, length of cable which connects pulley with winch lw, and all the other dynamic variables.

In this Section, the changes of these variables will be included into CWU system's mathematical model. By introducing this novel phenomenon, a new form of the mathematical model of the CWU system is obtained. Based on Figs. 1-6, for small changes of variables $\Delta(\theta \cdot R)$, Δy and Δlw one can write the following Eq.:

$$\Delta(\theta \cdot R) = -\Delta y - \Delta lw. \tag{1}$$

Since the increment of the product $\Delta(\theta \cdot R)$ can be written in the following form:

$$\Delta(\theta \cdot R) = \Delta\theta \cdot R + \theta \cdot \Delta R, \tag{2}$$

one can substitute Eq. (2) into Eq. (1) to obtain:

$$\Delta\theta \cdot R + \theta \cdot \Delta R = -\Delta y - \Delta lw. \tag{3}$$

In this way, the relation between increments Δy, $\Delta\theta$, Δlw and ΔR is obtained.

One can see that the left side of Eq. (3), $\Delta\theta \cdot R + \theta \cdot \Delta R$, is dependent on increments $\Delta\theta$ and ΔR, because these variables are changeable during implementation of the task of CWU system.

If Eq. (3) is divided with a small increment of time Δt, in the limit one can write:

$$\dot{\theta} \cdot R + \theta \cdot \dot{R} = -\dot{y} - \dot{lw}. \tag{4}$$

From Eq. (4) one can write the following equation for angular velocity of the winch:

$$\dot{\theta} = -\frac{\dot{y}}{R} - \frac{\dot{lw}}{R} - \frac{\theta \cdot \dot{R}}{R}. \tag{5}$$

From Eq. (5), one can see that angular velocity $\dot{\theta}$ is dependent on variables: $\dot{y}$, $\dot{R}$, $\dot{lw}$, θ and R. Equation (5) presents a kinematic relation between velocities of external y and internal coordinate θ and other dynamic variables.

3.1.1 The Kinematic Model of CWU System for Multi-Row Radial and Axial CWU Process

Kinematic model of CWU system for multi-row radial and axial CWU process is defined with Eqs. (1)-(5). For regular winding/unwinding of the cable on the winch, this system has a translator motion of the winch along its axis z_w as well. This motion is labelled as c_θ. As it was previously described, this motion is usually implemented with the same motor as angular motion θ. For achieving the translator motion, this system consists of an additional gear. Relation between angular and translator motion was given in [13], and in case considered in this paper it is:

$$\theta = \frac{2\pi}{d} \cdot c_\theta. \tag{6}$$

In Eq. (6), variable d presents the diameter of the cable, see Figs. 4-6. It is assumed that the cable has an idealized circular cross-section.

Equation (6) illustrates a linear relation between the angular and translator motion. After substituting Eq. (6) and its first derivative in Eq. (5), following equation for definition of translator velocity is achieved:

$$\dot{c}_\theta = \frac{-\frac{\dot{y}}{R} - \frac{l\dot{w}}{R} - \frac{\frac{2\pi}{d}\cdot c_\theta \cdot \dot{R}}{R}}{\frac{2\pi}{d}} = -\frac{d}{2\pi}\cdot\frac{\dot{y}}{R} - \frac{d}{2\pi}\cdot\frac{l\dot{w}}{R} - \frac{c_\theta \cdot \dot{R}}{R}. \tag{7}$$

Equation (7) shows that translator velocity of winch $\dot{c}_\theta$ along its axis z_w depends on variables: $\dot{y}$, $\dot{R}$, $l\dot{w}$, c_θ, R and d. Equation (7) presents another kinematic relation which is a result of Eq. (6). One can use either Eq. (5) or Eq. (7) to define the kinematic model of CWU system for multi-row radial and axial CWU process.

Special case of mathematical model of CWU system for multi-row radial and axial CWU process is a period when dynamic variables R and lw are constant and their first derivatives are zero. One of these cases is shown in Fig. 5 and that is the period when only the first layer of cable is wound on the winch. In this case a simplified kinematic model is achieved from Eq. (5):

$$\dot{\theta} = -\frac{\dot{y}}{R}. \tag{8}$$

In this case, kinematic model of translator motion in comparison to Eq. (7) is:

$$\dot{c}_\theta = -\frac{d}{2\pi}\frac{\dot{y}}{R}. \tag{9}$$

Now, it is mathematically clear that application of only the first layer of cable winding/winding on the winch, shown in Fig. 5, is much simpler then application of cable winding/unwinding on the second, third or n-th layer on the winch, See Figs. 6 and 4.

3.2 The Dynamic Model of CWU System

In the previous sub-section, the equations which describe the kinematic model of CWU system have been defined. The kinematic model of this system presents a prerequisite for performing dynamic analysis of the CWU system. In order to define the dynamic model of CWU system one needs to identify the resultant torque which acts on shaft of the CWU system. Lagrange virtual work principle is used in this paper to acquire the following equation:

$$F \cdot \dot{y} = M \cdot \dot{\theta}, \tag{10}$$

where F is the force acting on the load's weight m and M is the torque acting on the shaft of CWU system. By substituting Eq. (5) into Eq. (10), the following equation is obtained:

$$F \cdot \dot{y} = -M \cdot \left(\frac{\dot{y}}{R} + \frac{l\dot{w}}{R} + \frac{\theta \cdot \dot{R}}{R}\right). \tag{11}$$

From Eq. (11), one can write the equation for resultant torque M:

$$M = -\frac{F \cdot \dot{y}}{\frac{\dot{y}}{R} + \frac{l\dot{w}}{R} + \frac{\theta \cdot \dot{R}}{R}}. \tag{12}$$

Resultant torque M acting on the shaft of CWU system's winch includes the dynamics of the CWU process. It can be seen that the resultant torque M is dependent on geometry of the considered CWU system, Eqs. (10)-(12), i.e. it depends on: variables $\dot{y}$, $\dot{R}$, $l\dot{w}$, θ and R.

Now the final form of the mathematical model of CWU system, which is analysed in this paper, can be written. To define the dynamic model of CWU systems presented in Figs. 1-6, the well-known equation of motor will be used. The dynamic model of the CWU system is presented by:

$$u = G_v \cdot \ddot{\theta} + L_v \cdot \dot{\theta} + S_v \cdot M, \tag{13}$$

where: u – control signal (voltage) of the motor, G_v – motor's inertia characteristic, L_v –motor's damping characteristic, S_v – motor's geometric characteristics, while $\dot{\theta}$ and $\ddot{\theta}$ present the first and the second derivatives of angular position of the winch, respectively. Regardless that the motion of this system is influenced by $l\dot{w}$ and $\dot{R}$, the system does not get more complicated in the terms of number of DOFs.

3.2.1 The Dynamic Model of CWU System for Multi-Row Radial and Axial CWU Process

Dynamic model of CWU system for multi-row radial and axial CWU process is defined by Eqs. (10)-(13). These equations can be expressed through the winch's translator motion c_θ as well. If one substitutes the first derivative of Eq. (6) in Eq. (10), the following equation is determined:

$$F \cdot \dot{y} = M \cdot \frac{2\pi}{d} \cdot \dot{c}_\theta, \tag{14}$$

By substituting Eq. (7) into Eq. (14), the following equation is obtained:

$$F \cdot \dot{y} = -M \cdot \left(\frac{\dot{y}}{R} + \frac{l\dot{w}}{R} + \frac{2\pi}{d}\cdot\frac{c_\theta \cdot \dot{R}}{R}\right). \tag{15}$$

From Eq. (15) one can express the resultant torque M in form:

$$M = -\frac{F \cdot \dot{y}}{\frac{\dot{y}}{R} + \frac{l\dot{w}}{R} + \frac{2\pi}{d}\cdot\frac{c_\theta \cdot \dot{R}}{R}}. \tag{16}$$

Now the motor equation depending on the winch's translator motion c_θ can be written:

$$u = G_v \cdot \frac{2\pi}{d} \cdot \ddot{c}_\theta + L_v \cdot \frac{2\pi}{d} \cdot \dot{c}_\theta - S_v \cdot \frac{F \cdot \dot{y}}{\frac{\dot{y}}{R} + \frac{l\dot{w}}{R} + \frac{2\pi}{d} \cdot \frac{c_\theta \cdot \dot{R}}{R}}. \quad (17)$$

Special case of the mathematical model of CWU system for multi-row radial and axial CWU process is a period when dynamic variables R and lw are constant and their first derivatives are zero. One of these cases is shown in Fig. 5 and that is the period when only first layer of cable is wound on the winch.

In this case, the load torque is:

$$M = -F \cdot R. \quad (18)$$

Load torque from Eq. (18) can be achieved by using Eqs. (12) or (16).

The equations from this Section, define the general form of mathematical model of CWU system having variables R and lw changeable during the motion of the load along line h.

In order to prove the validity of the mathematical formulations defined in this Section, in the following Section a novel program package, named CWUSOFT, is presented.

4. THE PROGRAM PACKAGE CWUSOFT

Based on the mathematical model of CWU system, defined by Eqs. (1)-(18), a novel program package CWUSOFT has been synthesized. This program package contains several subroutines combined into one unit:

1. Subroutine for generation of the reference trajectory. In this case, the reference trajectory of the load in *x-y* space along the line h is defined. From *x-y* space, the internal coordinate, angular position of CWU system θ (as defined by Eq. (5)), is calculated. It should be noted that for CWU system for multi-row radial and axial CWU process it is possible to calculate the translator motion c_θ of the winch as well. This procedure includes the kinematic model of the CWU system.
2. Subroutine for generation of the dynamic response of the CWU system. In this subroutine, the influence of the changes of radius R and length between the winch and hanging point lw is included. This is defined through the resultant torque M which includes the dynamics of load's motion (force F) via Lagrange virtual work principle. Equation (12) includes the geometry of the mechanism, i.e. its kinematic model, as well. It can be seen that now the relation between the resultant torque M and force F is related via the following variables $\dot{y}$, $\dot{R}$, $l\dot{w}$, θ and R. It should be emphasized that for CWU system for multi-row radial and axial CWU process one can use the Eq. (16) as well.
3. Subroutine for control structure of the system. This routine assumes the creation of various control algorithms for the motion control of the load in *x-y* space along the line h.

Based on the analysis from Sections 3 and 4, next part of the paper will show the simulation results which illustrate the implementation of one trajectory of one type of the CWU system.

5. TESTING THE CWU SYSTEM – SIMULATION RESULTS

In this Section one example of CWU system's load motion is simulated. CWU system named system for smooth single – row radial multi-layered CWU process from Figs. 2(b) and 3 was chosen.

This system has constructive radii: $R_{i0} = 0.0136m$ and $\tilde{R}_{i0} = 0.014m$. The mathematical model of CWU system defined in Section 3 will be tested. For generation of all the results in this Section, the program package CWUSOFT from Section 4 is used. The considered CWU system has the carrying capacity of 0.064 *kg*.

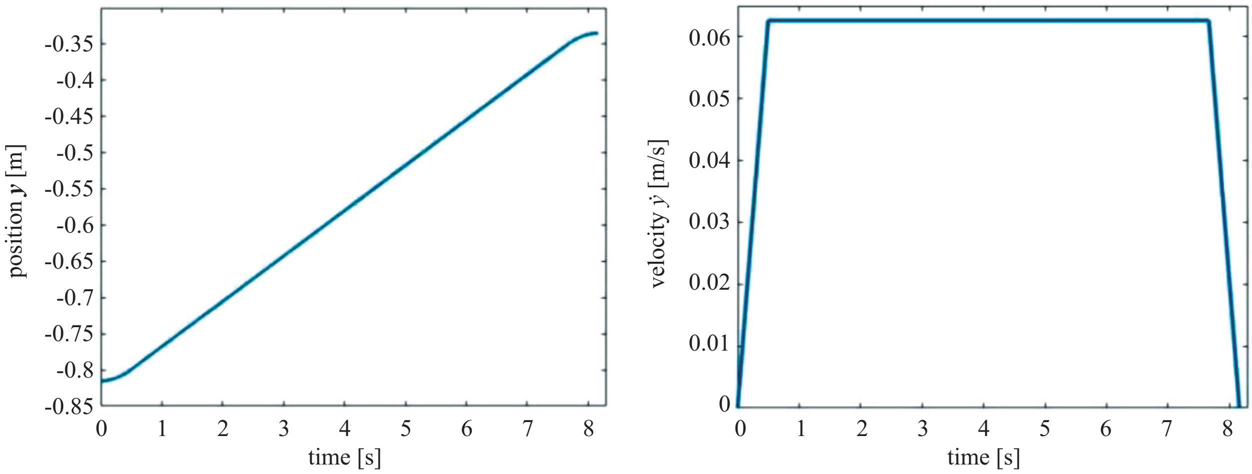
For the purpose of testing, the following reference trajectory has been used: the load needs to move along the line h from point $Ystart = -0.815m$ to point $Yend = -0.335m$. Total load carrying length is 0.48*m*. See Fig. 7(a). During the load lifting, cable is wound on the winch. Velocity of the load has a trapezoidal shape with maximal value of 0.063 m/s. See Fig. 7(b). For driving the winch, a DC motor with rated voltage of 12 V was used.

By using Eq. (5) and based on the geometry of examined CWU system, angular velocity of the CWU system $\dot{\theta}$ is determined and that is shown in Fig. 8(b). By integrating this variable, angular position of CWU system is achieved, see Fig. 8(a). Total angular motion of the CWU system during the task execution is from 0 to 28.82 *rad.* Maximal angular velocity is 4.235 *rad/s.*

Figure 9(a) shows the change of winding/unwinding radius R during the task implementation, while Fig. 9(b) shows its velocity $\dot{R}$. At the beginning, from 0 – 0.48s, the CWU system is winding inside the *con* area, where its radius R has a constant value of R = 0.0148*m*, while its first derivative is $\dot{R} = 0$. After this moment, from 0.48 – 1.26s, the CWU system is inside the *smvar* area and during this period of time radius R grows and its first derivative is $\dot{R} > 0$. As it can be seen from Fig. 9, during the load lifting, i.e. during the cable winding on a winch, a cyclical alternation of *con* and *smvar* areas occurs. This statement is confirmed by Fig. 9(b) where one can see the change of velocity $\dot{R}$ during the load lifting.

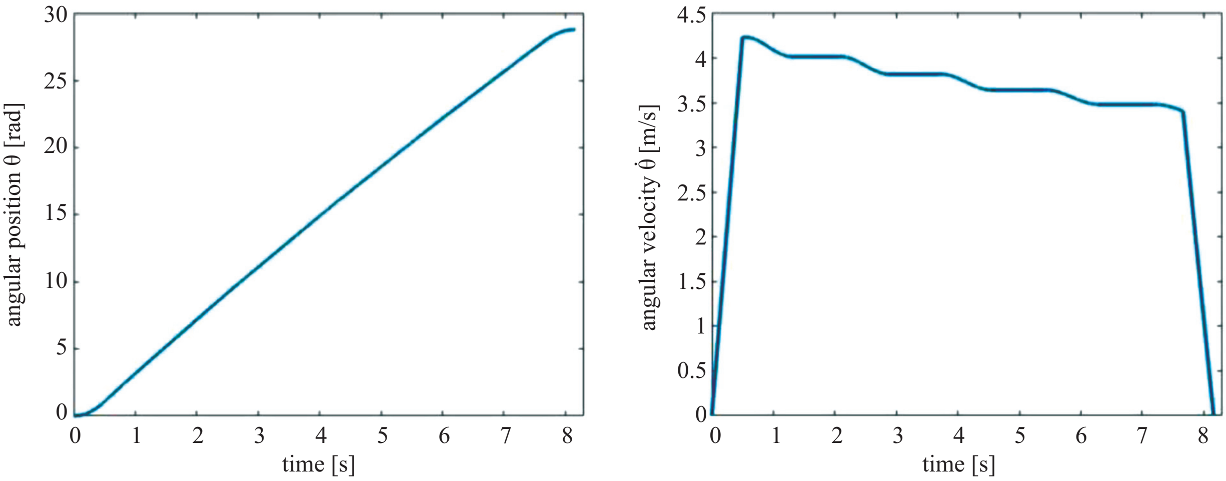
As radius R changes its value during the load lifting, length lw and angle γ change their values as well.

Figure 10(a) shows the change of the length lw, while Fig. 10(b) shows the first derivative of this variable – $l\dot{w}$. From these figures one can see that this variable has a smooth change in time. By comparing the Figs. 10(a) and 9(a), one can see that during the *con* period both R and lw are constant. Unlike the *con* period, when winch is in the *smvar* period during the cable winding, radius R is growing while length lw is decreas ing in the same period.



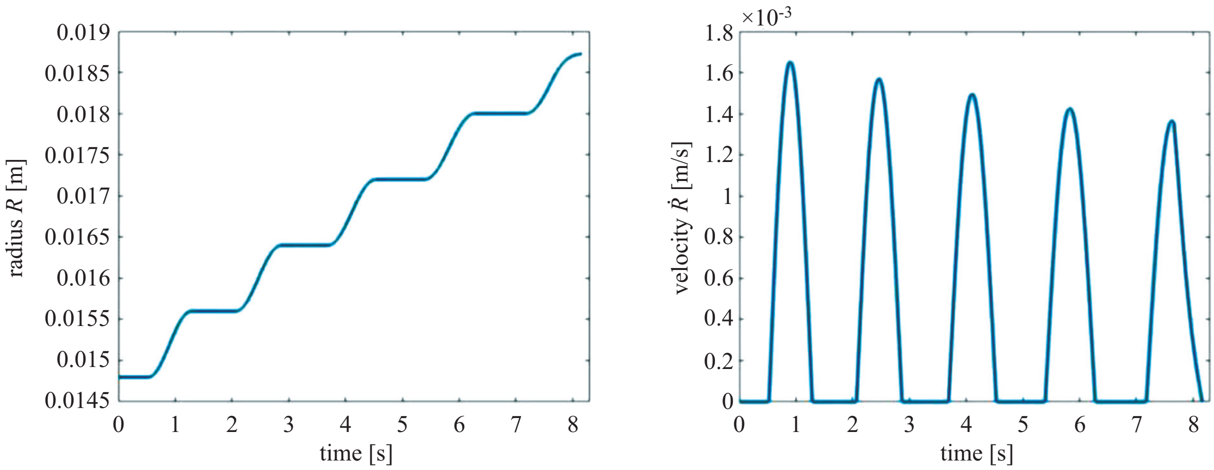
(a) (b)

Fig. 7 (a) position and (b) velocity of load's motion.



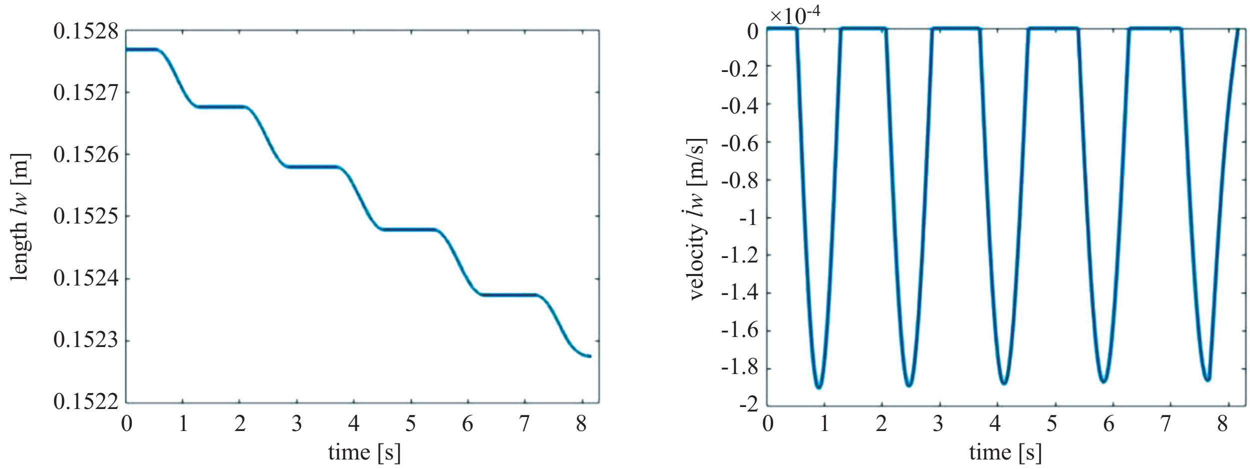
(a) (b)

Fig. 8 Angular (a) position and (b) velocity of CWU system.



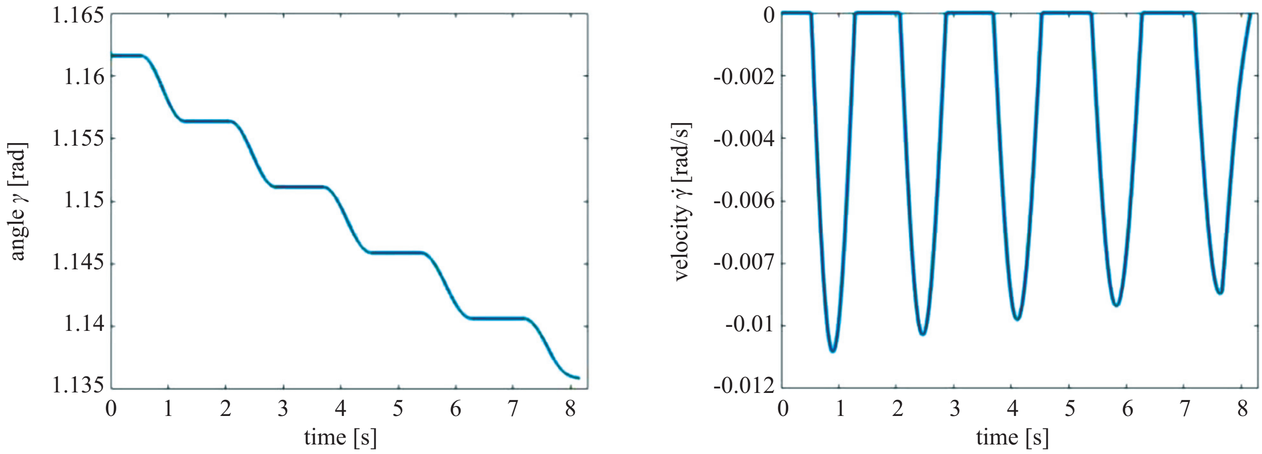
(a) (b)

Fig. 9 (a) Winding/unwinding radius R and (b) its velocity $\dot{R}$.



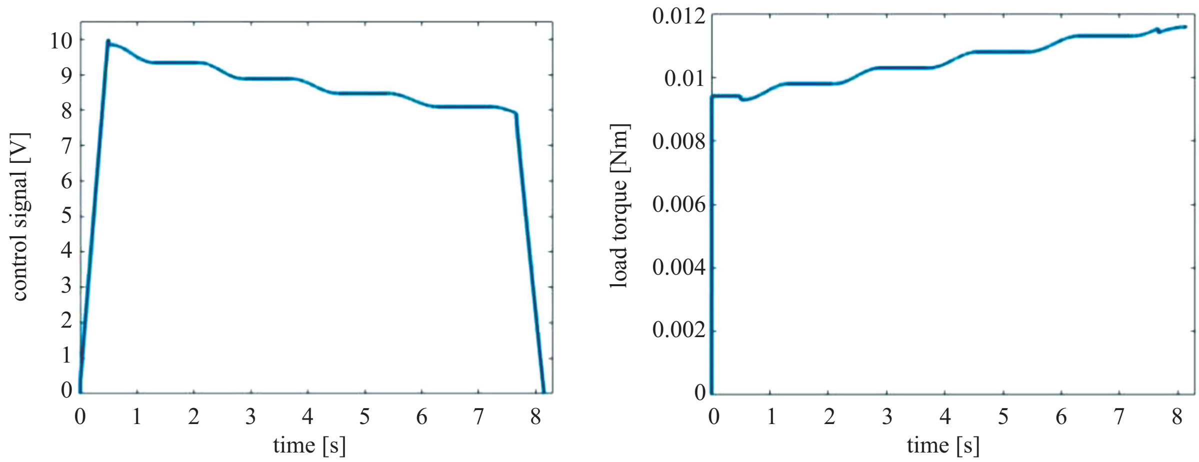
(a) (b)

Fig. 10 (a) Length lw and (b) its velocity $\dot{l}w$.



(a) (b)

Fig. 11 (a) Angle γ and (b) its velocity $\dot{\gamma}$.



(a) (b)

Fig. 12 (a) Control signal u and (b) load torque M.

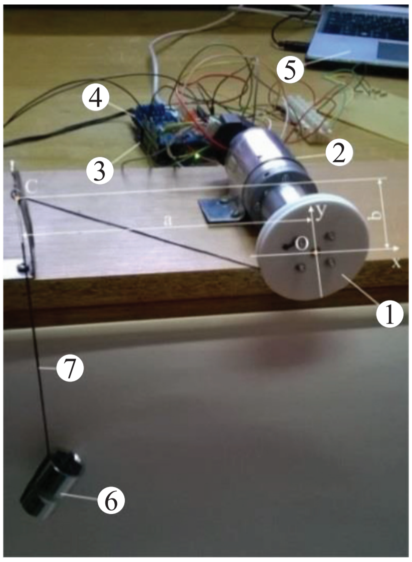


Fig. 13 The experimental set-up.

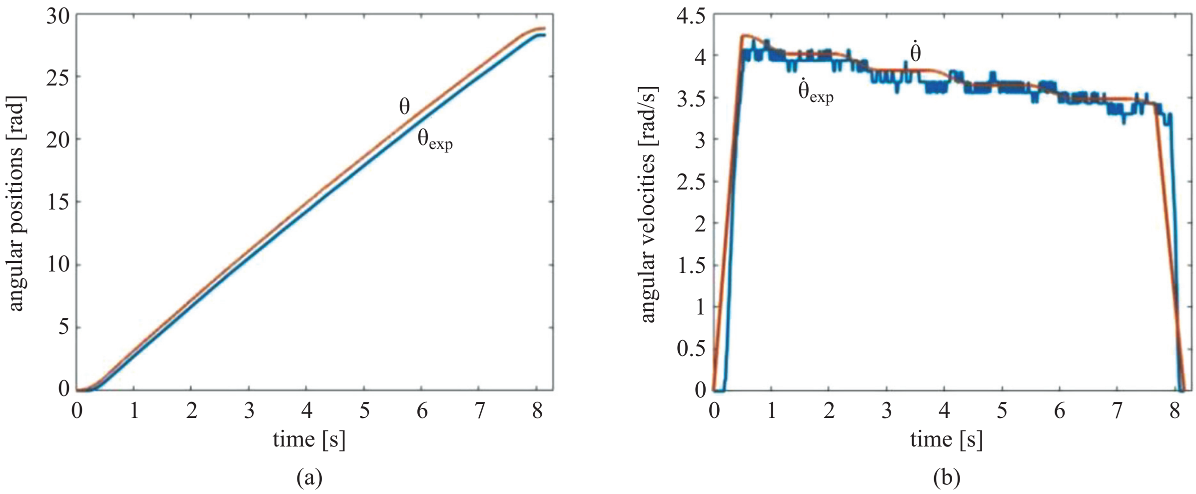


Fig. 14 Angular (a) position and (b) velocity of the CWU system.

In Fig. 11(a), change of the angle γ during the load lifting is presented, while Fig. 11(b) shows its first derivative $\dot{\gamma}$. From these figures, one can see that angle γ has the same change dynamics as length *lw* - when cable is winding both *lw* and γ decrease their values. When winch is in the *con* area, i.e. when $R = const$, angle γ also has a constant value, as well as length *lw*.

Figure 12(a) presents the control signal *u,* while Fig. 12(b) shows the load torque *M* which acts on the CWU system's shaft. One can see that the load toque is growing during the cable winding and that is due to the growth of radius *R* during the task implementation.

In this part of the paper, simulation results which show load lifting process were shown. During the load lifting process, cable is always winding on the winch. Opposite process of cable unwinding was not shown, because all the phenomena that occur during the process of winding occur during the process of unwinding as well only in opposite direction.

Next Section of the paper will present an experimental analysis which will confirm theoretical results from third Section and simulation results from this Section of the paper.

6. EXPERIMENTAL RESULTS

In this part of the paper, experimental confirmation of simulation and theoretical results from previous Sections will be given. Experiment was made on the set-up shown in Fig. 13.

The winch is in Fig. 13 labelled as position 1. The following components have been used: DC motor, 12V, 20W with encoder and gear head (position 2), Arduino UNO REV3 (position 3), Arduino Motor Shield REV3 (position 4) and a laptop computer (position 5). The mass of the load (position 6) which is lifted is 0.064 kg, while the total cable (position 7) winding length is 0.48m, lengths a and b are a = 0.147m, b = 0.045m. Duration of the motion is 8.2 s. Experiment was made on a CWU system which uses a winch from Fig. 2(b) whose constructive radii are: R_{i0} = 0.0136m and $\tilde{R}_{i0} = 0.014m$. All the parameters are the same as ones used for simulation results. Because of that, experimental and simulation results are comparable.

Figure 14(a) shows angular position of the CWU system during the task implementation. This figure presents results achieved by simulations θ and experiment θ_{exp}. One can see that θ_{exp} has a change from 0 to 27.3 *rad*, while θ has a change from 0 to 28.82 *rad*. It is clear that θ_{exp} can never achieve the value of θ, because of the problem of un-modelled frictions in CWU system. Figure 14(b) presents comparison between angular velocities of the CWU system achieved with simulation $\dot{\theta}$ and experiment $\dot{\theta}_{exp}$. In general, it can be said that angular velocity has decrease of value during the task execution which is caused by the growth of load torque acting on the CWU system's shaft. By comparing $\dot{\theta}$ and $\dot{\theta}_{exp}$, one can also conclude that because of unmodelled frictions $\dot{\theta}_{exp}$ on average always has a value which is under $\dot{\theta}$ (average value). This causes $\theta_{exp} < \theta$ for each moment during the task execution.

7. CONCLUSIONS

The general mathematical model of cable winding/unwinding (CWU) system is presented. CWU system includes motor, gear and the winch for CWU process. This model is defined in general form for CWU systems from Figs. 1-6.

The purpose of this research is pointing out the complexity of CWU systems. Even at these simple constructions shown in Figs. 1-6 where each of them contains motor, gear and winch one can see the influence of winding radius R and length lw on system's dynamics of response.

For the verification of the defined mathematical model, a novel program package CWUSOFT was defined. Simulation results are shown through relevant dynamic variables of CWU system: angular position θ, radius R, length lw and angle γ. Simulation results were performed for one novel type of CWU system, see Figs. 2(b) and 3. Simulation results were achieved by using the program package CWUSOFT. These simulation results as well as theoretical definitions were confirmed through the experimental analysis.

CWU systems can be sub-systems of more complex mechatronic systems and in that case the mathematical model of this complex system is much more complicated and one of the reasons is mutual coupling of several CWU sub-systems. Because of that, it is very important that dynamic characteristics which are indicated in this paper are included in the analysis and synthesis of these complex mechatronic systems. This will be a subject of future research of authors of this paper.

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NOMENCLATURE

DOF	degree of freedom
CWU	cable winding/unwinding
R	radius of cable winding/unwinding
lw	cable length
γ	inclination of the cable with respect to y_w axis
θ	rotation angle of the winch
c_θ	translator motion of the winch

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