THE ALGEBRAIC STRUCTURE OF PROPOSITIONS

A Dissertation

Submitted to the Graduate School
of the University of Notre Dame
in Partial Fulfillment of the Requirements
for the Degree of

Doctor of Philosophy

by

Geoffrey Hall

Jeff Speaks , Co-Director

Daniel Nolan , Co-Director

Graduate Program in Philosophy
Notre Dame, Indiana

July 2020

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Abstract

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This dissertation is about various puzzles that arise when we attempt to construct general theories of propositions, properties and relations. The goal of the dissertation is to make some progress on these puzzles by employing a broadly abductive approach to them. On my view, one feature of an abductive approach is working out the overall algebraic structure of the objects in question. On grounds of simplicity and elegance, theories admitting of more straightforward algebraic description are favored, at least initially. Theories not admitting of such a description must make up for this lack of elegance. This provides us with some guidance when our evidence and intuition falls short.

Chapter 1 concerns the relationship between theories of propositional fineness of grain and theories of belief. I argue that we should accept a relatively coarse grained account of propositions, Booleanism, and deny the principle of Distribution, according to which believing a conjunction implies believing both conjuncts. The principle of Distribution turns out to be surprisingly strong: for many theories of propositional fineness of grain, conjoining that theory with Distribution has implausible consequences. Indeed I will tentatively suggest that *no* theory of propositional fineness of grain can accommodate Distribution without some collateral damage.

Chapter 2 concerns the relationship between indefinite extensibility and theories of propositional explanation. In particular, I will argue that we cannot overcome

problems with the principle of sufficient reason by adopting a view on which contingent truth is indefinitely extensible. Given our best theories of indefinite extensibility, even if contingent truth turns out to be indefinitely extensible, the principle of sufficient reason still entails that all truths are necessary, given plausible background principles.

Chapter 3 concerns a puzzle of logical form that has been put forward recently by Kit Fine (2017). This chapter develops a solution to the puzzle by outline two notions of form and arguing that our intuitive notion of logical form does not privilege one over the other. The puzzle is then answered by showing that it rests on a conflation of these two notions of form.

Chapter 4 puts forward a theory of the representational properties of propositions that goes some way towards answering worries relating to the unity of the proposition. Contra many recent accounts in this vicinity, I will argue that the best theory here is a primivitist one. The theory makes use a novel operation I call *application*. I will argue that with this primitive, a very general theory of the representational properties of various sorts of abstract objects can be developed.

CONTENTS

ACKNOWLEDGMENTS

I have been incredibly lucky to have Jeff Speaks and Daniel Nolan as my thesis advisors. Both have been incredibly generous with their time, serving not only as my thesis advisors but also helping me through orals and several dissertation proposals for dissertations that were not to be. Both Daniel and Jeff have worked with me since I arrived at Notre Dame, and have been consistently supportive of my work.

I would like to thank Brian Cutter for helpful feedback throughout my time at Notre Dame. I can recall many times that I randomly dropped by his office looking for advice on this and that, and he always welcomed me graciously. I am also incredibly grateful to both Christopher Shields and Jc Beall, both of whom agreed to join my committee somewhat last minute despite me giving them little idea on what exactly the dissertation would be about. Thank you!

I would also like to thank the philosophy department and the University of Notre Dame for making my research possible. Thanks, also, to audiences from the Notre Dame at the Graduate Student Colloquium and the Dissertation Research Seminar for providing helpful comments.

I've learned an immense amount from my fellow graduate students at Notre Dame. While I will no doubt leave some people out, I want to thank in particular Ellen Lehet, Ben Middleton, Sebastian Murgueitio Ramirez, Jack Himelright, Alex Rausch and Mack Sullivan for both friendship and many, many conversations about Booleanism (sorry!). I want to thank Ellen Lehet in particular for reading drafts of all of my work and generally helping me out with everything.

CHAPTER 1

BOOLEANISM AND BELIEF

1.1 Introduction

It seems obvious that believing a conjunction requires believing each conjunct. Whenever one believes that it is Monday and it is morning, one thereby believes that it is Monday and believes that it is morning. The principle, which I'll call *Distribution*, that it is metaphysically necessary that one believes a conjunction only if one believes the conjuncts of that conjunction, is widely regarded as a truism.[1]

It is also widely held that belief is not closed under entailment. Only idealized agents in distant possibilities believe all the consequences of what they believe. For agents like us, failing to draw an inference is commonplace. Call the thesis that it is metaphysically necessary that one believes any proposition entailed by a proposition one believes *Closure* and its negation *NoClosure*.

It is somewhat surprising that Distribution, NoClosure and the thesis that propositions form a Boolean algebra under the operations of conjunction and negation, which I'll call *Booleanism*, are mutually inconsistent—all the more so given the wide acceptance of Booleanism in both philosophy and linguistics.[2,3]

[1] The principle of Distribution is often mentioned more in passing than defended outright; but here is a partial list of places in which the principle is either endorsed or presented favorably: Soames (1987), Dorr (2011, p. 957), Williamson (2000, p. 280), Speaks (2006, p. 443).

[2] Most authors in these areas accept Booleanism under the guise of the possible worlds theory of propositions. For further discussion see §2. The inconsistency only really holds given a further thesis about entailment. While this further thesis is often taken to be part of Booleanism, it could be divorced from it. However a restricted sort of Closure thesis does hold given Booleanism independently of one's views of entailment. For further discussion see §3.

[3] In what follows I will be solely concerned with *single premise* closure as opposed to multi-

The basic problem, in outline, is that in a Boolean theory of propositions, a proposition p entails a proposition q if and only if the conjunction of p and q is identical to p.[4] Thus if the proposition p entails q, and one believes p, then one believes the conjunction of p and q. And so if believing a conjunction requires believing its conjuncts, it follows that one believes q.

Some philosophers, following [92], reject Booleanism in light of this problem. These authors often take Distribution and NoClosure to be premises in a powerful argument, the *Conflation Argument*, for the structured theory of propositions. A visible minority, most notably [101], opt instead to accept Closure having been won over by the simplicity and strength of Booleanism and the purported obviousness of Distribution. Few philosophers reject Distribution. This paper is an exploration and (tentative) defense of the third way out—that of maintaining Booleanism and denying Distribution.

The methodology employed in this paper is abductive.[5] The package consisting of Booleanism and the negation of Distribution will be shown to have some distinctive benefits lacked by its competitors. This doesn't mean I'll dismiss the intuitive data in favor of Distribution. It just means that I don't think there is any better theory overall that accounts for this intuitive data. As I will argue, Booleanism can be combined with other principles related to Distribution that nevertheless explain our inclination to accept Distribution as obvious.

§1 contains background on the language that I will be using to discuss questions of propositional fineness of grain. In §2 I state the theory Booleanism and provide

premise closure. One can easily combine Booleanism (an in particular the possible worlds theory of propositions) Distribution and the negation of multi-premise closure without inconsistency. The fragmentalists provide one example of this sort of view, see for instance See Stalnaker (1984), Lewis (1988) and also Braddon-Mitchell and Jackson (2007).

[4]For discussion of why we should accept this claim, see §3.

[5]In the sense of Williamson (2016).

some examples of the theory in the literature. §3 develops the conflation argument againts Booleanism. The main goal of this section is to explain why Booleanism and Distribution jointly entail Closure. §4 shows how this kind of argument generalizes to challenge not only Booleanism, but certain quasi-structured views of propositions as well. I will also discuss some responses on behalf of the Boolean to some of these more fine-grained theories of propositions. In §5 I argue that the package consisting of the full structured theory of propositions and Distribution is not as well motivated by conflation type arguments than it has sometimes been supposed. §6 addresses what I take to be some of the main arguments for Distribution and argues that the package consisting of the negation of Distribution and Booleanism is able to provide plausible responses to these arguments. Finally, I end in §7 with a direct argument against Distribution. The conclusion I think we should draw is the following: while it it is initially rather unintuitive to give up Distribution, there are some good reasons to hold that the package consisting of Booleanism and the negation of Distribution fares better than any theory of propositional fineness of grain conjoined with Distribution.

1.2 Propositional Fineness of Grain

Booleanism is a thesis about *propositional fineness of grain*. The goal of a theory of propositional fineness of grain is to discern some of the general principles governing the interaction of various operations on propositions and the identity of propositions. For instance a theory of propositional fineness of grain should answer basic questions like "Is the proposition that p identical to the proposition that $\neg\neg p$?" and "Is the proposition that $p \wedge (q \wedge r)$ identical to the proposition that $(p \wedge q) \wedge r$?" It is also desirable that the theory answer slightly more involved questions involving the metaphysics of application and generalization. For instance, what is the relationship between the proposition that x is such that it is blue and white and the proposition that x is blue and x is white? Are they one proposition or two? The former proposition results from applying a complex property, being blue and white, to x. The latter results from conjoining two propositions, one of which results from applying blue to x and the other applying white to x. To hold that they are identical is thus to hold that there is no *unique* decomposition of propositions into the properties they predicate and the individuals they are about.[6]

Booleanism is an exceedingly simple and powerful theory of propositional fineness of grain.[7] At first pass, it says that the proposition that φ is identical to the proposition that ψ whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic. The view is so named since by identifying classically equivalent propositions we ensure that they form a Boolean algebra with respect to the operations of conjunction and negation. The view immediately entails some positive answers to some of the sample questions raised above: each proposition is identical to its own double negation, and the operation of conjunction is associative.

Recent discussions of propositional fineness of grain have tended to take place in the context of a more general discussion of the granularity of properties, relations, properties of properties, relations between relations, and so on. In this context, Booleanism can be formulated as a much more general theory, governing not just propositions, but properties and relations of all types.[8] So for instance we might formulate Booleanism for properties of individuals by stating that the property of being such that φ is the property of being such that ψ whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic.

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[6]There are some missing steps in this argument. Frank Ramsey (1925), for instance, accepted the identity of these propositions and accepted that propositions were uniquely decomposable into basic constituents and operations. He used these as premises to infer that there simply couldn't be such a property as the property of being blue and white. Thus according to Ramsey, all properties are logically simple.

[7]It may also deserve the title of the first systematic theory of propositional fineness of grain. After all, Boole (1854) explicitly developed the theory of Boolean algebras as an attempt to abstract away from the idiosyncracies of language in order to capture "the laws of thought."

[8]See Dorr(2016).

While my primary concern in this paper is *propositional* fineness of grain, it is my view that a fully abductive comparison of views of propositional fineness of grain is not possible without taking into account how they might interact with, or be extended to, views of the fineness of grain of properties and relations in addition. In order to formulate these views, we thus need a general framework in which competing hypotheses about the granularity of such entities can be formulated. The framework adopted in this paper is the language of higher-order logic. In particular, I will use the language of simple relational type theory with lambda abstraction. The rest of this section is devoted to describing some of the details of this language, its interpretation, and the background principles stated in this language I will be assuming.

The language consists of a collection of *terms* classified into *types* or *syntactic categories*. There is one basic type, e, for the syntactic category of *individual constants*. All other types are generated from the basic type e as follows. For any natural number n, including the number 0, and any types $\tau_1, \ldots, \tau_n$, the n-tuple $\langle \tau_1, \ldots, \tau_n \rangle$ is a type. A term R is of this type when, for some given terms $A_1, \ldots, A_n$, with A_i of type τ_i, $\ulcorner R(A_1, \ldots, A_n) \urcorner$ is a term of type $\langle\rangle$. Intuitively, then, R is an n-place predicate whose *ith* argument must be of type τ_i. The type $\langle\rangle$ is the type for the syntactic category of *formulas*. Thus formulas are treated, in effect, as nullary predicates.

The language includes a typed collection of *constants*: these are the basic terms of the language.[9] Among the constants I will explicitly include a constant $\equiv_\tau$ of type $\langle \tau, \tau \rangle$ meant to express identity for the relevant type of entity. So for instance if φ is 'water is water' and ψ is 'water is $H_2 0$' we might express 'the proposition that water is water is the proposition that water is $H_2 0$' by $\ulcorner \varphi \equiv_{\langle\rangle} \psi \urcorner$. In addition to these "identification constants" the language will have Boolean constants: $\top$ of type $\langle\rangle$ for the "simple tautology", $\neg$ of type $\langle\langle\rangle\rangle$ for negation, $\wedge$ of type $\langle\langle\rangle, \langle\rangle\rangle$ for conjunction,

[9]By which I mean that they are syntactically basic. It is a further requirement that they be interpreted by metaphysically basic entities.

and so on for the rest of the Boolean operators.[10] We also include a constants B of type $\langle\langle\rangle\rangle$ for *belief*; thus where φ stands for 'snow is white', $\ulcorner B\varphi \urcorner$ stands for 'one believes that snow is white'. Further specification of the constants will be added as needed.[11]

The language also includes a countably infinite collection of variables of each type. If $x_1, \ldots, x_n$ is a list of variables of types $\sigma_1, \ldots, \sigma_n$ respectively, and φ is a formula, then $(\lambda x_1 \ldots x_n.\varphi)$ is a term of type $\langle\sigma_1, \ldots, \sigma_n\rangle$. Where φ is the formula 'x is blue', with x a variable of type e, we might pronounce $\ulcorner(\lambda x.\varphi)\urcorner$ as 'is an x such that x is blue'. So for instance, if t is the term 'the Pacific ocean', $\ulcorner(\lambda x.\varphi)t\urcorner$ represents, roughly, 'the Pacific ocean is an x such that x is blue', or more simply 'the Pacific ocean is such that is is blue'.

Finally, to express generality at each type there is included, among the constants, quantifiers $\forall_\sigma$ and $\exists_\sigma$ of type $\langle\langle\tau\rangle\rangle$. Quantifiers combine with predicates to form sentences. Thus where F is a predicate of type $\langle\sigma\rangle$, $\ulcorner\forall_\sigma F\urcorner$ is a term of type $\langle\rangle$. In this language, quantifiers do not do the double work of expressing generality *and* binding variables. Variable binding is done by the lambda terms; generality is achieved by the quantifiers. So that the notation looks more familiar, however, we will often write $\forall_\sigma(\lambda x.\varphi)$ as $\forall_\sigma x\varphi$ (similarly $\exists_\sigma(\lambda x, \varphi)$ will be written $\exists x\varphi$).

I am going to informally speak of propositions, properties and relations in English in order to communicate claims whose desired content are those expressed by higher-order generalizations. So for instance, when I say in English "For any proposition, if one believes that propositions, one believes that one believes it", the claim I am intending to communicate is that $\forall p(Bp \to BBp)$. But I do not think we should

[10] I'm being a bit hand wavy here about which Boolean operators are taken as basic. Given a Boolean theory of propositions, it turns out not to really matter all that much. But one can imagine more fine-grained theories that hold that the material conditional is truth functionally equivalent to a complex operator defined in terms of disjunction and negation, it is not strictly identical to this operation.

[11] In general I will write formulas in infix notation (e.g., $\wedge(A, B)$ will be written as $(A \wedge B)$.

impose an understanding of higher-order logic via translation into English, but rather adopt it as a framework in which close analogue's of these questions can be precisely formulated.[12] By doing so we are able to avoid scholastic debates about whether terms like 'proposition' and 'property' admit multiple interpretations, in addition to distracting set theoretic issues concerning which collections of propositions form a set. That being said, I also do not want to impose an understanding of such quantification that rules out a first-order interpretation; perhaps ultimately all quantification is first-order quantification. In this paper I will largely remain neutral on this question.

One might worry that we have, in moving to a higher-order setting, somehow lost contact with the initial puzzle motivating this paper, which was supposed to concern *propositional* fineness of grain. This is a mistake. In general, theories of propositional fineness of grain are interesting because of the consequences they have for the non-propositional realm. For instance, oftentimes authors will reject the possible worlds conception of propositions on the grounds that it entails that one believes that $2+2=4$ if and only if one believes that $3+3=6$. This consequence does not explicitly concern propositions, and can be easily formulated in our higher-order language. Moreover, we can formulate specific theories in our higher-order language that have this as a consequence, and so can evaluate, in the higher-order setting, whether this consequence alone is deserving of rejection. More generally, we can often find higher-order replacements of first-order theories that have the same, or near enough, consequences for our non-propositional discourse. Since these *consequences* are usually how theories of propositions are evaluated, we do just as well by working in a higher-order setting.[13]

[12]For a defense of taking higher-order quantifiers as primitive see Prior (1971), Williamson (2003) and Williamson (2013, Section 5.9).

[13]This seems to me correct when the question at issue is about grain as opposed to ontology. There are of course lots of interesting debates about the *ontology* of propositions some of which cannot be formulated in a higher-order setting.

Unless indicated otherwise, I'll assume that all classical reasoning is sound in this language (e.g., modus ponens, all classical tautologies as well as standard classical principles governing the quantifiers). I'm also going to frequently appeal to the following principles governing identity of properties, propositions and relations.[14]

Ref $F \equiv_\sigma G$.

LL $F \equiv_\sigma G \to (\chi \to \chi[F/G])$, where $\chi[F/G]$ is obtained from χ by replacing one or more occurrences of F with G such that no variables free in F or G are bound in φ or $\varphi[F/G]$.

The principle **LL** is controversial given the presence of the belief operator. For instance, many authors might want to maintain that (1) while denying (2):

(1) All bachelors are bachelors $\equiv$ all bachelors are unmarried males.

(2) B(All bachelors are bachelors)$\leftrightarrow$ B(all bachelors are unmarried males).

A lot of work has been done detailing views on belief, propositions, and the basic logic governing identity on which (1) does not entail (2). But I have to admit that I find the entailment very compelling. If the proposition that all bachelors are bachelors *just is* the proposition that all bachelors are unmarried males, then it seems to me one must believe that all bachelors are bachelors if and only if all bachelors are unmarried males given the somewhat widespread assumption that B expresses a property of propositions.[15]

This paper investigates the interaction of the logic of belief and the theories of propositional fineness of grain under the assumption that the property expressed by B is transparent. This seems appropriate to me for the following reason. As I read things, the standard "Russellian" picture of propositions of the sort defended by

[14] See Dorr (2016) for further discussion.

[15] For instance, many authors accept 'believes' expresses a relation between individuals and propositions. If that is correct then 'one believes' should express a property of propositions: the property one gets by "plugging up" the first slot in the believes relation.

Soames is one on which the operator B is transparent. Indeed, it is a crucial assumption of the argument of Soames (1987) against coarse grained views like Booleanism that propositional attitudes *do not* create opaque contexts in the way that, for instance, Fregeans argue that they do. Since this paper constitutes something of a response to the Russellians, it seems to me appropriate to grant some of their background views. By endorsing **LL**, I am only making things harder *for my own position.* What will turn out to be distinctive to my approach is the manner in which I will assess the bearing of principles governing belief on propositional identity. The general methodology for those accepting **LL** has been to use intuitions about the truth values of belief ascriptions in order to arrive at very fine-grained views of propositions. This paper takes a slightly more removed perspective of propositional attitude psychology. Instead of focusing solely on which belief ascriptions seem true and which do not, I want to focus on the overall theoretical benefits of adding various psychological generalizations (in particular Distribution), to particular theories of propositional fineness of grain, and then evaluating the respective theories in terms of their overall virtues. I hope to convince the reader that the purported benefits of structured view of propositions with respect to the psychological data we'll be looking at is actually the result of a weakness in the overall theory, a weakness that it does not clearly make up for by accomodating the psychological data.

1.3 Booleanism

1.3.1 The basic theory

The theory Booleanism can be axiomatized by the following schema.

Schematic Propositional Booleanism $\varphi \equiv \psi$, whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic.[16]

[16]This formulation of the view is a special case of that given by Dorr (2016). At the end of this section I will briefly discuss the full formulation of the view.

In general I'll take the instances of a schema to be closed under generalization, so that if φ is an instance of a schema, so is $\ulcorner \forall p \varphi \urcorner$ for any propositional variable p. Thus, since $p \leftrightarrow \neg\neg p$ is a theorem of classical propositional logic, $\forall p\ p \equiv \neg\neg p$ is a theorem of **Schematic Propositional Booleanism**.

The theory admits of a somewhat more laborious definition that may nevertheless aid intuition.

Propositional Booleanism

Identity Laws	$p \wedge \top \equiv p$	$p \vee \bot \equiv p$
Complement Laws	$p \wedge \neg p \equiv \bot$	$p \vee \neg p \equiv \top$
Commutative Laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

The laws are understood to be implicitly universally quantified. Here $\bot$ abbreviates $\neg\top$. Clearly each of the above laws can be obtained as an instance of **Schematic Booleanism**. The fact that each instance of **Schematic Propositional Booleanism** can be proven from the laws of **Propositional Booleanism** follows from the fact that (i) the above laws axiomatize the class of Boolean algebras, and (ii) classical logic is sound and complete with respect to its Boolean valued semantics.[17] Thus going forward I will simply talk of **Booleanism** for the theory corresponding to these two equivalent descriptions.

1.3.2 Examples

Philosophers tend to be more familiar with particular examples of Booleanism rather than the theory itself. The simplest example of a Boolean theory is *extensionalism.*

[17] Halmos (1963) states (i) without proof, but I believe it was originally shown in Huntington (1904).

Extensionalism $p \leftrightarrow q \rightarrow p \equiv q$

According to this theory, materially equivalent propositions are identical. Since any proposition is either materially equivalent to $\top$ or materially equivalent to $\bot$, this theory entails that every proposition is either $\top$ or $\bot$:

$$\forall p(p \equiv \top \vee p \equiv \bot)$$

Thus according to this view every proposition is either a logical truth or a logical falsity. We might intuitively think of the view as stating that there are exactly two propositions, the true and the false.[18] The operation of negation takes the true to the false and the false to the true. The operation of conjunction takes a pair of propositions to the true proposition if they are the true propositions; otherwise it takes them to the false proposition. Other Boolean operations are given the obvious truth functional readings.

This model is not a very good model of propositions. Since it is impossible that something be distinct from itself, necessary that everything be identical to itself and contingent that something talk to itself, there should be at least three propositions: an impossible one, a necessary one and a contingent one. More precisely, where $\Box$ is a normal modal operator expressing metaphysical necessity, extensionalism entails *necessitarianism*:

$$\forall p(p \rightarrow \Box p)$$

To see this suppose that p. Then $p \leftrightarrow \top$ and so, by **Extensionalism**, $p \equiv \top$. Under the assumption that $\Box$ is normal, we can infer $\Box\top$. Hence by **LL**, we can infer $\Box p$. Since p was arbitary we infer $\forall p(p \rightarrow \Box p)$.[19]

[18]The theory that there is only one proposition is inconsistent in the sort of classical theory we are working. If $\forall p(p \equiv \top)$ then $\neg\top \equiv \top$. And so since $\top \leftrightarrow \top$, we could infer $\top \leftrightarrow \neg\top$.

[19]Extensionalist tend to respond to these sorts of arguments by distinguishing between proposi-

Extensionalism entails **Booleanism**, but the converse is false.[20] A more interesting example is *intensionalism*.

Intensionalism $\Box(p \leftrightarrow q) \to p \equiv q$.

Here $\Box$ should be interpreted as expressing metaphysical necessity. Under the assumptions that $\Box$ is normal and the background logic is classical, **Intensionalism** entails **Booleanism**.[21]

The theory **Intensionalism** is validated by, for instance, the theory that identifies propositions with sets of possible worlds, though the latter theory has a richer structure than the former. For instance, the possible worlds theory of propositions is usually taken to include the assumption that any arbitrary set of worlds is a proposition. In particular singletons of worlds are propositions. But the singletons of worlds have quite distinctive behavior: for any proposition they either entail that proposition or entail its negation. Thus the possible worlds theory validates the following principle:

$$\exists q\big(q \wedge \forall p\big(\Box(q \to p) \vee \Box(q \to \neg p)\big)\big)$$

Intensionalism itself, however, can consistently be combined with the negation of this principle.

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tions and their *presentations*. There are many different approaches here, depending on what one takes presentations to be: Fregean senses, sentences etc. These views respond to arguments like the above by holding that there is an equivocation in the application of **LL**: $\varphi \equiv \psi$ does not imply that $\Box\varphi \leftrightarrow \Box\psi$ because φ and ψ denote different things in the context of $\Box\varphi \leftrightarrow \Box\psi$ than they do in the context of $\varphi \equiv \psi$ (roughly, there are some subtle differences between the views that I am glossing over).

[20] Quine (1955) gives a perplexing putative proof that **Booleanism** entails **Extensionalism**. I'm not exactly sure what is happening there, but it *seems* to me that Quine's argument is a sort of slingshot argument. In any case, it is exceedingly simple to come up with models in which **Booleanism** is true and **Extensionalism** is false, so in the present framework they are clearly distinct views.

[21] Suppose our background modal logic is normal, it is a theorem that $\Box(\varphi \leftrightarrow \psi)$ whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic. Thus in the theory **Intensionalism** it immediately follows that $\varphi \equiv \psi$ is a theorem.

If truth and necessary truth coincide in the language of pure mathematics, **Intensionalism** entails that there are only two propositions expressible in the language of pure mathematics. This is often pressed as a major objection to **Intensionalism**. While fully evaluating this argument against **Intensionalism** would take us too far astray, let me briefly sketch a response before moving on.[22] Platonism is either true it is not true. If it is true, then there really are mathematical objects, and they really have the mathematical properties that our true mathematical theories ascribe to them. But assuming platonism is true, it's hard to see how some mathematical properties and relations could fail to be fundamental.[23] What metaphysical analysis of set membership is available that doesn't make recourse to other purely mathematical relations or operations on sets? Supposing that *is* correct, there seems to be good reason to reject the necessity of mathematics, since fundamental properties, as they are often understood, can be freely recombined (within limits). For instance if *being a set* is fundamental, plausibly the actual sets could fail to be sets. To say otherwise is to commit oneself to the claim that large swaths of mathematics consists of *brute necessities*.

Alternatively if Platonism is not true, then all of our mathematical theories are false on their straightforward readings anyways. This means that in order to properly capture mathematical practice we already need to give a sort of revisionary account of what mathematicians are up to. Given this revisionary account it's no longer clear that we should hold mathematical propositions to be necessary. For instance, if mathematical propositions implicitly are assertions about a useful and contingently

[22] One reason it is important to at least indicate a response here is that, as mentioned below, some authors have suggested that there is a strong argument that takes us from **Booleanism** to **Intensionalism**. Since I accept the former I was to point out what I might says if shown to be committed to the latter.

[23] Two possible challenges to this might be the theory developed by Maddy (1990) and the theory developed by Lewis (1991, 1993), both of which grant the real existence of sets but provide something like a theory in which they are not fundamental.

existing fiction, it shouldn't be obvious that mathematical propositions are necessary. Or if mathematical propositions are *really* propositions concerning what is derivable from what in what formal systems, then insofar as sentences and the like are contingently existing, perhaps so too are all mathematical propositions.

Intensionalism entails **Booleanism**, but the converse is false. Some philosophers think that the temporal can vary freely of the modal, so that what is possible today will be impossible tomorrow.[24] Such theories invalidate **Intensionalism** but not **Booleanism**. To see this let A be the normal tense operator "it is always the case that…".[25] Suppose that **Intensionalism** is true. We will show that $\forall p(\Box p \to Ap)$. Suppose that $\Box p$. Thus $\Box(p \leftrightarrow \top)$. By **Intensionalism**, $p \equiv \top$. Since A is a normal modal operator, $A\top$. And so by **LL** Ap. Thus $\forall p(\Box p \to Ap)$. So **Intensionalism** entails, given plausible background assumptions, that what's necessary is always true.[26] But the thesis that modal status of propositions changes is clearly compatible with Booleanism. For instance, instead of requiring that necessary propositions coincide, we might instead require that propositions that are necessarily always materially equivalent are identical:

$$\Box A(p \leftrightarrow q) \to p \equiv q$$

If we permit cases in which something is necessary but not necessarily always true, this theory diverges from **Intensionalism**.

The above discussion shows that that Booleanism is consistent with an extensional theory of propositions, an intensional theory of propositions and a hyperintensional

[24]This sort of view is predicted by the logics of tense outlined in both Kaplan (1989) and Fine (1977b) for instance.

[25]See Burgess (2002) for background on tense logic.

[26]I don't mean to suggest here that the tense operator A is the best translation of ordinary claims in English involving the word 'always'. King (2007) has pretty forcefully argued against this view. I am just using this view about tense to provide a possible example of a Boolean theory that rejects **Intensionalism**.

theory of propositions. In this paper I am going to remain neutral on these more specific theories. I will simply work in the theory Booleanism itself, without any added axioms. That being said, it is worth noting that Booleanism on its own predicts something very close to intensionalism. Consider the property of being identical to the tautologous proposition $\top$:

$$L := \lambda p\, p \equiv \top$$

It is a consequence of **Booleanism** that propositions equivalent according to L are identical:[27]

Boolean Equivalence $L(\varphi \leftrightarrow \psi) \to \varphi \equiv \psi$

Bacon (2018) shows that given plausible background assumptions, **Booleanism** predicts that L is the broadest necessity operator.[28] One might conclude from this that L would have to be metaphysical necessity since, as it is often conceived, metaphysical necessity is the broadest notion of necessity.[29] This would then collapse **Booleanism** into **Intensionalism**.

There are several ways to resist the argument. Consider the property of being *counterfactually necessary*:

$$\blacksquare := \lambda p. \neg p \mathrel{\Box\!\!\rightarrow} \bot$$

Daniel Nolan (1997) has provided intuitive examples which appear to show that $\Box$ is not broader than $\blacksquare$.[30] That is, he has given specific examples that support the

[27]In general I will be supposing that $\varphi \leftrightarrow \psi$ when φ and ψ are $\beta\eta$-equivalent. Thus $L\varphi \leftrightarrow \varphi \equiv \top$.

[28]For instance we could explicitly add the schemas $L(\varphi \equiv \varphi)$ and $L(\varphi \equiv \psi \to (\chi \to \chi[\varphi/\psi])$. See also Cresswell (1965) and Seszko (1975) for proposals of defining necessity in terms of identity.

[29]Dorr(2016) conditionally draws this conclusion; for further discussion of this conception of necessity see McFetridge (1990) and Hale (1996).

[30]See Nolan (2011) for further criticism of the idea that metaphysical necessity is the broadest necessity.

generalization:

$$\neg\forall p(\Box p \to \blacksquare p)$$

If that's right, then $\Box$ is not the broadest necessity and so $\Box \not\equiv_{\langle\langle\rangle\rangle} \blacksquare$.

This argument is slightly unstable in the current context, however. The general intuitive judgments that tell against $\Box$ as the broadest necessity cited by Nolan (1997, 2011) also look to tell against L as the broadest necessity. And so since L is provably the broadest necessity given **Booleanism**, Nolan's cases look to tell against **Booleanism**.[31] One might reply by finding a principled distinction between the cases that tell against $\Box$ as the broadest necessity and those that tell against L as the broadest necessity. I think there are some grounds for this kind of view. But for our purposes it suffices to note that there are other avenues for responding to the charge that **Booleanism** entails **Intensionalism**.

In particular, the notion of metaphysical necessity was not introduced *merely* as the broadest necessity, but was also introduced by Kripke (1980) with explicit paradigm cases. If we want our notion of metaphysical necessity to respect these paradigm examples, this may lead to an interpretation of metaphysical necessity in which it is not the broadest necessity, though perhaps the broadest necessity in some restricted class of "objective" necessities.[32] This is of course a difficult issue. For reasons of space I am going to set it aside for later discussion.

[31]One assumption we might give up here is that $\blacksquare$ *is* a necessity operator. This seems natural given the sorts of examples discussed in Nolan (1997): for instance we might want to simply reject the thesis that $\blacksquare\top$ (i.e., $\bot \,\Box\!\!\to \bot$).

[32]See Williamson (2016) for a view like this.

1.3.3 Structure and ontology

Booleanism itself is silent on the ontological category of propositions. It is consistent with the view that propositions are sets,[33] that they are properties,[34] that they are states of affairs,[35] that they are events or acts,[36] that they are facts,[37] and that they are their own *sui generis* category of thing.[38] Let an *ontological theory* of propositions be a theory that says which ontological category propositions belong to. A *structural theory* of propositions is a theory that tells us what the laws are that govern the interaction of identity and various operations on propositions. Using the familiar jargon, a structural theory of propositions is a theory of the granularity of propositions, it tells us how fine grained propositions are. Booleanism is a structural theory of propositions—a theory of propositional fineness of grain. It is not an ontological theory of propositions.

Ontological theories *on their own* will not in general constrain structural theories all that much. It is often one's theory of the operations on propositions rather than the ontology of propositions which provides the structural core of the theory. The possible worlds conception of propositions is not coarse grained because propositions are identified with sets of possible worlds but rather because negation is identified with complementation, conjunction is identified with intersection and being necessary is identified with being the set of all worlds.

There has been a tendency to raise the significance of ontological questions in philosophy over more structural metaphysical questions like questions of propositional

[33]See Lewis 1980.

[34]See Speaks 2014a, Lewis 1983, van Inwagen 2004.

[35]Chisholm 1970.

[36]See Hanks (2011, 2015) and Soames (2014, 2015).

[37]King 2007 and 2014.

[38]Merricks 2015.

fineness of grain. Some will object that **Booleanism** is merely a schema for a theory rather than a theory of propositions itself. A fully adequate theory would tells us what propositions *are*. I think this is a misguided criticism. It would be similar to objecting to ZFC as a theory of sets on the grounds that it fails to tell us what sets are. What is important about these theories is the *wealth* of significant information they give us about the relevant objects. It is this wealth of information that ZFC gives us about sets that makes it a mathematically significant theory. Similarly, I suggest, it is the wealth of information about propositions that makes Booleanism a metaphysically significant theory.

Another reason why this objection seems to me misguided is that it presupposes an account of higher-order quantification according to which it is reducible to first-order quantification. I have been pronouncing in English claims like $\exists pBp$ and $p \equiv q$ as "there is a proposition that one believes" and "the proposition that p is identical to the proposition that q." Propositions thus become objects in the domain of individuals; it is then natural, though not inevitable, to ask after their intrinsic properties. But if we instead take $p \equiv q$ to mean something like "for it to be the case that p is for it to be the case that q" and then take the quantifiers to be irreducibly higher-order, the official formulation of Booleanism will make no reference to any abstract objects called 'propositions'. In this kind of framework it is not at all obvious what an account of the "ontology" or 'intrinsic properties" of propositions would amount to.

The strength of the theory becomes even more apparent when extended not just to the propositional case, but to cover properties and relations in addition.

Full Schematic Booleanism $\lambda x_1 \ldots x_n.\varphi \equiv \lambda x_1 \ldots x_n.\psi$, whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic.[39]

So extended, the theory can be seen to embody a fully general account of the

[39]This formulation is taken from Dorr (2016).

structure of propositions, properties, relations, and higher-order properties and relations. Like the Boolean theory of propositions, the extended theory admits of an equivalent non-schematic formulation.

Full Booleanism

Identity Laws

$\lambda p.p \wedge \top \equiv \lambda p.p$	$\lambda p.p \vee \bot \equiv \lambda p.p$

Complement Laws

$\lambda pq.p \wedge \neg p \equiv \lambda pq.\bot$	$\lambda pq.p \vee \neg p \equiv \lambda pq.\top$

Commutative Laws

$\lambda pq.p \wedge q \equiv \lambda pq.q \wedge p$	$\lambda pq.p \vee q \equiv \lambda pq.q \vee p$

Distributive Laws

$\lambda pqr.p \wedge (q \vee r) \equiv \lambda pqr.(p \wedge q) \vee (p \wedge r)$	$\lambda pqr.p \vee (q \wedge r) \equiv \lambda pqr(p \vee q) \wedge (p \vee r)$

This formulation requires that terms like $\lambda pq.\top$ be well formed, however. Some developments of the language of higher-order will require that p be free in φ in order for $\lambda p.\varphi$ to be a well-formed term of the language. The fact that **Booleanism** can be naturally extended this way to provide a strong theory of all intensional entities demonstrates some of the elegance and power of the theory. We will later see the contrast when it comes to attempts to flesh out the structured theory of propositions, which looks to work in certain special cases, but is difficult to state in full generality.

Summing up, we've seen that Booleanism is a structural theory of propositions that has received widespread endorsement, albeit in slightly different forms, over the course of the past one hundred years or so. It is consistent with a wide variety of hypotheses about the ontology of propositions, but is also naturally interpreted in a theory that takes on higher-order quantification as fundamental. In the higher-order setting the view is naturally seen as a special case of a much more general hypothesis concerning the structure of all propositions, properties and relations. In the next section we'll look at how the view interacts with various psychological generalizations.

1.4 Booleanism and The Conflation Argument

Soames (1987) puts forward an argument, well known among certain crowds of philosophers, that propositions cannot be identical to sets of truth supporting circumstances. The crux of this argument is that *if* propositions are sets of truth supporting circumstances, then certain plausible psychological generalizations carry commitment to certain implausible psychological generalizations. We might call the argument the "conflation argument" since the objectionable feature of the views in question is that they conflate the plausible with the implausible.

In what follows I will present a version of this sort of argument against **Booleanism**. In my view, the version presented here really gets to the heart of the problem. It is not theses concerning the ontology of propositions that matter to the argument; rather it is the consequences these ontological theses are often taken to have for one's account of entailment and propositional fineness of grain that matter. I will thus present a slightly pared down version that abstracts away from specific ontological hypotheses.

1.4.1 Distribution and Boolean Closure

Recall the principle of Distribution.[40]

Distribution $\forall p \forall q \big(B(p \wedge q) \rightarrow (Bp \wedge Bq)\big)$

Distribution is intended as a psychological generalization about actual indi-

[40]Strictly speaking we should include an initial operator for metaphysical necessity in order to capture the modal dimension of the principle Distribution. But nothing much is added to the discussion by including modality. The topics I want to discuss are more cleanly presented without bringing in modal operators and so for now I'll focus on the extensional version of the principle.

viduals. On its intended interpretation, it tells us that for any proposition p and proposition q, if one believes that p and q, then one believes that p and one believes that q. The 'one' in 'one believes that' is supposed to be interpreted generically. Thus if **Distribution** is true, anyone who believes that grass is green and snow is white believes that grass is green and believes that snow is white.

Oftentimes authors who investigate doxastic principles like **Distribution** are more concerned with normative statements to the effect that one *should* believe a conjunction only if one believes that conjuncts of that conjunction. This reading is not the one that will concern me. After all, it is at least somewhat plausible that one should believe all of the consequences of what they believe. If **Booleanism** takes us from a norm of distribution to a norm of closure, that is not any evidence against the theory if there is already a norm of closure to begin with. If a theory takes us from the psychological generalization, **Distribution**, to the psychological generalization that one actually does believe the consequences of what one believes, that looks like some evidence against the theory. The descriptive readings of the relevant principle will be assumed in what follows.

In order to formulate Closure in the object language, we need some way of formulating what it is for one proposition to entail another. One approach would be to just take entailment to be necessitation: p entails q when $\Box(p \to q)$. One benefit of Booleanism is that it provides us with a plausible notion of entailment that is free from the somewhat unclear notion of metaphysical necessity. In particular, we can say that a proposition p *entails* a proposition q if $L(p \to q)$ (i.e., $p \to q \equiv \top$). More precisely we can define an entailment operation on propositions as follows:

$$\leq := \lambda pq.L(p \to q)$$

Below I will give some further reasons for why we might take $\leq$ to be *the* entailment relation between propositions. Before doing that, I will show how the conflation argument can be run in this general setting with $\leq$ as our entailment relation.

We can formulate closure in the object language as follows.

Boolean Closure $\forall p \forall q(p \leq q \wedge Bp \rightarrow Bq)$

Boolean Closure is a consequence of **Distribution** given **Booleanism**. To see this suppose that $p \leq q$ and Bp. It is a consequence of **Booleanism** that $p \leq q$ if and only if $p \wedge q \equiv p$. So from our assumptions and **LL**, it follows classically that $B(p \wedge q)$. And so from **Distribution**, it follows that Bq. Thus **Boolean Closure** holds. Since **Boolean Closure** obviously entails **Distribution**, given **Booleanism**, the result is that **Booleanism** conflates the two principles: from the perspective of the Boolean, closure under consequence is the same as distribution over conjunction.

Note that if ψ is a consequence of φ in classical propositional logic, then $\varphi \rightarrow \psi \leftrightarrow \top$ is a theorem of classical propositional logic. Thus **Booleanism** implies that $\varphi \leq \psi$. The result is that whenever ψ is a consequence of φ in classical propositional logic, **Booleanism** together with **Distribution** *proves* $B\varphi \rightarrow B\psi$. But it is important to recognize that the conflation argument is supposed to extend further than merely the fact that belief is closed under the consequence relation of classical propositional logic: as I see it, it is an argument that belief is closed under *any* entailment.

To bring out this perspective on the argument it might be helpful to first consider how the argument works in a certain special case. The argument is naturally seen as a generalization of the more familiar argument against the possible worlds conception of propositions. As this view is commonly developed, a proposition X is taken to *entail* a proposition Y, if X is a subset of Y. But, so the objection goes, a proposition X is a subset of a proposition Y if and only if the intersection of X and Y is identical to X. Hence if one believes X, and $X \subset Y$, one believes $X \cap Y$. Thus given Distribution, one believes Y. Given the definition of entailment as subset, this guarantees that belief is closed under classical propositional consequence. But it shows much more given our background knowledge. For instance, it is often thought that mathematical truths are necessary if true. Thus on this view, every proposition entails a mathematical proposition: so if X is a proposition that Y is a mathematical proposition, $X \cap Y = X$.

The notion of entailment as subset makes reference to membership and so is tied down by background set theoretic notions. But as noted, the condition that X is a subset of Y is equivalent to some purely algebraic conditions: for instance that $X \cap Y = X$ *and* that $X \rightarrow Y = W$. So the first step of translating this more familiar case to the more abstract setting is to replace our definition of entailment as subset with one of these algebraic definitions. So for instance suppose we say that p entails q if $p \wedge q \equiv p$. In the context of **Intensionalism**, p entails q, in this sense, if and only if $\Box(p \rightarrow q)$; in turn, $\Box(p \rightarrow q)$ if and only if $p \leq q$ (i.e. $p \rightarrow q \equiv \top$). Thus given the more restricted theory of **Intentionalism**, the broad notion of entailment $\leq$ *just is* the familiar strict entailment stated in terms of $\Box$. So, like **Booleanism**, **Intensionalism** with **Distribution** *entails* that belief is closed under classical propositional consequence. But together with our background knowledge, it entails much more. For instance, many of us will be inclined to accept:

$$\Box(Fa \rightarrow \exists x Fx)$$

Thus **Intensionalism** entails, together with our background knowledge

$$Fa \rightarrow \exists x Fx \equiv \top$$

It also entails

$$Fa \wedge \exists x Fx \equiv Fa$$

Similarly, many of us accept

$$\Box(\Box p \rightarrow p)$$

Thus **Intensionalism** entails with our background knowledge that

$$\Box p \wedge p \equiv \Box p$$

In other words, the operator $\leq$ in the setting of **Intensionalism** is a *global* entailment relation: it does not represent entailment in some restricted system or anything like that.

I think that when working in the more general setting of **Booleanism** we should continue to treat $\leq$ as a global entailment relation, just like it was in the setting of **Intensionalism**. Of course we have to be *somewhat* more strict about what entails what in this setting. For instance, we might deny that all necessary truths entail all other necessary truths. But there is no reason to give up, and I think plenty reason to accept, statements like $Fa \leq \exists x Fx$. Indeed I think that our ability to take $\leq$ as the relation of propositional entailment is one of the selling points of a coarse grained view of propositions like **Booleanism**.

First, note that the definition of entailment for propositions can be easily generalized to relations of arbitrary types:

$$R \leq_{\langle \tau_1,\ldots,\tau_n \rangle} S := \lambda x_1 \ldots x_n.(R(x_1,\ldots,x_n) \leq S(x_1,\ldots,x_n))$$

This provides us with a notion of property entailment *that does not make use of any quantifiers.* Dorr (2014) has emphasized the importance of such quantifier-free notions of property of entailment for current debates over quantifier variance. In terms of them, we can investigate the logic of the quantifiers using only identity and the truth functional connectives. Since there is plausibly no variance in meaning in the truth functional connectives, nor in identity, across communities whose patterns of reasoning with respect to these operations are the same, this gives us a stable way to investigate whether two communities who reason similarly with the quantifier expressions pick out the same quantifier by doing so.

There is also something to be said for the fact that the relation $\leq$ is spelled out in purely extensional terms: we only used the Boolean operators $\to$ and $\top$ in addition to the identity operator $\equiv$. Since the definition of intensional notions using extensional resources is usually thought to be a benefit, the identification of the intensional notion of entailment with $\leq$ suggests itself on these grounds.

The relation $\leq$ also *behaves* like the entailment relation. For instance, supposing **Booleanism** is true, we get the following law:

$$(\chi \leq \varphi \wedge \psi) \leftrightarrow (\chi \leq \varphi \wedge \chi \leq \psi)$$

That is, a proposition entails a conjunction if and only if it entails both its conjuncts. And this principle in fact *uniquely* characterizes the conjunctive proposition. For suppose that $\chi \leq \chi' \leftrightarrow \chi \leq \varphi \wedge \chi \leq \psi$. Then $\chi \leq \chi' \leftrightarrow \chi \leq \varphi \wedge \psi$. Since every proposition entails itself we get that $\chi' \leq \varphi \wedge \psi$ and $\varphi \wedge \psi \leq \chi'$. Hence $L(\chi' \leftrightarrow \varphi \wedge \psi)$. So $\chi' \equiv \varphi \wedge \psi$.

Analogous principles hold for other Boolean connectives. Thus from the perspective of the Boolean, we have this relation, $\leq$, that has a simple algebraic characterization that looks to have many of the hallmarks of propositional entailment. This suggests to me that $\leq$ *is* propositional entailment.

This leaves us with the following, simple argument against Booleanism.

The Conflation Argument

- P1 Distribution is true.
- P2 Closure is false.
- P3 If Booleanism is true, then Distribution is true if and only if Closure is true.
- C Therefore, Booleanism is false.

The justification for P3 is of course just the above proof, together with the further arguments that $\leq$ is the relation of propositional entailment. I won't have much to say about P2. For the purposes of this paper, I'm inclined to accept it without argument. Ultimately, my goal will be to cast doubt on P1. But my method of doing so will be rather indirect. What I want to show is that the theory **Booleanism** conjoined with the negation of **Distribution** is a better theory overall than more finegrained theories that validate **Distribution**.

1.4.2 Independent Distribution and Independent Boolean Closure

By examining the above proof, one sees that the problematic feature of **Booleanism** is that it permits a definition of entailment in *terms* of the binary connective $\wedge$. When the proposition that φ entails the proposition that ψ, believing that φ entails believing that $\varphi \wedge \psi$, since the proposition that φ *is* the proposition that $\varphi \wedge \psi$. And so given **Distribution**, believing that φ implies believing that ψ.

It has sometimes been suggested to me in conversation that the Boolean should, in response, reformulate **Distribution** so as to avoid these sorts of troubling cases. In particular, they should admit that in general **Distribution** is false, but accept it for those special cases in which the conjuncts are *independent*.

We'll write $\varphi \approx \psi$ if either $\varphi \leq \psi$ or $\psi \leq \varphi$. Thus $\varphi \approx \psi$ if and only if the proposition that φ entails the proposition that ψ or vice versa. Then the proposition that φ and the proposition that ψ are *independent* if $\neg\varphi \approx \psi$ (which we will abbreviate as $\varphi \not\approx \psi$). With this terminology in place we can formulate a restricted version of Distribution as follows.

Independent Distribution $\forall p \forall q\big(p \not\approx q \rightarrow (B(p \wedge q) \rightarrow (Bp \wedge Bq))\big)$.

Independent Distribution is still able to predict a good deal of the cases we ordinarily observe and so is a natural fallback for the Boolean. But I think there are some good grounds for rejecting it. Just as **Distribution** implies **Boolean Closure**,

Independent Distribution also implies some unwanted closure conditions on belief. The basic idea is that given **Independent Distribution** and **Booleanism**, we can show that whenever one believes that $\varphi \wedge \psi$, and φ and ψ are independent, one believes every consequence of ψ that is independent from φ in a certain restricted sense.

To spell this out precisely let's introduce a new relation $\prec$ of type $\langle\langle\rangle, \langle\rangle, \langle\rangle\rangle$ (i.e., a three place relation between propositions) defined by

$$\prec := (\lambda pqs.p \leq q \wedge (s \wedge p \equiv s \wedge q))$$

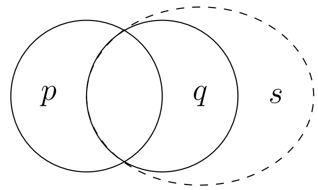
We will write $\prec (p, q, s)$ as $p \prec_s q$ and so the statement that $p \prec_s q$ is equivalent to $(p \leq q \wedge (s \wedge p \equiv s \wedge q))$. So $p \prec_s q$ if and only if the proposition that p entails the proposition that q, and conjoining the proposition that s with each proposition results in the same proposition. Metaphorically, p entails q and, from the point of view of s, p *is* q. We can then state the following closure condition on belief.

Independent Boolean Closure $\forall p \forall q \forall s(p \not\approx q \wedge q \prec_p s) \rightarrow (B(p \wedge q) \rightarrow Bs)$

A lot of theories might prove every instance of Independent Boolean Closure by proving that the antecedent is false. **Booleanism** allows for many true instances of the antecedent, however. Moreover, given **Independent Distribution**, one can prove Independent **Boolean Closure** given **Booleanism**.

To see this suppose that $p \not\approx q$ and $q \prec_q s$. Suppose, moreover, that $B(p \wedge q)$. Then $p \wedge s \equiv p \wedge q$ by $q \prec_p s$ and so $B(p \wedge s)$ by **LL**. Thus to show that Bs it suffices to show that $p \not\approx s$. If $s \leq p$ then since $q \leq s$ it follows that $q \leq p$, contradicting the assumption that $p \not\approx q$. If $p \leq s$ then $p \wedge s \equiv p$. And so since $p \wedge s \equiv p \wedge q$ we have $p \equiv p \wedge q$ (i.e. $p \leq q$), contradicting the fact that $p \not\approx q$. Thus $p \not\approx s$ and so it follows that Bs.

The argument may seem a bit opaque. But the situation is rather simple. We can visualize the fact that $p \prec_q s$ in terms of the following Venn diagram.



The left circle represents the proposition that p and the right the proposition that q. The dotted oval represents the proposition that s. The conjunction of two propositions is represented by the area in which they intersect. That one proposition entails another is represented by the fact that one is contained within the other. Thus we have that p and q are independent, since neither contains the other. And we have that q entails a proposition s that does not intersect p in any new places. So if one believes oneself to be in the area in which p and q intersect, one believes oneself to be in the area in which p and s intersect. Thus if one believes $p \wedge q$, one believes $p \wedge s$ and so believes s given the principle of **Independent Distribution**.

It's not entirely clear to me how plausible or implausible the principle of **Independent Closure** is. But it looks like a rather awkward principle to commit oneself to. On this sort of view belief will end up behaving in rather odd ways in those circumstances in which p entails q independently of χ, even though there doesn't seem to be any other reason to suppose it would other than the fact that its a consequence of one restriction on **Distribution**. This suggests to me that the principle of **Independent Distribution** is not a natural fallback position for the **Boolean**.

1.5 The Conflation Argument and Quasi-Structured Views

In this section I show that arguments that are analogous to the Conflation Argument can be used to challenge views that take propositions to more closely reflect the sentences that express them. By considering these views we will get a better sense of the trade offs between psychological generalizations like **Distribution** and various theories of propositional fineness of grain.

1.5.1 Aboutness, Subject Matter and Agglomerativism

I will first consider a recent view of propositional granularity that has been considered by Jeremy Goodman.[41] According to this view, logically equivalent propositions can be distinguished in virtue of being *about* different individuals.

To get a sense for the motivation for this kind of view consider one of the standard objections to the possible worlds conception of propositions. Since it is necessary that $2 + 2 = 4$ and necessary that $3 + 3 = 6$, it is necessary that $2 + 2 = 4$ if and only if $3 + 3 = 6$. Thus, according to the possible worlds theory, the proposition that $2 + 2 = 4$ is identical to the proposition that $3 + 3 = 6$. Many have found this consequence implausible. But why? Some will say that it is because the propositions play different roles in thought. Others will say that it is because the propositions differ in their constituents. But the most natural thing to say, it seems to me, is simply that they seem to be about different things. The proposition that $2 + 2 = 4$ is about 2 and 4, not 3 and 6, and the proposition that $3 + 3 = 6$ is about 3 and 6, not 2 and 4.

Boolean theories of propositions have a difficult time capturing aboutness. Every instance of $\varphi \equiv \varphi \wedge (\psi \vee \neg\psi)$ comes out true on a Boolean theory. So if Booleanism is true

$$(2 + 2 = 4) \equiv ((2 + 2 = 4) \wedge ((3 + 3 = 6) \vee \neg(3 + 3 = 6)))$$

But intuitively, the proposition expressed on the left is about different individuals than the proposition expressed on the right. Agglomerativism takes these intuitions seriously: for many propositions p and q, p is not identical to $p \wedge (q \vee \neg q)$ since p and

[41]Goodman (2019) describes but does not endorse the view. But Goodman (2017b) suggests that he does endorse a view like this.

$p \wedge (q \vee \neg q)$ are about different individuals.

We could present the argument by introducing a relation **About** of type $\langle\langle\rangle, e\rangle$ and stipulating some general principles governing it. For instance, the following principles seem quite natural given the target notion of aboutness (using p and q as variables of type $\langle\rangle$ and x as a variable of type e):

(i) $\forall p \forall q \forall x \big(\mathbf{About}(p \wedge q, x) \leftrightarrow \big(\mathbf{About}(p, x) \vee \mathbf{About}(q, x)\big)\big)$

(ii) $\forall p \forall q \forall x \big(\mathbf{About}(p \vee q, x) \leftrightarrow \big(\mathbf{About}(p, x) \vee \mathbf{About}(q, x)\big)\big)$

The combined effect of (i) and (ii) is that the conjunction or disjunction of two propositions is about whatever individuals its conjuncts or disjuncts are about. Given **Booleanism**, these principles entail that that any arbitrary proposition is about any arbitrary individual if some proposition is about that individual.

$$\forall x \forall p (\mathbf{About}(p, x) \to \forall q \mathbf{About}(q, x))$$

To see this suppose $\mathbf{About}(p, x)$. Let q be an arbitrary proposition. Then given **Booleanism** we have $q \equiv q \vee (p \wedge \neg p)$. By (i) $\mathsf{About}(p \wedge \neg p, x)$. And so by (ii), $\mathbf{About}(q \vee (p \wedge \neg p), x)$. Thus by **LL**, $\mathbf{About}(q, x)$.

The Boolean has several avenues of response. First it should be noted that there are some more sophisticated Boolean theories of aboutness, one's that wouldn't validate (i) or (ii), that are able to predict some of our intuitive judgments concerning what is about what.[42] A more flatfooted response is to insist that intuitions concerning what is about what ultimately rest on a confusion of properties of sentences and properties of propositions. It is sentences that can be about things. Propositions can only be said to be about this or that relative to a choice of presentation (for instance, a sentence expressing the proposition).

[42] See Fine (1977a).

What sorts of restrictions on **Booleanism** are predicted by taking aboutness seriously? Obviously the identification $q \equiv q \vee (p \wedge \neg p)$ has got to go. But what else? Goodman (2019) provides an algebraic characterization of those identifications that can be preserved on this sort of view. I am going to provide an account that is inspired by Goodman's and that *I think* is equivalent, though I haven't confirmed this. Even so, the account I present seems to capture the background idea quite well.

To state the theory we first need a preliminary definition. Where T is the set of terms of our language, a *content assignment* on T is a map from T into a bounded-join semilattice $\mathcal{S} = (S, \vee, 0)$.[43]

- $f(L) = 0$, whenever L is a logical constant (e.g., Boolean connectives and quantifiers).
- $f(R(A_1, \ldots, A_n)) = f(R) \vee f(A_1) \vee \cdots \vee f(A_n)$;
- $f(\lambda x_1, \ldots, x_n.\varphi)) = f(\varphi)$.

Intuitively we can think of S as consisting of sets of individuals such that $\emptyset$ is in S and whenever X and Y are in S, $X \cup Y$ is in S. The semilattice is then $(S, \cup, \emptyset)$. A content assignment is then simply an assignment of individuals to terms: the set of individuals the proposition, property or relation thereby expressed is about. Under this interpretation the conditions on content assignments are that $\top$ isn't about anything and the proposition that $R(A_1, \ldots, A_n)$ is about something if and only if R or A_i is about that thing, for some i; the property $(\lambda x_1 \ldots x_n.\varphi)$ is about whatever the proposition that φ is about.

Say that $\varphi \sim \psi$ if and only if $f(\varphi) = f(\psi)$ for every content assignment f. With these notions in place we formulate Agglomerativism as follows.

Agglomerativism $\varphi \equiv \psi$, whenever $\varphi \leftrightarrow \psi$ is a tautology and $\varphi \sim \psi$.

[43] That is, a tuple $(S, \vee, 0)$ equipped with an associative and commutative binary operation $\vee : S \times \to S$ and a distinguished element $0 \in S$, that is such that, for any element $s \in S$, $s \vee s = s$, and $s \vee 0 = s = 0 \vee s$.

Agglomerativism validates several of the Boolean laws we used to define **Booleanism**.[44]

Identity Laws	$p \wedge \top \equiv p$	$p \vee \bot = p$
Commutative Laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

Let's verify of couple of these just to get a feel for the view. First, note that $p \wedge \top \leftrightarrow p$ is a theorem of classical propositional logic. Thus to show (one half) of the identity laws, it suffices to show that $f(p \wedge \top) = p$, for any content assignment f. So let f be an arbitrary content assignment. Then $f(p \wedge \top) = f(\wedge) \vee f(p) \vee f(\top)$. Since $\wedge$ and $\top$ are logical constants, $f(\wedge) = 0 = f(\top)$. Thus $f(p \wedge \top) = 0 \vee f(p) \vee 0 = f(p)$. For the Distributive Laws it suffices to note that in a bounded join-semilattice, the join operation $\vee$ is *idempotent*: $x \vee x = x$ for any element. Thus the extra occurence of a variable p on the left hand side of the identity symbol in the Distributive Laws won't make any difference to the content assigned by a content assignment.

The theory is weaker than **Booleanism** since the Complement Laws are not theorems of **Agglomerativism**. (For example, consider any content assignment f in which $f(p) \neq 0$ for some propositional variable p. Then $f(p \vee \neg p) \neq 0 = f(\top)$. Thus $p \vee \neg p \not\equiv \top$.) Other laws that will fail are *absorption laws*, like $p \equiv p \wedge (p \vee q)$, for obvious reasons.

In order to study the interaction between various closure conditions on belief and **Distribution**, we again need to find some way to formulate claims about propositional entailment against the backdrop of an Agglomerative view of propositions. Like **Booleanism**, **Agglomerativism** admits of a natural, algebraic notion of entailment, though it differs slightly from the Boolean theory.

[44] See Goodman (2019) for further background and discussion.

The relation $\leq$ is no longer a plausible analysis of entailment against the backdrop of a view that takes aboutness seriously. We can see that this is so intuitively. For suppose that $\varphi \to \psi \equiv \top$. Then since $\top$ is not about anything, neither is $\varphi \to \psi$. But this entails that neither φ not ψ is about anything. Thus $\varphi \leq \psi$ only if neither φ nor ψ is about anything.

In the Boolean setting we were able to define a relation of propositional entailment that was such that, whenever ψ was a consequence of φ in classical propositional logic, **Booleanism** proved that the proposition that φ entailed the proposition that ψ. Thus **Booleanism**, in a sense, had the power to see classical propositional entailments among propositions. In the agglomerative setting, there turns out to be a relation that plays a similar role. In particular, instead of requiring that $\varphi \to \psi$ be identical to the purely qualitative tautology $\top$, we require that $\varphi \to \psi$ be identical to a tautology that, intuitively, is about the same things as $\varphi \to \psi$. The following seems like a natural choice:[45]

$$\varphi \preceq \psi := \varphi \to \psi \equiv (\varphi \to \psi) \to (\varphi \to \psi)$$

The relations $\leq$ and $\preceq$ agree assuming **Booleanism**, but differ in the Agglomerative setting. In the Agglomerative setting, $\preceq$ plays the natural role of entailment. For instance, just like in the Boolean setting, $\preceq$ interacts with the Boolean operations just as one would expect. On the one hand, it is not hard to see that if ψ is a classical propositional consequence of φ, then according to **Agglomeratvism** $\varphi \preceq \psi$. Just as in the Boolean setting though, it can be quite useful to allow for cases in which $\varphi \preceq \psi$ even when ψ is not a classical propositional consequence of φ. The relation $\preceq$ provides the Agglomerativist a simple algebraic characterization of propositional

[45]Goodman (2017b) proposes that $\varphi \equiv \varphi \to \varphi$ as an account of what it is for a proposition to be tautologous. Thus the proposal is that φ entails ψ if the material conditional $\varphi \to \psi$ is tautologous. This is thus analogous to the Boolean treatment of entailment.

entailment.

Unlike **Booleanism**, **Agglomerativism** together with **Distribution** does not predict the analogous closure condition:

Agglomerative Closure $\forall p \forall q(p \preceq q \to (Bp \to Bq)$.

For instance, $p \preceq q \vee \neg q$ is a theorem of **Agglomerativism**; but since one cannot infer from this that $p \wedge (q \vee \neg q) \equiv p$, there is no obvious way to get $B(q \vee \neg q)$ from Bp like there was in the Boolean setting. But there are some weaker principles concerning the closure of belief that **Agglomerativism** and **Distribution** *do* predict. In particular, I will argue that in an Agglomerative setting, one believes all of the *qualitative* consequences of what one believes, supposing that **Distribution** is true. In order to make this precise we first need to introduce the notion of a qualitative proposition.

Intuitively, a proposition is *qualitative* if it is not about any particular individual. It is *haecceitistic* if it is not qualitative.[46] The proposition that something is blue is qualitative whereas the proposition that Dory is blue is not. Further candidates for qualitative propositions are the propositions that necessarily every living thing has proper parts, that there were dinosaurs, that it is wrong to murder and that dogs have four legs.[47] Hence qualitative propositions can be modal, past tensed, normative and generic.[48]

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[46]The distinction between qualitative and haecceitistic propositions is an instance of the more general distinction between qualitative and haecceitistic properties and relations. I purposely avoid the terminology 'singular proposition' here since it has become, it seems to me, too bound up with a Russellian view on propositions. Furthermore, even for those who use 'singular proposition' but reject, or withold assent from, Russellianism, it is unclear whether or not singular propositions and haecceitistic propositions will coincide.

[47]These examples are of course open to dispute. For instance one might challenge the claim that natural kind predicates like 'dinosaur' and 'dog' express qualitative properties. Perhaps for instance they express properties whose definition makes reference to one particular animal in their evolutionary history.

[48]There are more controversial cases such as the proposition that the tall spy is sly. If, inspired

The notion of qualitativeness could be formalized directly using the relation **About**. However there turns out to be a more useful approximation of qualitativeness that can be formulated purely in terms of the Boolean operations and identity. The basic thought is that, if two propositions p and q are about the exactly same things, then $p \vee \neg p$ and $q \vee \neg q$ are about the exact same things. And so, if logically equivalent propositions that are about the same things are identical, $p \vee \neg p \equiv q \vee \neg q$. We can thus formalize the claim that p and q are about the same things, $p \sim^* q$, simply as $p \vee \neg p \equiv q \vee \neg q$.[49] A proposition is qualitative, then, if it is about the same things as the prototypical qualitative proposition $\top$. That is, p is qualitative, written $\mathcal{Q}p$, if $p \sim^* \top$.

With this notion in place we can formulate the following restricted Closure principle:

Aboutness Closure $\forall p \forall q(p \preceq q \wedge p \sim^* q \rightarrow (Bp \rightarrow Bq)$

Intuitively, if a propostion p entails a proposition q, and p and q are about the same things, believing p implies believing q. Note that as a special case, we get the principle of qualitative closure:

Qualitative Closure $\forall p \forall q(\mathcal{Q}p \wedge \mathcal{Q}q \rightarrow (p \preceq q \rightarrow (Bp \rightarrow Bq))$

Aboutness Closure, and thus **Qualitative Closure**, is deducible from **Agglomerativism** and **Distribution**. To see this suppose that $p \preceq q$, $p \sim^* q$ and Bp. The following is a theorem of **Agglomerativism**

(1) $p \equiv p \wedge (p \vee \neg p)$

Since $p \sim^* q$ we know that $p \vee \neg p \equiv q \vee \neg q$. Thus from (1) and **LL** we get (2)

by Russell (1904), we hold that this is just the proposition that there is a tall spy who is sly and identical to all tall spies then the proposition is qualitative even if it turns out that there is exactly one tall spy.

[49] See Goodman (2019) for further discussion of this point.

(2) $p \equiv p \wedge (q \vee \neg q)$

From the Distributive Laws we get

(3) $p \wedge (q \vee \neg q) \equiv (p \wedge q) \vee (p \wedge \neg q)$.

From **Agglomerativism**, we know that $p \wedge \neg q \equiv \neg(p \rightarrow q)$. Thus (2) and (3) entail

(4) $p \equiv (p \wedge q) \vee \neg(p \rightarrow q)$

Since $p \preceq q$, $(p \rightarrow q) \equiv (p \rightarrow q) \rightarrow (p \rightarrow q)$. And so from (4) we can infer (5):

(5) $p \equiv (p \wedge q) \vee \neg\big((p \rightarrow q) \rightarrow (p \rightarrow q)\big)$

But notice that $(p \wedge q)$ is logically equivalent to $(p \wedge q) \vee \neg\big((p \rightarrow q) \rightarrow (p \rightarrow q)\big)$. Moreover, any content assignment agrees on these formulas. Thus from **Agglomerativism** we have

(6) $p \wedge q \equiv (p \wedge q) \vee \neg\big((p \rightarrow q) \rightarrow (p \rightarrow q)\big)$

Finally (5) and (6) deliver (7)

(7) $p \equiv p \wedge q$

The argument from here is then simple. We have Bp and so from (7) we have $B(p \wedge q)$. Thus from **Distribution** we infer Bq.

What this argument shows is that, in an Agglomerative setting, belief can satisfy a distribution principle without satisfying a full closure principle; but it nevertheless satisfies a restricted qualitative closure principle. This allows us to run a kind of conflation argument against **Agglomerativism**. Simply put, **Distribution** is true and **Qualitative Closure** is false. But if **Agglomerativism** is true, **Distribution** is true only if **Qualitative Closure** is true. Hence, so the argument goes, **Agglomerativism** is false.

Now recall that ultimately I think conflation arguments like this fail since, as I will argue, **Distribution** should be rejected. The point here is just to show that they extend beyond **Booleanism** to quasi-structured views as well.

The argument actually extends quite a bit further than just views that take aboutness seriously. As Goodman (2019) points out, there are several other recent theories of propositional fineness of grain that are instances of **Agglomerativism**. For instance, many recent views of propositions that makes use of *subject matters* validate the theorems of **Agglomerativism**.[50] To see this notice that we could give an alternative interpretation of what content assignments are doing. Instead of taking a content assignment to be assigning sets of individuals to sentences, the individuals they are about, take them to be assigning *subject matters* to sentences. The join operation $\vee$ can then be interpreted as the operation of *fusion* defined over subject matters. The element 0 thus represents the trivial subject matter. The resulting view is one that, intuitively, says that two propositions are identical if and only if they are logically equivalent and have the same subject matter.[51]

What the previous argument shows is that recent views that try to move away from Booleanism by introducing notions of aboutness, whether in terms of being about individuals or having subject matters, cannot be motivated *just* on the grounds that they provide us with a better theory of the objects of belief.[52] This goes against Hawke et al (forthcoming) who present and agglomerative account of content as a solution to problems of logical omniscience.[53] As they note, their theory validates closure conditions for knowledge that are not immediately obvious. For instance their

[50]This applies, for instance to the sorts of views outlined in Yablo (2014) and Yalcin (2018).

[51]For more details see Goodman (2019).

[52]There are of course lots of other sorts of motivations that one might give for the view.

[53]They combine this sort of theory with a fragmentation account of knowledge in order to get a richer logic for knowledge.

theory validates

$$K(p \wedge q) \rightarrow K(\neg p \rightarrow q)$$

Their response? Basically just the standard Stalnakerian line: the reason this looks unintuitive is because one might fail to know that two sentences express the same propositions. What I don't understand is why this shouldn't be taken to completely undercut the motivation for their view. The agglomerative view is a weaker view than Booleanism. This weakening of the view must be justified. But it isn't justified if after all of the weakenings, not to mention the complications that come with fragmented views, we are left with essentially the same problem, and give essentially the same solution that the Boolean does.

Now to be fair there are many *other* reasons one might give in favor of agglomertive type views (for instance intuitions concerning aboutness mentioned above). I just don't see how it can be motivated on the grounds that it provides one with a better account of the objects of belief and knowledge.

1.5.2 Only Logical Circles

Cian Dorr (2016) puts forward a theory of the granularity of propositions, properties and relations that draws even finer distinctions than agglomerative views.[54] Dorr's view is based on a sort of "no circularity principle." Roughly speaking, the view says that if some relation R is a proper constituent of another relation S, then the other proper constituents of S must be logical constants. Officially, the view is formulated as a schema:

[54] As mentioned before, Dorr's theory is not put forward as an account of how fine-grained propositions are, but rather as an account of general principles governing the operator 'for it to be the case that ... is for it to be the case that ...' and related notions. I will continue to use proposition talk to express in English claims whose official formulation is in a higher-order language and remain netural on what the correct interpretation of such higher-order quantification is.

OLC $x \equiv_\tau \lambda\bar{v}R(y,x,\bar{v}) \to \mathbf{Logical}_\sigma(y)$[55]

For current purposes we can restrict our attention to what this theory predicts at the propositional level. The instance of the schema obtained by replacing τ with $\langle\rangle$ gives us

$$p \equiv R(f,p) \to \mathbf{Logical}_\sigma(f)$$

That is, if p is the proposition that f bears R to p, then f must be a logical constant.

To get a feel for this view let's look at some consequences.[56] Let p be a non-logical proposition and suppose that $p \equiv p \wedge p$. Thus $p \equiv (\lambda q.q \wedge p)p$. So from **OLC**, it follows that $(\lambda q.q \wedge p)$ is logical. But clearly this is wrong. So **OLC** predicts that p is distinct from $p \wedge p$.

As Dorr grants, distinguishing p from $p \wedge p$ is not exactly the most intuitive consequence. But the view is not motivated by this example. Rather it is motivated by examples like the following. Let $\varphi \vee_\gamma \psi$ abbreviate $(\varphi \wedge \gamma) \vee (\psi \wedge \neg\gamma)$. Intuitively no sequence of non-logical propositions provides a solution to the following system of equations:

$$\alpha \equiv \varphi \vee_\gamma \psi$$

$$\beta \equiv \psi \vee_\gamma \varphi$$

$$\varphi \equiv \alpha \vee_\gamma \beta$$

The first two equations tell us that φ is a "constituent" of α and β. But the first equation tells us that φ is itself built up from α and β via the operation $\vee_\gamma$. Dorr thinks that such a situation would be objectionably circular and so a theory of

[55] Dorr (2016, p. 74).

[56] In assessing the consequences of the view it is important to note that Dorr endorses the principle that a formula φ is materially equivalent to a formula φ^* whenever φ^* results from φ by replacing a constituent of the form $(\lambda\bar{x}.\psi)(\bar{a})$ with one of the form $\psi(\bar{a}/\bar{x})$ (provided that every variable in $\bar{x}$ occurs free in ψ). I will be supposing this is right for the purposes of this section.

propositions shouldn't allow it. The first two equations taken on their own, however, are unproblematic. However, given **Booleanism**, the first two equations entail the third.

This sort of objection goes back at least to Bealer (1982) though it is worked out in detail in Dorr (2016, p. 70-71). It seems perfectly coherent to suppose, for instance, that the words 'grue' and 'bleen' are introduced in such a way so as to verify the following two equations.

$$x \text{ is grue } \equiv x \text{ is blue } \vee_{x \text{ is spherical}} x \text{ is green}$$
$$x \text{ is bleen } \equiv x \text{ is green } \vee_{x \text{ is spherical}} x \text{ is blue}$$

But combining these two equations with the following seems circular.

$$x \text{ is blue } \equiv x \text{ is grue } \vee_{x \text{ is spherical}} x \text{ is bleen}$$

It is not just that the last statement sounds strange or has the appearance of being false. It is rather that the last equation has the appearance of *conflicting* with the first two.

The objection from circularity to **Booleanism**, like the above objection from aboutness, are metaphysical objections, and do not rest on any statements concerning cognitive significance or the like. Like the objection from aboutness, the Boolean can respond by challenging some of the judgments that they rest on. I'll briefly sketch a response like this. First, I'll show that a little bit of circularity in reality falls out of an extremely natural picture of the logic of "partial identifications". And second, I want to raise a couple of explicit counterexamples to Dorr's theory.

1.5.3 Nonlogical Circles

Let's add to our language a connective $>$ in which $p > q$ intuitively means "part of what it is for it to be the case that p is for it to be the case that q." We might think of $>$ as a kind of worldly entailment relation; whether or not it turns out to be *the* entailment relation is open to dispute. For now we can just think of it as a "partial identification" constant.

The following principles strike me as quite natural principles connecting partial identification and the Boolean constants.[57]

$\wedge$-adjunction $(r > (p \wedge q)) \leftrightarrow ((r > p) \wedge (r > q))$.

$\vee$-adjunction $((p \vee q) > r) \leftrightarrow ((p > r) \wedge (q > r))$

$\rightarrow$-adjunction $((r \wedge p) > q) \leftrightarrow (r > (p \rightarrow q))$.

Thus according to the first principle, part of what it is for it to be the case that snow is white is that grass is green and lemons are yellow if and only if part of what it is for it to be the case that snow is white is for it to be the case that grass is green, and part of what it is for it to be the case that snow is white is for it to be the case that grass is green. This principle strikes me as quite plausible given the intended reading of $>$; the $\vee$-adjunction principle is then just a sort of dual to this principle.

The principle **$\rightarrow$-adjunction** may require slightly more comment. Suppose that its being the case that $p \rightarrow q$ is part of its being the case that r. Then when we conjoin r with p, since the conditional $p \rightarrow q$ is already part of its being the case that q, this suggests to me that so too is q; nothing else needs to be added to r, as it were, in order to get p.

[57]The principles derive their names from category theory. In this framework, **$\wedge-$adjunction** states that conjunction is right adjoint to the "diagonal functor", **$\vee$-adjunctoin** states that disjunction is left adjoint to the diagonal functor (this cannot be formulated explicitly in our type theory since we don't have product types). The **$\rightarrow$-adjunction** states that, for any p, $(\lambda q.p \rightarrow q)$ is right adjoint to $(\lambda r.r \wedge p)$.

These principle provide something like object language descriptions of the standard introduction and elimination rules for $\wedge, \vee$ and $\to$. On their own they are not all that interesting. But it is somewhat natural to hold that such principles *uniquely* characterize the application of these Boolean operations to propositions. We can force this by adding the following two principles connecting partial identifications with full identification:[58]

Yoneda I $\forall r(r > p \leftrightarrow r > q) \to p \equiv q)$

Yoneda II $\forall r(p > r \leftrightarrow q > r) \to p \equiv q)$.

The basic idea behind these principles is a broadly structuralist one: the identity of a proposition is determined by those propositions that are "partially identical" to it and those propositions to which it is "partially identical". More precisely if its being the case that r is part of its being the case that p if and only if its being the case that r is part of its being the case that q, then its being the case that p *is* its being the case that q. The package consisting of the Yoneda principles together with the above Boolean adjunction principles is quite strong, since they suffice to uniquely characterize the application of these Boolean operations to propositions.

To see this first consider the case of conjunction. If $\forall r(r > s \leftrightarrow (r > p \wedge r > q))$ then by $\wedge$-**adjunction**, $\forall r(r > s \leftrightarrow r > p \wedge q)$. So by **Yoneda I** $s \equiv p \wedge q$. Using $\vee$-adjunction and **Yoneda II** we observe that disjunctive propositions are also uniquely pinned down. Finally using $\to$-**adjunction** note that if $\forall r(r > s \leftrightarrow r \wedge p > q)$ it follows that $\forall r(r > s \leftrightarrow r > p \to q)$. Thus with **Yoneda I** we also uniquely pin down conditional propositions.

So we have some very strong principles with some intuitive plausibility concerning the relationship of partial identification of propositions and full identification of propositions. I now want to show that these principles suffice to give quite

[58]These principles derive their name from the Yoneda lemma in category theory.

a bit of circularity. The simplest example is that the combination of **Yoneda I** and $\wedge$-**adjunction** entail that $\forall p(p \equiv p \wedge p)$. This is almost immediate since $\forall p(((r > p) \wedge (r > p)) \leftrightarrow (r > p))$ is a theorem of the background classical logic.

What's more interesting is that these principles also suffice to show that the operation of conjunction *preserves* the operation of disjunction; this gives us a more robust kind of circularity.

The argument goes as follows. Let r be an arbitrary proposition. From $\rightarrow$-**adjunction**:

(1) $\big(((p \vee q) \wedge s) > r\big) \leftrightarrow (\big(p \vee q) > (s \rightarrow r)\big)$.

And from $\vee$-**adjunction**:

(2) $\big((p \vee q) > (s \rightarrow r)\big) \leftrightarrow \big(p > (s \rightarrow r) \wedge q > (s \rightarrow r)\big)$

Then, from (1) and (2) we can infer (3):

(3) $\big(((p \vee q) \wedge s) > r\big) \leftrightarrow \big(p > (s \rightarrow r) \wedge q > (s \rightarrow r)\big)$

By $\rightarrow$-**adjunction** again we get (4):

(4) $\big(p > (s \rightarrow r) \wedge q > (s \rightarrow r)\big) \leftrightarrow \big((p \wedge s) > r \wedge (q \wedge s) > r\big)$

And from $\vee$-**adjunction** again we have (5)

(5) $\big((p \wedge s) > r \wedge (q \wedge s) > r\big) \leftrightarrow \big(((p \wedge s) \vee (q \wedge s)) > r\big)$

But (4) and (5) jointly (6):

(6) $\big(((p \vee q) \wedge s) > r\big) \leftrightarrow \big(((p \wedge s) \vee (q \wedge s)) > r\big)$.

Thus, since r was chosen arbitrarily, we can infer using **Yoneda II** that conjunction preserves disjunction:

$$(p \vee q) \wedge s \equiv (p \wedge s) \vee (q \wedge s)$$

So the idea that reality has some circularity to it can be motivated directly by some natural principles governing partial identifications. I don't think an argument like this takes us all the way to a view as strong as **Booleanism**. The sticking point is negation, whose Boolean behavior is difficult to motivate by reflection on partial identifications. I do think it makes more apparent why some amount of circularity in one's theory of propositional fineness of grain is in fact quite natural, at least given a certain "structuralist" perspective on the relation $>$.

This provides us some theoretical reason for endorsing circularity. I also think that there are some rather intuitive cases of circular phenomena. For instance, I find it natural to hold that for a set x to be hereditarily finite is for x to be a finite set all of whose members are hereditarily finite. This is how the notion of "hereditarily finite" is often introduced, afterall. But this seems to involve the sort of circularity ruled out by Dorr's theory. Explicitly, the following principle concerning the property of being hereditarily finite strikes me as intuitively plausible:

$$\mathbf{HF} \equiv_{\langle e \rangle} \lambda x(\mathbf{FinSet}(x) \wedge \forall y(y \in x \rightarrow \mathbf{HF}(y))$$

By substituting β-equivalent constituents on the right-hand side, this identification expands to:

$$\mathbf{HF} \equiv_{\langle e \rangle} \lambda x\big((\lambda X x.\mathbf{FinSet}(x) \wedge \forall y(y \in x \rightarrow X(y))(\mathbf{HF}, x)\big)$$

Thus from **OLC** we can infer

$$\mathbf{Logical}_{\langle\langle e \rangle, e\rangle}\big(\lambda x\big((\lambda X x.\mathbf{FinSet}(x) \wedge \forall y(y \in x \rightarrow X(y))\big)$$

But the idea that *that* property is logical strikes me as very implausible. Thus **OLC** seems to predict that being hereditarily finite is not being a finite set all of whose members are hereditarily finite.[59]

These are by no means knock down arguments. I've included them here just to indicate some metaphysical considerations that might tell in favor of a more coarse grained view like **Booleanism**.

One *could* attempt to motivate a view like **OLC** not on the basis of metaphysical considerations, but rather on the basis that it provides a better theory of the propositional attitudes.[60] In particular, one might attempt to motivate the view by showing that it can more naturally be combined with **Distribution** without, at the same time, running into any sort of conflation type argument. What I want to show now is that even a theory like **OLC** faces a sort of conflation argument, though it is quite a bit less robust than the previous one's considered.

1.5.4 A conflation argument for OLC

As with the previous examples, in order to run an argument like this, we first need to figure out whether there is a plausible algebraic notion of entailment definable in the theory in question. Neither of the defined operations $\leq$ nor $\preceq$ serve as adequate formulations of propositional entailment in the context of **OLC**. Dorr (2016, ft. 16) points out that there is, however, a definition of entailment that is well-behaved in certain extensions of his own theory. Say that propositions p and q are *logically equivalent*, written $p \sim_L q$ if $p \wedge q \equiv p \vee q$. Then we can say that p *entails* q, written

[59]Dorr does discuss cases like this. His response is to accept the consequence but note that in all non-controversial cases of apparently circular definitions like this, there is always available a definition which is not circular. But this doesn't really undercut the point I was intending to making which was merely to point out that there is some evidence against Dorr's theory. Booleanism can accomodate this evidence in a more flatfooted way than **OLC**.

[60]It is important to emphasize that this is not at all the kind of things that motivates Dorr since, on his view, the behavior of the belief operator does not act as an independent constraint on identifications.

$p \Longrightarrow q$, if $p \vee q \sim_L q$.[61]

In the context of **Agglomerativism**, $p \Longrightarrow q$ if and only if $p \preceq q$; thus in the context of **Booleanism** $p \leq q$ if and only if $p \Longrightarrow q$. Neither of these equivalences hold in Dorr's theory.

The relation $\Rightarrow$ acts as a natural entailment relation when Dorr's theory is strengthened by adding some of the Boolean identities that are not ruled out by his theory. Which one's are these? Some of the obvious ones are $p \equiv \neg\neg p$ and $p \wedge q \equiv q \wedge p$. But we can also define which Boolean identities to add in a more systematic way. Let a *complexity assignment*, $\#$, be a map form the set of terms T of the language to the natural numbers $\mathbb{N}$ such that

- $\#(t) = 0$ if t is a logical constant;
- $\#(R(A_1, \ldots A_n) = \#R + \#A_1 + \cdots + \#A_n$;
- $\#(\lambda x_1 \ldots x_n . \varphi) = \#\varphi$.

We write $\varphi \# \psi$ if for any complexity assignment $\#$, $\#(\varphi) = \#(\psi)$. Then, similarly to **Agglomerativism**, we can fill out the theory **OLC** by adding the following schema:

NC $\varphi \equiv \psi$, whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic and $\varphi \# \psi$.

NC entails that $p \equiv \neg\neg p$ since, for any complexity assignment, $\#(\neg\neg p) = \#\neg + \#\neg + \#p = 0 + 0 + \#p = \#p$. It also allows us to show that, whenever ψ is a classical consequence of φ, the proposition $\varphi \Longrightarrow \psi$ is a theorem of **NC**. To see this, note that if $\varphi \rightarrow \psi$ is a theorem of classical propositional logic, then $\big((\varphi \vee \psi) \wedge \psi\big) \leftrightarrow \big((\varphi \vee \psi) \vee \psi\big)$ is also a theorem. Moreover, it is equally clear that $\big((\varphi \vee \psi) \wedge \psi\big) \# \big((\varphi \vee \psi) \vee \psi\big)$. Thus from **NC**, it follows that $\varphi \vee \psi \sim_L \psi$ (i.e., $\varphi \Longrightarrow \psi$).

One cannot show that closure follows from **NC** together with **Distribution**.

[61] Goodman (2017b) also puts forward this account of entailment as the proper notion of entailment in Dorr's theory.

OLC Closure $\forall p \forall q(p \Longrightarrow q \rightarrow (Bp \rightarrow Bq)$.

But one is able to derive a kind of closure principle from **NC** and **Distribution**.

$\vee$**-Closure** $\forall p \forall q(p \Longrightarrow q \rightarrow (B((p \vee q) \vee q) \rightarrow Bq)$

To see this suppose that $p \Longrightarrow q$, Bp and $B((p \vee q) \vee q)$. Then, since $p \Longrightarrow q$, $(p \vee q) \vee q \equiv (p \vee q) \wedge q$. Thus by **LL**, $B((p \vee q) \wedge q)$. So from **Distribution**, Bq.

Intuitively we might think about this closure principle as follows. Suppose that you believe that p and p implies q. Then I can get you to believe q using the following strategy. First, I point out that since p, it's also the case that $p \vee q$. If you are being rational you should accept and thus believe $p \vee q$. I then point out that, since $p \vee q$, it's also the case that $(p \vee q) \vee q$. Again if you a rational, you should accept and so believe $(p \vee q) \vee q$. But given $\vee$**-Closure**, this immediately entails that you believe q, whether or not you are aware that p implies q. Thus, on this theory, we can get an individual to believe the consequence of what they believe merely by getting them to *consider* certain iterated disjunctions of those consequences with what they believe.

This is of course a much more restricted kind of closure. But it strikes me as being equally open to a sort of conflation argument since the principle $\vee$**-Closure** looks like a rather implausible psychological generalization. So it seems to me that, like Boolean and agglomerative views, Dorr's view combined with **Distribution** has odd consequences for one's theory of belief. If one were committed to **Distribution**, one might take this as grounds for rejecting all of these views. I'm inclined to take it to cast doubt on **Distribution**. In order to make this case, I want to now turn to the full structured theory of propositions and investigate how well motivated it might be by **Distribution**

1.6 The Structured View of Propositions

The above arguments show that theories of propositions that incorporate aspects of the structured view face analogous problems to the Boolean theory of propositions. While they do not validate full closure outright, they do entail somewhat restricted closure principles provided that belief distributes over conjunction. I think that this shows that such theories cannot be justified in the basis of providing us with a better theory of the objects of thought. Either they have to give up **Distribution**, or else accept these restricted closure principles. The problem they face, then, is pretty much analogous to those faced by **Booleanism**. It is thus not clear what the restrictions on **Booleanism** have gotten us. The gains one gets in one's theory of belief is minimal; the costs in the simplicity and strength of one's background theory of propositions seem to me to be no less minimal.[62] The methodology of discounting Booleanism on the basis of its theory of belief is misguided, since it seems that any theory capable of providing a simple, algebraic account of entailment will face some sort of unintuitive consequences once combined with **Distribution**.

The moral that Soames (1987) drew from reflecting on considerations like these is rather different; on his view, there is one view in particular that is motivated on the grounds that it provides a better account of the objects of belief: the *structured* or *Russellian* view. In this section I will formulate the structured theory of propositions and sketch a couple of reasons why the combination of the structured theory of propositions with **Distribution** is less attractive than it might initially appear to be.

[62]The theory **NC** is a restriction on Booleanism. But Dorr's theory **OLC** is not. So this point as stated doesn't quite apply to Dorr. This is part of the reason I spent some extra time responding in particular to Dorr's theory.

1.6.1 Basic Structure and Distribution

Roughly speaking, the structured view of propositions holds that propositions are structured like the sentences that express them. How are sentences structured? Perhaps the key structural property of sentences of a language is that they are *uniquely readable*. The sentence $\ulcorner Fa \urcorner$ is identical to the sentence $\ulcorner Gb \urcorner$ if and only if the predicate F is identical to the predicate G and the constant a is identical to the constant b. The natural thought then is that, if propositions are structured like a language, they should be *uniquely decomposable*. We can start with a restricted structured principle to get a feel for the view:

Basic Structure

(1) $R(A_1, \ldots, A_n) \equiv_{\langle\rangle} S(B_1, \ldots, B_n) \rightarrow R \equiv_{\langle \tau_1, \ldots, \tau_n \rangle} S \wedge \bigwedge_{1 \leq i \leq n} A_i \equiv_{\tau_i} B_i$

(2) $F(A_1, \ldots, A_n) \not\equiv_{\langle\rangle} G(B_1, \ldots, B_m)$

And instance of (1) is obtained by replacing A_i and B_i with terms of type τ_i and replacing R and S with *constants* of type $\langle \tau_1, \ldots, \tau_n \rangle$ (more on this restriction in a moment). An instance of (2) is obtained by replacing F and G with constants of different types and A_i and B_i with terms of the appropriate types. This seems to provide us with an approximation of the idea that propositions are "uniquely readable". This theory predicts, for instance, the following general principle:

$$\forall p \forall q (p \wedge q \not\equiv p \vee q)$$

To see this note that if it were the case that $p \wedge q \equiv p \vee q$ then we could use (1) to infer that $\wedge \equiv_{\langle\langle\rangle,\langle\rangle\rangle} \vee$, which we know to be false. In other words a proposition cannot be *both* the result of applying conjunction to two propositions *and* the result of applying disjunction to those propositions. The view also predicts, for instance, that the “De Morgan law” is universally false:

$$\forall p(\neg(p \wedge q) \not\equiv \neg p \vee \neg q)$$

This follows immediately from (2) since $\neg$ and $\vee$ are constants of different types.

Is **Basic Structure** motivated by conflation type considerations? Here is what Soames says about the “Russellian approach” to propositions, of which **Basic Structure** is plausibly an instance.

> The Russellian approach offers a welcome constrast. Given the intuition that whenever an individual satisfies ⌜x believes that A & B⌝ he also satisfies ⌜x believes that A⌝ and ⌜x believes that B⌝, the Russellian approach supplies a plausible explanation. Since the objects of belief reflect the logical structure of the sentences used to report those beliefs, whenever a belief is correctly reported using a conjunction the agent will believe a conjunctive proposition which includes the propositions expressed by the conjuncts as constituents. Since these constituent propositions are, so to speak, before his mind, no computation is required in order for him to arrive at beliefs in the conjuncts. (1989, 70)

The argument seems to be that (i) **Distribution** is true and (ii) the Russellian approach, which I will for now take to be the view **Basic Structure**, offers the best explanation for why.

It's not quite clear to me why we should accept (ii). For instance if believing a conjunction entails believing its conjuncts, one would have thought the best explanation would be simply that believing a conjunction *just is* believing its conjuncts.

$$B(p \wedge q) \equiv (Bp \wedge Bq)$$

This would provide an immediate explanation for *why* believing a conjunction entails believing its conjuncts. But rather than being predicted by the structured theory, this explanation is in fact inconsistent with it by (2) in **Basic Structure**.

Perhaps the thought is something like the following. The only way **Distribution** could fail is if one believes $p \wedge q$, but believes it under the guise of a sentence that carves up its structure differently. For instance if $p \wedge q \equiv \neg r$ for some proposition r, then one might believe $\neg r$, and so believe $p \wedge q$, but not realize that the proposition one believes is conjunctive, and so what should be an automatic inference from conjunction to conjuncts does not go through. But according to the **Basic Structure** there simply cannot be such a case. So there cannot be any counterexamples to **Distribution**.

While this argument may sound initially somewhat plausible, I think it ultimately rests on a conflation between the structure of propositions and the structure of the things we use to represent propositions (whether it be sentences, utterances, actions etc.) The theory **Basic Structure** is a theory about the structure of propositions, not a theory about what sorts of devices can be used to represent propositions. And there doesn't seem to be any theory that could rule out the basic worry of the kind raised above. That is, even if propositions are structured, there are plenty of ways to represent them that don't make evident what their structure *is*. So if we can exploit misleading representations to get individuals to believe, e.g., $p \wedge q$, without making evident what the structure of this proposition is, then perhaps there are counterexamples to **Distribution**. **Basic Structure** says nothing that could rule this out.

1.6.2 Basic Structure and Entailment

Sometimes the structured view is presented as one that is "fine-grained enough" to avoid the conflation argument. But presenting things this way I think misses a bit of the interworkings of the argument. As we saw above, in each case the argument only goes through given a certain analysis of propositional entailment. These analyses are all hopelessly implausible given the full structured view. But this isn't clearly a virtue. In fact, I'm inclined to view it as a weakness of the view. **Booleanism** and the other quasi-structured views considered, were able to give simple algebraic analyses of entailment for propositions, properties and relations. These analyses made use only of extensional resources: conjunction, disjunction, identity etc. The structured theory is not capable of providing account of entailment like this.

Of course, one might reply that the package consisting of **Basic Structure** and **Distribution** is still nevertheless a better overall theory than, for instance, the package consisting of **Booleanism** and the negation of **Distribution**. I will argue that it is not. My argument for this has several parts. First, I'll argue that the theory **Basic Structure** is both objectionably weak and objectionably arbitrary. However, strengthening the theory in a nonarbitrary way proves to be difficult. I will then argue that accepting the negation of **Distribution** is not at all as bizarre that its proponents often make out. I will also provide a couple of *direct* arguments against **Distribution** for good measure.

1.6.3 Structured Propositions and Russell-Myhill

The principle **Basic Structure** restricts its instances to only those in which the main connective is replaced by a constant. The result is that the theory fails to predict a lot of what a theory of structured propositions *should* predict. For instance, a structured proposition theorist will presumably want to reject theses like

$$p \equiv (\lambda p.p)p$$

and

$$p \wedge q \equiv (\lambda p.p \wedge q)p$$

Basic Structure does not predict this. There is another, related worry. When the principle is restricted to *just* to the constants, the view seems arbitrary. What we have in effect done is chosen some class of properties arbitrarily and said that the principle governs these properties. But if that is true then we should either expect the principle to govern all properties, or else we should be in a position to describe in a nonarbitrary manner *which* properties it governs and which ones it does not. Let's consider both of these options in turn.

Suppose that one thought that every instance of **Basic Structure** was true even when R and S are replaced by arbitrary terms. Then the following generalization would hold:[63]

Full Structure $\forall p \forall q \forall X \forall Y(Xp \equiv Yp \to (X \equiv Y \wedge p \equiv q))$.

According to full structure, for any proposition p and property X, the proposition that Xp is uniquely decomposable into the property X and the proposition p.

The problem with **Full Structure** is that it can shown to be inconsistent given very minimal principles. This problem has been well documented in the recent literature in higher-order metaphysics.[64] But I think it is worth some further discussion for several reasons. First, it is important to emphasize this problem in the overall abductive comparison of the theories we have been discussing. Second, the proofs that have been given in recent literature showing that **Full Structure** is inconsistent have all relied on *classical* principles. I think it is worth emphasizing, however, that the proof of inconsistency can be given *constructively*. This shows the problem is somewhat more robust than has been noted. Finally, I just want to emphasize that the problem is not one that is somehow implicitly having to do with cardinality issues. It's not exactly clear to me whether anyone thinks this. But sometimes the result is informally presented as a cardinality issue. I think to do this slightly obscures what

[63] Recall that the instances of a schema are closed under generalization.

[64] The problem can be traced back to Russell (1937) and Myhill (1958). There has been renewed attention to the problem by metaphysicians however. See for instance Dorr(2016) and Goodman (2017).

is actually happening in the argument.

Starting with the first task, let's first show that **Full Structure** can be shown to be inconsistent when we restrict our background logic to intuitionistically acceptable principles. We will need one further principle, a principle that I have been implicitly using up to this point but is good to state explicitly:

Extensional β-equivalence $(\lambda v_1 \dots v_n.\varphi)(A_1, \dots, A_n) \leftrightarrow \varphi[A_i/v_i]$.[65]

Thus according to this principle, $p \leftrightarrow (\lambda p.p)p$. This principle does *not* assert that these propositions are identical; just that they are materially equivalent.

With this in place we can give a constructive proof of the following claim:

$$\forall p \neg \forall X \forall Y (Xp \equiv Yp \to X \equiv Y)$$

The basic proof is similar to the classical one's that have appeared in the literature. The difference is that the constructive proof is given a reductio. Let r be an arbitrary term of type $\langle\rangle$ and let

$$O = \lambda p \exists X (Xr = p \wedge \neg Xp)$$

Thus O is the property of being a proposition that predicates a property of r that it does not instantiate. Then the following is an instance of **Extensional β-equivalence**:

1. $O(Or) \leftrightarrow \exists X (Xr \equiv Or \wedge \neg X(Or))$

From **Full Structure** and **LL**:

(2) $\exists X (Xr \equiv Or \wedge \neg X(Or)) \leftrightarrow \exists X (X \equiv O \wedge \neg X(Or))$

From **LL** and existential generalization:

(3) $\exists X (X \equiv O \wedge \neg X(Oq)) \leftrightarrow \neg O(Oq))$

[65] This formulation is from Dorr (2016).

And (1), (2) and (3) entail (4):

(4) $O(Or) \leftrightarrow \neg O(Or)$

Since (4) is inconsistent in intuitionistic logic we can infer (5) by reductio:

(5) $\neg\forall X\forall Y(Xr \equiv Yr \rightarrow X \equiv Y)$

And so since r was chosen arbitrarily, this establishes our claim.

The proof is constructive. And the assumptions needed to make it go through in addition to intuitionistic propositional logic are quite weak. This seems to me to be strong evidence against **Full Structure**.

It is tempting to think that the proof is at bottom a cardinality type argument. When presenting the argument, Dorr (2016) sums up the conclusion:

> The argument is essentially Cantorian: one can think of the conclusion as saying that the domain of properties of propositions is larger than the domain of propositions, so that there can be no one-one correspondence between the two domains. (p. 64)

It is also the case that the *proof* directly mirrors the constructive proof of the proposition that there is no injection from $\mathcal{P}(X)$ to X.[66] However I think it can nevertheless be a bit misleading to present the view as having anything to do with size. The *explanation* for why the two theorems are so similar is that they are both instances of diagonalization arguments; however as Bruno Whittle (2018) has argued, the fact that Cantor's theorem is an instance of such an argument shows, if anything, that Cantor's theorem itself has nothing *intrinsically* to do with size. This might sound like a somewhat radical conclusion to draw, but it is actually one that I think *most* mathematicians would accept. William Lawvere (1969) showed that Cantor's theorem was a straightforward corollary of a simple algebraic theorem; a theorem

[66] See for instance Bell (2004).

that holds in contexts much broader than set theory.[67] Indeed it can be shown to hold in contexts in which no notion of *size* can even be defined. In the case of sets, there happens to be a connection between Cantor's theorem and size; but this is *only* because, in the case of sets, the Schroeder-Bernstein theorem holds: if there are injections from X to Y and Y to X then there is a bijection from X to Y. It is this fact that allows us to get some well defined notion of "larger than" and "smaller than" in set theory. Without some analogue of this theorem, the connection between these sorts of diagonalization arguments and size becomes obscure.

In order to avoid the inconsistency, the structured proposition theorist would have to embrace much more radical weakenings of their background logic. It's not clear what would motivate this in the present context. The argument doesn't rely on any general principles concerning truth, or any other factive operators. This suggests to me that the costs of **Full Structure** are quite high; if the structured theorists wants to maintain **Basic Structure**, they will need some principled way of telling us which *instances* of **Full Structure** are acceptable and which instances are not. While there has been some work that is suggestive in this area, I do not know of any worked out answers to this question.

I want to briefly sum up where I think we are in this argument. First, we've shown that **Distribution** combined with a variety of strong and simple theories of propositional fineness of grain predicts implausible results. And while **Distribution** conjoined with the principle **Basic Structure** does not predict implausible results, the view suffers from both weakness in terms of what it can prove, and arbitrariness in terms of what it does prove. I then argued that the structured proposition theorist faces an uphill battle when attempting to correct those defects. What I now want to do is address head on the question of how bad it really is to deny **Distribution**

[67] In particular, it is a corollary of a theorem that holds in any cartesian closed category, of which the category of sets is an example.

in addition to addressing the question of whether some *other* theory we have not yet considered will be able to accomodate **Distribution** without any collateral damage. I will argue that the denial of **Distribution** is much less implausible than it is sometimes thought to be; I will also argue that in a certain precise sense, we shouldn't expect any theory to be able to accommodate **Distribution** unscathed.

1.7 The Denial of Distribution

This section assesses the costs of denying **Distribution**. I'll do this by looking at some of the motivations for accepting the principle, and seeing how well those motivations support it.

1.7.1 Distribution and Belief Ascriptions

Perhaps the main motivation for accepting **Distribution** comes from the philosophy of language. Our intuitive reaction to belief ascriptions like 'Jones believes that grass is green and snow is white' is that it can only be true if 'Jones believes that grass is green' and 'Jones believes that snow is white' are also true. As Scott Soames puts the point:

> For many propositional attitude verbs distribution over conjunction is a fact whereas closure under necessary consequence is not. (Soames 1989, 49)

This suggests that many of us, at least implicitly, accept the following principle as valid in English:

Linguistic Distribution For any sentences φ and ψ and individual x, if x satisfies $\ulcorner x$ believes that φ and $\psi\urcorner$ then x satisfies $\ulcorner x$ believes that $\varphi\urcorner$ and x satisfies $\ulcorner x$ believes that $\psi\urcorner$.

But it is hard to see why **Linguistic Distribution** would be true if **Distribution** were false. If some individual could believe a conjunction without believing the conjuncts, then surely we should be able to report that this is so in English. The idea that there are failures of **Distribution** that are nevertheless inexpressible does not seem credible.

I think the argument from **Linguistic Distribution** to **Distribution** is convincing. The **Distribution** denying Boolean should thus reject **Linguistic Distribution**. This might seem like bad news for the Boolean since most of the instances of **Linguistic Distribution**that immediately come to mind seem obviously true. But if we look closer at the Boolean view it becomes clearer what the kinds of instances that turn out false will look like. I'll go over one sort of example.

Plausibly, the proposition foxes run entails the proposition that vixens run. So suppose that one believes that foxes run but disbelieves that vixens run, due to some sort of confusion. Since the proposition that foxes run entails the proposition that vixens run, the Boolean view entails that the proposition that foxes run and vixens run *is* the proposition that foxes run. Hence they believe that foxes run and vixens run. But, by stipulation, they do not believe that vixens run. So in this example the individual in question satisfies 'x believes that foxes run and vixens run' but does not satisfy 'x believes that vixens run'. **Linguistic Distribution** thus has a false instance.

There is something distinctive about this case that bears mentioning. The false instance of **Linguistic Distribution** that we get from **Booleanism** is an instance where intuitively the individual in question doesn't satisfy the antecedent. That is to say, the inference that looks unintuitive here is not that one can believe the proposition that foxes run without believing that vixens run, which given **Booleanism** generates a false instance of **Linguistic Distribution**, but rather the inference from believing that foxes run to believing that foxes run *and* vixens run. Thus the sorts of counterexamples that **Booleanism** provides are not unintuitive because we think the individual actually does believe both conjuncts. They are unituitive because we think the individual doesn't believe the conjunction. This turns out to be actually good news for the Boolean.

The reason why I think this is good news is that we can appeal to general heuristics we use in assessing belief ascriptions to explain this sort of data, without appealing to the truth of **Distribution**. For instance, the following rule is plausibly one that speakers apply in attempting to figure out the truth values of belief ascriptions:

ND Do not assert 'S believes that φ and ψ' if S fails to believe that φ or S fails to believe that ψ.

An instance of **ND** is obtained by substituting a name N for 'S', declarative sentences S and S' for φ and ψ respectively, and replacing 'S believes that φ and ψ' with $\ulcorner N$ believes that S and $S'\urcorner$.

One reason we might expect there to be a norm like this is the following. If I use some sentence in specifying one's belief, and that individual fails to believe that the sentence is true, then they are likely to reject my characterization of their belief. So if I can specify their belief using only sentences that they believe to be true, I should do so if what I want is to facilitate communication. Moreover, if one were to use the sentence 'foxes run and vixens run' where the sentence 'foxes run' would do, this raises the question as to why one went through the extra effort. If one merely believed that foxes run without believing that vixens run, describing their belief using the conjunctive sentence is likely to cause confusion. Consequently **ND** can be motivated independently of whether belief actually does distribute.[68]

One might worry that if **Linguistic Distribution** really were false, we would have no explanation as to why it is difficult for English speakers to discriminate 'John believes that it is raining and it is Tuesday' from 'John believes that it is raining and

[68]These rules of cooperation are of course somewhat rough and are violated often; but I am suggesting them more as heuristics we use to quickly assess the truth values of belief ascriptions rather than hard norms of discourse. I do suspect that **ND** is something like a full fledged norm. But I think this norm is plausibly explained by our background heuristics like those mentioned above.

John believes that it is Tuesday'. These sentences will likely strike ordinary speakers as equivalent; one good explanation of this fact is that they *are* equivalent. But notice that were this explanation correct, it would also support a principle which many theorists are happy to deny:

Agglomeration $\forall p \forall q(Bp \wedge Bq \rightarrow B(p \wedge q))$

It is generally thought that this principle is false: an individual might be "fragmented" and so fail to put together some of their beliefs. I suspect the reason for the linguistic indiscriminability of the the two cases is better explained by **ND**: there would just never be an occasion in day to day life to utter the one without uttering the other.

1.7.2 Distribution and Assertion

Another argument one might give for **Distribution** appeals to analogues between assertion and belief. If belief is the inner analogue of assertion, one might claim, then assertion distributes only if belief does. But certainly, the proponent says, assertion *does* distribute. So belief distributes.

We needn't inquire into whether belief really is the inner analogue of assertion nor ask after what exactly that claim amounts to: assertion does not distribute over conjunction. Indeed, the Boolean must reject the thesis that assertion distributes over conjunction for the same reasons that they must reject the thesis that belief does. If assertion distributes over conjunction, it is closed under logical consequence. But since assertion is not closed under logical consequence, it must not distribute.

This might seem like very bad news for the Boolean. How could one assert a conjunction but fail to assert both conjuncts? In addressing this question we need to be very careful to distinguish between *assertions* and *assertive utterances*. The objects of assertions are propositions. The objects of assertive utterances are sentences. The reason it might seem unassailable that assertion distributes is that one is prone to confuse it with a principle that *is* unassailable:

(SD) For any sentences S and S', if one assertively utters $\ulcorner S$ and $S'\urcorner$ then one assertively utters S and one assertively utters S'.

An assertive utterance of 'snow is white and grass is green' *just is* an assertive utterance of 'snow is white' closely followed by an utterance of 'and' closely followed by an assertive utterance of 'grass is green'. As far as I can tell, there is no reason whatsoever to reject **(SD)**. Once we have distinguished assertions from assertive utterances it is natural to ask after principles relating the two notions. One principle (schema to be exact) is particularly useful for our purposes:

(SA) If one assertively utters 'φ and ψ' then one asserts that φ and one asserts that ψ.

An instance of this schema results from replacing φ and ψ with declarative sentences S and S' respectively and replacing 'φ and ψ' with $\ulcorner S$ and $S'\urcorner$. Nothing in this paper challenges **(SA)**; it is perfectly consistent with the negation of **Distribution** and **Booleanism**. Any instance of **(SA)** can be derived from **(SD)** together with a corresponding instance of the schema **(A)**:

(A) If one assertively utters 'φ' then one asserts that φ.

instances of which are obtained by replacing φ with a declarative sentence S and 'φ' with $\ulcorner S\urcorner$. Since both **(SD)** and **(A)** look pretty plausible, this provides a strong case for **(SA)**.

1.7.3 Distribution and the Language of Thought

When it comes to the distribution of assertion over conjunction, it is easy to come up with replacements that nevertheless capture much of the original intuition. Are there analogous replacements for the case of belief? The obvious strategy is to appeal to mental representations in place of sentences. To implement this strategy symmetrically first note that we can redescribe the schema **(SA)** as a first order generalization over sentences:

(SA$^+$) For any sentences S and S' if one assertively utters $\ulcorner S$ and $S'\urcorner$ then for any propositions p and q such that S means p and S' means q, one asserts p and one asserts q.

Now suppose that individuals have a language of thought (a certain system of mental representations) and that beliefs are had via internal tokenings (the analogue of utterances) of these sentences. More precisely, there is some relation, *acceptance*, such that an individual believes that p if and only if they accept a mental representation that means that p. One's system of mental representations contains logical connectives such as conjunction. I'll write $\ulcorner m \mathbin{\dot{\wedge}} m'\urcorner$ for the conjunction of mental representations m and m'. We can then formulate the analogue of **(SA$^+$)** for belief:

(SB) For any mental representations m and m' if one accepts $\ulcorner m \mathbin{\dot{\wedge}} m'\urcorner$ then for any propositions p and q such that m means that p and m' means that q, one believes that p and one believes that q.

Provided that there is a language of thought, **(SB)** is a natural replacement for **Distribution**. When we try to imagine counterexamples to **Distribution**, for instance, we presumably do so by way of trying to imagine an individual with a conjunctive *mental representation* that nevertheless fails to believe the conjuncts. **(SB)** explains why such cases cannot be found.

As in the case of assertion, one can accept **(SB)** without accepting **Distribution**. One might initially worry that **(SB)** itself provides strong evidence for **Distribution**. Why should **(SB)** be true were **Distribution** false? Given that one can believe a conjunctive proposition without believing its conjuncts, why couldn't one do so by way of a conjunctive mental representation?

But in the case at hand I just don't think these rhetorical questions carry much weight. It seems like a perfectly plausible and acceptable psychological principles governing mental representations that when conjunctive mental representations get in the belief box, so do their conjuncts. It may be for instance that conjunctive mental representations get in the belief box *only* by way of getting their conjuncts in the belief box. If that's right, then whether one believes $p \wedge q$ via a conjunctive mental representation is obviously relevant as to whether one's belief distributes.

I think that this shows that the failure of **Distribution** is not as bad as we might have initially thought. On the one hand, every counterexample to the principle according to the Boolean is a proposition that, according to the structured theorist, isn't a counterexample. That is, whenever the Boolean say that one believes that $p \wedge q$ without believing that p, the structured theorist will just say that one doesn't actually believe that $p \wedge q$. The problem thus reduces to a more familiar problem: explaining why certain belief ascriptions, in our case certain conjunctive belief ascriptions, are true despite appearing to be false. This is the sort of problem that structured theories themselves face. For instance Soames' (1987) view entails that one believes that Hesperus is Hesperus if and only if one believes that Hesperus is Phosphorus. And in general I think there are a lot of responses available to these sorts of worries. In our case, I have sketched two routes: one via a norm and the other via a principle concerning mental representations.

In the concluding section of this appear, I want to address one final worry one might have with the sort of argument I have been pushing in this paper. The worry is that, while it may be true that given our present state of knowledge, **Booleanism** without **Distribution** seems to offer a compelling theory, what if the structured theorist is able to answer the arbitrariness worries raised above? Then there might be a strong and simple theory that can be combined with **Distribution** without any collateral damage. Since I haven't shown that this isn't the case, shouldn't I be agnostic moving forward? I'm not quite sure if this is a convincing argument. Nevertheless, I want to respond by sketching an argument that shows that in a certain sense no theory can accept **Distribution** without at least a little bit of unintuitive consequences.

1.8 Distribution and Arbitrary Conjunctions

The principle of **Distribution** can be iteratively applied. Thus it entails each instance of the following schema:

n-**Distribution** $\forall p_1 \ldots \forall p_n(B(p_1 \wedge \cdots \wedge p_n) \to (Bp_1 \wedge \cdots \wedge Bp_n))$.[69]

One might take this as grounds itself for rejecting **Distribution**: while the inference from $p \wedge q$ to p seems obvious, perhaps the inference from $p_1 \wedge \cdots \wedge p_{1,000,000}$ to p_{500} will seem less obvious. In this concluding section, I want to press a different sort of argument against **Distribution**. The argument basically runs as follows. First, we should accept n-**Distribution** only if we accept that belief distributes over *arbitrary* conjunctions. But no one, not even those who accept structured views, should accept that belief distributes over arbitrary conjunctions. Thus we shouldn't accept $n-$**Distribution**. Since **Distribution** entails n-**Distribution**, we shouldn't accept **Distribution**.

One of the main premises of this argument is that no one should accept that belief distributes over arbitrary conjunctions. In order to make this case we first need to provide some way to *express* this idea. Here is one way to do this. Let us set our typed language aside and work in a simple two-sorted higher-order language that has both propositional variables $p, q, r, \ldots$ as well as plural propositional variables $pp, qq, rr, \ldots$ with quantifiers to bind them. We suppose that there are still the Boolean operators, the belief operator B and the propositions identity operator $\equiv$.

[69]In the Boolean setting $p \wedge \cdots \wedge p_n$ can be read unambiguously. But in a structured setting there may be many propositions we are intending to pick out depending on how we bracket the terms of the conjunction. On the intended reading of this schema, there is, for each way of bracketing the formulas, and instance in which the formulas are bracketed that way.

We also add two novel resources: an operator $p \prec pp$ which informally states that p is one of pp and an operator $\bigwedge pp$ which represents the conjunction of the propositions pp.[70] In this language, arbitrary distribution can be formulated as follows:

Arbitrary Distribution $\forall pp(B \bigwedge pp \to \forall p(p \prec pp \to Bp))$

I will sketch an argument against this principle. The argument will proceed informally, but I suspect we should be able to provide a fully formal version of the argument as well. For reasons of space I will only briefly indicate how this is to be done.

The argument I will give is inspired by a recent argument Peter Fritz (unpublished) gives against recent theories of ground. In this paper, Fritz argues that many grounding theories are committed to the following two, jointly inconsistent principles:

($S \bigwedge$) $\forall pp \forall qq(Tpp \wedge Tqq \to (\bigwedge pp \equiv \bigwedge qq \to pp \equiv qq)$.

($T \bigwedge$) $\forall pp(Tpp \to \bigwedge pp)$

Here Tpp abbreviates the claim that all of the propositions among pp are true (i.e. $\forall p(p \prec pp \to p)$ and $pp \equiv qq$ abbreviates the claim that something is in pp if and only if it is in qq (i.e., $\forall p(p \prec pp \leftrightarrow p \prec qq)$). The argument against these two principles is simply the Russell-myhill argument in plural form: if both of these were true, then $\bigwedge$ would define an "injection" from pluralities of truths to truths. But by a familiar argument, we know that there cannot be any such injection. As Fritz notes, turning this argument into a fully deductive one is just a matter of translating Russell's (1903, Appendix B) argument into plural form.

Now consider a variant of this argument. Let's say that a proposition p is *non-conjunctive* if there are no propositions pp such that $p \equiv \bigwedge pp$ (i.e., $\neg\exists pp(p \equiv \bigwedge pp)$. We write this as $\mathbf{NC}(p)$. Some propositions pp are nonconjunctive if every one of

[70] For a much more leisurely description of this language see Hall (forthcoming) as well as Fritz (unpublished).

them is nonconjunctive (i.e. $\forall p(p \prec pp \to \mathbf{NC}(p))$. We write this as $\mathbf{NC}(pp)$. Now consider the following two principles:

Non-Conjunctive Structure $\forall pp \forall qq(\mathbf{NC}(pp) \wedge \mathbf{NC}(qq) \to (B \bigwedge pp \equiv B \bigwedge qq \to pp \equiv qq))$

Non-Conjunctive Belief $\forall pp(\mathbf{NC}(pp) \to \mathbf{NC}(B \bigwedge pp))$

By an argument anologous to Fritz's, these principles are plausibly inconsistent. If both were true, then the complex operator $B \bigwedge$ would define an injection from pluralities of non-conjunctive propositions to non-conjunctive propositions. But by general diagonal type arguments, we should expect this to fail.

Okay so suppose that is correct. Any structured view should, it seems to me, accept **Non-Conjuntive Belief**. For instance this sort of view would be predicted by some analogy of **Basic Structure** adapted to our present context, which, as we saw, was a core part of their view. Thus a structured theorist is committed to rejecting **Non-Conjunctive Structure**. So for two distinct collections of *non-conjunctive* propositions pp and qq, believing $\bigwedge pp$ just is believing $\bigwedge qq$. But this fact, together with **Arbitrary Distribution** looks to entail implausible closure conditions on belief. Without loss of generality, we can suppose that there is some proposition q in qq that is not one of pp. Now suppose that one comes to believe $\bigwedge pp$ by explicitly considering each of the nonconjunctive propositions in pp and inferring $\bigwedge pp$. Then it follows that one believes $\bigwedge qq$. So by **Arbitrary Distribution**, one believes each $q \prec qq$. But there is some proposition $q \prec qq$ that one did not explicitly consider yet, since it is not among pp. Moreover, since none of the $p \prec pp$ were conjunctive, one cannot have aquired the belief in q by believing some conjunctive proposition in pp of which q was a conjunct. So it looks like the principle of **Arbitrary Distribution**, together **Non-Conjunctive Belief** entail undesired closure condition on belief *all on their own*. This casts doubt on **Arbitrary Distribution** and so, it seems to me, cast doubt on **Distribution**.

1.9 Conclusion

This paper presented an extended argument for the theory comprised of Booleanism and the negation of **Distribution**. It was shown that a variety of quasi-structured accounts of propositions face analogous challenges that **Booleanism** does when combined with **Distribution**. Moreover, articulating the structured theory in a non-arbitrary way without lapsing into inconsistency turned out to be challenging. This suggests that the hope of providing an account of propositional fineness of grain that accommodates **Distribution** without collateral damage may be a false hope. Indeed in the final section of this paper, I presented an argument that in a certain sense, this can be shown to be right. Since, as argued in §6, the combination of **Booleanism** with negation of **Distribution** turned out to be far less implausible than it might have initially seemed, this seems to me to provide us with good reason to accept that package of views.

CHAPTER 2

INDEFINITE EXTENSIBILITY AND THE PRINCIPLE OF SUFFICIENT REASON

2.1 Introduction

The principle of sufficient reason (PSR) is the thesis that every contingent truth has a sufficient reason.[1] Peter van Inwagen (1983, pp. 202-204) has shown that the PSR collapses modal distinctions against a backdrop of plausible principles governing explanation, necessity, and their interaction. (Jonathan [15] (p. 115) presents a similar argument.) If the PSR is true, then, it would appear, there are no contingent truths and so all truths are necessary (i.e., *necessitarianism* is true).[2]

Recently, Samuel Levey has proposed an *extensibilist* response to van Inwagen's argument.[3] Just as the indefinite extensibility of the concept set would, according to the extensibilist, resolve the set theoretic antimonies, so too the indefinite extensibility of the concept of contingent truth would, according to the extensibilist, block van Inwagen's argument. If correct, extensibilism offers the rationalist a novel way to maintain their commitment to the PSR while recognizing genuine contingency in reality.

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[1]Some authors state a stronger version of the PSR which says that all truths have sufficient reasons. In Appendix B I show that the stronger version not only guarantees modal collapse, but is in fact inconsistent with commonly accepted principles.

[2]Given some further assumptions, one can then conclude that the true, the possible and the necessary coincide.

[3]See [63] and for some criticisms see [73].

While extensibilist responses to the set theoretic paradoxes are familiar, apart from [63], there has not been much discussion concerning the viability of an extensibilist response to van Inwagen's argument. This paper argues that there is an important asymmetry between extensibilist responses to set theoretic antimonies and extensibilist responses to van Inwagen's argument. Extensibilist responses to the set theoretic antimonies can be given a modal characterization that does not involve changing background logical principles; extensibilist responses to van Inwagen's argument, I will argue, cannot. Thus whereas extensibilism in the set theoretic case can be developed by a classically minded philosopher, extensibilism in the present case requires a more constructivist outlook.

The following provides a rough overview of the paper. In the first couple of sections we work towards a rigorous statement of van Inwagen's argument. We will see that an important property of this argument is that it contains a comprehension principle (in this case a sort of higher-order analogue of plural comprehension) and a collapse premise (the principle of sufficient reason). In this way the argument is analogous to plural formulations of Russell's paradox. In response to Russell's paradox, extensibilists will either restrict plural comprehension or restrict the collapse premise. Many philosophers have thought that restricting plural comprehension is the less plausible of these two implementations. I agree with this sentiment and will provide some arguments that the relevant higher order analogue of comprehension should be preserved. I then outline the extensibilist response to van Inwagen's argument that involves restricting the PSR (analogous to the way extensibilists restrict the principle that every plurality of things forms a set). I will argue that given classical principles, this extensibilist restriction fails to avoid the consequence of necessitarianism.

2.2 The Argument

Since my main opponent is Levey I will work with his formulation of van Inwagen's argument:

> Let C be the conjunction of all contingent truths. Then C itself is a contingent truth, for no necessary truth can have a contingent truth as a conjunct. By PSR, there is an explanatory ground G that is a sufficient reason for C. G entails C and explains C. Is G itself a contingent truth? If so, then G is in C. But then in explaining C, G would explain itself, and no contingent truth can explain itself. If G is not a contingent truth but a necessary truth, then because G entails C, it follows that C is a necessary truth, contrary to hypothesis. So, given PSR, there can be no conjunction C of all contingent truths. If there is no conjunction C of all contingent truths, then it must be that there are no contingent truths. Therefore, PSR entails that there are no contingent truths. (2016, 399-400)

The goal in what follows will be to assess the bearing indefinite extensibility has on this argument. In order to do so, it will prove useful to provide a more precise formulation. Only by doing so, I suggest, will we be able to properly assess whether the extensibilist response can be carried out. Below I offer one such formalization. The formalization will have another benefit worth mentioning. Several authors have claimed that ultimately the paradox rests on some controversial claim about the structure of a certain kind of abstract object: propositions, states of affairs or facts perhaps.[4] The language in which I formalize the paradox will not make use of *any* first order quantifiers and so will be strictly speaking neutral as to whether there are any of these objects. In this sense, the formulation that I offer can be seen as a "pure" version of the paradox of sufficient reason.

2.2.1 Formalizing the Argument

One natural setting in which to formalize the paradox is a higher order language that permits, in addition to quantification into sentence position, a kind of quan-

[4] See for instance [86].

tification that stands to quantification into sentence position as plural quantification stands to ordinary first order quantification. With this language in place we will be able to formally simulate both the singular and plural quantification over truths involved in the paradox of sufficient reason.

I'll suppose we have a countable collection of propositional variables, $p_1, p_2, \ldots,$ Boolean operators $\neg$ for negation and $\wedge$ for conjunction, a unary modal operator $\Box$ for metaphysical necessity, and a universal quantifier $\forall$ binding propositional variables. Each propositional variable is a well formed formula, and in addition to the standard clauses for building complex well formed formulas we stipulate that if p_i is a propositional variable and φ a well formed formula, $\forall p_i \varphi$ is a well formed formula. Other Boolean and modal operators are treated as abbreviations.

In order to formalize the PSR, I'll add a further connective $<$ to the language so that, intuitively, $\varphi < \psi$ formalizes that the proposition that φ is a sufficient reason for the proposition ψ. For a given formula φ and propositional variable q not free in φ, $C\varphi$ abbreviates $\varphi \wedge \Diamond\neg\varphi$ (it is contingent that φ) and $E\varphi$ abbreviates $\exists q(q < \varphi)$ (φ has a sufficient reason). The PSR can now be stated as follows:

(PSR) $\quad \forall p\big(Cp \rightarrow Ep\big).$

While the language so defined has the expressive resources to *state* the PSR, it is not yet expressive enough to state the argument against the PSR. The argument against the PSR involves forming the conjunction of *all* contingent truths. For all we know, there are infinitely many contingent truths. Thus we will need some way to talk about arbitrary conjunctions in our language.[5] While there may be several ways to do this, the approach I take will facilitate comparison of the paradox of sufficient

[5]We could introduce the conjunction of contingent truths as the unique truth that necessitates each contingent truth and is necessitated by everything that necessitates every contingent truth. The problem is that this builds in the assumption that there *is* such a unique truth. Without supposing that necessarily equivalent propositions are identical, this assumptions looks unjustified.

reason with more familiar set theoretic paradoxes such as Russell's paradox.[6]

To this end let us extend the language with a countable collection of new *plural* propositional variables $pp_1, pp_2, \ldots$ together with a universal quantifier—also written $\forall$—to bind them. By analogy with plural logic, I will also suppose that we have a binary logical operator with propositional variables going in the first place and plural propositional variables going in the second, $p_i \prec pp_i$, which may be read "the proposition that p_i is one of the propositions that pp_i." Lastly, to make generalizations about arbitrary conjunctions I'll suppose that we have a monadic operator that combines with plural propositional variables, $\bigwedge pp_i$, which may be read as "the conjunction of the propositions that pp_i."

The definition of well formed formula is then extended as follows: $p_i \prec pp_i$ and $\bigwedge pp_i$ are well formed formulas whenever p_i is a propositional variable and pp_i is a plural propositional variable. Additionally when pp_i is a plural propositional variable and φ a well formed formula, $\forall pp_i \varphi$ is a formula. And we say that t is a *term* if t is either a formula or a plural propositional variable. Call the resulting language $\mathcal{L}_{HP}$.

How should this language be interpreted? I have suggested informal pronunciations for $\forall p \ldots$ and $\forall pp \ldots$ as "for any proposition $p \ldots$ and "for any propositions $pp \ldots$'. Arguably, however, we needn't rely on any first order theory of propositions to make sense of this sort of quantification. Rather, the higher order quantifiers can be taken as new fundamental ideology. So interpreted, even a nominalist, one who denies that there are any propositions, can make use of the language in order to understand the paradox of sufficient reason.[7]

[6]An alternative route is to formulate the argument in a simply typed λ-calculus and treat the arbitrary conjunction operator as an operator that combines with sentential operators to form sentences. So for instance given the truth operator λpp we form the conjunction of all truths as $\wedge\lambda pp$. In a functionally typed setting, $\wedge$ is of type $(t \to t) \to t$. For further discussion see section 3.

[7]This is of course a controversial stance to take on higher order quantification and the ideas presented in this paper do not require such an interpretation. For defenses of the idea of taking higher order quantification as fundamental see [80] and [111].

There is an additional interpretive issue. Should $\exists pp \ldots$ be informally understood as "there are one or more propositions..." or "there are two or more propositions ..." or what? Since it is technically simpler, I will make use of a different reading on which it is to be read as 'there are zero or more propositions ...".[8] The universal quantifier $\forall pp$ can then be read as asserting "however some propositions pp may be ...". I do not mean to commit myself to any controversial metaphysical thesis in doing so. The purpose of permitting "empty pluralities" is mere technical convenience.[9]

Turning now from grammar to logic, I will suppose that we have all of classical logic in addition to the analogies of classical axioms and rules of inference governing propositional and plural propositional quantification. Where x is either a propositional variable or a plural propositional variable:

A1 Any substitution instance of a propositional tautology.

A2 $\forall x\varphi \rightarrow \varphi[t/x]$ (where t is a term of the appropriate sort free for x in φ).

A3 $\forall x(\varphi \rightarrow \psi) \rightarrow (\forall x\varphi \rightarrow \forall x\psi)$.

R1 $\varphi, \varphi \rightarrow \psi/\psi$.

R2 $(\varphi \rightarrow \psi)/(\varphi \rightarrow \forall x\psi)$ if x is not free in φ.

Since $\Box$ is meant to formalize metaphysical necessity it would be natural to suppose that it obeyed the logic of **S5**. For our purposes, nothing so strong is required. The argument goes through even in the weakest normal modal logic **K**:

A4 $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$.

R3 $\varphi/\Box\varphi$.

This covers the logical principles for the quantifiers, Boolean connectives and modal operators. The only other logical constant is $\prec$. In the first order case,

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[8] Arguably this has become the standard reading of the plural quantifiers. See for instance [20], [103] and [70] among others.

[9] For discussion see [21], 154-156.

the standard principles governing this connective are *plural comprehension* and *extensionality*. I will suppose for now that we can appeal to the analogue of plural comprehension:

A5 $\exists qq \forall p(p \prec qq \leftrightarrow \varphi(p))$ (provided that p does, and qq does not, have free occurrence in $\varphi(p)$).

Let the *background logic*, **BL**, be the set comprised of all instances of **A1-A5** and closed under **R1-R3**. I'll write $\vdash_{\mathbf{BL}} \varphi$ for $\varphi \in \mathbf{BL}$. We can extend $\vdash$ to a finitary consequence relation by letting $\varphi_1, \ldots, \varphi_n \vdash \psi$ abbreviate $\vdash \varphi_1, \ldots, \varphi_n \to \psi$. I will now move to formalizing the argument. I will attempt to stay as close as possible to the actual principles that Levey appeals to in his formulation of the argument; some of these principles will plausibly follow from more fundamental principles concerning sufficient reasons, entailment, and conjunction.

2.2.2 The Paradox of Sufficient Reason

Let's start at the beginning:

> Let C be the conjunction of all contingent truths. Then C itself is a contingent truth, for no necessary truth can have a contingent truth as a conjunct.

There are two principles implicitly being appealed to here. First if the conjunction of some propositions is necessary, then each of those propositions is necessary:

(Inheritance) $\quad \forall pp\big(\Box \bigwedge pp \to \forall p(p \prec pp \to \Box p)\big).$

The principle **Inheritance** does not entail that the conjunction of contingent truths is contingent since it does not entail that the conjunction of some truths is *true*. Since this principle is completely uncontroversial it is easy to skip over:[10]

[10]Strictly speaking, as we will see, these two principles are also not jointly sufficient to derive that the conjunction of contingent truths is contingent since it may turn out, indeed it does turn out if the argument is correct, that there are no contingent truths. The difference between my formulation

(Truth) $\forall pp\big(\forall p(p \prec pp \to p) \to \bigwedge pp\big)$.

Next:

> By PSR, there is an explanatory ground G that is a sufficient reason for C. G entails C and explains C. Is G itself a contingent truth? If so, then G is in C. But then in explaining C, G would explain itself, and no contingent truth can explain itself.

I have already formulated the PSR. Two other principles are appealed to here. First, nothing (or at least no contingent truth) explains itself:

(Irreflexivity) $\forall p(p \not< p)$.[11]

And second, if p is a sufficient reason for q then p *entails* q:

(Sufficiency) $\forall p \forall q\big(p < q \to \Box(p \to q)\big)$.[12]

(**Sufficiency** is *not* being proposed as a principle that governs explanation more generally. Rather, it is supposed to govern that specific sort of explanation that authors within this literature are concerned with when they say that one proposition is a sufficient reason for another. **Sufficiency** is a standard assumption in the literature concerning sufficient reason.)

These are not the only principles appealed to in the passage; Levey also assumes that if G explains C then "G would explain itself." Why should this be so? The reasoning seems to be twofold: that if G explains and entails C then G is contingent, and that if G explains a conjunction, it explains each conjunct. The latter principle is:

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of the argument and Levey's is that I am allowing empty pluralities whereas he does not appear to be. The differences are not deep however.

[11] Recently several philosophers, such as [54], [113] and [114], have argued against this principle (or against analogous principles).

[12] Some philosophers reject this, for instance, [3]. Moreover, if $<$ is interpreted as grounding, the principle becomes quite a bit more controversial.

(Distribution) $\forall p \forall qq(p < \bigwedge qq \to \forall q(q \prec qq \to p < q))$.[13]

As for the former, suppose that q is contingent and that $p < q$. Then from **S** it follows that $\Box(p \to q)$. And so if $\Box p$ we know that $\Box q$. Thus **S** guarantees that if p explains a contingent truth then p is not necessary. Thus in order to guarantee that p is contingent, we need only add that it is *true*. I will suppose the stronger principle that whenever p explains q both p and q are true. Thus $<$ is *factive*:

(Factivity) $\forall p \forall q(p < q \to (p \wedge q))$.[14]

Let the *theory of sufficient reason*, **TSR**, be the theory (set) comprised of **PSR**, **Inheritance**, **Irreflexivity**, **Sufficiency**, **Distribution**, **Truth** and **Factivity**. Define *necessitarianism* to be the following principle:

(Necessitarianism) $\forall p(p \to \Box p)$.

We can then show the following:

Theorem 2.1. **TSR** $\vdash_{\mathbf{BL}}$ **Necessitarianism**.

The proof of theorem 2.1 is almost immediate given the following lemma:

Lemma 2.2. **TSR** $\vdash_{\mathbf{BL}} \forall pp\big((\forall p(p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)) \to \Box \bigwedge pp\big))$

The proof of lemma 2.2 is given in Appendix A. One might think this Lemma a bit odd. Didn't we explicitly set up the principles in **TSR** so that they would entail

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[13]Not all formulations of the puzzle make use of this principle.
The formulation of the puzzle in [104] does not make use of **Distribution** but rather of the principle that if $\Box(p \leftrightarrow q)$ then neither $p < q$ nor $q < p$. But it has not been standard practice to follow van Inwagen in this respect since many authors would like to allow the possibility of explanations that hold between necessarily materially equivalent propositions. Thus most formulations of the argument contain **Distribution** or something like **Distribution** (see for instance [82]). Nevertheless **Distribution** is certainly one of the more controversial principles in the derivation. If $<$ is read as grounding, many authors will reject it. See [26].

[14]Note that **Factivity** could be derived from $\forall p \forall q(p < q \to p)$, **Sufficiency** and the principle that $\forall p(\Box p \to p)$.

that the conjunction of contingent truths is *contingent*? Not exactly. Here is how to think of the situation: the conjunction of some truths entails each of those truths and is true if each of them are true. So indeed the conjunction of the contingent truths must be contingent *provided there are contingent truths*. But if there are no contingent truths, the conjunction of the contingent truths is the conjunction of the "empty plurality." Informally, think of the conjunction of some things as the greatest lower bound under entailment of those things. This means that the conjunction of the empty plurality must (i) entail every member of that plurality and (ii) be entailed by everything that entails every member of that plurality. But since the plurality has no members *every* truth entails every member of the plurality. And so the conjunction of the empty plurality must be entailed by everything; this is just another way of saying that it must be necessary.

Once this lemma is in place the proof of Theorem 2.1 is straightforward. Since some details of the proof will become philosophically relevant in the next section I will state it here (what follows is more of an informal explanation of how to produce the proof. The main purpose of including it is to highlight the role that **A5** plays in the derivation):

Proof of Theorem. The principle

$$\forall pp\big(\forall p(p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)) \rightarrow \Box \bigwedge pp\big)$$

together with the principle **Inheritance** immediately implies (in **BL**) the principle

$$\forall pp\big(\forall p(p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)) \rightarrow \forall p(p \rightarrow \Box p)\big)$$

And so given Lemma 3.1 **Necessitarianism** then follows in **BL** from the following instance of plural comprehension **A5**:

$$\exists pp \forall p \big((p \prec pp \leftrightarrow (p \land \Diamond \neg p) \big)$$

$\square$

Necessitarianism is extremely implausible.[15] For instance, the following application of universal instantiation delivers an absurd conclusion: If for any P, P only if it is metaphysically necessary that P then if I raise my hand it is metaphysically necessary that I raise my hand. Since that instance of necessitarianism is clearly false, the principle itself should be rejected. Thus the above puzzle strikes me as genuine in the following sense: plausible principles can be precisely formulated so as to entail an absurd conclusion. In order to avoid the absurd conclusion one must either reject some part of the theory of sufficient reason or weaken the background logic. What is the extensibilist response?

2.3 Extensibilism and Plural Comprehension

A concept is *indefinitely extensible* if no matter what some instances of that concept may be, we are in a position to define, by reference to them, a new instance of that concept that is not one of them. Recently [63] has argued that by recognizing the indefinite extensibility of the concept of contingent truth, we are in a position to respond to the above argument. In his words:

> [I]f *contingent truth* is indefinitely extensible, then Completeness [the thesis that if there are any contingent truths, there is such a thing as all

[15] The higher order version of necessitarianism should be sharply distinguished from views sometimes called 'necessitarianism' that are formulated in terms of first order quantification over facts or propositions or states of affairs (see for instance Schaffer 2012). Perhaps all facts conceived of as first order entities of a given sort are necessary. But this would just mean that a view that attempted to provide an analysis of higher order quantification in terms of first-order quantification over facts would face an uphill battle. The negation of **Necessitarianism** seems to me to be a datum.

contingent truths] is incorrect: there can be contingent truths and yet no such thing as all contingent truths and so no conjunction C of all contingent truths. And thus — on the extensibilist interpretation — the van Inwagen-Bennet argument falls through. (2016, 403)

How should we interpret Levey's response?

The following strikes me as a plausible reading. Levey is advocating the following two theses: first, indefinitely extensible concepts provide untrue instances of plural comprehension. And second the condition '$p \wedge \Diamond\neg p$' is indefinitely extensible. These two theses jointly entail that the following instance of plural comprehension is untrue:

(IE) $\exists pp \forall p\big((p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)\big)$

And this suffices to defuse van Inwagen's argument since **IE** plays an ineliminable role in the proof of Theorem 2.1.

There are difficult interpretative issues surrounding the concept of indefinite extensibility. But I am willing to grant for the sake of argument that *contingent truth* is indefinitely extensible in whatever sense of that term that applies to concepts such as *set* and *ordinal* (according to the extensibilist). Why should we think that indefinitely extensible concepts provide untrue instances of plural comprehension?

Levey says surprisingly little about this. And without saying more this solution is not completely satisfying. Consider the following analogy. Suppose that you have just presented your friend with the liar paradox for the first time, and suppose that they respond as follows:

> There is no paradox here. The false premise is simply that the sentence $L =$ 'L is not true' satisfies the T-schema. It is just not the case that 'L is not true' is true if and only if L is not true. And since this premise is false, the reasoning that leads to a contradiction in the liar paradox is blocked.

This would be a rather unsatisfying response, I think, since we are told nothing about what the underlying difference is between sentences that obey disquotational reasoning and sentences that do not. Any viable classical solution to the liar paradox should contain such an explanation.

Similarly, to merely *assert* that '$p \wedge \Diamond\neg p$' provides an untrue instance of plural comprehension is no solution to the paradox of sufficient reason since it does not give us any explanation of the underlying difference between the conditions that provide true instances and those that provide false instances of plural comprehension. As Gabriel Uzquiano says in a slightly different setting:

> [T]o claim that some conditions fail to determine some objects as all and only the objects that satisfy the condition is no better than "to wield the big stick" without offering an explanation. (2015, 149)

What might a principled restriction of plural comprehension look like? One possible route to restrict plural comprehension is to restrict it to all and only those truths that have a common explanation:

A5$^<$ $\forall p \exists qq \forall q \big(q \prec qq \leftrightarrow (p < q \wedge \varphi(q))\big)$

There are several reasons why this is a natural restriction. First it is analogous to a restriction one might place on first-order plural comprehension in light of the set theoretic antimonies. There, the restriction is that for any condition, there are all and only those things that satisfy that condition provided they are all contained within some set. And second, it is designed to block the derivation of **Necessitarianism**. In order to conjoin some truths there must be those truths. But given **A5**$^<$ there are those truths only provided there is some single truth that explains each of them. This truth in turn is naturally taken to be a sufficient reason for the conjunction of those truths.

There are serious costs in rejecting **A5** however. First let us note that **A5**$^<$ is a radically weak principle. For instance, given the factivity of $<$, the only pluralities of propositions guaranteed to exist will be true propositions. But given some truths pp it is natural to suppose that there are all of the negations of propositions in pp, which would then be comprised of falsities. Even if some propositions lack a common explanation, it might have been that they had a common explanation. Cases like this might arise when p is true and q is false but were q true there would have been some r that explained both p and q.

There is a second worry that is perhaps slightly more difficult to pin down. In order to know whether there are some truths that are the truths that satisfy some condition, we need to know whether those truths have a common explanation. In the finite case, this might just be a matter of taking note that r explains p and r explains q. But if we are wondering whether some things satisfying some condition have a common explanation when infinitely many things satisfy that condition, it would appear that we *first* need to know whether there are those things and then consider the explanation of their conjunction (provided the conjunction is contingent). This is at least one plausible way to come to know that there is a common explanation. There is thus a kind of epistemic circularity. In order to know whether there are some propositions meeting a certain condition we must first know that everything meeting that condition has a common explanation. In order to know that everything meeting that condition has a common explanation we must know that there are all and only those propositions meeting the condition (so that we can conjoin them and infer by PSR that they have a common explanation).

Putting this restriction aside, how credible is any restriction of plural comprehension? An initial motivation for plural comprehension is that it is a plausible, strong and simple principle. That is a reason to think that it is true. But the proponent of PSR might respond that the PSR too has such virtues. While that may be true, I don't think the virtues of PSR will be enough to convert a philosopher who starts out with more "extensionalist" sympathies. For instance, suppose that we are not reasoning about metaphysical necessity, or sufficient reasons or any other such metaphysical exotica, but are rather simply reasoning about what is the case, and Boolean combinations of these states of affairs. We will inevitably find ourselves appealing to principles like the following:

$$\exists qq \forall p(p \prec qq \leftrightarrow p)$$

and

$$\exists qq \forall p(p \prec qq \leftrightarrow \neg\neg p)$$

and

$$\exists qq \forall p\big((p \prec qq \leftrightarrow \exists q \exists r(p \leftrightarrow q \wedge r)\big)$$

The worry is that once we give up the instance of plural comprehension for '$p \wedge \Diamond\neg p$' we will have to give up these instances as well. For instance, if there are all and only the truths, surely there are also all and only the contingent truths, since the contingent truths are among the truths. And if there are all and only those propositions whose double negations are true, then presumably there are all those propositions whose double negations are contingently true. And if there are all and only those propositions materially equivalent to conjunctions then there are all and only those propositions materially equivalent to conjunctions of the form '$p \wedge \Diamond\neg p$'. By giving up plural comprehension, not only is our theory of modal metaphysics weakened, but also principles in extensional metaphysics are weakened.

Perhaps there is a plausible theory that allows us to appeal to comprehension for extensionally definable pluralities but not intensionally definable pluralities. Levey's proposed solution does not provide this. This seems to be significant since without a criterion for distinguishing true and untrue instances of plural comprehension, it is hard to evaluate how radical the extensibilist position is.

There is a further consequence of the rejection of plural comprehension that is worth pointing out. I have formulated the argument using quantification into sentence position together with plural quantification into sentence position. On the current interpretation of Levey, he is suggesting that while there are contingent truths, there are no things that are *the* contingent truths. But he is not suggesting that there is no such thing as *being contingently true*. But just as ordinary plural quantification can be traded out for second order quantification, so too plural quantification into sentence position can be traded out for third order quantification. Thus if we can make sense of the infinitary conjunction of the condition of being contingently true independently of the instances of that property, then the argument can be reformulated without quantifying over pluralities of propositions but instead by quantifying over properties of propositions. Once this is done, we see that a rejection of plural comprehension at the level of plural sentential variable inevitably leads to a rejection of comprehension at the level of variables occupying the position of a monadic sentential operator. In other words, there is an inherent tension between there being some property of being contingently true while there being no things that are the contingent truths.

A final potentially worrying consequence of the rejection of plural comprehension is that it looks to be at odds with some of the motivations behind the PSR itself. One historically popular use of the principle was to infer that there was some explanation of every member of some descending chain of contingent explanations $\ldots p_2 < p_1 < p_0$. But without plural comprehension, there isn't a guarantee that there are those propositions that are the propositions in the chain. Here again what the extensibilist needs is either some further argument that indefinitely extensible concepts must provide untrue instances of plural comprehension or else some other formulation of what the indefinite extensibility of contingent truth amounts to. In the next section, I will consider one such argument suggested by [70] and [115] in the context of the iterative conception of set. I will argue that Linnebo's and Yablo's arguments do not generalize to the context of the paradox of sufficient reason. Rather, the considerations that they mention motivate a restriction on the PSR.

2.4 Set Theory and Indefinite Extensibility

The concept of indefinite extensibility is often appealed to as a response to Russell's paradox. One standard way of formulating the paradox is in *plural-first order logic*. In this setting, the paradox can be seen as an inconsistency between (first order) plural comprehension and the principle that any things form a set, where some things are said to form a set if and only if there is a set whose members are precisely those things. The first order principle of plural comprehension can be written in a two sorted first order language with a predicate $\prec$:

(Comprehension) $\exists xx \forall u(u \prec xx \leftrightarrow \varphi(u))$

An instance of **Comprehension** results from replacing $\varphi(u)$ with a formula in which u occurs free and the plural variable xx does not occur free. The principle can be shown to be inconsistent with a naive conception of set formation:

(Collapse) $\forall xx \exists y \forall u(u \prec xx \leftrightarrow u \in y)$

By replacing $\varphi(u)$ with $u \notin u$ in **Comprehension** one obtains:

1. $\exists xx \forall u(u \prec xx \leftrightarrow u \notin u)$

Then by **Collapse** and uncontroversial reasoning one can infer

(2) $\exists y \forall u(u \in y \leftrightarrow u \notin u)$

But (2) is inconsistent in classical first order logic.

Yablo has suggested that we reject **Comprehension** in light of the inconsistency:

> The condition $\phi(u)$ that (I say) fails to define a plurality can be a perfectly determinate one; for any object x, it is a determinate question whether x satisfies $\phi(u)$ or not. How then can it fail to be a determinate matter what are *all* the things that satisfy $\phi(u)$? I see only one answer to this. Determinacy of the ϕ's follows from
>
> (i) Determinacy of $\phi(u)$ in connection with particular candidates,

(ii) Determinacy of the pool of candidates.

If the difficulty is not with (i), it must be with (ii). ([115], 151-152)[16]

Yablo goes on to suggests that in the case of sets, there is indeterminacy in the pool of candidates. The indeterminacy has to do with the iterative conception of sets. The universe of sets is built up in stages. At each stage all of the sets are formed from objects at previous stages. Nevertheless, at no stage is this process complete. At any stage, it is possible to go on and form new sets.

[70] develops this response in more detail. On Linnebo's reading of Yablo, the failure of plural comprehension in the context of set theory is attributed to the implicit modal character of the set theorist's quantifiers. When the set theorist quantifies over, for instance, all sets, they are to be read as quantifying over all of the sets *no matter what sets one goes on to form.* To capture this "implicitly modal character" of the set theorist's quantifiers, he introduces a new primitive modal operator $\blacksquare$ together with its dual $\blacklozenge$. Informally, $\blacksquare\phi$ is interpreted as "no matter what sets we go on to form it will remain the case that ϕ" and $\blacklozenge\phi$ as "it is possible to go on to form sets so as to make it the case that ϕ." The set theorist's quantifiers, $\forall$ and $\exists$, are then read as $\blacksquare\forall$ and $\blacklozenge\exists$. So interpreted **Comprehension** becomes

(Potential Comprehension) $\blacklozenge\exists xx\blacksquare\forall u(u \prec xx \leftrightarrow \varphi(u))$

and **Collapse** becomes:

(Potential Collapse) $\blacksquare\forall xx\blacklozenge\exists y\forall u(u \prec xx \leftrightarrow u \in y)$

But **Potential Comprehension**, Linnebo contends, is false:

> This scheme translates as the claim that, given any formula $\phi(u)$ it is possible for there to be some objects xx such that *no matter what sets we go on to form* xx are all and only the ϕ's. It is not hard to see that this claim is problematic. (2010, 157)

[16]The quotation has been altered to fit the notation of this paper.

But what if we insist on reading the quantifiers in their normal, "non-potentialized" manner? On that reading, Linnebo is clear, **Comprehension** is true and **Collapse** is false: it is only on the suitably "potentialized" reading of **Collapse** that we get a truth. For notice that if we insist on the implicitly potential reading of the quantifiers, all that this shows is the tenability of **Potential Collapse**; it does not show the tenability of collapse on its flat footed reading.

One could read Linnebo's account as a defense of rejecting **Comprehension**. Alternatively, we can read it as a way of rejecting **Collapse** and replacing it with **Potential Collapse**. The principle **Potential Collapse** provides one with a way of preserving the intuition that any things whatsoever form a set: what our intuitions are tracking is merely that wherever we are in the set theoretic hierarchy, the things at that level will go on to form sets at some further level (indeed they will go on to form sets as soon as possible in the hierarchy, as it were). What I do want to do is look at whether we can pursue a similar strategy in the context of the paradox of sufficient reason.

2.5 PSR and Indefinite Extensibility

Just as the set theoretic antimonies can be conceived as a tension between **Comprehension** and **Collapse**, the paradox of sufficient reason can be conceived as a tension between the **PSR** and the higher order version of plural comprehension (the axiom **A5** of the **BL**). The principle of **Collapse** says that for each plurality there is a set. The **PSR** says that for each contingent truth, there is a sufficient reason. In the one case, pluralities are collapsed into sets; in the other, contingent truths are collapsed into explanans. We might thus try to model failures of comprehension in the context of the paradox of sufficient reason on the failures of comprehension in the set theoretic case.[17]

One might object that **PSR** and **Collapse** are not analogous. Whereas **Collapse** truly collapses *some* things to sets, **PSR** just pairs each contingent truth with an explanation. The analogy can be made stronger by either looking at a plural corollary of **PSR** or by looking at a non-plural basis for **Collapse**. Let Cp abbreviate $p \wedge \Diamond\neg p$. The following principle is derivable from **TSR**:

$$\forall pp(\forall p(p \prec pp \to Cp) \to \exists q \forall p(p \prec pp \to q < p)$$

In prose, if some propositions are all contingent, then there is some single sufficient reason for each of them.[18] Thus pluralities of contingent truths are collapsed into their grounds. We might call this the *principle of explanatory ground.* It is simpler to conceive of the tension as being between **PSR** and **A5** than between the principle of explanatory ground and **A5**. If one does set up the tension between **A5** and the principle of explanatory ground, one quickly realizes that what one says about the tension will ultimately rest on what one says about **PSR**.

There is another motivation for modeling a solution to the paradox of sufficient reason off a solution to the set theoretic paradoxes. Both the relation of set formation and the relation of *being a sufficient reason for* are what [16] calls "building relations."

[17] There is one important disanology that merits further discussion. The set theoretic paradoxes can be recast as a tension between second-order comprehension and the principle that any property has an extension. But in the presence of second-order comprehension, not only is the principle that any property has an extension inconsistent, but so too is the appropriate modalized version of it. It matters quite a bit to the contours of the problem whether it is given plural or second-order formulations. As mentioned in section 3, an alternative formulation of van Inwagen's argument eschews plurals and with it plural comprehension in terms of higher-order comprehension (or some form of λ-conversion if lambdas are used). The tension is then conceived as between there being the property of being a contingent truth and the principle of sufficient reason. What's interesting is that it does not seem to matter as much to van Inwagen's argument whether it is formulated in terms of plurals or using higher-order resources.

[18] To construct the proof take some contingent truths pp. If they are non-empty, then $\bigwedge pp$ is contingent by **Truth** and **Sufficiency** and so by **PSR** there is a sufficient reason q for $\bigwedge pp$. Then by **Distribution** q is a sufficient reason for each p. If pp are empty then the consequent of the conditional holds vacuously.

The exact definition of 'building relation' needn't concern us here. What matters is that they fall into a natural class of relations. Both the paradox of sufficient reason and the set theoretic paradoxes might then be viewed as arguments to the effect that building relations, or building relations of a certain sort, must meet certain substantive constraints: given plausible principles governing some sort of entities over which the building relation is defined, it cannot be that everything is built up by something according to that building relation. Perhaps, though, it can be that *potentially* everything is built by something according to that building relation. If similar principles governing similar relations lead to similar problems, we should expect similar solutions.

How then should we model a response to the paradox of sufficient reason off of the response given to the set theoretic paradox? Suppose we start by trying to mirror Yablo's response:

> The condition 'Cp' fails to define a plurality despite being a perfectly determinate condition; for any proposition p, it is a determinate question whether p satisfies 'Cp' or not. How then can it fail to be a determinate matter what *all* the things that satisy 'Cp'? I see only one answer to this. Determinacy of 'Cp' follows from:
>
> (i) Determinacy of 'Cp' in connection with particular candidates,
> (ii) Determinacy of the pool of candidates.
>
> If the difficulty is not with (i), it must be with (ii).

But what are the pool of candidates? In setting up the language, I introduced the higher order quantifiers as irreducible and absolutely unrestricted. In the case of set theory, it is clear that one's quantifiers are restricted (implicitly or explicitly) by the non-logical predicate 'set'. In the case of the paradox of sufficient reason, there does not appear to be any analogous restriction.

The proponent of the PSR might take up a particularly strong stance: there is indeterminacy in reality itself, as it were. But how should this response be understood and developed? Here is one suggestion. Let us for the moment conceive of the higher order quantifiers as ranging over some special abstract objects, *propositions*. Talk of reality is to be understood as talk of everything that is the case, or alternatively, as talk of the collection of all true propositions. The proponent of the PSR sees indeterminacy in this pool of candidates by seeing reality as a hierarchy with no bottom level: given any collection of true propositions, there is some more fundamental level at which those propositions are jointly grounded. In the set theoretic case, the sets at higher levels are generated by collecting together sets at lower levels. In the present case, propositions at lower levels are "generated" by taking sufficient reasons for conjunctions of propositions at higher levels. The concept of contingent truth is indefinitely extensible because the contingent truths are indefinitely *explicable*.

We needn't take this talk of levels of reality as ontologically serious. Just as in the case of the extensibilist response to the set theoretic antimonies we can introduce a new modal operator $\blacksquare$ as before together with its dual $\blacklozenge$. Here $\blacksquare\varphi$ is to be read, intuitively, as "no matter what contingent truths get explained it will remain the case that φ" and $\blacklozenge\varphi$ as "it is possible to go on to explain some contingent truths so that φ." If we then insist on reading our initial quantifiers on their "unpotentialized readings" the **PSR** is rejected in favor of the principle of *potential* sufficient reason:

(PSR$^{\blacksquare}$) $\blacksquare\forall p(Cp \to \blacklozenge\exists q(q < p))$

We of course want to ensure that no matter what stage of explanation we are at, all of the principles in **TSR** apply. Thus in addition to replacing the **PSR** with the **PSR$^{\blacksquare}$** we ought also to replace the other principles **TSR** with the result of prefixing $\blacksquare$ to them. Let then the *theory of potential sufficient reason*, **TPSR**, be the result of prefixing $\blacksquare$ to each principle in **TSR** and then replacing the **PSR** with the **PSR$^{\blacksquare}$**.

Alternatively, if a potentialized reading of the quantifiers is in play, the principle **A5** is dropped and replaced with:

A5$^{\blacksquare}$ $\blacklozenge\exists qq\blacksquare\forall p(p \prec qq \leftrightarrow \varphi(p))$

The principle **A5**$^{\blacksquare}$ might be rejected on the grounds that there is no level of reality, or stage of explanation, at which all contingent truths from all stages are collected together: at every stage, one generates a new contingent truth by taking the sufficient reason for the conjunction of contingent truths at that stage. Notice though that this does not give us a reason to reject the original comprehension principle, **A5**, since that principle is now being read as asserting that at each level, one can collect together the instances of a given condition at that level.

Similarly, **PSR**$^{\blacksquare}$ can be maintained since at each level, every truth can go on to be explained at some more fundamental level. But **PSR** on its flatfooted reading should be rejected: the conjunction of all contingent truths is not explained at a given level but only at some more fundamental level.

It is important here to not get confused by talk of "levels of explanation." This is just a way of informally stating claims fronted by $\blacklozenge$. But it is crucial for the success of this model of indefinite extensibility that $\blacklozenge$ be a prophylactic operator. Thus an assertion of $\blacklozenge\exists x\varphi$ should not in general commit one to what an assertion of $\exists x\varphi$ commits them to. In particular our understanding of $\blacklozenge$ should guarantee that there are failures of the inference from $\blacklozenge\exists x\varphi$ to $\exists x\varphi$. The unpotentialized quantifier $\exists$ is supposed to be absolutely unrestricted. Nevertheless one could go on to explain things so that there would be propositions doing things that they are not actually doing.

As I mentioned above, I prefer a reading of Linnebo's response on which it is **Collapse** that is being rejected while **Comprehension** is maintained. I think that in the present case, the analogous move is a natural one to make. The background logic should be suitable for many metaphysical contexts, not just special purpose generalizations about the hierarchy of explanation. If we thus insist that the unrestricted flatfooted reading is in play, the theory **TSR** should be replaced by **TPSR**.

What are the appropriate logical principles for the newly introduced operator $\blacksquare$? While I do not make any claims of completeness, I suggest that *at least* $\blacksquare$ should have the modal logic **T**. Thus we should assume the following principles:

A6 $\blacksquare(\varphi \to \psi) \to (\blacksquare\varphi \to \blacksquare\psi)$

A7 $\blacksquare\varphi \to \varphi$

R4 $\varphi/\blacksquare\varphi$

In the contexts of set theory, [70] supposes that $\blacksquare$ has the logic of **S4.2**; the above principles will do for my purposes. It will also not matter for present purposes what principles govern the interaction of $\blacksquare$ and $\Box$. Let us then extend **BL** with the axioms **A6** and **A7** together with the rule of inference **R4**. Call the result the *extended background logic*, $\mathbf{BL}^+$.

The official response from indefinite extensibility is then this: in order to reflect the indefinite extensibility of the concept of contingent truth, we ought to expand our language with $\blacksquare$ in order to reason explicitly about the potential stages of explanation. The proper theory of sufficient reason is **TPSR** and the proper background logic at least *includes* $\mathbf{BL}^+$.

Does the proposed response avoid modal collapse? Surprisingly no. Or perhaps it is better to say, *almost* no. We need to make one assumption that has not been explicitly made at this point. And this is that the finitary version of distribution holds:

(Finite Distribution) $\blacksquare\forall p\forall q\forall r(r < p \land q \to (r < p \land r < q))$.

Note that Levey himself appeals to the more general version of this principle, and so he is hardly in a position to reject it. Moreover if we add an identity operator to our language, then there is no need to appeal to **Finite Distribution** since we can mimic its effect with **Distribution** by defining pluralities comprised of the conjuncts of some finite conjunction, e.g., $r \in rr \leftrightarrow r = p \vee r = q$. The principle **Distribution** then guarantees that something explains $\bigwedge rr$ only if it explains both p and q.

We can then show the following:

Theorem 2.1. TPSR $\cup$ {Finite Distribution} $\vdash_{\mathbf{BL}^+}$ Necessitarianism

A proof of theorem 2.1 is given in Appendix A. Informally the idea is just that if p is an unexplained contingent fact then it is an inexplicable, but contingent, fact that p is an unexplained fact. And so given the sort of classical principles we have in the background logic, if all contingent truths are potentially explicable, all contingent truths are explicable. This combined with other easily verifiable facts shows that the theory of potential sufficient reasons entails every member of the theory of sufficient reason. And so since the extended background logic includes all of the background logic, the theory of potential sufficient reason entails **Necessitarianism** if the theory of sufficient reason does.

With a bit of reflection on the proposed response, it is not all that surprising that it fails. The main part of the proof is exactly analogous to the the proof that "weak-verificationism", the principle that all truths are knowable, entails "strong verificationism", the principle that all truths are known.[19] If we let Ep abbreviate $\exists q(q < p)$ then one can show that the following two facts hold:

$$Ep \rightarrow p$$

$$E(p \wedge q) \rightarrow (Ep \wedge Eq)$$

The proof of the first principle makes use of the principle **Factivity**, the proof of the second makes use of **Finite Distribution**. Thus E is what we might call a "Fitch operator." But with respect to such Fitch operators, together with classical principles, the following two principles are equivalent:

[19] For discussion see [110]. For the original presentation of the proof see [37].

1. $\forall p(p \to Op)$
2. $\forall p(p \to \Diamond Op)$,

where O is a Fitch operator and $\Diamond$ is a normal modal operator. That weak verificationism entails strong verificationism is an instance of this fact. That the principle of potential sufficient reason entails the principle of sufficient reason is also an instance of this fact.

Given this fact, potential responses to the argument can be developed by analogy with responses to Fitch's argument. For instance, [107] argues that the derivation of strong verificationism from weak verificationism does not go through if one's background logic is intuitionistic. As [109] states the conclusion, Fitch's argument provides the weak-verificationist with a "reason to revise logic." A similar conclusion is suggested here concerning the extensibilist. Theorem 2.1. shows that when one models the putative indefinite extensibility of contingent truth on standard models of the indefinite extensibility of the concept of set, one is still able to deduce necessitarianism. This combined with the points made in §4 shows that the extensibilist proponent of the PSR has a reason to revise logic: either by rejecting plural comprehension, or else by rejecting classical propositional logic in favor of intuitionisitic logic.

2.6 Conclusion

The paradox of sufficient reason, as Levey calls it, can be formalized precisely in a two sorted higher order language that permits in addition to quantification into sentence position, plural quantification into sentence position. When the paradox is precisely set out in this language, one sees that it can naturally be described as a tension between a higher order version of plural comprehension and the principle of sufficient reason; the tension is very much analogous to the tension between a crucial set collapse principle and first order plural comprehension. It is thus natural to wonder whether an extensibilist solution to the one extends to the other. The main take away of this paper is that extensibilist solutions may require constructivist commitments over and above those required by the extensibilist solutions in the set theoretic case.

CHAPTER 3

TWO THEORIES OF FORM

3.1 Introduction

According to one historically popular conception of logic, logic is the science of *logical form*.[1] Despite its historical prominence, explicit *metaphysical* theories of these objects, logical forms, are rarely offered. One recent exception is Fine (2017), who provides a theory of logical form based on his (1985) theory of arbitrary objects.[2]

Fine takes his theory to be motivated by a puzzle of logical form. There are three principles that, each taken on its own, intuitively governs logical forms, but when taken together, look to be unsatisfiable.

> *Existence*: For each formula there is an object that is the form of the formula.
>
> *Identity*: The forms of two formulas are the same if and only if the formulas are alphabetic variants.
>
> *Structural Similarity*: The form of a negative formula is the "negation" of a form... and the form of a conjunctive formula is the "conjunction" of two forms.
>
> (Fine 2017, p. 515)

Let p and q be propositional variables. From the principle *Existence* it follows that they have forms. And from the principle *Identity* it follows that their forms are

[1] Arguably this view traces back to Kant (see [71].) It is of course more common to characterize logic as the science of *logical consequence*. But it is also not uncommon to say that logical consequence is consequence that holds in virtue of the form of the argument. In turn the form of an argument is specified in terms of the forms of its constituents.

[2] See also [34] for relevant background to [36].

the same, since any two propositional variables are alphabetic variants. The formulas $p \wedge p$ and $p \wedge q$ on the other hand are not alphabetic variants. From the principles of *Existence* and *Identity* it follows that they have distinct forms. The problem is now that from the principle *Structural Similarity* it follows that $p \wedge p$ and $p \wedge q$ are each the conjunction of two forms. And there appears to be only one option in each case. The form of $p \wedge p$ is the form of p conjoined with the form p. The form of $p \wedge q$ is the form of p conjoined with the form of q. But the form of p *is* the form of q. So the form of p conjoined with the form of p *is* the form of p conjoined with the form of q. Therefore the form of $p \wedge p$ is the form of $p \wedge q$. We have reached a contradiction.

Fine responds that that the form of $p \wedge p$ is a conjunction of forms, but it is not the form of p conjoined with the form of p. In order to make sense of this proposal he develops a theory of logical forms according to which they are *arbitrary formula*. An arbitrary formula is, roughly, a formula defined by a "let-clause." There is an arbitrary formula that could be any formula: it is the formula defined by the clause "let x be a formula." This is an example of an *independent* arbitrary formula. It is defined without reference to any other arbitrary formulas. Another example of independent arbitrary formula is the arbitrary conjunctive formula, the formula defined by the let-clause "let x be a formula such for for some formulas φ and ψ, $x = \varphi \wedge \psi$." But there are also dependent arbitrary formulas. Where x is the arbitrary conjunctive formula, a dependent arbitrary formula might be defined by the let clause "let y be a left-conjunct of x" or by the clause "let y be a right-conjunct of x." It is these dependent arbitrary formulas that Fine thinks serves as the conjuncts of conjunctive forms.

This paper develops a position on logical form according to which the principle *Identity* fails. We will see that there is a natural alternative to Fine's view that allows one to maintain *Structural Similarity* on its most plausible reading according to which the form of $p \wedge p$ is the form of p conjoined with itself. It may be that there are two distinct but perfectly intelligible notions of logical form. One obeys the principle *Identity* and the other the principle *Structural Similarity*. The resolution of the puzzle is then achieved by disambiguation. On no reading of 'logical form' are all of the principles true (at least on their most plausible interpretations), but each principle is true on some interpretation of 'logical form'.

I want to briefly say something about the context of this paper. Fine's (2017) focus on logical forms is partly to illustrate the role that a theory of arbitrary objects might play in an account of the structure of types more broadly.[3] While I will have less to say about this question, I too want to treat logical forms as a sort of case study by which we might draw some broader lessons. One of my overaraching goals, only gestured at in this paper, is to argue that the algebraic notion of a freely generated object can shed light on the somewhat obscure notion of an arbitrary object. An *arbitrary group*, for instance, might be thought of as a free group in the category of groups; and arbitrary Boolean algebra could be a free Boolean algebra in the category of Boolean algebras. On this conception of arbitrariness, the space of formulas themselves turn out to be a kind of arbitrary object. This raised the question of how the space of forms of formula should be thought to relate to the formulas themselves. The problem, however, is that it is hard to find any sort of natural characterization of the space of forms of formulas on Fine's view. This led me to consider an alternative conception of form that admits a more natural characterization. The contents of this paper lay out that notion of form.

In §1 we formulate Fine's puzzle as a sort of "no-go theorem" in the context of universal algebra. What this theorem shows is that logical forms, as Fine conceives them, cannot have certain features that we might have intuitively supposed them to have. The algebraic perspective on the puzzle leads naturally to an alternative

[3]This sort of approach to arbitrary object theory has some similarity with Leon Horsten's recent theory.

conception of logical form that deviates from the principle of *Identity* just enough to secure *Structural Similarity*, and so provides a conception of form that has some of the features that fine's conception of forms lack. In §2, the alternative conception of form is developed and shown to have several nice features. But it also has some features that are not so nice. In §3 an argument is provided that forms on this alternative conception lack certain features that intuitively forms ought to have. Forms, on Fine's conception, however, do not face this argument. I suggest in light of this that we abandon the idea there is any single notion of *the* form of a formula and instead hold that there are many forms of formulas.

3.2 Fine's Puzzle

There is a simple theorem underlying Fine's puzzle. We would like sameness of form to be a congruence relation on the space of formulas.[4] The problem is that the relation that two formulas stand in when they are alphabetic variants is not a congruence relation. Our intuitive conception of form is pushing us in two opposite directions. To bring out this perspective on the puzzle, we start with some background.

A *logical algebra* is a triple $(A, \wedge, \neg)$ such that A is a set, $\wedge$ is a binary operation on A and $\neg$ is a unary operation on A. There is no constraint whatsoever on what the elements of the set A are nor on what the operations $\wedge$ and $\neg$ are, other than that they have arities two and one respectively. The fact that we write the operations as $\wedge$ and $\neg$ of course indicates that we are going to be thinking of logical algebras as representing objects over which some notion of conjunction and negation are defined. But notice that there are many logical algebras for which this characterization may be misleading. For instance the algebra $(\mathbb{Z}, +, -)$, where $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

[4] Intuitively a congruence relation on formulas is an equivalence relation that respects the Boolean operations.

are the integers, $+$ maps two integers to their sum and $-$ maps each integer to its additive inverse, is a logical algebra.

Among the logical algebras there are certain distinguished ones—*term algebras*—that we can use to represent formulas of a given propositional language. We define these as follows.

Definition 3.1 (Term Algebra). *A* term algebra *is a logical algebra* $(A, \wedge, \neg)$ *admitting of a map* $i : X \to A$ *such that*

- i, $\wedge$ *and* $\neg$ *are all injections.*
- *The images of* $i, \wedge$ *and* $\neg$ *are all disjoint.*
- A *is the result of closing* $i(X)$ *under* $\wedge$ *and* $\neg$.

The set X *is said to* generate *the algebra* $(A, \wedge, \neg)$.[5]

Let $(A, \wedge, \neg)$ be a term algebra generated by the set X. We can view the set X as a set of atomic formula, or propositional variables, and the map $i : X \to A$ as the inclusion of X into the set of formula generated by X. The assumption that $i, \wedge$ and $\neg$ be injections with disjoint images is an algebraic characterization of the familiar property of unique readability. In what follows

One of the questions motivating Fine's puzzle is: how similar are forms of objects, in structure, to the objects of which they are forms. Restricted to the special case of logical forms, we want to investigate the degree to which logical forms resemble their instances. As has become standard, we can study the structural relationships between objects by looking at the behavior of various structure preserving mappings in and out of those objects. Let $\mathcal{A} = (A, \wedge, \neg)$ and $\mathcal{B} = (B, \wedge', \neg')$ be two logical algebras. A *homomorphism* from $(A, \wedge, \neg)$ to $(B, \wedge', \neg')$ is a map $f : A \to B$ such

[5]See Bergman (2015, p. 391) for a more general definition and discussion. It is well known that such term algebras *exist*. The proof proceeds basically by constructing explicitly the language with X as a set of propositional variables. See Bergman (2015, p. 392) Theorem 9.3.3.

that $f(x \wedge y) = f(x) \wedge' f(y)$ and $f(\neg x) = \neg' f(x)$, for any $x, y \in A$. An *isomorphism* from $\mathcal{A} \to \mathcal{B}$ is a homomorphism $f : \mathcal{A} \to \mathcal{B}$ that is *invertible*: there exists a homomorphism $g : \mathcal{B} \to \mathcal{A}$ such that $g \circ f = 1_{\mathcal{A}}$ and $f \circ g = 1_{\mathcal{B}}$ (where $1_{\mathcal{A}}$ and $1_{\mathcal{B}}$ are the identity maps on $\mathcal{A}$ and $\mathcal{B}$ respectively). An *endomorphism* is a homomorphism from a term algebra to itself. An *automorphism* is an endomorphism that is in addition an isomorphism.

In what follows we will call a *formula* an element of the countably infinite term algebra $\mathcal{F} = (F, \neg, \wedge)$.[6] We will write V for the set that freely generates $\mathcal{F}$ and call elements of V *propositional variables* or simply *variables*. A *substitution* on formulas is an endomorphism $f : \mathcal{F} \to \mathcal{F}$. The following universal property of $\mathcal{F}$ will be useful to appeal to in what follows.

Proposition 3.1. *Let $\mathcal{B} = (B, \neg^B, \wedge^B)$ be a logical algebra. Let $f : V \to B$ be a function. Then there is a unique homomorphism $\bar{f} : \mathcal{F} \to \mathcal{B}$ with the property that*

$$\bar{f} \circ i_V = f$$

where $i_V : V \to F$ is the inclusion of V into F.[7]

In particular, given any function $f : V \to F$ there is exactly one substitution $\bar{f} : \mathcal{F} \to \mathcal{F}$ whose restriction to V agrees with f. Hence substitutions on $\mathcal{F}$ correspond one-to-one with functions $f : V \to F$. Thus in order to know how a given substitution acts on formulas, it suffices to know where it sends the propositional variables.footnoteFor further backgroun see [22] and [17].

It is worth pointing out here a relationship between the metaphysical notion of an arbitrary object of a given sort, and the algebraic notion of a term algebra.

[6]The assumption of uniqueness is justified by the fact that any two term algebras of the same cardinality are isomorphic.

[7]This fact is well known, but for a specific proof see Bergman(2015, p. 391); this proposition is an immediate corollary of Lemma 9.3.2

What proposition 3.1 shows is that, in effect, term algebras are *arbitrary logical algebras* since they can be homomorphically mapped into any logical algebra. Since an arbitrary logical algebra doesn't really have any interesting features, this fact is perhaps not super interesting. But when it comes to more interesting categories of algebras (groups say), there are groups that stand to all other groups as term algebras stand to logical algebras (the free groups).This suggests that in certain special cases, the notion of an arbitrary F can be given a precise mathematical description. I will say a bit more about this point later on.

Following Fine, we will restrict our attention to the forms of formulas of propositional logic (i.e., elements of $\mathcal{F}$). Ultimately a theory of form should of course be extended to account for difference in form between, for instance, formulas with quantifiers and those without. Since Fine's puzzle already arises at the level of propositional logic, including quantifiers would be a needless distraction.

3.2.1 Alphabetic Variants

Two formulas, Fine tells us, are "*alphabetic variants* if they differ merely in the identity of their sentence letters." (Fine 2017, p. 515) Say that a formula φ is *interpretable* in a formula ψ, written $\varphi \leq \psi$, if there is a substitution f on $\mathcal{F}$ with the property that $f(\varphi) = \psi$. Formulas φ and ψ are *bi-interpretable* if $\varphi \leq \psi$ and $\psi \leq \varphi$. When φ and ψ are bi-interpretable, we write $\varphi \sim \psi$.

Proposition 3.2. *$\sim$ is an equivalence relation on F.*

Proof. The relation $\sim$ is automatically symmetric. It is reflexive since the identity map is a substitution. And it is transitive since the composite of two substitutions is a substitution. □

It is natural to define alphabetic variants as bi-interpretable formulas. We recognize the formulas $p \wedge q$ and $r \wedge s$ as alphabetic variants because each can be obtained from the other by substitution. The formula $p \wedge p$ and $p \wedge (r \wedge s)$ are not alphabetic variants since, while $p \wedge (r \wedge s)$ can be obtained from $p \wedge p$ by substitution, there is no substitution mapping $p \wedge (r \wedge s)$ to $p \wedge p$.

There is another natural definition of alphabetic variants that may be slightly more intuitive. Say that formula φ is *isomorphic* to a formula ψ if there exists an automorphism of $\mathcal{F}$ that maps φ to ψ. This relation is evidently an equivalence relation. And it is also a plausible account of alphabetic variants. To see this let $Aut(\mathcal{F})$ be the automorphism group of $\mathcal{F}$ and let $Sym(V)$ be the symmetric group of V.[8]

Proposition 3.3. *$Aut(\mathcal{F})$ is isomorphic (as a group) to $Sym(V)$.*

Proof. First we show that the restriction map

$$f \mapsto f|_V : Aut(\mathcal{F}) \to Sym(V)$$

is a group homomorphism (i.e., the map that sends each $f : \mathcal{F} \to \mathcal{F}$ to the map $f|_V : V \to V$ obtained by restricting f to V). To show that this is well defined, we must show that the range of $f|_V$ is identical to V whenever f is an automorphism. Let $f \in Aut(\mathcal{F})$ and let $p \in V$. Then there is some $\varphi \in F$ such that $f(\varphi) = p$ (since f is an automorphism). If $\varphi = \neg\psi$ or $\varphi = \psi \wedge \chi$ for any formulas ψ or χ, then $p = \neg f(\psi)$ or $p = f(\psi) \wedge f(\chi)$. This contradicts the fact that the images of $\neg$ and $\wedge$ are disjoint from V. It follows that $\varphi \in V$. So for each $p \in V$, there is some $q \in V$ such that $f(q) = p$. Since f is automatically injective on V, it follows that $f|_V \in Sym(V)$. It only remains to show that it respects composition. And this is

[8] That is, $Aut(\mathcal{F})$ is the set of automorphisms of $\mathcal{F}$ equipped with operation $\circ$ of function composition and $Sym(V)$ is the set of bijections from V to V also equipped with function composition.

easy since for any $f, g \in Aut(\mathcal{F})$ and $p \in V$ we have

$$(g \circ f)|_V(p) = (g \circ f)(p) = g(f(p)) = g_V(f|_V(p)) = g|_V \circ f|_V(p)$$

Hence $f \mapsto f|_V$ is a homomorphism of groups. To show that it is an isomorphism, we finds its inverse.

For each $g \in Sym(V)$ let $\bar{g} : \mathcal{F} \to \mathcal{F}$ be the unique homomorphism such that $\bar{g} \circ i = g$. We show that the map

$$g \mapsto \bar{g} : Sym(V) \to Aut(\mathcal{F})$$

is a group homomorphism. Let $p \in V$ and $f, g \in Sym(V)$. Then

$$\begin{aligned}(\bar{g} \circ \bar{f}) \circ i(p) &= \bar{g} \circ \bar{f}(p) \\ &= (\bar{g} \circ i) \circ (\bar{f} \circ i)(p) \\ &= g \circ f(p)\end{aligned}$$

So $\bar{f} \circ \bar{g} \circ i = g \circ f$. Since $\overline{g \circ f}$ is the unique substitution with that property, $\overline{g \circ f} = \bar{g} \circ \bar{f}$. For each $g \in Sym(V)$, $\bar{g}$ has an inverse given by $\overline{g^{-1}}$ since

$$\bar{g} \circ \overline{g^{-1}} = \overline{g \circ g^{-1}} = \overline{1_V} = 1_{\mathcal{F}} = \overline{1_V} = \overline{g^{-1} \circ g} = \overline{g^{-1}} \circ \bar{g}$$

So $g \mapsto \bar{g}$ is a homomorphism of groups.

To finish the proof we note that $f \mapsto f|_V$ and $g \mapsto \bar{g}$ are mutually inverse. For $f \in Sym(V)$ and $p \in V$, we have

$$(\bar{f})|_V(p) = \bar{f} \circ i(p) = f(p)$$

Hence $(\bar{f})|_V = f$. And for any $g \in Aut(g)$ and $p \in V$ we have

$$g \circ i(p) = g|_V(p)$$

Hence $g \circ i = g|_V$ and so $\overline{(g|_V)} = g$. □

From this proposition we see that automorphisms of $\mathcal{F}$ are essentially permutations of propositional variables. So if φ and ψ are isomorphic, one can get to ψ from φ by permuting the variables in φ. And this seems to be exactly what alphabetic variation amounts to.

This provides us with two ways of thinking of alphabetic variation: isomorphism and bi-interpretability. But there is only one relation corresponding to these two ways of thinking, since it can be shown that that two notions coincide.

Theorem 3.4. *$\varphi \sim \psi$ if and only if φ and ψ are isomorphic.*

To prove theorem 3.4 it is helpful to first prove a lemma concerning the relationship between substitutions and subformulas. Given that φ and ψ are bi-interpretable, we want to somehow construct an automorphism of formulas that takes φ to ψ. Say that a substitution f *fixes* a formula φ if $f(\varphi) = \varphi$. The key to proving the theorem is that in $\mathcal{F}$, a substitution fixes a formula if and only if it fixes all of its subformulas.

Lemma 3.5. *Let f be a substitution and φ a formula. Then f fixes φ if and only if it fixes every subformula of φ.*

Proof. We argue for the nontrivial direction by induction on φ. The base case $\varphi \in V$ is trivial. Let $\varphi = \neg\psi$ and let f be a substitution. Suppose that $f(\varphi) = \varphi$. Hence $\neg f(\psi) = f(\neg\psi) = \neg\psi$. Since $\neg$ is an injection $f(\psi) = \psi$. Since the subformulas of φ are the subformulas of ψ plus φ itself, it then follows by induction that $f(\chi) = \chi$ for each subforula χ of φ. The conjunctive case follows by analogous reasoning. □

With this lemma in place the theorem is proved as follows.

Proof of theorem. The right to left direction follows immediately from the fact that every automorphism has an inverse. To show the other direction let f and g be substitutions such that $f(\varphi) = \psi$ and $g(\psi) = \varphi$. Then $g \circ f(\varphi) = \varphi$ and so by lemma 3.5, $g \circ f(p) = p$ for each propositional variable p in φ. And so since p is not in the range of $\neg$ or $\wedge$, $f(p)$ must be a propositional variable. Moreover, whenever p and q are distinct propositional variables occurring in φ, $f(p)$ and $f(q)$ are distinct since $g \circ f(p) = p$ and $g \circ f(q) = q$. Define $h : V \to V$ to map p to $f(p)$ if p occurs in φ and to itself otherwise. Then h is a bijection and so from proposition 3.3 there is an automorphism $\bar{h}$ that agrees with h on propositional variables. Where $V(\varphi)$ are the propositional variables occurring in φ, it follows by a trivial induction that $\bar{h}$ and f agree on any formula all of whose propositional variables occur in $V(\varphi)$. In particular it follows that $\bar{h}(\varphi) = f(\varphi) = \psi$. □

Two formulas are isomorphic if and only if they are bi-interpretable. This is evidence that the precise notion of bi-interpretability captures the informal notion of alphabetic variation since we've seen that the two intuitive ways of making alphabetic variation precise coincide.

3.2.2 Structural Similarity

Bi-interpretability provides a precise account alphabetic variants and with it the principle of *Identity*. *Structural Similarity* also has an algebraic formulation. First, we require that the class of forms admit notions of "negation" and "conjunction". In other words, there should be a *logical algebra*, $\mathcal{U} = \langle U, \neg, \wedge \rangle$, where U is the set of forms of formula in F. And second, this logical algebra ought to preserve the structure of the formulas of which they are forms. In other words, there should be a homomorphism $f : \mathcal{F} \to \mathcal{U}$ with the property that $f(\varphi)$ is the form of φ.

It is sometimes assumed that sameness of structure requires the existence of an isomorphism rather than simply a homomorphism. But it would be misguided in the present setting to demand that there be an isomorphism between $\mathcal{F}$ and $\mathcal{H}$ since it would be misguided to demand that distinct formulas get assigned distinct forms. Logical forms are forms shared by many formula. Thus the weaker requirement of a homomorphism seems more appropriate here.

It should be noted though that the requirement of a homomorphism between formulas and their forms goes beyond what *Structural Similarity* actually says. According to structural similarity, the form of, for instance, $\varphi \wedge \psi$ should itself be a conjunction. The requirement of homomorphism says that it must be the conjunction of the forms of φ and ψ. It is this extra condition, we'll see, that leads to inconsistency with other plausible conditions on forms and is dropped in Fine's theory. However it seems to me to be a plausible requirement insofar as *Structural Similarity* is a plausible requirement. We'll return to this point when discussing Fine's view.

So let's suppose that the the requirement of *Structural Similarity* demands a homomorphism from $\mathcal{F}$ to $\mathcal{H}$ and the requirement of identity demands that two formulas have the same form if and only if they are bi-interpretability. Putting these together results in the requirment that there be a homomorphism between $\mathcal{F}$ and $\mathcal{H}$ that sends bi-interpretable formulas to the same element of $\mathcal{H}$.

In general, given a homomorphism $f : \mathcal{A} \to \mathcal{B}$, we call the *kernel* of f, $\ker f$, that binary relation on A that relates a and a' if f maps them to the same element (i.e., $\ker f = \{(a, a') \in A \times A \mid f(a) = f(a')\}$). Stated with this terminology, the joint effect of *Identity* and *Structural Similarity* is that $\sim$ should be the kernel of the homomorphism mapping formulas to their forms.

3.2.3 Fine's Puzzle

Putting all of this together we get the following precise formulation of the Fine's requirements:

EXISTENCE: There is a function $f : F \to H$ that associates each formula $\varphi \in F$ with its form $f(\varphi) \in H$;

STRUCTURAL SIMILARITY: This function $f : F \to H$ is a homomorphism of logical algebras $f : \mathcal{F} \to \mathcal{H}$;

IDENTITY: The kernel of this function is the relation $\sim$.

The puzzle is then that while these constraints each seem plausible on their own they cannot be jointly satisfied:

Theorem 3.6. *There does not exist a logical algebra $\mathcal{B}$ and homomorphism $f : \mathcal{F} \to \mathcal{B}$ with the property that $\sim = \ker f$.*[9]

Proof. Suppose otherwise. Let φ and ψ be distinct formulas such that $\varphi \sim \psi$. Then $f(\varphi) = f(\psi)$ and so $f(\varphi) \wedge f(\varphi) = f(\varphi) \wedge f(\psi)$. By assumption f is a homomorphism. Therefore $f(\varphi \wedge \varphi) = f(\varphi \wedge \psi)$. Since $\sim = \ker f$, $\varphi \wedge \varphi \sim \varphi \wedge \psi$. By theorem 3.4 there is an automorphism $g \in Aut(\mathcal{F})$ with the property that $g(\varphi \wedge \varphi) = \varphi \wedge \psi$. So $g(\varphi) \wedge g(\varphi) = \varphi \wedge \psi$. Since $\wedge$ is an injection, $g(\varphi) = \varphi$ and $g(\varphi) = \psi$. Therefore $\varphi = \psi$. Contradiction. $\square$

Given theorem 3.6, there is simply no way to construct a consistent theory of logical form satisfying the requirements of EXISTENCE, IDENTITY and STRUCTURAL SIMILARITY. If sameness of form coincides with bi-interpretability, then the space of forms is rather unlike the space of formulas of which they are forms.

But this is puzzling. We often speak as if forms had linguistic structure. We say that $p \wedge q$ has a conjunctive form whereas $\neg p$ does not. What is it to have a conjunctive form if not to be a form that is a conjunction? In the next section we'll explore views that reject *Identity* and preserve *Structural Similarity*, finding one such alternative particularly compelling as an account of form.

[9]The proof of this is almost immediate given the well known fact that $\sim = \ker f$ only if $\sim$ is a congruence. I will introduce the concept of congruence in the next section.

3.3 Denying Identity

Theorem 3.6 shows that if the forms of formula have a similar structure to the formulas themselves, then it is false that all and only bi-interpretable formulas have the same form. The challenge for the IDENTITY denying theorist of logical form is to construct a theory of sameness of form that replaces bi-interpretability with a relation that allows for STRUCTURAL SIMILARITY to be satisfied.

There are three kinds of relations that might act as replacements for bi- interpretability. A *liberal* theory holds that all bi-interpretable formulas have the same form, but in addition some formulas have the same form without being bi-interpretable. A *conservative* theory holds that formulas have the same form only if they are bi-interpretable, but there are some bi-interpretable formulas that differ in form. Lastly an *orthogonal* theory holds that bi-interpretability is neither necessary nor sufficient for sameness of form.

Conservative theories can be shown to be non-starters. The only relation contained in $\sim$ that can consistently be combined with Structural Similarity is the identity relation (this is made precise and demonstrated below). However there is an interesting liberal theory that is far from trivial. The main goal of this section is to develop this theory.

In order to develop this view more precisely we start with the following definition. Let $\mathcal{A} = (A, \wedge, \neg)$ be a logical algebra. A *congruence* on $\mathcal{A}$ is an equivalence relation on A such that for any $a_1, a_2, b_1, b_2 \in A$, if $a_1 \equiv a_2$ and $b_1 \equiv b_2$ then $\neg a_1 \equiv \neg a_2$ and $a_1 \wedge b_1 \equiv a_2 \wedge b_2$. The following is well known.

Proposition 3.1. *Let $\mathcal{A} = (A, \wedge, \neg)$ be a logical algebra and $\equiv$ an equivalence relation on A. Then $\equiv$ is a congruence if and only if it is the kernel of a homomorphism $f : \mathcal{A} \to \mathcal{B}$ for some logical algebra $\mathcal{B}$.*[10]

[10]See for instance Burris and Sankappanavar (1981).

A consequence of proposition 3.1 is that if Structural Similarity holds, the relation of sameness of form is a congruence relation. The goal is then to find a plausible conception of form according to which it is a congruence.

3.3.1 Conservative theories of form

A theory is conservative if sameness of form is included in $\sim$. In order for the conservative to maintain Structural Similarity they must find some congruence relation $\approx$ contained in $\sim$. Unfortunately there is no plausible candidate.

Theorem 3.2. *The only congruence contained in $\sim$ is the identity relation*

Proof. Let $\approx$ be a congruence relation contained in $\sim$ and suppose that $\varphi \approx \psi$. Since $\approx$ is an equivalence relation, we have $\varphi \approx \varphi$. And so since $\approx$ is a congruence relation

$$(\varphi \wedge \varphi) \approx (\varphi \wedge \psi)$$

By assumption $\approx \subset \sim$. Therefore

$$(\varphi \wedge \varphi) \sim (\varphi \wedge \psi)$$

By theorem 2.5 there is an automorphism f such that

$$f(\varphi) \wedge f(\varphi) = f(\varphi \wedge \varphi) = \varphi \wedge \psi$$

Since $\wedge$ is injective, $\varphi = f(\varphi) = \psi$. So $\varphi = \psi$. $\square$

Clearly any conception of logical form according to which φ is identical to its own form is inadequate. If we want a more plausible *identity* denying theory, we have to look elsewhere.

3.3.2 Liberal theories of form

The conservative theory collapses form and identity and so can be safely ignored. However there is a plausible liberal theory that can consistently be combined with *Structural Similarity* on its most plausible reading while maintaining quite a bit of intuitive appeal. The basic idea behind this theory is that two formulas should be regarded as having the same form if, ignoring repeated occurrences of propositional variables, the formulas are isomorphic. So for instance, the formulas $p \wedge p$ and $p \wedge q$ will have the same form, according to this theory, because if we ignore the fact that $p \wedge p$ has repeated occurrences of the propositonal variable p, say by replacing one occurrence of p with an occurrence of a distinct propositional variable r to get the formula $p \wedge r$, the formulas are isomorphic.

We'll say that a formula is *differentiated* if no propositional variable occurs more than once within it.[11] So, for instance, whenever p and q are distinct propositional variables, $p \wedge q$ is differentiated and $p \wedge p$ is not. The idea of "ignoring" repeated occurrences of propositional variables can be explained in terms of associating with each formula φ a differentiated formula φ^* for which there exists a proper sort of substitution taking φ^* to φ. Intuitively this should be a substitution that preserves all of the logical structure of φ^* except perhaps the number of occurrences of propositional variables. Let $f : \mathcal{F} \to \mathcal{F}$ be a substitution. Then f is *simple* if the image of V is contained in V (i.e., $f(V) \subset V$)). Simple substitutions map propositional variables to propositional variables. Every automorphism is automatically simple but the converse is false. The substitution that maps every propositional variable to one

[11] For each propositional formula $p \in V$ we define a map $\#_p : \mathcal{F} \to \mathbb{N}$ that keeps track of p's occurrences in formulas by induction. In particular $\#_p$ is the smallest function such that

(i) $\#_p(p) = q$ and $\#_p(q) = 0$ (for $q \neq p$).

(ii) $\#_p(\neg\varphi) = \#_p(\varphi)$.

(iii) $\#_p(\varphi \wedge \psi) = \#_p\varphi + \#_p\psi$.

Thus φ is differentiated if if $\#_p\varphi = 1$ or $\#_p\varphi = 0$ for each variable $p \in V$

propositional variable is simple, but not invertible.

With all of this in place we come to the main definition. The intuitive description of the relation we are after suggests that two formula φ and ψ will stand in this relation if there are isomorphic differentiated formula φ^* and ψ^* together with simple substitutions taking φ^* and ψ^* to φ and ψ respectively. However note that if f is an automorphism taking φ^* to ψ^* and g is a simple substitution taking ψ^* to ψ, then $f \circ g$ is a simple substitution taking φ^* to ψ. Thus we can require the superficially stronger condition that φ^* and ψ^* actually be identical, not only isomorphic. This leads to the following definition.

Definition 3.2 (Relative Bi-Interpretability)**.** *Formulas φ and ψ are* relatively bi-interpretable*, $\varphi \equiv \psi$, if there is a differentiated formula χ and a pair of simple substitutions f and g such that $f(\chi) = \varphi$ and $g(\chi) = \psi$.*

We can picture the relative bi-interpretability of two formulas φ_1 and φ_2 as being witnessed by a wedge:

$$\begin{tikzcd}
\varphi_1 & & \varphi_2 \\
& \varphi \arrow[ul, "f_1"'] \arrow[ur, "f_2"] &
\end{tikzcd}$$

Here φ is differentiated and f_1 and f_2 are simple substitutions. It is fairly natural to think of such wedges as "generalized isomorphisms" between formula. Thought of this way, two formulas φ_1 and φ_2 are relatively bi-interpretable if and only if there is a generalized isomorphism connecting φ_1 and φ_2. It is not hard to see that there is a generalized isomorphism connecting each formula to itself and that if there is a generalized isomorphism connecting φ_1 to φ_2 there is a generalized isomorphism connecting φ_2 to φ_1 (so $\equiv$ is both reflexive and symmetric).

It is a bit more difficult to verify the transitivity of $\equiv$. What needs to be shown is that we can compose these generalized isomorphisms in a natural way so that if we have a diagram like the following,

$$
\begin{tikzcd}
\varphi_1 & & \varphi_2 & & \varphi_3 \\
& \varphi \arrow[ul, "f_1"'] \arrow[ur, "f_2"'] & & \psi \arrow[ul, "g_1"] \arrow[ur, "g_2"'] &
\end{tikzcd}
$$

we can find a generalized isomorphism taking φ_1 to φ_3. It suffices to show that whenever we have a situation like the above, we get an automorphism η taking φ to ψ. We then obtain a generalized isomorphism between φ_1 and φ_2 by composing the simple substitution g_2 with the isomorphism η:

$$
\begin{tikzcd}
\varphi_1 & & \varphi_3 \\
& \varphi \arrow[ul, "f_1"'] \arrow[ur, "g_2 \circ \eta"'] &
\end{tikzcd}
$$

Explicitly, we'll prove the following lemma.

Lemma 3.3. *For any differentiated formulas φ and ψ and simple substitutions f and g, if $f(\varphi) = g(\psi)$ then there is an automorphism h such that $h(\varphi) = \psi$.*

Proof. If ψ is a propositional variable, $f(\varphi) = g(\psi)$ implies φ is a propositional variable and so $h(\varphi) = \psi$ is immediate. Fixing φ, we prove the lemma by induction on ψ.

The base case is already done. Let $\psi = \neg\psi'$ and suppose $f(\varphi) = g(\psi)$. Since f is simple, and the images of $i : V \to F$ and $\neg : F \to F$ are disjoint, φ is not a propositional variable. Since the images of $\wedge$ and $\neg$ a disjoint, $f(\varphi_1) \wedge f(\varphi_2) = f(\varphi_1 \wedge \varphi_2)$ is distinct from $\neg g(\psi') = g(\psi)$, for any formulas $\varphi_1 \wedge \varphi_2$. And since F is generated by V under $\neg$ and $\wedge$ we conclude that $\varphi = \neg\varphi'$, for some formula φ'. Therefore, $\neg f(\varphi') = f(\varphi) = g(\psi) = \neg g(\psi')$. Since $\neg$ is an injection $f(\varphi') = g(\psi')$. From the induction hypothesis $h(\varphi') = \psi'$ for an automorphism h. Therefore $h(\varphi) = \neg h(\varphi') = \neg\psi' = \psi$.

Now let $\psi = (\psi_1 \wedge \psi_2)$ and $f(\varphi) = g(\psi)$. By analogous reasoning, $\varphi = (\varphi_1 \wedge \varphi_2)$ and so $f(\varphi_1) = g(\psi_1)$ and $f(\varphi_2) = g(\psi_2)$. Applying the induction hypothesis, there are automorphisms h_1 and h_2 mapping φ_1 to ψ_1 and φ_2 to ψ_2 respectively. By assumption φ is differentiated and so φ_1 and φ_2 do not share propositional variables. We can thus define an automorphism h mapping φ to ψ by

$$h(p) = \begin{cases} h_1(p) \text{ if } p \in V(\varphi_1) \\ h_2(p) \text{ if } p \in V(\varphi_2) \\ p \text{ otherwise} \end{cases}$$

□

From this lemma we observe the following proposition.

Lemma 3.4. *The relation $\equiv$ of relative bi-interpretability is an equivalence relation.*

Being an equivalence relation, however, doesn't suffice to show that it provides us with a workable notion of sameness of form. In order to show that, we must show that $\equiv$ is a congruence relation.

Proposition 3.5. *$\equiv$ is a congruence relation.*

Proof. Given lemma 3.5, it suffices to show that $\equiv$ preserves $\wedge$ and $\neg$. Suppose that $\varphi_1 \equiv \varphi_2$. Thus there is a generalized isomorphism between φ_1 and φ_2:

$$\begin{tikzcd} \varphi_1 & & \varphi_2 \\ & \varphi \arrow[ul, "f_1"] \arrow[ur, "f_2"'] & \end{tikzcd}$$

Since $\neg\varphi$ is differentiated whenever φ is, and $f(\varphi) = \varphi_i$ if and only if $f(\neg\varphi) = \neg\varphi_i$, it follows that there is a generalized isomorphism between $\neg\varphi_1$ and $\neg\varphi_2$:

$$\begin{tikzcd} \neg\varphi_1 & & \neg\varphi_2 \\ & \neg\varphi \arrow[ul, "f_1"] \arrow[ur, "f_2"'] & \end{tikzcd}$$

For the conjunctive case, let $\varphi_1 \equiv \varphi_2$ and $\psi_1 \equiv \psi_2$. Choose any generalized isomorphism betweeen φ_1 and φ_2:

$$\begin{tikzcd}[column sep=small] \varphi_1 & & \varphi_2 \\ & \varphi \arrow[ul, "f_1"] \arrow[ur, "f_2"'] & \end{tikzcd}$$

Without loss of generality we can choose a differentiated formula ψ that does not overlap in its propositional variables with φ and g_1 and g_2 simple substitutions that together constitute a generalized isomorphism between ψ_1 and ψ_2.

$$\begin{tikzcd}[column sep=small] \psi_1 & & \psi_2 \\ & \psi \arrow[ul, "g_1"] \arrow[ur, "g_2"'] & \end{tikzcd}$$

Since φ and ψ are differentiated and do not overlap in their propositional variables, $\varphi \wedge \psi$ is differentiated. Define h_1 to be that substitution such that $h_1(p) = f_1(p)$ if p occurs in φ and $h_1(p) = g_1(p)$ otherwise. And define $h_2(p) = f_2(p)$ if p occurs in φ and $h_2(p) = g_2(p)$ otherwise. Then the following generalized isomorphism witnesses $(\varphi_1 \wedge \psi_1) \equiv (\varphi_2 \wedge \psi_2)$:

$$\begin{tikzcd}[column sep=small] \varphi_1 \wedge \psi_1 & & \varphi_2 \wedge \psi_2 \\ & \varphi \wedge \psi \arrow[ul, "h_1"] \arrow[ur, "h_2"'] & \end{tikzcd}$$

□

By replacing bi-interpretability with relative bi-interpretability in the principle of *Identity* we obtain the following principle.

> *Weak Identity:* The form of φ and ψ is the same if and only if $\varphi \equiv \psi$.

More generally, relative bi-interpretability gives rise to the following theory.

> Existence: There is a function $f : F \to H$ that associates each formula $\varphi \in F$ with its form $f(\varphi) \in H$;
>
> Structural Similarity: This function $f : F \to H$ is a homomorphism of logical algebras $f : \mathcal{F} \to \mathcal{H}$;
>
> Weak Identity: The kernel of this function is the relation $\equiv$.

What Proposition 3.6 shows is that these principles are consistent.

3.3.3 Logical forms

One promising feature of this account of logical form is that it allows us to simply and literally speak of a form H being the conjunction of forms I and J. We can in fact strengthen the theory with the principle that the space of forms $\mathcal{H}$ be a term algebra and so satisfy unique readability.

To see this it is helpful to first explicitly construct an algebra that verifies that Structural Similarity. We do this by constructing a quotient algebra.

Definition 3.3. *Let* $[\varphi] = \{\psi \mid \varphi \equiv \psi\}$. *Let* $F/_{\equiv} = \{[\varphi] \mid \varphi \equiv \psi\}$. *We define the quotient algebra as follows. The quotient algebra* $\mathcal{F}/_{\equiv}$ *has domain* $F/_{\equiv}$ *and operations defined by*

$$\neg'[\varphi] = [\neg\varphi].$$

$$[\varphi] \wedge' [\psi] = [\varphi \wedge \psi]$$

The map $\varphi \mapsto [\varphi]$ is a homomorphism from $\mathcal{F}$ to $\mathcal{F}/_{\equiv}$ whose kernel is $\equiv$. The consistency of Weak Identity with Structural Similarity and Existence can then be demonstrated by identifying the form of φ with its $\equiv$-equivalence class $[\varphi]$. By identifying forms with elements of $\mathcal{F}/_{\equiv}$ we obtain a strong theory of forms according to which forms themselves have linguistic structure.

For each set X let $F(X)$ be the term algebra generated by X (so $\mathcal{F} = F(V)$).

Proposition 3.6. *Let* 1 *be a singleton. Then* $\mathcal{F}/_{\equiv}$ *is isomorphic to* $F(1)$.

Proof. It suffices to show that $\mathcal{F}/_{\equiv}$ is a term algebra generated by $[p]$ for $p \in V$. Let us first show that the maps $\neg'$ and $\wedge'$ are injections. Let φ and ψ be formulas. Suppose that $\neg'[\varphi] = \neg'[\psi]$. So $[\neg\varphi] = [\neg\psi]$. Therefore $\neg\varphi \equiv \neg\psi$. Let χ be differentiated and f and g be simple substitutions with $f(\chi) = \neg\varphi)$ and $g(\chi) = \neg\psi$. Since simple substitutions map variables to variables, negations to negations and conjunctions to conjunctions, it follows that $\chi = \neg\chi'$, for some formula χ'. Since χ is differentiated,

$\neg\chi'$ is differentiated. Then $\neg f(\chi') = f(\neg\chi') = \neg\varphi$ and $\neg g(\chi') = g(\neg\chi') = \neg\psi$. Since $\neg$ is an injection, $f(\chi') = \varphi$ and $g(\chi') = \psi$. Hence $\varphi \equiv \psi$ and so $[\varphi] = [\psi]$. This shows that $\neg'$ is an injection. The argument that $\wedge$ is an injection is the same and so is omitted.

The operations $\neg'$ and $\wedge'$ have disjoint images that do not contain $[p]$ since simple substitutions always map variables to variables, negations to negations, and conjunctions to conjunctions. To complete the proof we need only show that $\mathcal{F}_\equiv$ is generated by $[p]$. Let $\Omega([p])$ be the result of closing $\{[p]\}$ under the operations of $\neg'$ and $\wedge'$. Let $\varphi(q_1, \ldots, q_n)$ be an arbitrary formula whose propositional variables are exactly $\{q_1, \ldots, q_n\}$. Let φ' be a differentiated formula with a simple substitution $f : \varphi' \to \varphi(q_1, \ldots q_n)$. Finally let $\varphi(p, \ldots p)$ be the result of replacing q_i with p. Then there is a simple substitution from φ' to $\varphi(p, \ldots, p)$. Thus $\varphi(q_1, \ldots, q_n) \equiv \varphi(p, \ldots, p)$. Hence $\varphi(p, \ldots, p) \in [\varphi(q_1, \ldots, q_n)]$ and so $[\varphi(q_1, \ldots, q_n)] = [\varphi(p, \ldots, p)] = \varphi([p], \ldots, [p])$. Since $\varphi([p], \ldots, [p]) \in \Omega([p])$ we are done. $\square$

The picture is thus the following. There is a single simple form P that is the form of all propositional variables. Anything that can be freely obtained by conjoining and negating a form is a form. Hence we have forms like $P \wedge P$ and $\neg(P \wedge P)$ and $P \wedge (P \wedge \neg P)$ and so on.

Notice that it is not required that we *identify* forms with equivalence classes of formulas to get this result. Rather we just require that the space of forms be isomorphic to $\mathcal{F}/_\equiv$. This is enough to secure the fact that forms possess linguistic structure of some kind.

This shows that weak forms provide candidate forms that allow us to consistently maintain Structural Similarity. But might there be some other notion of form that is more closely aligned with the intuitive notion of bi-interpretability that also validates structural similarity? We can actually show that the answer to this question is negative since $\equiv$ can be shown to be the *smallest* congruence extending $\sim$. To show this it is helpful to first prove the following lemma.

Lemma 3.7. *For any congruence $\sim'$ extending $\sim$, $\varphi \sim' f(\varphi)$ for any formula φ and simple substitution f.*

Proof. We prove this by induction on φ. If φ is is a propositional variable, then $\varphi \sim f(\varphi)$ for any simple substitution f. Let $\varphi = \neg\psi$ and let f be any simple substitution. By induction $\psi \sim' f(\psi)$. So $\neg\psi \sim' \neg f(\psi)$ since $\sim'$ is a congruence. Similarly, if $\varphi = (\varphi_1 \wedge \varphi_2)$ and f is a simple substitution, then $\varphi_1 \sim' f(\varphi_1)$ and $\varphi_2 \sim' f(\varphi_2)$ by induction. Therefore $(\varphi_1 \wedge \varphi_2) \sim' f(\varphi_1) \wedge f(\varphi_2) = f(\varphi_1 \wedge \varphi_2)$ since $\sim'$ is a congruence. $\square$

Using this lemma we now prove our claim:

Theorem 3.8. *The relation $\equiv$ is the smallest congruence relation containing $\sim$.*

Proof. Let χ be a differentiated formula and f and g simple substitutions mapping χ to φ and χ to ψ. Let $\sim'$ be any congruence extending $\sim$. By lemma 3.7, $\chi \sim' \varphi$ and $\chi \sim' \psi$. Since $\sim'$ is an equivalence relation, $\varphi \sim' \psi$. Therefore $\equiv$ is contained in $\sim'$, for any arbitrary congruence that contains $\sim$. $\square$

Summing up, the relation $\equiv$ of relative bi-interpretability is the smallest congruence extending $\sim$. Thus a theory stating that $\equiv$ captures sameness of form can be seen as the theory that diverges from the standard account in the minimal way required in order to satisfy *Structural Similarity*.

3.3.4 Meta-forms

If forms have linguistic structure, shouldn't they themselves have forms? One might worry we are on our way to something like a third man argument, since it would now appear that p and the form of p, $[p]$, themselves share a form. In this section I'll argue that there is no regress since the form of any form of a formula is just the form of that formula.

For the moment we will suppose forms are formulas belonging to $F(p) = F(\{p\})$ for some $p \in V$. Note that the relation $\equiv|_{F(p)}$ is simply the singleton $\{(p,p)\}$ since the only differentiated formula in $F(p)$ is p. But this fact seems to me not very deep, for there is another relation that coincides with $\equiv$ on $\mathcal{F}$ but whose restriction to $F(p)$ is less trivial.

Say that a formula φ is *uniform* if there is exactly one propositional variable p that is a subformula of φ. Say that φ and ψ are *uniformly bi-interpretable* if for some uniform formula χ, and simple substitutions f and g, $f(\varphi) = \chi$ and $g(\psi) = \chi$.

Proposition 3.9. *Two formulas φ and ψ are relatively bi-interpretable if and only if they are uniformly bi-interpretable.*

Proof sketch. If φ and ψ are relatively bi-interpretable, then there is a differentiated formula χ with simple substitutions f and and g taking χ to φ and χ to ψ respectively. But then there will be simple substitutions taking φ and ψ to the result of substituting out all propositional variables in χ for a single propositional variable p. Similarly if φ and ψ are uniformly bi-interpretable, there is a uniform formula χ and simple substitutions mapping φ to χ and ψ to χ. Then if we change each variable in χ so that it is differentiated, there will be simple substitutions taking this transformed formula to φ and ψ respectively. $\square$

Thus in order to study the space of meta-forms, we can look at how the relation of uniform bi-interpretability behaves over $F(p)$. The main result is the following:

Theorem 3.10. *Let $\varphi, \psi \in F(p)$. Then the following are equivalent.*

1. *φ and ψ are isomorphic with respect to $F(p)$*
2. *φ and ψ are uniformly bi-interpretable with respect to $F(p)$.*
3. *φ and ψ are identical.*

Proof. Obviously (3) entails (1). To show that (1) entails (2), suppose that φ and ψ are isomorphic. Then there is a simple substitution mapping φ to ψ. So since every formula in $F(p)$ is uniform, it immediately follows that φ and ψ are uniformly bi-interpretable with respect to $F(p)$. To complete the proof we need to show that (2) implies (3). We will show this by induction. For the base case, note that anything uniformly bi-interpretable with p is p, since p is the only formula in $F(p)$ that is not the result of applying $\neg$ or $\wedge$ to some other formulas. Suppose that anything uniformly bi-interpretable with φ is φ and suppose that ψ is uniformly bi-interpretable with $\neg\varphi$. Then since φ and ψ are both uniform, there is a simple substitution f such that $f(\neg\varphi) = \neg f(\varphi) = \psi$. Since φ is uniformly bi-interpretable with $f(\varphi)$, $\varphi = f(\varphi)$. Hence $\neg\varphi = \neg f(\varphi) = \psi$. The case of conjunction is proved similarly. $\square$

The result is that the space of meta-forms, on the current conception, is isomorphic to the space of forms. The natural development of the theory is then to hold that while the theory of forms of formula is non-trivial, since no form is identical to the formula of which it is a form, the theory of meta-forms is trivial, since every meta-form is identical to the form of which it is a form.

It's unclear to me whether this should be seen as a criticism of this conception of form or not. The question of whether the form of a form is that form itself strikes me as something we have little independent grip on. Indeed as indicated above, it might be a benefit. On this conception of forms, we are not committed to an ever increasing hierarchy of meta-forms.

In the final section we will return to the question of the nature of forms. There, an interpretation of Fine's theory of forms is provided which takes *arbitrary formulas* to be elements of a clone corresponding to $\mathcal{F}$ (equivalently, forms can be be thought of as arrows of a certain Lawvere theory corresponding to this clone). We'll see that the forms singled out by $\equiv$ are naturally identified as a kind of "basis" of the forms singled out by $\sim$. But before we do that, I want to first look at a reason why one might think the forms singled out by $\equiv$ are insufficient as an account of *the* logical form of a given formula.

3.4 An Objection

There is an important objection to the idea that relative bi-interpretability serves as a notion of sameness of form. We can formulate the objection as argument:

P1 If φ and ψ have the same logical form, then φ is logically valid if and only if ψ is logically valid.

P2 For any formula φ, there is some differentiated formula ψ such that $\varphi \equiv \psi$.

P3 No differentiated formula is logically valid.

P4 Some formula is logically valid.

C There are formulas φ and ψ such that $\varphi \equiv \psi$, but φ and ψ do not have the same form.

Premise 1 seems to be a natural requirement on sameness of form: sameness of form preserves validity. Premise 2 is easily verified from the definitions. Premise 3 strikes me as plausible, though the specific argument will depend a bit on how the term 'logical validity' is understood. If by 'logically valid formula' one means a theorem of classical logic, the premise becomes a corollary of the following proposition.

Proposition 3.1. *No differentiated formula is a theorem of classical logic.*

Proof. A formula φ is a theorem of classical logic if and only if φ is true in every model v (where a model is a homomorphism $v : \mathcal{F} \to 2 = (\{0, 1\}, 1 - \cdot, \min\{\cdot, \cdot\}))$. To prove the proposition we show by induction that for any differentiated formula φ, there is a pair of models (t, f) such that $t(\varphi) = 1$ and $f(\varphi) = 0$. The induction is mostly trivial. The conjunctive case can be shown as follows. If $(\varphi \wedge \psi)$ is a differentiated formula then φ and ψ are differentiated. By induction there are pairs (t_1, f_1) and (t_2, f_2) mapping φ and ψ, respectively, to truth and falsity, respectively.

Note that since $f_1(\varphi) = 1$, $f_1(\varphi \wedge \psi) = \min\{f_1(\varphi), f_1(\psi)\} = 0$. Thus it suffices to show that $(\varphi \wedge \psi)$ is true under some interpretation. Since $(\varphi \wedge \psi)$ is differentiated, φ and ψ do not overlap in propositional variables. We can thus define a homomorphism t so that it agrees with both t_1 and t_2 on propositional variables in φ and ψ. Then

$$t(\varphi \wedge \psi) = \min(t(\varphi), t(\psi)) = \min(t_1(\varphi), t_2(\psi)) = 1$$

$\square$

Similar sorts of arguments can be made for logics that are not classical. For instance, in intuitionistic logic the argument can be made by replacing homomorphisms into 2 with homomorphisms into the variety of Heyting algebras. How far exactly the argument extends is somewhat outside of the scope of this paper.

Given that premises 2-4 are on solid footing, the only option is to deny premise 1. But how can this be plausible at all? Here is one natural thought. When we look at the space of formulas $\mathcal{F}$ from above, we are viewing formulas independently of context. And it is not only form but context that account for validity. Automorphisms preserve both context and form. But the relation of relative bi-interpretability only preserves only form. Thus, premise 1 fails because it fails to take into account the context that the form is placed in.

I think there may be a way to develop that account into a plausible account. I want to suggest another lesson, however. It seems to me that what this argument and Fine's puzzle bring out is that it is not correct to suppose that there is one item that is *the* form of a formula. Instead, a formula has several forms. Some of these forms track validity; others tracks logical complexity. In particular, there is a kind of form corresponding to bi-interpretability and a kind of form corresponding to relative bi-interpretability. Some of our intuitions concerning logical form go with relative bi-interpretability, and some with bi-interpretability. The solution to Fine's puzzle, then, is to disambiguate: the principle *identity* is true of one of our notions of form, while the principle *Structural Similarity* is true of our other notion of form.

3.5 Conclusion

This paper represents only a preliminary investigation into the metaphysics of form. The goal was to identify two different notions of form in terms of their structure and use them to dissolve a puzzle of form. Further investigation is required if we are to figure out what the *nature* of these forms are, and how the theory might be generalized to provide an account of the forms of other objects, both abstract and concrete.

CHAPTER 4

UNITY AND APPLICATION

4.1 Introduction

Propositions are (or can be) *about* individuals and *predicate* properties of them. The proposition that two is prime is about two, for instance, and predicates being prime of it. Many philosophers think we need a *reductive* theory of propositions in order to account for the representational features of propositions.[1] This paper challenges this claim. I'll develop a primivitist account of the representational features of propositions and argue that it is no less elegant, simple, or unifying than any of the reductive accounts currently on offer.

The theory I develop takes as its starting place some notions from algebraic theories of propositions.[2] The main posit of the theory is a primitive operation, an operation I will call *application* that maps properties and individuals to propositions.[3] I will argue that the sense in which propositions are formed from individuals, properties and relations, should be explained in terms of this primitive operation; moreover, a theory with this primitive can be developed in which the representational properties of propositions are explained in an analogous way to the representational features

[1] See [50], [59], [60], [61] and [97]. For some recent exceptions see [75] and [79].

[2] See Bealer (1979, 1982, 1998), McMichael and Zalta (1980), Zalta (1983) and Menzel ( ) As we will see the theory I develop differs in several important ways from these theories. In particular I will not suppose any strong decomposition principles for propositions, in a sense to be explained. I will also be advocating for a theory that makes use of types, whereas those in the algebraic tradition are partly motivated by a desire to avoid typed theories.

[3] The algebraic theory, as developed by Bealer, makes use of a similar operation that he called *predication.*

of a whole host of other abstract objects that philosophical theories quantify over. In outline, representational phenomena will be ultimately explained in terms of the inputs and outputs of the application operation, and in terms of our relationship to these inputs and outputs.

Like philosophers working in the algebraic tradition of theorizing about propositions, it seems to me that the right level generality at which a theory of propositions should be developed is within a theory of propositions, properties and relations more broadly. There are a couple of reasons for this. One is that many of the key structural features of propositions are also structural features of properties and relations: just as we can conjoin, negate and believe propositions, we can conjoin, negate and ascribe properties and relations. It would be surprising if our accounts of these phenomena for propositions made no contact with our account for properties. A more important reason: many of the key structural features of propositions concern how propositions are related to properties and relations. As I will argue, propositions are, by their very nature, *applications* of relations to relata.[4] Thus propositions are what you get by combining a relation of a given type with some relata of the appropriate types, where the mode of combination in question is a primitive kind of combination, distinct from fusion or set formation.

Section 1 introduces the main primitive of the theory (application). Section 2 outlines postulates on application. Section 3 further situates the theory in the literature by showing how many recent reductive theories can be construed as providing reductive accounts of application. In section 4, I argue that the disagreement between the sort of primivitism outlined in section 1 and the reductive theories introduced in section 2, is a disagreement about what the appropriate primitives of a theory of propositions should be. I'll then provide some considerations in favor of my chosen primitives.

[4] See §3 for an elaboration on this idea.

4.2 The Minimal Theory of Application

This section and the next develop a primivitist account of propositions with explanatory ambition. At the core of the theory is the notion of *application*. Application is an operation that takes a property of an individual and an individual and delivers a proposition. Below I'll provide some ways in which we can get a grip on this notion without providing anything like a definition of it. Instead, I'll provide a collection of postulates on application. These postulates play the dual role of connecting application to more familiar notions and providing axioms in terms of which the representational features of propositions can be explained.

4.2.1 Application

A stack of plates stands to the individual plates in the stack in the same way that a pile of bricks stands to the individual bricks in the pile. Arguably, the stack of plates is not merely the plates that are stacked. The stack of plates is one thing whereas the plates are many things. The analogous point holds for the pile of bricks. What is this relation that the stack of plates bears to the plates and the pile of bricks bears to the bricks? The standard answer is that the stack of plates is the *fusion* of the plates and the pile of bricks is the *fusion* of the bricks. On this view, there is operation—fusion—such that applying it to the bricks gives you the pile, and applying it to the plates gives you the stack.[5]

A similar situation arises in the theory of propositions. The proposition that two is prime stands to two and being prime as the proposition that three is odd stands to three and being odd. That two is prime is not merely the plurality consisting of two and being prime: the proposition is one thing whereas two and being prime are two things. The analogous point holds for the proposition that three is odd. What is

[5] And so the *relation* at issue is the unique relation r such that for any x and xx, for x to bear r to xx is for x to be identical to the fusion of xx.

this relation that the proposition that two is prime bears to two and being prime and the proposition that three is odd bears to three and being odd? While there is no standard answer, it is natural to develop an answer by analogy with the above case. In particular, it is natural to postulate some operation such that applying it to two and the property of being prime is the proposition that two is prime and applying it to three and the property of being odd is the proposition that three is odd. On this way of thinking, the proposition that two is prime is identical to $App(f, x)$ where App is an operation—which I'll call *application*—f is the property of being prime and x is the number two.[6]

Generalizing from the above example, the operation of application should be understood so that every instance of the following schema comes out true.

> For any individual x and property f, if f is the property of being F, then $App(f, x)$ is the proposition that that x is F.

An instance of this schema is a sentence that results from replacing the capital 'F' by a predicate (making appropriate adjustments for grammaticality). The schema is not a definition or analysis of App. But it provides us with nontrivial information about its "extension". For example, we can infer from it that if f is the property of being blue, then the proposition that x is blue is identical to $App(f, x)$ for any individual x (assuming standard disquotational reasoning).

We can further tighten our grip on application by analogizing it to *function application*. A function $f : A \to B$ can be applied to an element a of A to get an element $f(a)$ of B. For instance the successor function $s : \mathbb{N} \to \mathbb{N}$ can be applied to any number $n \in \mathbb{N}$ to get its successor $s(n) = n + 1 \in \mathbb{N}$. Hence the successor function is a function of type *natural number to natural number* that when applied

[6] As will become more clear in the next section, the theory is inspired by models of typed lambda calculus that make use of a typed function, often called application, that allow us to combine entities from certain types to get entities of other types.

to a natural number delivers its successor.[7] Similarly, given a property of a certain type, say a property of individuals, and an individual, we can apply that property to the individual to get a proposition. So the property of being blue can be thought of as being a property of type *individual to proposition* that is such that, when applied to an individual x delivers the proposition that x is blue.[8]

The analogy is only partial. Property application is highly constrained in a way that function application is not. There is a function $f : \mathbb{N} \rightarrow \mathbb{N}$ that maps every number to 113. More generally, for any functional relation $R \subset \mathbb{N} \times \mathbb{N}$ there is a function whose graph is that relation. The analogous behavior plausibly fails for properties. Plausibly, there is no property whose application to any individual is the proposition that snow is white. Suppose we had in our language some predicate F such that ⌜the property of being F⌝ denoted this property. Then ⌜$\forall x App(f, x) =$ that x is F⌝ would be true given the schema by which application was introduced. But then ⌜$\forall x$ that snow is white = that x is F⌝ would be true since $App(f, x)$ was stipulated to be the proposition that snow is white. But the sentence ⌜that I am F⌝ does not express the same proposition as 'that snow is white' for any F since the former is about me whereas the latter is not. It is not the case that for any functional relation r between individuals and propositions, there is a property f such that x bears r to $App(f, x)$.[9]

---

[7]It is important to mindful of typing considerations here. There is no operation whose domain includes *all* functions since there is no set of all functions. Let A and B be sets and $A \rightarrow B$ the type of functions whose domain is A and whose co-domain is B. Then $App_{A,B}$ is a function of type $((A \rightarrow B) \times A) \rightarrow B$ that takes each $f \in A \rightarrow B$ and $a \in A$ and maps it to $f(a) \in B$. Talk of application should be understood as talk of a family of operations indexed to some type hierarchy. Similar typing considerations apply in the case of properties. In the next section I will more explicitly introduce typing considerations.

[8]The root of the idea that properties can be applied to individuals to get propositions comes from the work of Frege (1891), (1892).

[9]This is of course a controversial point but seems to me well supported by the above example. Some authors take properties to be functions from possible worlds to extensions. Propositions are functions from worlds to truth values and monadic properties functions from worlds to sets of individuals. Suppose that there is a constant domain D of individuals. Then to any property

I think this gives us some grip on the notion of application. Some will demand an account of *what it is* for a proposition to be an application of a property to an individual. Application is not plausibly fusion. Let $f + x$ denote the fusion of f and x. The proposal that $App(f, x) = f + x$ entails that the proposition that I am walking is the fusion of me and the property of walking. But this proposal has some counterintuitive consequences. For example, since parthood is transtive, it entails that all of my parts are parts of the proposition that I am walking. The issues become worse when we consider the applications of relations to relata (for more on this see the next section). Consider an n-ary relation R and its application $App(R, x_1, \ldots, x_n)$ to n relata. The obvious generalization of the fusion theory is to define $App(R, x_1, \ldots, x_n)$ as $R + x_1 + \cdots + x_n$. But this theory conflates the proposition that x_1 loves x_2 (the application of loving to x_1 and x_2 in that order) and the proposition that x_2 loves x_1.

There is a response to these objections that solves both at once. Instead of defining $App(R, x_1, \ldots, x_n)$ to be the fusion $R+x_1+\cdots+x_n$ of a relation and relata, we could define it to be the fusion $R + \langle x_1, \ldots, x_n\rangle$ of a relation and a sequence of relata. When R is unary the result is that the proposition that I am walking is the fusion of the property of walking and the one-termed sequence whose sole term is me. Since this sequence is plausibly a simple (or if one likes set theoretic reductions, its only parts are other sets), this avoids the transitivity argument and the "forgetting order" problem. But I do not see any particular reason to believe it unless one is already committed to using the fusion operation in one's theory of the combinatorial

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$f : W \to \mathcal{P}(D)$ there is a corresponding propositional function $\bar{f} : D \to (W \to 2)$ defined so that $f(d)(w) = 1$ iff $d \in f(w)$. Conversely given a propositional function $g : D \to (W \to 2)$ there is a corresponding property $\bar{g} : W \to \mathcal{P}(D)$ defined so that $\bar{g}(w) = \{d \in D \mid g(d)(w) = 1\}$ (using a horizontal bar for the correspondence in both directions is, I think, a harmless ambiguity). It is not hard to verify that $\bar{\bar{f}} = f$ and $\bar{\bar{g}} = g$. So the identication of properties with intensions $f : W \to \mathcal{P}(W)$ is equivalent to the identification of properties with propositional functions $g : D \to (W \to 2)$. So if the applicative behavior of properties is constrained in the way that I have argued, there are intensions that do not correspond to any properties.

features of propositions. This will become even more apparent when developing the theory of application; in many cases the fusion theory would only add unecessary complication to the theory.[10]

As I see things, there is no more basic operation in terms of which application can be defined. It is not function application: since it is a manner of combining things, its behavior is highly constrained in a way that function application is not. And it is not fusion: there doesn't seem to be any notion of parthood standing to application as our ordinary notion of part stands to fusion. I suggest we take application as a primitive manner of combining elements and see where it gets us.

4.2.2 Application and Algebraic Theories of Propositions

Philosophers in the algebraic tradition of theorizing of propositions have often made use of operations similar to application. As Bealer (1998, p. 10) says, it is a truism that "The proposition that Fx is the predication of the property F of x." This "predication" operation is essentially my "application" operation, though we'll see that it plays rather different roles in our respective theories. In this section, I want to emphasize several ways in which the approach I develop differs from that in the algebraic tradition. Before doing this, it is important to point out one thing: the main point of this paper will be to explain various representational notions in terms of application. This is not something that algebraic theorists often do in their theories. So there is a certain sense in which the main parts of our respective theories are not in competition with one another. For instance one could attempt to develop a theory very much like the one I develop *within*, e.g., Bealer's approach if one preferred. But I think the framework I sketch here provides a better foundation for the theory that

[10] For instance, as we will see, the theory I prefer is ultimately a typed theory. This doesn't immediately preclude a theory that makes of fusion, but it does multiply the theoretical possibilities in developing such a theory. For further problems with mereological accounts see [57] and [75] ch. 4.

I go on to develop.

Algebraic theories of propositions often locate propositions as the 0-ary case of n-ary relations more broadly. The basic idea behind these theories, the reason why they are called *algebraic*, is that propositions, properties and relations fit into a certain kind of *algebraic structure*.[11] The algebraic structure consists of the disjoint union of a family of *domain*, $D, R_0, R_1, \ldots$ where D is the family of individuals, R_0 the family of propositions, R_1 the family of properties andR_n the family of n-ary relations for $n \geq 2$.[12] In addition, this disjoint union is equipped with a collection of (partial) operations. There is an operation $\neg$ (negation) that maps each proposition $p \in R_0$ to its *negation* $\neg p \in R_0$. Other Boolean operations are taken as primitive operations on propositions. There is also included an application operation, *App*, that takes an element $f \in R_1$ and elements x in any domain, and produces an element $App(f, x)$ in R_0.[13]

On this approach, application is treated on a par with notions of conjunction, negation and other logical operations. Moreover application is treated as a *global* notion: we can apply a given property f not just to individuals, but also to properties and relations more generally. My preferred approach diverges from this approach in two ways. First, on the approach I prefer, application is treated as an operation, whereas the Boolean operations are treated as (higher-order) properties and relations. Thus the sole primitive *operation* of the theory is application. Second, the theory I prefer is typed, whereas authors working in the algebraic tradition tend to prefer untyped theories (like that sketched above). Here is a rough sketch of how this would

[11] These structures are not the familiar sorts of algebras one might study in a course in universal algebra. But they are closely related to several notions one might come across in more advanced study, such as partial algebras, clones and algebraic theories (in the categorical sense).

[12] It is natural then to identify R_0 and R_1 with 0-ary and 1-ary relations respectively. I'm not sure if much hangs on this though.

[13] As mentioned above this operation is called *pred* in Bealers theory.

go. The collection of types includes a basic type e and for any finite sequence of types $\sigma_1, \ldots, \sigma_n$, a derived type $\langle \sigma_1, \ldots, \sigma_n \rangle$. We then assign entities types as follows. Individual are type e. Anything that combines with things of type $\sigma_1, \ldots, \sigma_n$, in that order, is of type $\langle \sigma_1, \ldots, \sigma_n \rangle$. Thus for instance propositions are of type $\langle \rangle$, properties of individuals are of type $\langle e \rangle$, properties of propositions are of type $\langle \langle \rangle \rangle$.

What do we mean by entities that *combine* with other entities in a given order? In my view this is where the notion of application comes in, and must be treated as a primitive *operation* (as opposed to a relation of some type). In particular we suppose that for any type $\langle \sigma_1, \ldots, \sigma_n \rangle$, there is an operation $App_{\langle \sigma_1, \ldots, \sigma_n \rangle}$ such that $App_{\langle \sigma_1, \ldots, \sigma_n \rangle}(R, x_1, \ldots, x_n)$ is a proposition. Thus the application operation that we introduced in the first section of this paper is the application operation of type $\langle e \rangle$: it "combines" properties and individuals to get propositions.

In this framework, conjunction and negation can be treated as themselves certain kinds of properties and relations between propositions. Let $\wedge$ be the conjunction relation. Let p be the proposition that snow is white and q the proposition that grass is green. Then $App_{\langle \rangle}(\wedge, p, q)$ is the proposition that grass is green and snow is white. Quantifiers can similarly be treated as higher-order properties. Where f is the property of being prime, and $\exists$ is the higher-order property of "being instantiated", a property of type $\langle \langle e \rangle \rangle$, we can think of $App_{\langle \langle e \rangle \rangle}(\exists, f)$ as the proposition that something is prime.

The framework is quite obviously inspired by models of typed lambda calculus. The move made in this paper is to treat application as "representationally significant", in the sense of taking it to correspond to a real live operation out in the world; a theory of this operation, I will argue, can help us make progress with certain problems concerning the representational status of propositions (as well as other abstract objects). The view described provides something like an answer to the question: What is a proposition? The answer is: a propositions is an application of a relation to some relata. More precisely, a proposition is, for some type $\tau_1, \ldots, \tau_n$ and some relata $A_1, \ldots, A_n$ of types $\tau_1, \ldots, \tau_n$ respectively, an application of a relation R of type $\langle \tau_1, \ldots, \tau_n \rangle$ to $A_1, \ldots, A_n$:

$$App(R, A_1, \ldots, A_n).^{14}$$

Fully comparing a typed view of this kind with the untyped view of Bealer is beyond the scope of this paper. The typed theory will mostly rest in the background in what follows since I will be primarily concerned with the operation $App_{\langle e \rangle}$ that takes properties of individuals and individuals to propositions. The reason for restricting my focus is that it is here that propositions make "contact" with the concrete world, as it were. There may be interesting things to say about higher-type entities, but for the purposes of this paper I will mostly ignore them.[15]

One might wonder whether this account is really a *primivitist* account of propositions: afterall, haven't we just defined propositions as the output of application? We have, but in explaining what application was, I made ineliminable use of the word 'proposition'. Thus the theory does not provide anything like a reductive analysis of propositions. At least in one sense of 'primitivist', I take the theory to be a primivitist view. Nothing of great importance seems to rest on this fact though.

One might also object to describing this view as one that takes *the* operation of application as primitive: really the theory has posited a typed collection of application operations and so has posited many new primitive operations. One might even object on these grounds that he view should be rejected on the grounds of being

[14]If this claim is to be included as part of the official theory, it would be desirable to treat it as short hand for a infinitary disjunction instead of a claim that explicitly quantifies over types. In general the actual *principles* of the theory I put forward will not make use of quantification over types. The type theory merely acts as a background framework in which the theory is developed.

[15]There are no doubt quite few questions raised by this typed approach. I can't hope to settle the debate between typed and untyped theories here.

"ideologically complex".

I have two things to say in response. First, a typed collection of operations could easily be traded in for one single partially defined operation. The types then merely let us keep track of where the operation is defined. Second, just as ontological complexity is more of how varied in kind items in one's ontology are, as opposed to mere number of things, so too ideological complexity should be taken as a measure of how heterogeneous one's ideology is, as opposed to merely the numbers of items included in one's ideology. Since I'm inclined to regard all of the typed application operations as being of the same in kind, I'm inclined to think that the theory is not very ideologically complex at all.

Contra some algebraic theories, I will not assume any strong decomposition principles for propositions like the following:

Structure If $App(f, x) = App(g, y)$ then $f = g$ and $x = y$.

It seems to me that allowing for a bit more freedom in the behavior of the operation *App* can lead to some genuine explanatory advances; in the final section of this paper I will sketch one such case in particular. There is also a worry about inconsistency; given moderate resources, STRUCTURE seems to be inconsistent. For instance, fix a proposition q and suppose that f is the property of being a proposition p such that for some property of propositions h, $p = App_{\langle\rangle}(h, q)$ and p does not instantiate h. Consider the proposition $App(f, q)$. If $App(f, q)$ does not instantiate f, then then for every property h such that $App(f, q) = App(h, q)$, q instantiates h. Thus, since $App(f, q) = App(f, q)$, q instantiates f. So if q doesn't intantiate f, it does instantiate f. Classically, this entails that q instantiates f. So for some h, $App(f, q) = App(h, q)$ and q does not instantiate h. But if STRUCTURE *were* true, any such h would have to be f, which we've already shown q does not have; so STRUCTURE is false.[16]

[16]This is of course, one version of the Russell-Myhill argument. See [**?**], Appendix B.

One might worry that given the falsity of Structure there is no sense in which *App* can be regarded as a way of *combining* things. But I don't see why this should be so. Ordinary fusion is idempotent: the fusion of x and x is x. Thus even the ordinary fusion wouldn't satisfy something as strict as Structure. Since the theory I'll develop explicitly denies that *App* is fusion, we are free to posit even more radical failures of structure: for instance, we might allow for certain "freely absorbable" or "non-structure creating" properties: properties for which $App(f, -)$ is the identity map on objects, for instance. If there are explanatory gains for allowing such properties, we should. One possible example of such freely absorbable properties and relations is the relation corresponding to the operation of application. For instance, we might suppose that there is a relation a of type $\langle\langle e\rangle, e\rangle$, the application *relation*, such that applying it to a property f and individual x is the same as applying f to x:

$$App_{\langle\langle e\rangle,e\rangle}(a, f, x) = App_{\langle e\rangle}(f, x)$$

These non-structure creating relations have some grounds for being called *logical*. Under that criterion, the application relation is itself a logical relation. We might then view the theory I will put forward as one that attempts to account for the representational features using only broadly logical resources. I will return to this point below. I now want to begin to develop a theory of the representational features of propositions within this framework.

4.3 Application and Representation

In what follows I will write $App(f, x)$ for $App_{\langle e\rangle}(f, x)$; when higher sorts of application become relevant I will make the type in question explicit. The goal of this section is to outline some of the ways in which the representational features of propositions can be derived from their applicative nature. I will do so by outline some postulates on application.

4.3.1 Aboutness and Predication

Some of the key representational features of propositions is that propositions are about individuals and predicate properties of them. We can capture this fact with the following principles:

Rigidity Necessarily for any f and x and any proposition p if $p = App(f, x)$ then necessarily if f and x exist $p = App(f, x)$.

Aboutness For a proposition p to be about x is for there to be some f such that $p = App(f, x)$.

Predication For a proposition p to predicate f of an individual is for there to be some x such that $p = App(f, x)$.

The principle Rigidity tells us that '$App(f, x)$' is to be read as a *rigid designator*. Aboutness and Predication are proposed as *analyses* of a propositions' representational features in terms of application. Together these principles entail some plausible facts concerning the representational features of propositions. For instance, the proposition that two is prime is necessarily about two. The theory provides the following explanation. The proposition that two is prime is the application of being prime to two (by the schema by which application was introduced). So by Rigidity it is necessarily the application of being prime to two. And so by existential generalization, necessarily there is some property f such that it is the application of f to the number two. And finally by Aboutness it follows that necessarily it is about two. Similarly, since it is the application of being prime to two, it is, for some individual x, the application of being prime to x; hence by Predication it predicates the property of being prime (necessarily so by Rigidity).

It might be worth briefly mentioning how propositions expressed using definite descriptions fit into this theory. On my view the proposition that the present kind of France is bald does not predicate baldness. One might worry that this means the view has lost contact with any pretheoretic notion of 'predication' since, surely on a pretheoretic sense, this proposition *does* predicate baldness.

In response I want to say two things. First, I'm not sure I have a grip on what it would mean for a proposition to predicate a property but not predicate that property of anything. To predicate, on my view, is to predicate *of*. If that's right the pretheoretic data may be a bit murkier than the objection makes out. And second, I'm not really all that concerned with capturing *all* of the pretheoretic data: I reject the idea that one's theory either aligns with the pretheoretic data or else it is revisionary. An abductive approach looks for the joint carving notions in the vicinity of the pretheoretic data without being hostage to them. In the present case, I think there are good theoretical reasons for adopting my approach to predication rather than one that takes the proposition that king of France is bald to predicate baldness. The basic reasons are just those that motivated Russell (1904) to treat propositions like the proposition that the present King of France is bald as being qualitative.[17] The proposition is not used to pick something out, and predicate something of it. Rather the proposition is ultimately quantificational. One could follow Russell in taking it to express a more complicated proposition built up from universal and existential quantifiers. Or one could take 'the' to express a primitive higher-order relation, ι, and identify the proposition that the present king of France is bald as being identical to $App_{\langle\langle e\rangle,\langle e\rangle\rangle}(\iota, f, g)$ for some properties f and g. On these sorts of views it is probably more accurate to say that the proposition that the present king of France is bald predicates* the relation expressed by 'the' of the properties expressed by 'present king of France' and 'is bald', where "predicates*" is some higher-order analogue of first order predication. Expressing this idea on English is admittedly a bit difficult.

[17]Well not purely qualitative since there is reference to France. But in general the idea of treating definite descriptions quantificationally is well motivated, but of course open to question.

4.3.2 Predication, truth and instantiation

Predication bears an intimate relation to instantiation. A proposition that predicates f of x is necessarily true if and only if x *instantiates* f. What accounts for this fact? One reason that this question has proved difficult to answer is that authors have tended to look for explanations of a propositions' truth in terms of which objects instantiate which properties. But once we have the operation of application in hand it becomes natural to reverse the order of explanation:

Instantiation For x to instantiate f is for $App(f, x)$ to be true.

The principle Instantiation is proposed as an *analysis* of instantiation in terms of truth. This reverses the traditional order of explanation. To many it will look like a hopelessly confused attempt to analyze the *noumenon* in terms of the *phenomenon*, or less grandiosely, to say how things are in terms of how they are represented to be. But the issues here are delicate. The proposed analysis is consistent with how things are being prior to how they are represented to be. Contrast the following two statements:

1. The proposition that two is prime is true because two is prime.
2. The proposition that two is prime is true because two has the property of being prime.

Ordinarily we might not distinguish these statements. But when doing metaphysics it is important that we recognize the coherence of the position that accepts (1) while rejecting (2). On the sort of view I am imagining, not only is the truth of propositions explained by how things are, but so too is the instantiation of properties. That is, in addition to accepting (1) and rejecting (2), this kind of theorist accepts (3):

(3) Two has the property of being prime because two is prime.

This puts propositions and properties on equal footing by treating both as explanatory posterior to how things are. Propositions are true or false whereas properties are true or false *of* things.[18] And both truth and truth *of* are explained by some prior notion of how things are.

I prefer a different view. Instead of taking both properties and propositions to be explanatorily posterior to a prior notion of how things are, I think we should both properties and relations to be constitutive of how things are: propositions correspond to distinctions in reality, not to distinctions in how reality is represented. Similarly, properties correspond to distinctions among individuals in reality, not to how individuals in reality are represented.[19] Less metaphorically, what it is for the proposition that two is prime to be true is for two to be prime, and what it is for two to have the property of being prime is for it to be prime.[20] Several authors have recently argued that identifications like 'for it to be the case that... is for it to be the case that ...' obey analogous principles to ordinary identity predicates. In particular they obey the obvious analogues of transitivity and symmetry.[21] If that it is right then it immediately follows that for two to instantiate being prime is for the proposition that two is prime to be true. And this sort of argument generalizes. Schematically:

P1 For the proposition that x is F to be true is for x to be F.

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[18]This view, or something like it, has been defended by [?] and [27] among others.

[19]Some authors prefer views that posit both propositions and another sort of entity they call "states of affairs". States of affairs are supposed to correspond to distinction in reality whereas propositions correspond to distinctions in how states of affairs are represented. I have a hard time seeing how such a view differs substantively from Fregeanism, which I reject. In any case the idea that there are *two* different kinds of entities, propositions and states affairs, is hardly a datum. Perhaps if one were already committed to a sort of truth maker view it might seem natural to propose this sort of two tiered picture. But even this seems overly complicated; a distinction between fundamental and non-fundamental propositions could do a lot of the work done by a truth maker theory. Instead of looking for the state of affairs that make the proposition true, we look for a specification of the truth conditions of the proposition that makes use of only fundamental propositions, for instance.

[20]See Rayo (2013) for a further defense of these sorts of identifications.

[21]See [29] and [85].

P2 For x to instantiate being F is for x to be F.

C Hence, for x to instantiate being F is for the proposition that x is F to be true.

This provides confirmation to the principle of INSTANTIATION since every instance of C can be inferred from it together with the schema by which the notion of application was introduced.

4.3.3 Application and cognition

One thing a theory of propositions is supposed to provide is an account of why, for instance, thinking that two is prime entails thinking *about* two, and why thinking that two is prime entails *ascribing* the property of being prime to two. The following two principles strike me as quite natural:

Attitude For x to think about y is for x to entertain $App(f, y)$ for some f.

Ascription For x to ascribe f to something is for x to believe $App(f, x)$ for some x.[22]

To think about something is to entertain a proposition about that thing and to ascribe a property is to believe a proposition that predicates that property. These principles unify various norms on belief and ascription. Truth is a norm of belief just as instantantiation is a norm of ascription. One should believe p only if p is true and one should ascribe f only if f is instantiated. Given the proposed theory the latter norm follows from the former. For any x, one should believe $App(f, x)$ only if $App(f, x)$ is true. So by Instantiation one should believe $App(f, x)$ only if x instantiates f. And then by the principle Ascription, it follows that one should ascribe f only if f is instantiated.

[22] Some authors hold that there is a neutral sense of 'ascribe' according to which one can ascribe blueness to an object without thereby believing the object to be blue. This sense of ascription can plausibly be captured by substituting 'entertain' for 'believe' in the principle.

It's worth pausing here for a moment to say something about my treatment of propositional attitudes. Peter Hanks (2015, p. 45) has recently objected to primivitist views of propositions on the grounds that primivitism about propositions inevitably leads to primivitism about propositional attitudes:

> [I]f propositions are simple and unstructured, we cannot take this act of endorsement [judgment] to consist in a mental operation performed on the constituents of a proposition. Furthermore... we cannot say that to endorse a proposition is to accept it as true... If accept p as true is to judge that p is true then we've analyzed one judgment, judging that p, in terms of another, judging that p is true. This leads to regress... it looks as though [the primivitist] is going to have to view judgment as a primitive attitude one can bear to a proposition.

I find this argument very unconvincing. On the one hand, the reasoning behind the argument is just hard to make out. For instance, suppose that one sees an electron. Then one bears the seeing relation to a simple item that lacks any internal structure. Does this mean that *seeing* must be simple, and unalyzable. Of course not. In general, having simple relata has nothing to do with whether a relation is analyzable. The connection that Hanks sees between these remains a bit mysterious.

Now Hanks does propose a couple of analyses and points out that they fail; one on general grounds and the other putatively requires propositions to be structured (though it's unclear to me whether this is really so). But he fails to show that many of our *leading* account of propositional attitudes conflict with primivitism: functionalism, interpretationism, causal theories, optimal conditions accounts etc. As far as I can see, *all* of these accounts are perfectly consistent with primivitism. Indeed since many of these accounts have been developed under the assumption the propositions are sets of possible worlds, some of them are actually more naturally combined with views on which propositions are sets of possible worlds, since they generally do not make clear in virtue of what attitudes could differ in fine grained content. Since the view that propositions are sets of possible worlds does not differ structurally from the view on which propositions are primitive and form a complete, atomic Boolean algebra, it is hard to see how there are going to be any in principle problems with primitivism when it comes to the propositional attitudes.

4.3.4 Laws and action

The theory offered thus far shows how to define various representational properties and relations from truth and application. One way to broaden the explanatory ambitions of the theory is to show that aboutness and predication as they arise in other domains can be accounted for in the theory of propositions. Suppose, for instance, that we regard the thesis that φ, the fact that φ, the law that φ and the act that φ as the proposition that φ under different guises. If that's correct we can immediately account for any aboutness or predication these entities exhibit in terms of the theory of application. I will look at two examples.

Suppose that the law that φ is simply the proposition that φ. This follows from the plausible theory that propositions are the referents of 'that'-clauses. Some people have maintained that laws of nature are *purely general.* Laws of nature do not mention any particular individuals. We can formulate this thesis more precisely in the present framework as follows: for any law l, there is no individual x and property f such that $l = App(f, x)$ (or perhaps some generalization of this idea). This provides a language independent account of the generality that laws exhibit.

Another example comes from the theory of action. Suppose that you and I both pick up a pen. There is a sense in which we have done the same thing and a sense in which we have not done the same thing. What are these senses? Well suppose that actions are propositions—things we make true. I made the following proposition true: that I pick up the pen. And you made the following proposition true: that you pick up the pen. The sense in which we've both done the same thing is that we've both made propositions true that predicate the property of picking up the pen. Then sense in which we've done different things is that that the proposition you've made true is about you whereas the proposition that I've made true is about me. The current proposal thus allows us to dispense with any ontological distinction between types of actions and token actions. We say that p is the same action (same "token") as q if p is an action and $p = q$. We say that p is the same type of action as q if p and q are both actions and p predicates f iff q predicates f for any property f.

4.3.5 The aboutness of properties

A more radical extension of the theory attempts to explain the distinction between *qualitative* properties and *haecceitistic* properties in terms of propositional aboutness. Consider the property of being identical to John. We can recognize *some* sense in which this property is *about* John. At the very least it is more closely related to John than the property of being identical to the person wearing the blue shirt, even provided that John is wearing the blue shirt. Even if one were disinclined to accept that being identical to John is about John, the property nevertheless seems to stand to John in the same way that the proposition that John is identical John stands to John. The proposition that something is identical to John is plausibly the *existential generalization* of the property of being identical to John. Now consider the proposition that x is identical to John, for an arbitrary individual x. If we are not too fine-grained about the individuation of propositions, we can take this proposition to be an application of the following complex property to John: the property of being a y such that x is identical to y. If that is correct, then we can say that property of being identical to John is about John because its application to an arbitrary individual is about John. More generally, I propose the following theory of property aboutness:

Property Aboutness For a property f to be about x is for $App(f, y)$ to be about x, for all y.

With a notion of property aboutness in hand, we can say that a property is *qualitative* if it is not about any particular thing and haecceitistic otherwise. So for instance, being blue is qualitative because, plausibly, there is no x such that for any individual y, the proposition that y is blue is about x. Being identical to John is not qualitative because the proposition that y is identical to John is about John, for any individual y.

Call the theory just outlined the *minimal theory of application*. The minimal theory of application provides analyses of many of the representational properties of propositions and agents in terms of application and so demonstrates some of the potential explanatory power a primitivist view that makes us of application can have. The theory is of course incomplete in many ways. A full theory would generalize application to n-ary relations and show that it can be consistently combined with one's desired theory of propositional fineness of grain. I won't do that here. Instead, in the next section, I will situate the minimal theory within the literature to get a better sense of how it compares with more recent attempts to account for the representational dimension of propositions. In the final section I will evaluate this theory against these other theories and argue that it is to be preferred on broadly abductive grounds.

4.4 Application and the Metaphysics of Propositions

In the previous section I outlines a theory of application. The theory does not take to form of an analysis—it does not tell us what it is for a proposition to be an application of a property to an individual—it does provide axioms that accounts for the representational properties of propositions. In this section I want to first relate the theory developed to the problem of the unity of the proposition and then situate it within the literature.

4.4.1 The problem of the unity of the proposition

As is often acknowledged, there is no single clear problem that is "the problem of the unity of the proposition," but rather a family of related problems. I want to first suggest that at least *one* of the problems that has gone under this heading can be formulated in terms of application. The basic idea is that once we acknowledge that propositions are applications of properties to individuals, we certainly want some account of this operation that explains the distinctive traits of its outputs. In particular we want an account of application that explains the representational features of propositions.

We might conceive of this problem by analogy with the *general composition problem*.[23] Pace composition as identity theorist, I am not merely my parts. I am one thing whereas my parts are many. But I am the result of applying some operation to my parts: I am the *fusion of* my parts. The operation of fusion takes some things—my parts—and delivers one thing, me. The general composition problem is essentially that of providing an illuminating account of fusion. Hence the problem of the unity of the proposition, as I am conceiving of it, stands to application as the general composition problem stands to fusion.

We can be a bit more precise about the analogy. Peter van Inwagen calls the general composition question the question "What is it for some xx to compose y?" Call the *general application question* the question "What is it for p to be the application of f to x?" The theory I proposed is that it is *primitive* and so no answer to this question can be given that invokes more basic notions. But just like we can answer the question "What is it to be a set?" by providing some postulates on being a set, so too we can answer the general application question, I would argue, by providing postulates on application.

[23] See [105] ch. 4.

Recall that the *special composition question*, as opposed to the general one, is the question, "What are the necessary and sufficient conditions for some things to compose something?"[24] More precisely, it is the problem of finding some relation r such that such that xx compose something if and only if r holds of xx (or rather finding some informative description of r). Just as we can draw an analogy between the general composition question and the general application question, we can draw an analogy between the special composition question and the special application question. Say that a property f *applies* to an individual x iff $App(f, x)$ exists. Then we can ask for the necessary and sufficient conditions for a property f to apply to an object x. More precisely, the special application problem is that of finding some relation r such that f applies to x if and only if r holds of f and x (or rather finding some informative description of r).

Oftentimes when the problem of the unity is posed, it is posed as if it were the problem of finding an answer to the special application question.[25] For instance Peter Hanks says in describing the problem:

> Since the proposition [that Clinton is eloquent] is one thing, and the constituents [Clinton and eloquence] are two things, there must be something about the proposition that joins [Clinton] and [eloquence] together into a single thing. The constituents must bear a relation to one another that unifies them into a proposition.
>
> Hanks (2015, p. 43)

He then goes on to introduce the problem of unity as that of finding this relation

[24]See [105], ch. 2.

[25]Most authors do not use the terminology of application but rather talk about the constituents of a proposition. I prefer the terminology of application since it is consistent with, but does not immediately suggest, that if p is the application of f to x then p has f and x has *parts*. The sense in which applications are *formed* from what they are about and predicate could just be spelled out in the following way: $App(f, x)$ is formed from x and f in the sense that necessarily if $p = App(f, x)$ then necessarily $p = App(f, x)$; and so $App(f, x)$ being some way entails that f and x are some way. For instance, if $App(f, x)$ is true, then x has the property of being an x such that $App(f, x)$ is true.

that "unifies" f and x into a proposition. Similarly, Jeff King says

> Presumably the constituents of a proposition are related somehow in that proposition, with the relation imposing structure on them. I'll put this by saying the relation *holds the constituents together*. Answering [the unity question] requires saying which relations hold the constituents of propositions together.
>
> King (2009, p. 259)

Although neither King nor Hanks makes explicit use of application in formulating these questions, the questions formulated seem closer to the special application question as opposed to the general application question. The metaphor of "holding together" suggests that we are looking for a relation that holds of the the constituents of a proposition if and only if they form that proposition. And this is just the special application problem.

Despite these appearances, I will argue that the *theories* that Hanks and King both give, and many others for that matter, do *not* provide answers to the special application question. Rather, they provide answers to the general application question. And this is as it should be, since, I will also argue, the special application question is the *wrong* question to ask.[26]

4.4.2 Why the special application question is the wrong question.

To see that the special application question is the wrong question to ask, it is helpful to reflect for a minute on what Peter van Inwagen says concerning the relation between the special and general composition questions:

> What singular terms might be appropriate substituends for 'y' in the 'the xs compose y', given that *Contact* [xx compose something iff xx are

[26]King's theory, while primarily an answer to the general application question, inadvertently provides an answer to the special application question, at least on one interpretation of it. I will argue that this fact actually counts against his theory (again, on at least one interpretation of it).

> in contact] is the correct answer to the Special Composition Question? There is no way of answering this question, for neither *Contact* nor any other answer to the Special Composition Question tells us anything about the identity, or even the qualitative properties, of any composite object. Moreover, no answer to the Special Question Composition will tell us what composition *is*.
>
> Van Inwangen (1990, p. 38)

Van Inwagen is not putting forward any kind of controversial theory of the relation between the two questions in this passage but is making a straightforward logical point. To say that xx compose something iff they are in contact is consistent with any hypothesis concerning the kind of thing they compose. For instance, it is consistent to say that these two blocks compose something iff they are in contact, and what they compose is the entirety of the earth; it is consistent to say that two blocks compose something iff they are in contact and what they compose is the number π; it is consistent to say that two blocks compose something iff they are in contact and what they compose is the block on the left. The answer one gives to the special composition question on its own tells you *absolutely nothing* about the entity they compose. All it tells you is *when* (in a modally robust sense of 'when') some things compose.[27]

This point applies equally to the special application question. Suppose for definiteness that one thought that the application of being prime to two existed if and

[27]This claim needs to be qualified to deal with counterexamples that involve "cheating". For instance, suppose one put forward the following answer to the special composition question: two things compose something if and only if they are in contact and for any things, if they compose something, then they compose something that is a material object that is located roughly where they are located. In other words, if one builds into the description of the relation certain generalizations about the qualitative properties of what composite objects are like and how they relate to their parts, then of course one's answer to the special composition question will entail facts about the qualitative roles about the thing that, say, two blocks compose is like. I take it as self-evident that these kind of cheat answers are not worth taking seriously. One might put the point this way: if we demand that any answer to the special composition (or special application) questions must invoke non-gerrymandered relations, then no answer to the special application question will constrain one's account of composite objects (or propositions).

only if there is a state of affairs having the property of being prime as its universal component and the number two as its singular component. In brief: the application of being prime to two exists iff two and being prime are in contact in a state of affairs. This theory is logically consistent with any hypothesis whatsoever concerning what kind of thing the application is. For instance, it is consistent to say that the application of being prime to two exists iff two and being prime are in contact in some state of affairs and the application is identical to me; it is consistent to say that the application exists iff they are in contact in a state of affairs and the application is identical to the property of being prime. The answer one gives to the special application question on its own tells you absolutely nothing about what the application is. All it provides is the modal profile of the application.

This point can be easily overlooked but it is significant. Recall that one thing we want out of an answer to the question of unity is an account of the distinctive representational behavior of propositions. But since any answer to the special application question is logically consistent with any hypothesis whatsoever concerning the qualitative properties of propositions, any answer to the special application question is logically consistent with any hypothesis concerning the representational properties of propositions. Those who have been attempting to account for the representational properties of propositions merely by providing an account of what unifies the constituents of a proposition have been attempting the impossible. Thus, insofar as we want an account of certain qualitative properties of propositions, we *should not* be attempting to answer the special application question.

Now one might respond that really what we want is an answer to *both* the special application question and the general application question. A general theory of propositions will tell us what it is for p to be the application of f to x and also tell us when the application of f to x exists. This more general theory will hope to account for the representational properties in terms of its analysis of application rather than the modal profile it assigns to applications. But I think even this more nuanced approach embodies a mistake. I'll make this argument again by drawing another analogy to the case of composition.

Suppose that one started out accepting *universalism* about composition, according to which for any xx necessarily xx compose something. What would one say to the special composition question? I'm inclined to think one should *dismiss* the question as having no answer at all. There is no relation that makes it the case that some things compose because all some things need to do to compose is exist. This is in fact the way Peter van Inwagen *introduces* universalism:

> It is impossible for one to bring it about, [according to the universalist], that something is such that the xs compose it, because, necessarily. . . , something is such that the xs compose it. . . One can't bring it about that the xs compose something because they already do; they do so "automatically."
>
> Van Inwagen (1990, p. 72)

But we are in a similar situation with respect to propositions. Most authors grant that it is metaphysically necessary that whenever the property of being blue exists and the cup exists the proposition that the cup is blue exists. In my preferred terminology, necessarily whenever f is a property and x is an individual, $App(f, x)$ exists. There is nothing that one can do to bring it about that the application of a property to an object exists; it exists "automatically." Hence just as the universalist has no need to answer the special composition question, *no* theorist has any need to answer the special application question, since *all* theorists are universalists about application.

The search for some relation that "unifies" being prime and two into the proposition that two is prime is thus confused twice over. Since they form a proposition automatically, there is no such relation to be found. Moreover, since what we really want to know is why the proposition has certain qualitative features, the search for such a relation turns out to be completely irrelevant to what we really care about.

4.4.3 Reductive answers to the general application question.

This leaves us with something of a puzzle. Both King and Hanks and many others have formulated problems that on their face appear to be the special application question. They then go on to provide what they say are answers to this question and *also* claim of these answers that they account for the representational features of propositions. What is going on? When one looks more closely at proposed answers to the special application question, one quickly sees that they are not, after all, answers to this question, but are rather answers to the general application question. It is common in the literature for someone to highlight some relation r and *call* it the unifying relation. But it is never the case that according to their theories the application of f to x exists if and only if f bears r to x; rather, the proposed theory says that there is *some other* operation O such that the application of f to x is identical to $O(r, f, x)$. That is, the theories invariably are just *analyses* of application in terms of a further relation and a further operation. I'll give three examples of this.

Consider first Jeff Speaks' recent theory of propositions.[28] According to Speaks, the proposition that two is prime is the property of being such that two instantiates being prime. The application of f to x can exist without x instantiating f. Instantiation does not unify the constituents of a proposition. Rather the theory is that there is some three-place operation, *the property of being such that x bears r to y*, whose application to two, instantiatation, and being prime, delivers the proposition that two is prime. And this is just an answer to the general application question:

Speaks $App(f, x) =$ the property of being such that x instantiates f.[29]

[28] [97].

[29] Or if one prefers, $p = App(f, x)$ if and only if $p =$ the property of being such that x instantiates f.

The views of Peter Hanks and Scott Soames' view can be similarly formulated.[30] According to Soames (roughly) the proposition that two is prime is the act of ascribing primehood to two. Since the proposition that two is prime can exist even if no one actually ascribes being prime to two, ascription is not the relation that unifies the constituents of a proposition. Rather there is some operation, *the act of ascribing f to x*, whose application to being prime and two, delivers the proposition that two is prime. And this is just an answer to the general application question:

Soames $App(f, x) =$ the act of ascribing f to x.

Finally consider the position put forward by King.[31] According to King the proposition that two is prime is the fact that two bears a certain relation r to being prime. This relation r is quite complex and is defined by quantifying over linguistic items and their meanings; the details needn't concern us here. So on King's view, the proposition that two is prime is the result of applying some three place operation to r, being prime, and two. It is the operation denoted by 'the fact that x bears y to z'. So King provides an answer to the general application question:

King $App(f, x) =$ the fact that x bears r to f.[32]

Appearances to the contrary, many recent theories of propositions are thus better construed as answers to the general application question as opposed to the special application question. And once these theories are presented this way, it becomes clear that they are in competition with my own primivitist account. For instance, each of Speaks, Soames and King takes as primitive one or more notions that the minimal

[30]There are important differences between these views. The differences between them will not matter for present purposes.[94].

[31][61].

[32]Since the x bears r to f iff it is a fact that x bears r to f, King's view also entails an answer to the special composition question. This is actually a bit of a cost since it appear incompatible with the universalist answer to the special application question.

theory of application provides analyses of. Speaks makes use of instantiation; Soames makes use ascriptions and actions; and King makes use of facts. In this respect, the minimal theory of application recommends itself on the basis of its unifying power. The phenomena of aboutness and predication as they arise in the theory of properties, beliefs, actions and facts, on this view, are unified by the notion of application.[33] This strikes me as a point in its favor. In the next section I will further develop this argument for my theory.

4.5 A Defense of Primivitism

According to the theory I have proposed, the manner in which individuals and properties are formed into propositions is primitive. Application is not defined or explained in more basic terms. Moreover, propositions themselves are taken as primitive. No hypothesis concerning the kind of thing that propositions are is put forward by the theory.

Many authors have considered and dismissed primivitist views of propositions. Hanks (2015, p. 43) asserts that primitivism "does not advance our understanding" and that we should first "look for other ways of explaining how we represent the world in making judgments." In a similar spirit Soames (2015, p. 16) claims that we lack any understanding of "what such primitively representational entities are" and "why *our* cognizing them in the required way results in *our* representing things as bearing properties." King (2009, p. 260) confesses that he "just can't see how

[33] I do not mean to suggest that each of their views lack the resources for unification. In particular we could combine Speaks' view with every principle of the minimal theory of application apart from the principle Instantiation (or at least one couldn't offer this as a reductive account of instantiation). But in place of Instantiation Speaks could offer a reduction of truth to the properties of being a propositions and the relation of instantiation: for a property to be true is for it to be a proposition that is instantantiated. He might be able to provide a full reduction of truth to instantiation if the view was combined with the following analysis of being a proposition: for a property to be a proposition is for the following to be the case: for something to instantiate it just is for everything to instantiate it. That principle may have some hard edge cases (e.g. being blue only if blue), but I don't see that as decisive.

propositions or anything else could represent the world as being a certain way *by their very natures and independently of minds and languages.*" Primivitist views are widely held to be inferior to reductive ones.

How should we decide which theory of propositions to accept? Clearly any theory incompatible with our evidence is ruled out. But primivitism is not plausibly incompatible with our evidence. The evidence we have concerning propositions is that they play various roles: they are the objects of belief, the bearers of truth values and modal properties and the relata of entailment and explanation. More importantly, they are about things and predicate properties of things. Not only is primivitism compatible with all these facts, but as shown in section I, a primivitist view cast in terms of application provides simple and unifying explanations of the fact that propositions play some of these roles.

Perhaps we should disbelieve primivitism regardless of our evidence: that primivitism is false is a *default reasonable belief.*[34] Jeff Speaks endorses something like this thought:

> If one or more reductive theories succeeds in identifying entities suitable to play the theoretical roles of propositions, then we should reject the primivitist view.
>
> [Speaks Forthcoming b., p. 4]

While Speaks offers no independent argument for this principle, it is, as he notes, widely held. It might be supported on the grounds that views according to which the world is a relatively homogeneous place are preferable to those according to which the world is a relatively heterogeneous place. A reductive theory of propositions will attempt to reduce propositions to an entity of some sort we all already believe in. Take Speaks' view according to which propositions are monadic properties. I believe that there are monadic properties. But I don't believe propositions are monadic

[34] This phrase is due to [30].

properties. So according to my view, there are (at least) two disjoint categories of things, propositions and monadic properties. Supposing it is correct that there are enough distinctions among monadic properties to capture the distinctions we want to make using propositions, my theory appears overly complicated. Since my ontology already contains entities that can do the needed work, there is no reason to posit some extra ontological category of things to do that work.

There are two problems with this argument. First, while the minimal theory of application is a primivitist theory of propositions, it is a reductive theory of other things: aboutness, predication, instantiation, facts, and acts. My opponents on the other hand provides no reductive account of these things but rather take them as primitive. Speaks takes instantiation as primitive; Soames takes ascriptions and actions as primitive; and King takes facts as primitive. The demand for a more homogeneous views does not obviously decide between our theories. I'll return to this point below.

The second problem with this argument is that it fetishizes homogeneous theories to the detriment of other theoretical virtues. If a theory obtains homogeneity by *ad hoc* means that involve arbitrary choices, there is no obvious reason to prefer it over an elegant and unified theory that happens to have a more heterogeneous ontology. Theories of propositions should be evaluated on the basis of a broad range of virtues such as strength, elegance, simplicity and unifying power. As far as I can see, there is no *a priori* reason to expect reductive theories to score better than non-reductive theories on this criteria.

There is, in fact, a general reason to think that the sorts of reductive theories philosophers tend to offer will score *worse* by these criteria. Many proposed reductive theories will show that one kind of thing can play the role of another kind of thing. But they ensure that they play these roles only by treating what look like joint carving properties of the entity being reduced to gerrymandered properties of the entities doing the reducing. Conversely, what look like joint carving properties of the entities doing the reducing play absolutely no role in the theory of the entities being reduced. Reductive theories tend to not preserve the naturalness of the properties of the entities being reduced.

Here is a simple example of this phenomenon. Suppose one proposed a reduction of propositions to *sequences* and a reduction of application to the operation of pairing. On this view the proposition that two is prime is the pair whose first coordinate is two and whose second coordinate is being prime. By treating propositions as pairs, we gain some theoretical understanding simply because the theory of ordered sequences is established and well understood. Moreover, there are enough distinctions in the theory of ordered sequences to capture all of the distinctions we want to draw with a theory of propositions. But these distinctions are captured in a way that make the theory of propositions objectionably *arbitrary*. The most natural operation on sequences—concatenation—plays almost no role in the theory of propositions since the concatenation of two propositions will not in general be a proposition. Moreover, while properties like being about and predicating *can* be analyzed in this framework, this can only be achieved by what looks like *arbitrary choices*. For instance we could say that a proposition that I walk is a pair whose first coordinate is the property and whose second coordinate is me and then analyze aboutness by saying the proposition is about its second coordinate. But we could also say that it is a pair whose first coordinate is me and whose second coordinate is the property and say that it is about its first coordinate. Nothing in our linguistic practice seems to decide between these two theories. Finally, there does not appear to be any natural family of operations on ordered sequences that corresponds to the operations of negation, conjunction, disjunction and so on.

Whether this charge applies to recent reductive theories is debatable.[35] For our

[35]Williamson (2016) argues that one way to measure the overall elegance and simplicity of a

purposes, the important point is that theoretical virtues do not automatically favor reductive theories, and so nonreductive theories shouldn't be dismissed outright.

There is also a more positive case to be made in favor of my view over some recent competitors. As mentioned above, it would be somewhat misleading to designate my view primivitist and the views we have been considering above a reductive: each theory takes some things as primitive and analyzes other things in terms of those primitives. This suggests that in order to compare our respective views, we should figure out what the appropriate primitives are in a theory of propositions. Here is one reason to favor my chosen primitives. Recent reductive theories of propositions appeal to entities that exhibit features that are very much *like* the representational features that propositions exhibit. The fact that two is prime is plausibly about two; the property of being two *concerns* two is a way that seems quite analogous to how the proposition that two is prime concerns two. Moreover, the act of predicating something of two would appear to concern two in much the same way that the property of being two does. For instance, were there no number two, we would have no way of specifying the relevant fact, act or property. We specify these entities in terms of their relations to other entities. On the theory I favor, all of this is to be ultimately be explained in terms of application and truth. Application and truth are the common factors that unify the representational dimension of these various entities. This allows the view to achieve a generality that is lacking from competing views. The theories of Soames, King, Hanks or Speaks all treat the representational dimension of facts, properties and acts as somehow fundamentally different from

theory is to look at how well it handles evidence it was not explicitly designed to account for. Speaks (forthcoming) argues that the theories of King and Soames has some difficulty handling what he calls *easy transititions* between propositional attitudes. One way to think of this point is that theorizing in philosophy of perception, epistemology and philosophy of language involves generlizations that connect various propositional attitudes and these generalizations are harder to account for on King's and Soames' views. The point is not that their theories are refuted by such generalizations. Rather, when viewed as new evidence, they appear more difficult to accommodate and and such that the theories lack simplicity or elegance capable.

propositions.

Some will of course object to the idea of taking truth as primitive. Hanks and Soames would certainly object to this since for them, the problem of unity just is that of providing an account of how propositions have truth conditions. It's not clear to me what it is that needs to be explained. One might suggest that what needs to be explained is why, for instance, the proposition that grass is green is true if and only if grass is green. But this demand for explanation seems to me misguided. The proposition that grass is green is such that for it to be true is for grass to be green. This provides us with all the explanation we need. Consider an analogy. There is not any particular problem of explaining the instantiation *conditions* of a property. We know why grass has the property of being green if and only if grass is green since we know that the property of being green is such that for grass to have it is for grass to be green. So it is unclear why exactly truth conditions are supposed to be particularly troubling provided that instantiation conditions are not.

Soames further clarifies the explanatory challenge:

> [T]he triviality of routine instances of the propositional T-schema. . . approaches the triviality of routine instances of the instantiation schema for properties . . . . But the underlying question. *What sort of things must properties be in order to have instantiation conditions?* is itself trivial in a way in which the question *What sort of things must propositions be in order to have truth conditions?* is not. Properties are ways things are or could be. . . . For a *way something could be* to be instantantiated is for something to be that way. . . . There is no similarly obvious answer to the question *What must propositions be?* in order for them to have truth conditions. . .

This seems to me to be mistaken. Soames' explanation of why properties have instantiation conditions seems to me to be on equal footing with the following explanation of why propositions have truth conditions. Propositions are things that are or could be the case. For a proposition to be true is just for it to be the case. Both are equally obvious. And both seem correct. On the account of instantiation I mentioned above, this should come as no surprise. Properties are ways; to instantiate them is to be that way. And to be that way is just for it to be the case that you are that way (i.e., for it to be true that you are that way).

There is a further reason why truth and application strike me as appropriate primitives of a theory of propositions: both notions are broadly logical in character. As mentioned above, the application *relation* is plausibly not structure creating: applying application to a property and an individual is the same as applying the property to the individual. I'm inclined to accept a similar view when it comes to truth: applying truth to a proposition just delivers that same propositions back. That is

$$App_{\langle\langle\rangle\rangle}(t, p) = p$$

where t is the property of being true. On this sort of view the proposition that is is true that P *is* the proposition that P. We might even claim this as a *definition* of propositional truth: propositional truth is the unique property t such that applying it to a proposition gives you that proposition back. If that's right, then the sort of primitivism developed here can eliminate talk of truth in terms of Russellian definition descriptions of the truth role. Thus not only are predication and aboutness explained in terms of application, but so too is truth.

4.6 Conclusion

Many authors have reached for ontology in order to explain some of the distinctive traits of propositions. This paper argued that instead of ontological reduction we can construct a plausible theory of the representational aboutness by making use of some novel ideology. In particular, using the operation of application we are able to provide plausible, general accounts of various representational features of propositions and their kin.

APPENDIX A

PROOFS

The following appendix contains proofs of lemma 2.2 and theorem 2.1.

For convenience the principles of **TSR** and **BL** are listed again:

A1 Any substitution instance of a propositional tautology.

A2 $\forall x\varphi \to \varphi[t/x]$ (where t is a term of the appropriate sort free for x in φ).

A3 $\forall x(\varphi \to \psi) \to (\forall x\varphi \to \forall x\psi)$.

A4 $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$.

R1 $\varphi, \varphi \to \psi/\psi$.

R2 $(\varphi \to \psi)/(\varphi \to \forall x\psi)$ if x is not free in φ.

R3 $\varphi/\Box\varphi$.

(I) $\forall pp\big(\Box \bigwedge pp \to \forall p(p \prec pp \to \Box p)\big)$.

(T) $\forall pp\big(\forall p(p \prec pp \to p) \to \bigwedge pp\big)$.

(S) $\forall p\forall q\big(p < q \to \Box(p \to q)\big)$.

(Ir) $\forall p(p \not< p)$.

(D) $\forall p\forall qq(p < \bigwedge qq \to \forall q(q \prec qq \to p < q))$.

(F) $\forall p\forall q(p < q \to (p \wedge q))$.

(P) $\forall p\big(Cp \to Ep\big)$.

Let $C(pp)$abbreviate $\forall p(p \prec pp \leftrightarrow (p \wedge \Diamond\neg p))$. Then we want to show that **TSR** $\vdash_{\mathbf{BL}} \forall pp(C(pp) \to \Box \bigwedge pp)$. The following derivation provides a sketch of how the proof goes:

(1)	$(\mathbf{D}) \vdash p < \bigwedge pp \rightarrow (p \prec pp \rightarrow p < p)$	$\mathbf{A1}, \mathbf{A2}, \mathbf{R1}$
(2)	$(\mathbf{Ir}) \vdash p \not< p$	$\mathbf{A2}$
(3)	$(\mathbf{D}), (\mathbf{Ir}) \vdash (p < \bigwedge pp \rightarrow p \not\prec pp)$	$(1, 2)$
(4)	$(\mathbf{D}), (\mathbf{Ir}) \vdash (C(pp) \rightarrow (p < \bigwedge pp \rightarrow (p \rightarrow \Box p)))$	(3)
(5)	$(\mathbf{F}) \vdash p < \bigwedge pp \rightarrow p$	$\mathbf{A1}, \mathbf{A2}, \mathbf{R1}$
(6)	$(\mathbf{D}), (\mathbf{Ir}), (\mathbf{F}) \vdash (C(pp) \rightarrow \forall p(p < \bigwedge pp \rightarrow \Box p))$	$(4, 5)$
(7)	$(\mathbf{S}) \vdash \forall p(p < \bigwedge pp \rightarrow \Box p) \rightarrow (E \bigwedge pp \rightarrow \Box \bigwedge pp)$	$\mathbf{A1}, \mathbf{A2}, \mathbf{A3}, \mathbf{A4}, \mathbf{R1}$
(8)	$(\mathbf{P}) \vdash \neg E \bigwedge pp \rightarrow (\bigwedge pp \rightarrow \Box \bigwedge pp)$	$\mathbf{A1}, \mathbf{A2}, \mathbf{R1}$
(9)	$(\mathbf{T}) \vdash C(pp) \rightarrow \bigwedge pp$	$\mathbf{A1}, \mathbf{A2}, \mathbf{R1}$
(10)	$(\mathbf{D}), (\mathbf{Ir}), (\mathbf{F}), (\mathbf{S}) \vdash C(pp) \rightarrow (E \bigwedge pp \rightarrow \Box \bigwedge pp)$	$(6, 7)$
(11)	$(\mathbf{P}), (\mathbf{T}) \vdash C(pp) \rightarrow (\neg E \bigwedge pp \rightarrow \Box \bigwedge pp)$	$(8, 9)$
(12)	$(\mathbf{D}), (\mathbf{Ir}), (\mathbf{F}), (\mathbf{T}), (\mathbf{P}), (\mathbf{S}) \vdash \forall pp(C(pp) \rightarrow \Box \bigwedge pp))$	$(10, 11)$

Turning now to theorem 2.1 recall that the axioms and rules of inference of $\mathbf{BL}^+$ are those of $\mathbf{BL}$ plus

A6 $\blacksquare(\varphi \rightarrow \psi) \rightarrow (\blacksquare\varphi \rightarrow \blacksquare\psi)$

A7 $\blacksquare\varphi \rightarrow \varphi$

R4 $\varphi/\blacksquare\varphi$

We have already shown that $\mathbf{TSR} \vdash_{\mathbf{BL}} \mathbf{N}$. And so since $\mathbf{BL}^+ \supset \mathbf{BL}$ it suffices to show that for each $\varphi \in \mathbf{TSR}$, $\mathbf{TPSR} \vdash_{\mathbf{BL}^+} \varphi$. For most $\varphi \in \mathbf{TSR}$, that sequent has a one line proof that sites $\mathbf{A7}$. The only difficult case is showing that $\mathbf{TPSR} \vdash_{\mathbf{BL}^+} \mathbf{P}$.

For readability and convenience I'll introduce the following abbreviations:

($\mathbf{P}^{\blacksquare}$) $\blacksquare\forall p(Cp \to \blacklozenge\exists q(q < p))$

($\mathbf{F}^{\blacksquare}$) $\blacksquare\forall p\forall q\forall q(p < q \to (p \wedge q))$

($\mathbf{D}^{\blacksquare}$) $\blacksquare\forall p\forall q\forall r(p < q \wedge r \to (p < q \wedge p < r))$

Then the following lemma suffices to prove the theorem:

Lemma 1.1. $(\mathbf{P}^{\blacksquare}), (\mathbf{F}^{\blacksquare}), (\mathbf{D}^{\blacksquare}) \vdash_{\mathbf{BL}^+} (\mathbf{P})$

Proof.

(1)	$(\mathbf{P}^{\blacksquare}) \vdash_{\mathbf{BL}^+} C(p \wedge \neg Ep) \to \blacklozenge E(p \wedge \neg Ep))$	**A2**, **A7**
(2)	**Finite Distribution** $\vdash_{\mathbf{BL}^+} q < (p \wedge \neg Ep) \to (q < p \wedge q < \neg Ep)$	**A2**
(3)	$\mathbf{F} \vdash_{\mathbf{BL}^+} (q < \neg Ep) \to \neg Ep$	**A2**
(4)	**Finite Distribution**, $\mathbf{F} \vdash_{\mathbf{BL}^+} q < (p \wedge \neg Ep) \to (q < p \wedge \neg Ep)$	$(2, 3)$
(5)	$\vdash_{\mathbf{BL}^+} \neg(q < p \wedge \neg Ep)$	**A2**
(6)	**Finite Distribution**, $\mathbf{F} \vdash_{\mathbf{BL}^+} \neg(q < (p \wedge \neg Ep))$	$(4, 5)$
(7)	**Finite Distribution**, $\mathbf{F} \vdash_{\mathbf{BL}^+} \neg E(p \wedge \neg Ep))$	(6), **R2**
(8)	$(\mathbf{D}^{\blacksquare}), (\mathbf{F}^{\blacksquare}) \vdash_{\mathbf{BL}^+} \blacksquare\neg E(p \wedge \neg Ep)$	(7), **A6**, **R4**
(9)	$(\mathbf{P}^{\blacksquare}), (\mathbf{D}^{\blacksquare}), (\mathbf{F}^{\blacksquare}) \vdash_{\mathbf{BL}^+} \neg C(p \wedge \neg Ep)$	$(1, 8)$
(10)	$(\mathbf{P}^{\blacksquare}), (\mathbf{D}^{\blacksquare}), (\mathbf{F}^{\blacksquare}) \vdash_{\mathbf{BL}^+} Cp \to \neg C(p \wedge \neg Ep)$	(9)
(11)	$\vdash_{\mathbf{BL}^+} (Cp \wedge \neg C(p \wedge \neg Ep)) \to Ep$	**A1**, **A4** **R1**, **R3**
(12)	$(\mathbf{P}^{\blacksquare}), (\mathbf{D}^{\blacksquare}), (\mathbf{F}^{\blacksquare}) \vdash_{\mathbf{BL}^+} Cp \to Ep$	$(10, 11)$
(13)	$(\mathbf{P}^{\blacksquare}), (\mathbf{D}^{\blacksquare}), (\mathbf{F}^{\blacksquare}) \vdash_{\mathbf{BL}^+} (\mathbf{P})$	(12), **R2**

$\square$

APPENDIX B

A CONSISTENCY RESULT

When formulating van Inwagen's argument [63] makes use of a somewhat stronger version of the the **PSR**:

(Full PSR) $\forall p(p \rightarrow Ep)$

Let **Full TSR** be **TSR** $\cup$ {**Full PSR**}. Then we can show the following:

Proposition 2.1. Full TSR $\vdash_{\mathbf{BL^+}} \bot$

Proof. The conjunction of all truths is true and so has a sufficient reason t. By **Factivity** t is also a truth. Applying **Distribution** gives us that $t < t$ violating **Irreflexivity**. □

Interestingly, **Full TSR** without **Distribution**, however, is consistent. In fact we can show something a bit stronger. First we require that explanation be transitive:

(Transitivity) $\forall p \forall q \forall r(((p < q) \wedge (q < r)) \rightarrow p < r)$

Let the *basic theory of sufficient reason* **BTSR** be the set comprised of **Full PSR**, **Irreflexivity**, **Transitivity**, **Inheritance**, **Truth** and **Sufficiency**. Then we can show the following theorem:

Theorem 2.2. BTSR $\nvdash$ **Necessitarianism**

As an immediate corollary we also have that **BTSR** is consistent. We might take this as evidence for the thesis that it is **Distribution** that is to blame rather the indefinite extensibility of contingent truth.

To prove the theorem we will define a class of structures with the following properties: (i) **BL** is true in every member of this class, (ii) if $\varphi \to \psi$ and φ are true in a structure, ψ is true in that structure and (iii) **BTSR** is true in a structure that **Necessitarianism** is not. Exhibiting such a class suffices to prove theorem 2.2.

Let a *relational structure* be a triple $\langle W, R, E, \alpha\rangle$ such that W is a set, $\alpha \in W$, $R \subset W \times W$ and $E \subset \mathcal{P}(W) \times \mathcal{P}(W)$. An *assignment* function v maps propositional variables into $\mathcal{P}(W)$ and plural propositional variables into $\mathcal{P}(\mathcal{P}(W)$. A function $[\![\cdot]\!]^v$ from formulas of $\mathcal{L}_{HP}$ to subsets of W is defined as follows:

$$[\![p_i]\!]^v = v(p_i)$$

$$[\![p_i \prec pp_j]\!]^v = \{w \in W \mid v(p_i) \in v(pp_i)\}$$

$$[\![\bigwedge pp_i]\!]^v = \bigcap v(pp_i)$$

$$[\![\neg\varphi]\!]^v = W \setminus [\![\varphi]\!]^v$$

$$[\![\varphi < \psi]\!]^v = \{w \in W \mid [\![\varphi]\!]^v E [\![\psi]\!]^v\}$$

$$[\![\varphi \wedge \psi]\!]^v = [\![\varphi]\!]^v \cap [\![\psi]\!]^v$$

$$[\![\Box\varphi]\!]^v = \{w \in W \mid \forall w'(wRw' \to w' \in [\![\varphi]\!]^v)\}$$

$$[\![\forall p_i\varphi]\!]^v = \{w \in W \mid w \in [\![\varphi]\!]^{v[p_i/X]}, \forall X \subset W\}$$

$$[\![\forall pp_i\varphi]\!]^v = \{w \in W \mid w \in [\![\varphi]\!]^{v[pp_i/X]}, \forall X \subset \mathcal{P}W\}$$

Here $v[x/X]$ is the function like v except that it maps x to X. A formula φ is true on an assignment v in $\langle W, R, E, \alpha\rangle$ if $\alpha \in [\![\varphi]\!]^v$. A closed sentence is true in a relational structure if and only if it is true on some assignment. A theory is true in a relational structure if and only if each member of that theory is true in that relational structure.

It is not hard to verify that the class of relational structures has properties (i) and (ii) described above. Thus we need only show that it has property (iii).

Proposition 2.3. *It's not the case that* **Necessitarianism** *is true in every relational structure that* **BTSR** *is true in.*

Proof. Let ω be the first infinite von Neumann ordinal and let α and β be any distinct objects that are not in ω nor in $\mathcal{P}(\omega)$. We define a relational structure $\langle W, \sim, E, \alpha\rangle$ such that $W = \{\alpha, \beta\} \cup \omega$, $\sim$ is the equivalence relation corresponding to the partition $\{\{\alpha, \beta\}, \omega\}$ and E is the transitive closure of the smallest relation E^- satisfying the following principles:

1. For any $x \subset \omega$, $\langle \{\alpha\}, \{\alpha, \beta\} \cup x\rangle \in E^-$.
2. If $x \subset \omega$ is infinite then $\langle \{\alpha\}, \{\alpha\} \cup x\rangle \in E^-$.
3. If $x \subset \omega$ is finite, then $\langle \{\alpha\} \cup n, \{\alpha\} \cup x\rangle \in E^-$, where $n = \sup x + 1$.

Then **BTSR** is true in $\langle W, \sim, E, \alpha\rangle$ but **Necessitarianism** is not[1]

□

Let me end briefly with some philosophical remarks. One way of thinking about the above relational structure is this. Here α is the actual world and β a possible world. The finite ordinals represent the space of impossible worlds. Thus the model requires that propositions be individuated more finely than necessary equivalence. Nevertheless, it is still a *Boolean* model of propositional granularity since the higher order quantifiers are interpreted over (complete and atomic) Boolean algebras. Thus the above construction shows that the theory of sufficient reasons without distribution is consistent even when interpreted over a relatively coarse grained conception of propositions.

[1]One should be careful in thinking of this model. The finite ordinals $n < \omega$ are worlds in the model. But since each finite ordinal n is the set of its predecessors, each finite ordinal is also a proposition of the model; thus each finite ordinal is also a plurality of propositions in the model.

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