BOOLEANISM AND BELIEF

Abstract. Booleanism is the thesis that the set of propositions forms a Boolean algebra under the operations of conjunction and negation. If Booleanism is true, then belief distributes over conjunction if and only if it is closed under entailment. Many authors have rejected Booleanism on this basis. I argue that we ought to reject the thesis that belief distributes over conjunction.

Introduction

It seems obvious that believing a conjunction requires believing each conjunct. Whenever one believes that it is Monday and it is morning, one thereby believes that it is Monday and believes that it is morning. The principle, which I'll call *Distribution*, that it is metaphysically necessary that one believes a conjunction only if one believes the conjuncts of that conjunction, is widely regarded as a truism.[1]

It is also widely held that belief is not closed under entailment. Only idealized agents in distant possibilities believe all the consequences of what they believe. For agents like us, failing to draw an inference is commonplace. Call the thesis that it is metaphysically necessary that one believes any proposition entailed by a proposition one believes *Closure* and its negation *NoClosure*.

It is somewhat surprising that Distribution, NoClosure and the thesis that propositions form a Boolean algebra under the operations of conjunction and negation, which I'll call

[1]The principle of Distribution is often mentioned more in passing than defended outright; but here is a partial list of places in which the principle is either endorsed or presented favorably: Soames (1987), Dorr (2011, p. 957), Williamson (2000, p. 280), Speaks (2006, p. 443).

Booleanism, are mutually inconsistent—all the more so given the wide acceptance of Booleanism in both philosophy and linguistics.[2,3]

The basic problem, in outline, is that in a Boolean theory of propositions, a proposition p entails a proposition q if and only if the conjunction of p and q is identical to p.[4] Thus if the proposition p entails q, and one believes p, then one believes the conjunction of p and q. And so if believing a conjunction requires believing its conjuncts, it follows that one believes q.

Some philosophers, following **?**, reject Booleanism in light of this problem. These authors often take Distribution and NoClosure to be premises in a powerful argument, the *Conflation Argument*, for the structured theory of propositions. A visible minority, most notably **?**, opt instead to accept Closure having been won over by the simplicity and strength of Booleanism and the purported obviousness of Distribution. Few philosophers reject Distribution. This paper is an exploration and (tentative) defense of the third way out—that of maintaining Booleanism and denying Distribution.

The methodology employed in this paper is abductive.[5] The package consisting of Booleanism and the negation of Distribution will be shown to have some distinctive benefits lacked by its competitors. This doesn't mean I'll dismiss the intuitive data in favor of Distribution. It just means that I don't think there is any better theory overall that accounts for this intuitive data. As I will argue, Booleanism can be combined with other principles related to Distribution that nevertheless explain our inclination to accept Distribution as obvious.

[2]Most authors in these areas accept Booleanism under the guise of the possible worlds theory of propositions. For further discussion see §2. The inconsistency only really holds given a further thesis about entailment. While this further thesis is often taken to be part of Booleanism, it could be divorced from it. However a restricted sort of Closure thesis does hold given Booleanism independently of one's views of entailment. For further discussion see §3.

[3]In what follows I will be solely concerned with *single premise* closure as opposed to multi-premise closure. One can easily combine Booleanism (an in particular the possible worlds theory of propositions) Distribution and the negation of multi-premise closure without inconsistency. The fragmentalists provide one example of this sort of view, see for instance See Stalnaker (1984), Lewis (1988) and also Braddon-Mitchell and Jackson (2007).

[4]For discussion of why we should accept this claim, see §3.

[5]In the sense of Williamson (2016).

§1 contains background on the language that I will be using to discuss questions of propositional fineness of grain. In §2 I state the theory Booleanism and provide some examples of the theory in the literature. §3 develops the conflation argument againts Booleanism. The main goal of this section is to explain why Booleanism and Distribution jointly entail Closure. §4 shows how this kind of argument generalizes to challenge not only Booleanism, but certain quasi-structured views of propositions as well. I will also discuss some responses on behalf of the Boolean to some of these more fine-grained theories of propositions. In §5 I argue that the package consisting of the full structured theory of propositions and Distribution is not as well motivated by conflation type arguments than it has sometimes been supposed. §6 addresses what I take to be some of the main arguments for Distribution and argues that the package consisting of the negation of Distribution and Booleanism is able to provide plausible responses to these arguments. Finally, I end in §7 with a direct argument against Distribution. The conclusion I think we should draw is the following: while it it is initially rather unintuitive to give up Distribution, there are some good reasons to hold that the package consisting of Booleanism and the negation of Distribution fares better than any theory of propositional fineness of grain conjoined with Distribution.

1. Propositional Fineness of Grain

Booleanism is a thesis about *propositional fineness of grain*. The goal of a theory of propositional fineness of grain is to discern some of the general principles governing the interaction of various operations on propositions and the identity of propositions. For instance a theory of propositional fineness of grain should answer basic questions like "Is the proposition that p identical to the proposition that $\neg\neg p$?" and "Is the proposition that $p \wedge (q \wedge r)$ identical to the proposition that $(p \wedge q) \wedge r$?" It is also desirable that the theory answer slightly more involved questions involving the metaphysics of application and generalization. For instance, what is the relationship between the proposition that x is such that it is blue and white and the proposition that x is blue and x is white? Are they one proposition or two? The former proposition results from applying a complex property, being blue and white, to x. The latter results from conjoining two propositions, one of which results from applying blue to x and the other applying white to x. To hold that they are identical is thus to hold that there is no *unique* decomposition of propositions into the properties they predicate and the individuals they are about.[6]

Booleanism is an exceedingly simple and powerful theory of propositional fineness of grain.[7] At first pass, it says that the proposition that φ is identical to the proposition that ψ whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic. The view is so named since by identifying classically equivalent propositions we ensure that they form a Boolean algebra with respect to the operations of conjunction and negation. The view immediately entails some positive answers to some of the sample questions raised above: each proposition is identical to its own double negation, and the operation of conjunction is associative.

Recent discussions of propositional fineness of grain have tended to take place in the context of a more general discussion of the granularity of properties, relations, properties of properties, relations between relations, and so on. In this context, Booleanism can be formulated as a much more general theory, governing not just propositions, but properties and relations of all types.[8] So for instance we might formulate Booleanism for properties of individuals by stating that the property of being such that φ is the property of being such that ψ whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic.

While my primary concern in this paper is *propositional* fineness of grain, it is my view that a fully abductive comparison of views of propositional fineness of grain is not possible without taking into account how they might interact with, or be extended to, views of the fineness of grain of properties and relations in addition. In order to formulate these views, we thus need a general framework in which competing hypotheses about the granularity of

[6]There are some missing steps in this argument. Frank Ramsey (1925), for instance, accepted the identity of these propositions and accepted that propositions were uniquely decomposable into basic constituents and operations. He used these as premises to infer that there simply couldn't be such a property as the property of being blue and white. Thus according to Ramsey, all properties are logically simple.

[7]It may also deserve the title of the first systematic theory of propositional fineness of grain. After all, Boole (1854) explicitly developed the theory of Boolean algebras as an attempt to abstract away from the idiosyncracies of language in order to capture "the laws of thought."

[8]See Dorr(2016).

such entities can be formulated. The framework adopted in this paper is the language of higher-order logic. In particular, I will use the language of simple relational type theory with lambda abstraction. The rest of this section is devoted to describing some of the details of this language, its interpretation, and the background principles stated in this language I will be assuming.

The language consists of a collection of *terms* classified into *types* or *syntactic categories*. There is one basic type, e, for the syntactic category of *individual constants*. All other types are generated from the basic type e as follows. For any natural number n, including the number 0, and any types $\tau_1, \ldots, \tau_n$, the n-tuple $\langle \tau_1, \ldots, \tau_n \rangle$ is a type. A term R is of this type when, for some given terms $A_1, \ldots, A_n$, with A_i of type τ_i, $\ulcorner R(A_1, \ldots, A_n) \urcorner$ is a term of type $\langle\rangle$. Intuitively, then, R is an n-place predicate whose *ith* argument must be of type τ_i. The type $\langle\rangle$ is the type for the syntactic category of *formulas*. Thus formulas are treated, in effect, as nullary predicates.

The language includes a typed collection of *constants*: these are the basic terms of the language.[9] Among the constants I will explicitly include a constant $\equiv_\tau$ of type $\langle \tau, \tau \rangle$ meant to express identity for the relevant type of entity. So for instance if φ is 'water is water' and ψ is 'water is $H_2 0$' we might express 'the proposition that water is water is the proposition that water is $H_2 0$' by $\ulcorner \varphi \equiv_{\langle\rangle} \psi \urcorner$. In addition to these "identification constants" the language will have Boolean constants: $\top$ of type $\langle\rangle$ for the "simple tautology", $\neg$ of type $\langle\langle\rangle\rangle$ for negation, $\wedge$ of type $\langle\langle\rangle, \langle\rangle\rangle$ for conjunction, and so on for the rest of the Boolean operators.[10] We also include a constants B of type $\langle\langle\rangle\rangle$ for *belief*; thus where φ stands for 'snow is white', $\ulcorner B\varphi \urcorner$ stands for 'one believes that snow is white'. Further specification of the constants will be added as needed.[11]

[9]By which I mean that they are syntactically basic. It is a further requirement that they be interpreted by metaphysically basic entities.

[10]I'm being a bit hand wavy here about which Boolean operators are taken as basic. Given a Boolean theory of propositions, it turns out not to really matter all that much. But one can imagine more fine-grained theories that hold that the material conditional is truth functionally equivalent to a complex operator defined in terms of disjunction and negation, it is not strictly identical to this operation.

[11]In general I will write formulas in infix notation (e.g., $\wedge(A, B)$ will be written as $(A \wedge B)$.

The language also includes a countably infinite collection of variables of each type. If $x_1, \ldots, x_n$ is a list of variables of types $\sigma_1, \ldots, \sigma_n$ respectively, and φ is a formula, then $(\lambda x_1 \ldots x_n.\varphi)$ is a term of type $\langle \sigma_1, \ldots, \sigma_n \rangle$. Where φ is the formula 'x is blue', with x a variable of type e, we might pronounce $\ulcorner(\lambda x.\varphi)\urcorner$ as 'is an x such that x is blue'. So for instance, if t is the term 'the Pacific ocean', $\ulcorner(\lambda x.\varphi)t\urcorner$ represents, roughly, 'the Pacific ocean is an x such that x is blue', or more simply 'the Pacific ocean is such that is is blue'.

Finally, to express generality at each type there is included, among the constants, quantifiers $\forall_\sigma$ and $\exists_\sigma$ of type $\langle\langle\tau\rangle\rangle$. Quantifiers combine with predicates to form sentences. Thus where F is a predicate of type $\langle\sigma\rangle$, $\ulcorner\forall_\sigma F\urcorner$ is a term of type $\langle\rangle$. In this language, quantifiers do not do the double work of expressing generality *and* binding variables. Variable binding is done by the lambda terms; generality is achieved by the quantifiers. So that the notation looks more familiar, however, we will often write $\forall_\sigma(\lambda x.\varphi)$ as $\forall_\sigma x\varphi$ (similarly $\exists_\sigma(\lambda x, \varphi)$ will be written $\exists x\varphi$).

I am going to informally speak of propositions, properties and relations in English in order to communicate claims whose desired content are those expressed by higher-order generalizations. So for instance, when I say in English "For any proposition, if one believes that propositions, one believes that one believes it", the claim I am intending to communicate is that $\forall p(Bp \to BBp)$. But I do not think we should impose an understanding of higher-order logic via translation into English, but rather adopt it as a framework in which close analogue's of these questions can be precisely formulated.[12] By doing so we are able to avoid scholastic debates about whether terms like 'proposition' and 'property' admit multiple interpretations, in addition to distracting set theoretic issues concerning which collections of propositions form a set. That being said, I also do not want to impose an understanding of such quantification that rules out a first-order interpretation; perhaps ultimately all quantification is first-order quantification. In this paper I will largely remain neutral on this question.

[12]For a defense of taking higher-order quantifiers as primitive see Prior (1971), Williamson (2003) and Williamson (2013, Section 5.9).

One might worry that we have, in moving to a higher-order setting, somehow lost contact with the initial puzzle motivating this paper, which was supposed to concern *propositional* fineness of grain. This is a mistake. In general, theories of propositional fineness of grain are interesting because of the consequences they have for the non-propositional realm. For instance, oftentimes authors will reject the possible worlds conception of propositions on the grounds that it entails that one believes that $2+2=4$ if and only if one believes that $3+3=6$. This consequence does not explicitly concern propositions, and can be easily formulated in our higher-order language. Moreover, we can formulate specific theories in our higher-order language that have this as a consequence, and so can evaluate, in the higher-order setting, whether this consequence alone is deserving of rejection. More generally, we can often find higher-order replacements of first-order theories that have the same, or near enough, consequences for our non-propositional discourse. Since these *consequences* are usually how theories of propositions are evaluated, we do just as well by working in a higher-order setting.[13]

Unless indicated otherwise, I'll assume that all classical reasoning is sound in this language (e.g., modus ponens, all classical tautologies as well as standard classical principles governing the quantifiers). I'm also going to frequently appeal to the following principles governing identity of properties, propositions and relations.[14]

Ref: $F \equiv_\sigma G$.

LL: $F \equiv_\sigma G \to (\chi \to \chi[F/G])$, where $\chi[F/G]$ is obtained from χ by replacing one or more occurrences of F with G such that no variables free in F or G are bound in φ or $\varphi[F/G]$.

The principle **LL** is controversial given the presence of the belief operator. For instance, many authors might want to maintain that (1) while denying (2):

(1) All bachelors are bachelors $\equiv$ all bachelors are unmarried males.

[13] This seems to me correct when the question at issue is about grain as opposed to ontology. There are of course lots of interesting debates about the *ontology* of propositions some of which cannot be formulated in a higher-order setting.

[14] See Dorr (2016) for further discussion.

(2) B(All bachelors are bachelors)$\leftrightarrow$ B(all bachelors are unmarried males).

A lot of work has been done detailing views on belief, propositions, and the basic logic governing identity on which (1) does not entail (2). But I have to admit that I find the entailment very compelling. If the proposition that all bachelors are bachelors *just is* the proposition that all bachelors are unmarried males, then it seems to me one must believe that all bachelors are bachelors if and only if all bachelors are unmarried males given the somewhat widespread assumption that B expresses a property of propositions.[15]

This paper investigates the interaction of the logic of belief and the theories of propositional fineness of grain under the assumption that the property expressed by B is transparent. This seems appropriate to me for the following reason. As I read things, the standard "Russellian" picture of propositions of the sort defended by Soames is one on which the operator B is transparent. Indeed, it is a crucial assumption of the argument of Soames (1987) against coarse grained views like Booleanism that propositional attitudes *do not* create opaque contexts in the way that, for instance, Fregeans argue that they do. Since this paper constitutes something of a response to the Russellians, it seems to me appropriate to grant some of their background views. By endorsing **LL**, I am only making things harder *for my own position*. What will turn out to be distinctive to my approach is the manner in which I will assess the bearing of principles governing belief on propositional identity. The general methodology for those accepting **LL** has been to use intuitions about the truth values of belief ascriptions in order to arrive at very fine-grained views of propositions. This paper takes a slightly more removed perspective of propositional attitude psychology. Instead of focusing solely on which belief ascriptions seem true and which do not, I want to focus on the overall theoretical benefits of adding various psychological generalizations (in particular Distribution), to particular theories of propositional fineness of grain, and then evaluating the respective theories in terms of their overall virtues. I hope to convince the reader that the purported benefits of structured view of propositions with respect to the psychological

[15]For instance, many authors accept 'believes' expresses a relation between individuals and propositions. If that is correct then 'one believes' should express a property of propositions: the property one gets by "plugging up" the first slot in the believes relation.

data we'll be looking at is actually the result of a weakness in the overall theory, a weakness that it does not clearly make up for by accomodating the psychological data.

2. Booleanism

2.1. The basic theory.

The theory Booleanism can be axiomatized by the following schema.

Schematic Propositional Booleanism: $\varphi \equiv \psi$, whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic.[16]

In general I'll take the instances of a schema to be closed under generalization, so that if φ is an instance of a schema, so is $\ulcorner \forall p \varphi \urcorner$ for any propositional variable p. Thus, since $p \leftrightarrow \neg\neg p$ is a theorem of classical propositional logic, $\forall p\, p \equiv \neg\neg p$ is a theorem of **Schematic Propositional Booleanism**.

The theory admits of a somewhat more laborious definition that may nevertheless aid intuition.

Propositional Booleanism:

Identity Laws	$p \wedge \top \equiv p$	$p \vee \bot \equiv p$
Complement Laws	$p \wedge \neg p \equiv \bot$	$p \vee \neg p \equiv \top$
Commutative Laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

The laws are understood to be implicitly universally quantified. Here $\bot$ abbreviates $\neg\top$. Clearly each of the above laws can be obtained as an instance of **Schematic Booleanism**. The fact that each instance of **Schematic Propositional Booleanism** can be proven from the laws of **Propositional Booleanism** follows from the fact that (i) the above laws axiomatize the class of Boolean algebras, and (ii) classical logic is sound and complete with respect to its Boolean valued semantics.[17] Thus going forward I will simply talk of **Booleanism** for the theory corresponding to these two equivalent descriptions.

[16]This formulation of the view is a special case of that given by Dorr (2016). At the end of this section I will briefly discuss the full formulation of the view.

[17]Halmos (1963) states (i) without proof, but I believe it was originally shown in Huntington (1904).

2.2. **Examples.** Philosophers tend to be more familiar with particular examples of Booleanism rather than the theory itself. The simplest example of a Boolean theory is *extensionalism.*

Extensionalism: $p \leftrightarrow q \rightarrow p \equiv q$

According to this theory, materially equivalent propositions are identical. Since any proposition is either materially equivalent to $\top$ or materially equivalent to $\bot$, this theory entails that every proposition is either $\top$ or $\bot$:

$$\forall p(p \equiv \top \vee p \equiv \bot)$$

Thus according to this view every proposition is either a logical truth or a logical falsity. We might intuitively think of the view as stating that there are exactly two propositions, the true and the false.[18] The operation of negation takes the true to the false and the false to the true. The operation of conjunction takes a pair of propositions to the true proposition if they are the true propositions; otherwise it takes them to the false proposition. Other Boolean operations are given the obvious truth functional readings.

This model is not a very good model of propositions. Since it is impossible that something be distinct from itself, necessary that everything be identical to itself and contingent that something talk to itself, there should be at least three propositions: an impossible one, a necessary one and a contingent one. More precisely, where $\Box$ is a normal modal operator expressing metaphysical necessity, extensionalism entails *necessitarianism*:

$$\forall p(p \rightarrow \Box p)$$

To see this suppose that p. Then $p \leftrightarrow \top$ and so, by **Extensionalism**, $p \equiv \top$. Under the assumption that $\Box$ is normal, we can infer $\Box\top$. Hence by **LL**, we can infer $\Box p$. Since p was arbitary we infer $\forall p(p \rightarrow \Box p)$.[19]

[18] The theory that there is only one proposition is inconsistent in the sort of classical theory we are working. If $\forall p(p \equiv \top)$ then $\neg\top \equiv \top$. And so since $\top \leftrightarrow \top$, we could infer $\top \leftrightarrow \neg\top$.

[19] Extensionalist tend to respond to these sorts of arguments by distinguishing between propositions and their *presentations*. There are many different approaches here, depending on what one takes presentations to be: Fregean senses, sentences etc. These views respond to arguments like the above by holding that there is an equivocation in the application of **LL**: $\varphi \equiv \psi$ does not imply that $\Box\varphi \leftrightarrow \Box\psi$ because φ and ψ denote

Extensionalism entails **Booleanism**, but the converse is false.[20] A more interesting example is *intensionalism*.

Intensionalism: $\Box(p \leftrightarrow q) \to p \equiv q$.

Here $\Box$ should be interpreted as expressing metaphysical necessity. Under the assumptions that $\Box$ is normal and the background logic is classical, **Intensionalism** entails **Booleanism**.[21]

The theory **Intensionalism** is validated by, for instance, the theory that identifies propositions with sets of possible worlds, though the latter theory has a richer structure than the former. For instance, the possible worlds theory of propositions is usually taken to include the assumption that any arbitrary set of worlds is a proposition. In particular singletons of worlds are propositions. But the singletons of worlds have quite distinctive behavior: for any proposition they either entail that proposition or entail its negation. Thus the possible worlds theory validates the following principle:

$$\exists q\big(q \wedge \forall p\big(\Box(q \to p) \vee \Box(q \to \neg p)\big)\big)$$

Intensionalism itself, however, can consistently be combined with the negation of this principle.

If truth and necessary truth coincide in the language of pure mathematics, **Intensionalism** entails that there are only two propositions expressible in the language of pure mathematics. This is often pressed as a major objection to **Intensionalism**. While fully evaluating this argument against **Intensionalism** would take us too far astray, let me briefly sketch a response before moving on.[22] Platonism is either true it is not true. If it is true, then there

different things in the context of $\Box\varphi \leftrightarrow \Box\psi$ than they do in the context of $\varphi \equiv \psi$ (roughly, there are some subtle differences between the views that I am glossing over).

[20]Quine (1955) gives a perplexing putative proof that **Booleanism** entails **Extensionalism**. I'm not exactly sure what is happening there, but it *seems* to me that Quine's argument is a sort of slingshot argument. In any case, it is exceedingly simple to come up with models in which **Booleanism** is true and **Extensionalism** is false, so in the present framework they are clearly distinct views.

[21]Suppose our background modal logic is normal, it is a theorem that $\Box(\varphi \leftrightarrow \psi)$ whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic. Thus in the theory **Intensionalism** it immediately follows that $\varphi \equiv \psi$ is a theorem.

[22]One reason it is important to at least indicate a response here is that, as mentioned below, some authors have suggested that there is a strong argument that takes us from **Booleanism** to **Intensionalism**. Since I accept the former I was to point out what I might says if shown to be committed to the latter.

really are mathematical objects, and they really have the mathematical properties that our true mathematical theories ascribe to them. But assuming platonism is true, it's hard to see how some mathematical properties and relations could fail to be fundamental.[23] What metaphysical analysis of set membership is available that doesn't make recourse to other purely mathematical relations or operations on sets? Supposing that *is* correct, there seems to be good reason to reject the necessity of mathematics, since fundamental properties, as they are often understood, can be freely recombined (within limits). For instance if *being a set* is fundamental, plausibly the actual sets could fail to be sets. To say otherwise is to commit oneself to the claim that large swaths of mathematics consists of *brute necessities*.

Alternatively if Platonism is not true, then all of our mathematical theories are false on their straightforward readings anyways. This means that in order to properly capture mathematical practice we already need to give a sort of revisionary account of what mathematicians are up to. Given this revisionary account it's no longer clear that we should hold mathematical propositions to be necessary. For instance, if mathematical propositions implicitly are assertions about a useful and contingently existing fiction, it shouldn't be obvious that mathematical propositions are necessary. Or if mathematical propositions are *really* propositions concerning what is derivable from what in what formal systems, then insofar as sentences and the like are contingently existing, perhaps so too are all mathematical propositions.

Intensionalism entails **Booleanism**, but the converse is false. Some philosophers think that the temporal can vary freely of the modal, so that what is possible today will be impossible tomorrow.[24] Such theories invalidate **Intensionalism** but not **Booleanism**. To see this let A be the normal tense operator "it is always the case that…".[25] Suppose that **Intensionalism** is true. We will show that $\forall p(\Box p \to Ap)$. Suppose that $\Box p$. Thus $\Box(p \leftrightarrow \top)$.

[23]Two possible challenges to this might be the theory developed by Maddy (1990) and the theory developed by Lewis (1991, 1993), both of which grant the real existence of sets but provide something like a theory in which they are not fundamental.
[24]This sort of view is predicted by the logics of tense outlined in both Kaplan (1989) and Fine (1977b) for instance.
[25]See Burgess (2002) for background on tense logic.

By **Intensionalism**, $p \equiv \top$. Since A is a normal modal operator, $A\top$. And so by **LL** Ap. Thus $\forall p(\Box p \rightarrow Ap)$. So **Intensionalism** entails, given plausible background assumptions, that what's necessary is always true.[26] But the thesis that modal status of propositions changes is clearly compatible with Booleanism. For instance, instead of requiring that necessary propositions coincide, we might instead require that propositions that are necessarily always materially equivalent are identical:

$$\Box A(p \leftrightarrow q) \rightarrow p \equiv q$$

If we permit cases in which something is necessary but not necessarily always true, this theory diverges from **Intensionalism**.

The above discussion shows that that Booleanism is consistent with an extensional theory of propositions, an intensional theory of propositions and a hyperintensional theory of propositions. In this paper I am going to remain neutral on these more specific theories. I will simply work in the theory Booleanism itself, without any added axioms. That being said, it is worth noting that Booleanism on its own predicts something very close to intensionalism. Consider the property of being identical to the tautologous proposition $\top$:

$$L := \lambda p\, p \equiv \top$$

It is a consequence of **Booleanism** that propositions equivalent according to L are identical:[27]

Boolean Equivalence: $L(\varphi \leftrightarrow \psi) \rightarrow \varphi \equiv \psi$

Bacon (2018) shows that given plausible background assumptions, **Booleanism** predicts that L is the broadest necessity operator.[28] One might conclude from this that L would have

[26] I don't mean to suggest here that the tense operator A is the best translation of ordinary claims in English involving the word 'always'. King (2007) has pretty forcefully argued against this view. I am just using this view about tense to provide a possible example of a Boolean theory that rejects **Intensionalism**.
[27] In general I will be supposing that $\varphi \leftrightarrow \psi$ when φ and ψ are $\beta\eta$-equivalent. Thus $L\varphi \leftrightarrow \varphi \equiv \top$.
[28] For instance we could explicitly add the schemas $L(\varphi \equiv \varphi)$ and $L(\varphi \equiv \psi \rightarrow (\chi \rightarrow \chi[\varphi/\psi])$. See also Cresswell (1965) and Seszko (1975) for proposals of defining necessity in terms of identity.

to be metaphysical necessity since, as it is often conceived, metaphysical necessity is the broadest notion of necessity.[29] This would then collapse **Booleanism** into **Intensionalism**.

There are several ways to resist the argument. Consider the property of being *counterfactually necessary*:

$$\blacksquare := \lambda p. \neg p \mathrel{\Box\!\!\rightarrow} \bot$$

Daniel Nolan (1997) has provided intuitive examples which appear to show that $\Box$ is not broader than $\blacksquare$.[30] That is, he has given specific examples that support the generalization:

$$\neg \forall p (\Box p \rightarrow \blacksquare p)$$

If that's right, then $\Box$ is not the broadest necessity and so $\Box \not\equiv_{\langle\langle\rangle\rangle} \blacksquare$.

This argument is slightly unstable in the current context, however. The general intuitive judgments that tell against $\Box$ as the broadest necessity cited by Nolan (1997, 2011) also look to tell against L as the broadest necessity. And so since L is provably the broadest necessity given **Booleanism**, Nolan's cases look to tell against **Booleanism**.[31] One might reply by finding a principled distinction between the cases that tell against $\Box$ as the broadest necessity and those that tell against L as the broadest necessity. I think there are some grounds for this kind of view. But for our purposes it suffices to note that there are other avenues for responding to the charge that **Booleanism** entails **Intensionalism**.

In particular, the notion of metaphysical necessity was not introduced *merely* as the broadest necessity, but was also introduced by Kripke (1980) with explicit paradigm cases. If we want our notion of metaphysical necessity to respect these paradigm examples, this may lead to an interpretation of metaphysical necessity in which it is not the broadest necessity, though perhaps the broadest necessity in some restricted class of "objective" necessities.[32]

[29]Dorr(2016) conditionally draws this conclusion; for further discussion of this conception of necessity see McFetridge (1990) and Hale (1996).
[30]See Nolan (2011) for further criticism of the idea that metaphysical necessity is the broadest necessity.
[31]One assumption we might give up here is that $\blacksquare$ *is* a necessity operator. This seems natural given the sorts of examples discussed in Nolan (1997): for instance we might want to simply reject the thesis that $\blacksquare\top$ (i.e., $\bot \mathrel{\Box\!\!\rightarrow} \bot$).
[32]See Williamson (2016) for a view like this.

This is of course a difficult issue. For reasons of space I am going to set it aside for later discussion.

2.3. **Structure and ontology.** Booleanism itself is silent on the ontological category of propositions. It is consistent with the view that propositions are sets,[33] that they are properties,[34] that they are states of affairs,[35] that they are events or acts,[36] that they are facts,[37] and that they are their own *sui generis* category of thing.[38] Let an *ontological theory* of propositions be a theory that says which ontological category propositions belong to. A *structural theory* of propositions is a theory that tells us what the laws are that govern the interaction of identity and various operations on propositions. Using the familiar jargon, a structural theory of propositions is a theory of the granularity of propositions, it tells us how fine grained propositions are. Booleanism is a structural theory of propositions—a theory of propositional fineness of grain. It is not an ontological theory of propositions.

Ontological theories *on their own* will not in general constrain structural theories all that much. It is often one's theory of the operations on propositions rather than the ontology of propositions which provides the structural core of the theory. The possible worlds conception of propositions is not coarse grained because propositions are identified with sets of possible worlds but rather because negation is identified with complementation, conjunction is identified with intersection and being necessary is identified with being the set of all worlds.

There has been a tendency to raise the significance of ontological questions in philosophy over more structural metaphysical questions like questions of propositional fineness of grain. Some will object that **Booleanism** is merely a schema for a theory rather than a theory of propositions itself. A fully adequate theory would tells us what propositions *are*. I think this is a misguided criticism. It would be similar to objecting to ZFC as a theory of sets on the grounds that it fails to tell us what sets are. What is important about these theories is the

[33]See Lewis 1980.
[34]See Speaks 2014a, Lewis 1983, van Inwagen 2004.
[35]Chisholm 1970.
[36]See Hanks (2011, 2015) and Soames (2014, 2015).
[37]King 2007 and 2014.
[38]Merricks 2015.

wealth of significant information they give us about the relevant objects. It is this wealth of information that ZFC gives us about sets that makes it a mathematically significant theory. Similarly, I suggest, it is the wealth of information about propositions that makes Booleanism a metaphysically significant theory.

Another reason why this objection seems to me misguided is that it presupposes an account of higher-order quantification according to which it is reducible to first-order quantification. I have been pronouncing in English claims like $\exists pBp$ and $p \equiv q$ as "there is a proposition that one believes" and "the proposition that p is identical to the proposition that q." Propositions thus become objects in the domain of individuals; it is then natural, though not inevitable, to ask after their intrinsic properties. But if we instead take $p \equiv q$ to mean something like "for it to be the case that p is for it to be the case that q" and then take the quantifiers to be irreducibly higher-order, the official formulation of Booleanism will make no reference to any abstract objects called 'propositions'. In this kind of framework it is not at all obvious what an account of the "ontology" or 'intrinsic properties" of propositions would amount to.

The strength of the theory becomes even more apparent when extended not just to the propositional case, but to cover properties and relations in addition.

> **Full Schematic Booleanism:** $\lambda x_1 \dots x_n.\varphi \equiv \lambda x_1 \dots x_n.\psi$, whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic.[39]

So extended, the theory can be seen to embody a fully general account of the structure of propositions, properties, relations, and higher-order properties and relations. Like the Boolean theory of propositions, the extended theory admits of an equivalent non-schematic formulation.

> **Full Booleanism:**

[39] This formulation is taken from Dorr (2016).

Identity Laws

$\lambda p.p \wedge \top \equiv \lambda p.p$	$\lambda p.p \vee \bot \equiv \lambda p.p$

Complement Laws

$\lambda pq.p \wedge \neg p \equiv \lambda pq.\bot$	$\lambda pq.p \vee \neg p \equiv \lambda pq.\top$

Commutative Laws

$\lambda pq.p \wedge q \equiv \lambda pq.q \wedge p$	$\lambda pq.p \vee q \equiv \lambda pq.q \vee p$

Distributive Laws

$\lambda pqr.p \wedge (q \vee r) \equiv \lambda pqr.(p \wedge q) \vee (p \wedge r)$	$\lambda pqr.p \vee (q \wedge r) \equiv \lambda pqr(p \vee q) \wedge (p \vee r)$

This formulation requires that terms like $\lambda pq.\top$ be well formed, however. Some developments of the language of higher-order will require that p be free in φ in order for $\lambda p.\varphi$ to be a well-formed term of the language. The fact that **Booleanism** can be naturally extended this way to provide a strong theory of all intensional entities demonstrates some of the elegance and power of the theory. We will later see the contrast when it comes to attempts to flesh out the structured theory of propositions, which looks to work in certain special cases, but is difficult to state in full generality.

Summing up, we've seen that Booleanism is a structural theory of propositions that has received widespread endorsement, albeit in slightly different forms, over the course of the past one hundred years or so. It is consistent with a wide variety of hypotheses about the ontology of propositions, but is also naturally interpreted in a theory that takes on higher-order quantification as fundamental. In the higher-order setting the view is naturally seen as a special case of a much more general hypothesis concerning the structure of all propositions, properties and relations. In the next section we'll look at how the view interacts with various psychological generalizations.

3. Booleanism and The Conflation Argument

Soames (1987) puts forward an argument, well known among certain crowds of philosophers, that propositions cannot be identical to sets of truth supporting circumstances. The crux of this argument is that *if* propositions are sets of truth supporting circumstances, then certain plausible psychological generalizations carry commitment to certain implausible psychological generalizations. We might call the argument the "conflation argument" since the objectionable feature of the views in question is that they conflate the plausible with the implausible.

In what follows I will present a version of this sort of argument against **Booleanism**. In my view, the version presented here really gets to the heart of the problem. It is not theses concerning the ontology of propositions that matter to the argument; rather it is the consequences these ontological theses are often taken to have for one's account of entailment and propositional fineness of grain that matter. I will thus present a slightly pared down version that abstracts away from specific ontological hypotheses.

3.1. **Distribution and Boolean Closure.** Recall the principle of Distribution.[40]

Distribution: $\forall p \forall q \big(B(p \wedge q) \rightarrow (Bp \wedge Bq)\big)$

Distribution is intended as a psychological generalization about actual individuals. On its intended interpretation, it tells us that for any proposition p and proposition q, if one believes that p and q, then one believes that p and one believes that q. The 'one' in 'one believes that' is supposed to be interpreted generically. Thus if **Distribution** is true, anyone who believes that grass is green and snow is white believes that grass is green and believes that snow is white.

Oftentimes authors who investigate doxastic principles like **Distribution** are more concerned with normative statements to the effect that one *should* believe a conjunction only if one believes that conjuncts of that conjunction. This reading is not the one that will concern me. After all, it is at least somewhat plausible that one should believe all of the consequences of what they believe. If **Booleanism** takes us from a norm of distribution to a norm of closure, that is not any evidence against the theory if there is already a norm of

[40]Strictly speaking we should include an initial operator for metaphysical necessity in order to capture the modal dimension of the principle Distribution. But nothing much is added to the discussion by including modality. The topics I want to discuss are more cleanly presented without bringing in modal operators and so for now I'll focus on the extensional version of the principle.

closure to begin with. If a theory takes us from the psychological generalization, **Distribution**, to the psychological generalization that one actually does believe the consequences of what one believes, that looks like some evidence against the theory. The descriptive readings of the relevant principle will be assumed in what follows.

In order to formulate Closure in the object language, we need some way of formulating what it is for one proposition to entail another. One approach would be to just take entailment to be necessitation: p entails q when $\Box(p \to q)$. One benefit of Booleanism is that it provides us with a plausible notion of entailment that is free from the somewhat unclear notion of metaphysical necessity. In particular, we can say that a proposition p *entails* a proposition q if $L(p \to q)$ (i.e., $p \to q \equiv \top$). More precisely we can define an entailment operation on propositions as follows:

$$\leq := \lambda pq.L(p \to q)$$

Below I will give some further reasons for why we might take $\leq$ to be *the* entailment relation between propositions. Before doing that, I will show how the conflation argument can be run in this general setting with $\leq$ as our entailment relation.

We can formulate closure in the object language as follows.

Boolean Closure: $\forall p \forall q(p \leq q \land Bp \to Bq)$

Boolean Closure is a consequence of **Distribution** given **Booleanism**. To see this suppose that $p \leq q$ and Bp. It is a consequence of **Booleanism** that $p \leq q$ if and only if $p \land q \equiv p$. So from our assumptions and **LL**, it follows classically that $B(p \land q)$. And so from **Distribution**, it follows that Bq. Thus **Boolean Closure** holds. Since **Boolean Closure** obviously entails **Distribution**, given **Booleanism**, the result is that **Booleanism** conflates the two principles: from the perspective of the Boolean, closure under consequence is the same as distribution over conjunction.

Note that if ψ is a consequence of φ in classical propositional logic, then $\varphi \to \psi \leftrightarrow \top$ is a theorem of classical propositional logic. Thus **Booleanism** implies that $\varphi \leq \psi$. The result is that whenever ψ is a consequence of φ in classical propositional logic, **Booleanism** together with **Distribution** *proves* $B\varphi \to B\psi$. But it is important to recognize that the conflation argument is supposed to extend further than merely the fact that belief is closed under the consequence relation of classical propositional logic: as I see it, it is an argument that belief is closed under *any* entailment.

To bring out this perspective on the argument it might be helpful to first consider how the argument works in a certain special case. The argument is naturally seen as a generalization of the more familiar argument against the possible worlds conception of propositions. As this view is commonly developed, a proposition X is taken to *entail* a proposition Y, if X is a subset of Y. But, so the objection goes, a proposition X is a subset of a proposition Y if and only if the intersection of X and Y is identical to X. Hence if one believes X, and $X \subseteq Y$, one believes $X \cap Y$. Thus given Distribution, one believes Y. Given the definition of entailment as subset, this guarantees that belief is closed under classical propositional consequence. But it shows much more given our background knowledge. For instance, it is often thought that mathematical truths are necessary if true. Thus on this view, every proposition entails a mathematical proposition: so if X is a proposition that Y is a mathematical proposition, $X \cap Y = X$.

The notion of entailment as subset makes reference to membership and so is tied down by background set theoretic notions. But as noted, the condition that X is a subset of Y is equivalent to some purely algebraic conditions: for instance that $X \cap Y = X$ *and* that $X \to Y = W$. So the first step of translating this more familiar case to the more abstract setting is to replace our definition of entailment as subset with one of these algebraic definitions. So for instance suppose we say that p entails q if $p \wedge q \equiv p$. In the context of **Intensionalism**, p entails q, in this sense, if and only if $\Box(p \to q)$; in turn, $\Box(p \to q)$ if and only if $p \leq q$ (i.e. $p \to q \equiv \top$). Thus given the more restricted theory of **Intentionalism**, the broad notion of entailment $\leq$ *just is* the familiar strict entailment stated in terms of $\Box$. So, like **Booleanism**, **Intensionalism** with **Distribution** *entails* that belief is closed under classical propositional consequence. But together with our background knowledge, it entails much more. For instance, many of us will be inclined to accept:

$$\Box(Fa \to \exists x Fx)$$

Thus **Intensionalism** entails, together with our background knowledge

$$Fa \to \exists x Fx \equiv \top$$

It also entails

$$Fa \wedge \exists x Fx \equiv Fa$$

Similarly, many of us accept

$$\Box(\Box p \to p)$$

Thus **Intensionalism** entails with our background knowledge that

$$\Box p \wedge p \equiv \Box p$$

In other words, the operator $\leq$ in the setting of **Intensionalism** is a *global* entailment relation: it does not represent entailment in some restricted system or anything like that.

I think that when working in the more general setting of **Booleanism** we should continue to treat $\leq$ as a global entailment relation, just like it was in the setting of **Intensionalism**. Of course we have to be *somewhat* more strict about what entails what in this setting. For instance, we might deny that all necessary truths entail all other necessary truths. But there is no reason to give up, and I think plenty reason to accept, statements like $Fa \leq \exists x Fx$. Indeed I think that our ability to take $\leq$ as the relation of propositional entailment is one of the selling points of a coarse grained view of propositions like **Booleanism**.

First, note that the definition of entailment for propositions can be easily generalized to relations of arbitrary types:

$$R \leq_{\langle \tau_1, \ldots, \tau_n \rangle} S \coloneqq \lambda x_1 \ldots x_n.(R(x_1, \ldots, x_n) \leq S(x_1, \ldots, x_n))$$

This provides us with a notion of property entailment *that does not make use of any quantifiers*. Dorr (2014) has emphasized the importance of such quantifier-free notions of property of entailment for current debates over quantifier variance. In terms of them, we can investigate the logic of the quantifiers using only identity and the truth functional connectives. Since there is plausibly no variance in meaning in the truth functional connectives, nor in identity, across communities whose patterns of reasoning with respect to these operations are the same, this gives us a stable way to investigate whether two communities who reason similarly with the quantifier expressions pick out the same quantifier by doing so.

There is also something to be said for the fact that the relation $\leq$ is spelled out in purely extensional terms: we only used the Boolean operators $\rightarrow$ and $\top$ in addition to the identity operator $\equiv$. Since the definition of intensional notions using extensional resources is usually thought to be a benefit, the identification of the intensional notion of entailment with $\leq$ suggests itself on these grounds.

The relation $\leq$ also *behaves* like the entailment relation. For instance, supposing **Booleanism** is true, we get the following law:

$$(\chi \leq \varphi \wedge \psi) \leftrightarrow (\chi \leq \varphi \wedge \chi \leq \psi)$$

That is, a proposition entails a conjunction if and only if it entails both its conjuncts. And this principle in fact *uniquely* characterizes the conjunctive proposition. For suppose that $\chi \leq \chi' \leftrightarrow \chi \leq \varphi \wedge \chi \leq \psi$. Then $\chi \leq \chi' \leftrightarrow \chi \leq \varphi \wedge \psi$. Since every proposition entails itself we get that $\chi' \leq \varphi \wedge \psi$ and $\varphi \wedge \psi \leq \chi'$. Hence $L(\chi' \leftrightarrow \varphi \wedge \psi)$. So $\chi' \equiv \varphi \wedge \psi$.

Analogous principles hold for other Boolean connectives. Thus from the perspective of the Boolean, we have this relation, $\leq$, that has a simple algebraic characterization that looks to have many of the hallmarks of propositional entailment. This suggests to me that $\leq$ *is* propositional entailment.

This leaves us with the following, simple argument against Booleanism.

The Conflation Argument:

P1 Distribution is true.

P2 Closure is false.

P3 If Booleanism is true, then Distribution is true if and only if Closure is true.

C Therefore, Booleanism is false.

The justification for P3 is of course just the above proof, together with the further arguments that $\leq$ is the relation of propositional entailment. I won't have much to say about P2. For the purposes of this paper, I'm inclined to accept it without argument. Ultimately, my goal will be to cast doubt on P1. But my method of doing so will be rather indirect. What I want to show is that the theory **Booleanism** conjoined with the negation of **Distribution** is a better theory overall than more finegrained theories that validate **Distribution**.

3.2. **Independent Distribution and Independent Boolean Closure.** By examining the above proof, one sees that the problematic feature of **Booleanism** is that it permits a definition of entailment in *terms* of the binary connective $\wedge$. When the proposition that φ entails the proposition that ψ, believing that φ entails believing that $\varphi \wedge \psi$, since the proposition that φ *is* the proposition that $\varphi \wedge \psi$. And so given **Distribution**, believing that φ implies believing that ψ.

It has sometimes been suggested to me in conversation that the Boolean should, in response, reformulate **Distribution** so as to avoid these sorts of troubling cases. In particular, they should admit that in general **Distribution** is false, but accept it for those special cases in which the conjuncts are *independent*.

We'll write $\varphi \approx \psi$ if either $\varphi \leq \psi$ or $\psi \leq \varphi$. Thus $\varphi \approx \psi$ if and only if the proposition that φ entails the proposition that ψ or vice versa. Then the proposition that φ and the proposition that ψ are *independent* if $\neg\varphi \approx \psi$ (which we will abbreviate as $\varphi \not\approx \psi$). With this terminology in place we can formulate a restricted version of Distribution as follows.

Independent Distribution: $\forall p \forall q \big(p \not\approx q \to (B(p \wedge q) \to (Bp \wedge Bq))\big)$.

Independent Distribution is still able to predict a good deal of the cases we ordinarily observe and so is a natural fallback for the Boolean. But I think there are some good grounds for rejecting it. Just as **Distribution** implies **Boolean Closure**, **Independent Distribution** also implies some unwanted closure conditions on belief. The basic idea is that given **Independent Distribution** and **Booleanism**, we can show that whenever one believes that $\varphi \wedge \psi$, and φ and ψ are independent, one believes every consequence of ψ that is independent from φ in a certain restricted sense.

To spell this out precisely let's introduce a new relation $\prec$ of type $\langle\langle\rangle, \langle\rangle, \langle\rangle\rangle$ (i.e., a three place relation between propositions) defined by

$$\prec := (\lambda pqs.p \leq q \wedge (s \wedge p \equiv s \wedge q))$$

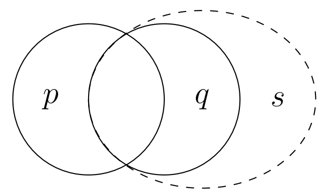
We will write $\prec (p, q, s)$ as $p \prec_s q$ and so the statement that $p \prec_s q$ is equivalent to $(p \leq q \wedge (s \wedge p \equiv s \wedge q))$. So $p \prec_s q$ if and only if the proposition that p entails the proposition that q, and conjoining the proposition that s with each proposition results in the same proposition. Metaphorically, p entails q and, from the point of view of s, p *is* q. We can then state the following closure condition on belief.

Independent Boolean Closure: $\forall p \forall q \forall s(p \not\approx q \wedge q \prec_p s) \to (B(p \wedge q) \to Bs)$

A lot of theories might prove every instance of Independent Boolean Closure by proving that the antecedent is false. **Booleanism** allows for many true instances of the antecedent, however. Moreover, given **Independent Distribution**, one can prove Independent **Boolean Closure** given **Booleanism**.

To see this suppose that $p \not\approx q$ and $q \prec_q s$. Suppose, moreover, that $B(p \wedge q)$. Then $p \wedge s \equiv p \wedge q$ by $q \prec_p s$ and so $B(p \wedge s)$ by **LL**. Thus to show that Bs it suffices to show that $p \not\approx s$. If $s \leq p$ then since $q \leq s$ it follows that $q \leq p$, contradicting the assumption that $p \not\approx q$. If $p \leq s$ then $p \wedge s \equiv p$. And so since $p \wedge s \equiv p \wedge q$ we have $p \equiv p \wedge q$ (i.e. $p \leq q$), contradicting the fact that $p \not\approx q$. Thus $p \not\approx s$ and so it follows that Bs.

The argument may seem a bit opaque. But the situation is rather simple. We can visualize the fact that $p \prec_q s$ in terms of the following Venn diagram.



The left circle represents the proposition that p and the right the proposition that q. The dotted oval represents the proposition that s. The conjunction of two propositions is represented by the area in which they intersect. That one proposition entails another is represented by the fact that one is contained within the other. Thus we have that p and q are independent, since neither contains the other. And we have that q entails a proposition s that does not intersect p in any new places. So if one believes oneself to be in the area in which p and q intersect, one believes oneself to be in the area in which p and s intersect. Thus if one believes $p \wedge q$, one believes $p \wedge s$ and so believes s given the principle of **Independent Distribution**.

It's not entirely clear to me how plausible or implausible the principle of **Independent Closure** is. But it looks like a rather awkward principle to commit oneself to. On this sort of view belief will end up behaving in rather odd ways in those circumstances in which p entails q independently of χ, even though there doesn't seem to be any other reason to suppose it would other than the fact that its a consequence of one restriction on **Distribution**. This suggests to me that the principle of **Independent Distribution** is not a natural fallback position for the **Boolean**.

4. The Conflation Argument and Quasi-Structured Views

In this section I show that arguments that are analogous to the Conflation Argument can be used to challenge views that take propositions to more closely reflect the sentences that express them. By considering these views we will get a better sense of the trade offs between psychological generalizations like **Distribution** and various theories of propositional fineness of grain.

4.1. **Aboutness, Subject Matter and Agglomerativism.** I will first consider a recent view of propositional granularity that has been considered by Jeremy Goodman.[41] According to this view, logically equivalent propositions can be distinguished in virtue of being *about* different individuals.

To get a sense for the motivation for this kind of view consider one of the standard objections to the possible worlds conception of propositions. Since it is necessary that $2+2=4$ and necessary that $3+3=6$, it is necessary that $2+2=4$ if and only if $3+3=6$. Thus, according to the possible worlds theory, the proposition that $2+2=4$ is identical to the proposition that $3+3=6$. Many have found this consequence implausible. But why? Some will say that it is because the propositions play different roles in thought. Others will say that it is because the propositions differ in their constituents. But the most natural thing to say, it seems to me, is simply that they seem to be about different things. The proposition that $2+2=4$ is about 2 and 4, not 3 and 6, and the proposition that $3+3=6$ is about 3 and 6, not 2 and 4.

Boolean theories of propositions have a difficult time capturing aboutness. Every instance of $\varphi \equiv \varphi \wedge (\psi \vee \neg\psi)$ comes out true on a Boolean theory. So if Booleanism is true

$$(2+2=4) \equiv ((2+2=4) \wedge ((3+3=6) \vee \neg(3+3=6)))$$

But intuitively, the proposition expressed on the left is about different individuals than the proposition expressed on the right. Agglomerativism takes these intuitions seriously: for many propositions p and q, p is not identical to $p \wedge (q \vee \neg q)$ since p and $p \wedge (q \vee \neg q)$ are about different individuals.

We could present the argument by introducing a relation About of type $\langle\langle\rangle, e\rangle$ and stipulating some general principles governing it. For instance, the following principles seem quite natural given the target notion of aboutness (using p and q as variables of type $\langle\rangle$ and x as a variable of type e):

[41]Goodman (2019) describes but does not endorse the view. But Goodman (2017b) suggests that he does endorse a view like this.

(i) $\forall p \forall q \forall x \big(\mathsf{About}(p \wedge q, x) \leftrightarrow \big(\mathsf{About}(p, x) \vee \mathsf{About}(q, x)\big)\big)$

(ii) $\forall p \forall q \forall x \big(\mathsf{About}(p \vee q, x) \leftrightarrow \big(\mathsf{About}(p, x) \vee \mathsf{About}(q, x)\big)\big)$

The combined effect of (i) and (ii) is that the conjunction or disjunction of two propositions is about whatever individuals its conjuncts or disjuncts are about. Given **Booleanism**, these principles entail that that any arbitrary proposition is about any arbitrary individual if some proposition is about that individual.

$$\forall x \forall p (\mathsf{About}(p, x) \rightarrow \forall q \mathsf{About}(q, x))$$

To see this suppose $\mathsf{About}(p, x)$. Let q be an arbitrary proposition. Then given **Booleanism** we have $q \equiv q \vee (p \wedge \neg p)$. By (i) $\mathsf{About}(p \wedge \neg p, x)$. And so by (ii), $\mathsf{About}(q \vee (p \wedge \neg p), x)$. Thus by **LL**, $\mathsf{About}(q, x)$.

The Boolean has several avenues of response. First it should be noted that there are some more sophisticated Boolean theories of aboutness, one's that wouldn't validate (i) or (ii), that are able to predict some of our intuitive judgments concerning what is about what.[42] A more flatfooted response is to insist that intuitions concerning what is about what ultimately rest on a confusion of properties of sentences and properties of propositions. It is sentences that can be about things. Propositions can only be said to be about this or that relative to a choice of presentation (for instance, a sentence expressing the proposition).

What sorts of restrictions on **Booleanism** are predicted by taking aboutness seriously? Obviously the identification $q \equiv q \vee (p \wedge \neg p)$ has got to go. But what else? Goodman (2019) provides an algebraic characterization of those identifications that can be preserved on this sort of view. I am going to provide an account that is inspired by Goodman's and that *I think* is equivalent, though I haven't confirmed this. Even so, the account I present seems to capture the background idea quite well.

[42] See Fine (1977a).

To state the theory we first need a preliminary definition. Where T is the set of terms of our language, a *content assignment* on T is a map from T into a bounded-join semilattice $\mathcal{S} = (S, \vee, 0)$.[43]

- $f(L) = 0$, whenever L is a logical constant (e.g., Boolean connectives and quantifiers).
- $f(R(A_1, \ldots, A_n)) = f(R) \vee f(A_1) \vee \cdots \vee f(A_n)$;
- $f(\lambda x_1, \ldots, x_n.\varphi)) = f(\varphi)$.

Intuitively we can think of S as consisting of sets of individuals such that $\emptyset$ is in S and whenever X and Y are in S, $X \cup Y$ is in S. The semilattice is then $(S, \cup, \emptyset)$. A content assignment is then simply an assignment of individuals to terms: the set of individuals the proposition, property or relation thereby expressed is about. Under this interpretation the conditions on content assignments are that $\top$ isn't about anything and the proposition that $R(A_1, \ldots, A_n)$ is about something if and only if R or A_i is about that thing, for some i; the property $(\lambda x_1 \ldots x_n.\varphi)$ is about whatever the proposition that φ is about.

Say that $\varphi \sim \psi$ if and only if $f(\varphi) = f(\psi)$ for every content assignment f. With these notions in place we formulate Agglomerativism as follows.

Agglomerativism: $\varphi \equiv \psi$, whenever $\varphi \leftrightarrow \psi$ is a tautology and $\varphi \sim \psi$.

Agglomerativism validates several of the Boolean laws we used to define **Booleanism**.[44]

Identity Laws	$p \wedge \top \equiv p$	$p \vee \bot = p$
Commutative Laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

Let's verify of couple of these just to get a feel for the view. First, note that $p \wedge \top \leftrightarrow p$ is a theorem of classical propositional logic. Thus to show (one half) of the identity laws, it suffices to show that $f(p \wedge \top) = p$, for any content assignment f. So let f be an arbitrary content assignment. Then $f(p \wedge \top) = f(\wedge) \vee f(p) \vee f(\top)$. Since $\wedge$ and $\top$ are logical constants,

[43]That is, a tuple $(S, \vee, 0)$ equipped with an associative and commutative binary operation $\vee : S \times \to S$ and a distinguished element $0 \in S$, that is such that, for any element $s \in S$, $s \vee s = s$, and $s \vee 0 = s = 0 \vee s$.
[44]See Goodman (2019) for further background and discussion.

$f(\wedge) = 0 = f(\top)$. Thus $f(p \wedge \top) = 0 \vee f(p) \vee 0 = f(p)$. For the DISTRIBUTIVE LAWS it suffices to note that in a bounded join-semilattice, the join operation $\vee$ is *idempotent*: $x \vee x = x$ for any element. Thus the extra occurence of a variable p on the left hand side of the identity symbol in the DISTRIBUTIVE LAWS won't make any difference to the content assigned by a content assignment.

The theory is weaker than **Booleanism** since the COMPLEMENT LAWS are not theorems of **Agglomerativism**. (For example, consider any content assignment f in which $f(p) \neq 0$ for some propositional variable p. Then $f(p \vee \neg p) \neq 0 = f(\top)$. Thus $p \vee \neg p \not\sim \top$.) Other laws that will fail are *absorption laws*, like $p \equiv p \wedge (p \vee q)$, for obvious reasons.

In order to study the interaction between various closure conditions on belief and **Distribution**, we again need to find some way to formulate claims about propositional entailment against the backdrop of an Agglomerative view of propositions. Like **Booleanism**, **Agglomerativism** admits of a natural, algebraic notion of entailment, though it differs slightly from the Boolean theory.

The relation $\leq$ is no longer a plausible analysis of entailment against the backdrop of a view that takes aboutness seriously. We can see that this is so intuitively. For suppose that $\varphi \to \psi \equiv \top$. Then since $\top$ is not about anything, neither is $\varphi \to \psi$. But this entails that neither φ not ψ is about anything. Thus $\varphi \leq \psi$ only if neither φ nor ψ is about anything.

In the Boolean setting we were able to define a relation of propositional entailment that was such that, whenever ψ was a consequence of φ in classical propositional logic, **Booleanism** proved that the proposition that φ entailed the proposition that ψ. Thus **Booleanism**, in a sense, had the power to see classical propositional entailments among propositions. In the agglomerative setting, there turns out to be a relation that plays a similar role. In particular, instead of requiring that $\varphi \to \psi$ be identical to the purely qualitative tautology $\top$, we require that $\varphi \to \psi$ be identical to a tautology that, intuitively, is about the same things as $\varphi \to \psi$.

The following seems like a natural choice:[45]

$$\varphi \preceq \psi := \varphi \to \psi \equiv (\varphi \to \psi) \to (\varphi \to \psi)$$

The relations $\leq$ and $\preceq$ agree assuming **Booleanism**, but differ in the Agglomerative setting. In the Agglomerative setting, $\preceq$ plays the natural role of entailment. For instance, just like in the Boolean setting, $\preceq$ interacts with the Boolean operations just as one would expect. On the one hand, it is not hard to see that if ψ is a classical propositional consequence of φ, then according to **Agglomeratvism** $\varphi \preceq \psi$. Just as in the Boolean setting though, it can be quite useful to allow for cases in which $\varphi \preceq \psi$ even when ψ is not a classical propositional consequence of φ. The relation $\preceq$ provides the Agglomerativist a simple algebraic characterization of propositional entailment.

Unlike **Booleanism**, **Agglomerativism** together with **Distribution** does not predict the analogous closure condition:

Agglomerative Closure: $\forall p \forall q(p \preceq q \to (Bp \to Bq)$.

For instance, $p \preceq q \vee \neg q$ is a theorem of **Agglomerativism**; but since one cannot infer from this that $p \wedge (q \vee \neg q) \equiv p$, there is no obvious way to get $B(q \vee \neg q)$ from Bp like there was in the Boolean setting. But there are some weaker principles concerning the closure of belief that **Agglomerativism** and **Distribution** *do* predict. In particular, I will argue that in an Agglomerative setting, one believes all of the *qualitative* consequences of what one believes, supposing that **Distribution** is true. In order to make this precise we first need to introduce the notion of a qualitative proposition.

Intuitively, a proposition is *qualitative* if it is not about any particular individual. It is *haecceitistic* if it is not qualitative.[46] The proposition that something is blue is qualitative

[45]Goodman (2017b) proposes that $\varphi \equiv \varphi \to \varphi$ as an account of what it is for a proposition to be tautologous. Thus the proposal is that φ entails ψ if the material conditional $\varphi \to \psi$ is tautologous. This is thus analogous to the Boolean treatment of entailment.

[46]The distinction between qualitative and haecceitistic propositions is an instance of the more general distinction between qualitative and haecceitistic properties and relations. I purposely avoid the terminology 'singular proposition' here since it has become, it seems to me, too bound up with a Russellian view on propositions. Furthermore, even for those who use 'singular proposition' but reject, or withold assent from, Russellianism, it is unclear whether or not singular propositions and haecceitistic propositions will coincide.

whereas the proposition that Dory is blue is not. Further candidates for qualitative propositions are the propositions that necessarily every living thing has proper parts, that there were dinosaurs, that it is wrong to murder and that dogs have four legs.[47] Hence qualitative propositions can be modal, past tensed, normative and generic.[48]

The notion of qualitativeness could be formalized directly using the relation About. However there turns out to be a more useful approximation of qualitativeness that can be formulated purely in terms of the Boolean operations and identity. The basic thought is that, if two propositions p and q are about the exactly same things, then $p \vee \neg p$ and $q \vee \neg q$ are about the exact same things. And so, if logically equivalent propositions that are about the same things are identical, $p \vee \neg p \equiv q \vee \neg q$. We can thus formalize the claim that p and q are about the same things, $p \sim^* q$, simply as $p \vee \neg p \equiv q \vee \neg q$.[49] A proposition is qualitative, then, if it is about the same things as the prototypical qualitative proposition $\top$. That is, p is qualitative, written $\mathcal{Q}p$, if $p \sim^* \top$.

With this notion in place we can formulate the following restricted Closure principle:

Aboutness Closure: $\forall p \forall q (p \preceq q \wedge p \sim^* q \rightarrow (Bp \rightarrow Bq)$

Intuitively, if a propostion p entails a proposition q, and p and q are about the same things, believing p implies believing q. Note that as a special case, we get the principle of qualitative closure:

Qualitative Closure: $\forall p \forall q (\mathcal{Q}p \wedge \mathcal{Q}q \rightarrow (p \preceq q \rightarrow (Bp \rightarrow Bq))$

Aboutness Closure, and thus **Qualitative Closure**, is deducible from **Agglomerativism** and **Distribution**. To see this suppose that $p \preceq q$, $p \sim^* q$ and Bp. The following is a theorem of **Agglomerativism**

(1) $p \equiv p \wedge (p \vee \neg p)$

[47]These examples are of course open to dispute. For instance one might challenge the claim that natural kind predicates like 'dinosaur' and 'dog' express qualitative properties. Perhaps for instance they express properties whose definition makes reference to one particular animal in their evolutionary history.

[48]There are more controversial cases such as the proposition that the tall spy is sly. If, inspired by Russell (1904), we hold that this is just the proposition that there is a tall spy who is sly and identical to all tall spies then the proposition is qualitative even if it turns out that there is exactly one tall spy.

[49]See Goodman (2019) for further discussion of this point.

Since $p \sim^* q$ we know that $p \vee \neg p \equiv q \vee \neg q$. Thus from (1) and **LL** we get (2)

(2) $p \equiv p \wedge (q \vee \neg q)$

From the Distributive Laws we get

(3) $p \wedge (q \vee \neg q) \equiv (p \wedge q) \vee (p \wedge \neg q)$.

From **Agglomerativism**, we know that $p \wedge \neg q \equiv \neg(p \to q)$. Thus (2) and (3) entail

(4) $p \equiv (p \wedge q) \vee \neg(p \to q)$

Since $p \preceq q$, $(p \to q) \equiv (p \to q) \to (p \to q)$. And so from (4) we can infer (5):

(5) $p \equiv (p \wedge q) \vee \neg\big((p \to q) \to (p \to q)\big)$

But notice that $(p \wedge q)$ is logically equivalent to $(p \wedge q) \vee \neg\big((p \to q) \to (p \to q)\big)$. Moreover, any content assignment agrees on these formulas. Thus from **Agglomerativism** we have

(6) $p \wedge q \equiv (p \wedge q) \vee \neg\big((p \to q) \to (p \to q)\big)$

Finally (5) and (6) deliver (7)

(7) $p \equiv p \wedge q$

The argument from here is then simple. We have Bp and so from (7) we have $B(p \wedge q)$. Thus from **Distribution** we infer Bq.

What this argument shows is that, in an Agglomerative setting, belief can satisfy a distribution principle without satisfying a full closure principle; but it nevertheless satisfies a restricted qualitative closure principle. This allows us to run a kind of conflation argument against **Agglomerativism**. Simply put, **Distribution** is true and **Qualitative Closure** is false. But if **Agglomerativism** is true, **Distribution** is true only if **Qualitative Closure** is true. Hence, so the argument goes, **Agglomerativism** is false.

Now recall that ultimately I think conflation arguments like this fail since, as I will argue, **Distribution** should be rejected. The point here is just to show that they extend beyond **Booleanism** to quasi-structured views as well.

The argument actually extends quite a bit further than just views that take aboutness seriously. As Goodman (2019) points out, there are several other recent theories of propositional fineness of grain that are instances of **Agglomerativism**. For instance, many recent views of propositions that makes use of *subject matters* validate the theorems of **Agglomerativism**.[50] To see this notice that we could give an alternative interpretation of what content assignments are doing. Instead of taking a content assignment to be assigning sets of individuals to sentences, the individuals they are about, take them to be assigning *subject matters* to sentences. The join operation $\vee$ can then be interpreted as the operation of *fusion* defined over subject matters. The element 0 thus represents the trivial subject matter. The resulting view is one that, intuitively, says that two propositions are identical if and only if they are logically equivalent and have the same subject matter.[51]

What the previous argument shows is that recent views that try to move away from Booleanism by introducing notions of aboutness, whether in terms of being about individuals or having subject matters, cannot be motivated *just* on the grounds that they provide us with a better theory of the objects of belief.[52] This goes against Hawke et al (forthcoming) who present and agglomerative account of content as a solution to problems of logical omniscience.[53] As they note, their theory validates closure conditions for knowledge that are not immediately obvious. For instance their theory validates

$$K(p \wedge q) \rightarrow K(\neg p \rightarrow q)$$

Their response? Basically just the standard Stalnakerian line: the reason this looks unintuitive is because one might fail to know that two sentences express the same propositions. What I don't understand is why this shouldn't be taken to completely undercut the motivation for their view. The agglomerative view is a weaker view than Booleanism. This

[50]This applies, for instance to the sorts of views outlined in Yablo (2014) and Yalcin (2018).

[51]For more details see Goodman (2019).

[52]There are of course lots of other sorts of motivations that one might give for the view.

[53]They combine this sort of theory with a fragmentation account of knowledge in order to get a richer logic for knowledge.

weakening of the view must be justified. But it isn't justified if after all of the weakenings, not to mention the complications that come with fragmented views, we are left with essentially the same problem, and give essentially the same solution that the Boolean does.

Now to be fair there are many *other* reasons one might give in favor of agglomertive type views (for instance intuitions concerning aboutness mentioned above). I just don't see how it can be motivated on the grounds that it provides one with a better account of the objects of belief and knowledge.

4.2. **Only Logical Circles.** Cian Dorr (2016) puts forward a theory of the granularity of propositions, properties and relations that draws even finer distinctions than agglomerative views.[54] Dorr's view is based on a sort of "no circularity principle." Roughly speaking, the view says that if some relation R is a proper constituent of another relation S, then the other proper constituents of S must be logical constants. Officially, the view is formulated as a schema:

OLC: $x \equiv_\tau \lambda\bar{v}R(y, x, \bar{v}) \to \mathsf{Logical}_\sigma(y)$[55]

For current purposes we can restrict our attention to what this theory predicts at the propositional level. The instance of the schema obtained by replacing τ with $\langle\rangle$ gives us

$$p \equiv R(f, p) \to \mathsf{Logical}_\sigma(f)$$

That is, if p is the proposition that f bears R to p, then f must be a logical constant.

[54]As mentioned before, Dorr's theory is not put forward as an account of how fine-grained propositions are, but rather as an account of general principles governing the operator 'for it to be the case that ... is for it to be the case that ...' and related notions. I will continue to use proposition talk to express in English claims whose official formulation is in a higher-order language and remain netural on what the correct interpretation of such higher-order quantification is.
[55]Dorr (2016, p. 74).

To get a feel for this view let's look at some consequences.[56] Let p be a non-logical proposition and suppose that $p \equiv p \wedge p$. Thus $p \equiv (\lambda q.q \wedge p)p$. So from **OLC**, it follows that $(\lambda q.q \wedge p)$ is logical. But clearly this is wrong. So **OLC** predicts that p is distinct from $p \wedge p$.

As Dorr grants, distinguishing p from $p \wedge p$ is not exactly the most intuitive consequence. But the view is not motivated by this example. Rather it is motivated by examples like the following. Let $\varphi \vee_\gamma \psi$ abbreviate $(\varphi \wedge \gamma) \vee (\psi \wedge \neg\gamma)$. Intuitively no sequence of non-logical propositions provides a solution to the following system of equations:

$$\alpha \equiv \varphi \vee_\gamma \psi$$

$$\beta \equiv \psi \vee_\gamma \varphi$$

$$\varphi \equiv \alpha \vee_\gamma \beta$$

The first two equations tell us that φ is a "constituent" of α and β. But the first equation tells us that φ is itself built up from α and β via the operation $\vee_\gamma$. Dorr thinks that such a situation would be objectionably circular and so a theory of propositions shouldn't allow it. The first two equations taken on their own, however, are unproblematic. However, given **Booleanism**, the first two equations entail the third.

This sort of objection goes back at least to Bealer (1982) though it is worked out in detail in Dorr (2016, p. 70-71). It seems perfectly coherent to suppose, for instance, that the words 'grue' and 'bleen' are introduced in such a way so as to verify the following two equations.

$$x \text{ is grue } \equiv x \text{ is blue } \vee_{x \text{ is spherical}} x \text{ is green}$$

$$x \text{ is bleen } \equiv x \text{ is green } \vee_{x \text{ is spherical}} x \text{ is blue}$$

[56] In assessing the consequences of the view it is important to note that Dorr endorses the principle that a formula φ is materially equivalent to a formula φ^* whenever φ^* results from φ by replacing a constituent of the form $(\lambda\bar{x}.\psi)(\bar{a})$ with one of the form $\psi(\bar{a}/\bar{x})$ (provided that every variable in $\bar{x}$ occurs free in ψ). I will be supposing this is right for the purposes of this section.

But combining these two equations with the following seems circular.

$$x \text{ is blue } \equiv x \text{ is grue } \vee_{x \text{ is spherical}} x \text{ is bleen}$$

It is not just that the last statement sounds strange or has the appearance of being false. It is rather that the last equation has the appearance of *conflicting* with the first two.

The objection from circularity to **Booleanism**, like the above objection from aboutness, are metaphysical objections, and do not rest on any statements concerning cognitive significance or the like. Like the objection from aboutness, the Boolean can respond by challenging some of the judgments that they rest on. I'll briefly sketch a response like this. First, I'll show that a little bit of circularity in reality falls out of an extremely natural picture of the logic of "partial identifications". And second, I want to raise a couple of explicit counterexamples to Dorr's theory.

4.3. **Nonlogical Circles.** Let's add to our language a connective $>$ in which $p > q$ intuitively means "part of what it is for it to be the case that p is for it to be the case that q." We might think of $>$ as a kind of worldly entailment relation; whether or not it turns out to be *the* entailment relation is open to dispute. For now we can just think of it as a "partial identification" constant.

The following principles strike me as quite natural principles connecting partial identification and the Boolean constants.[57]

$\wedge$**-adjunction:** $(r > (p \wedge q)) \leftrightarrow ((r > p) \wedge (r > q))$.

$\vee$**-adjunction:** $((p \vee q) > r) \leftrightarrow ((p > r) \wedge (q > r))$

$\rightarrow$**-adjunction:** $((r \wedge p) > q) \leftrightarrow (r > (p \rightarrow q))$.

Thus according to the first principle, part of what it is for it to be the case that snow is white is that grass is green and lemons are yellow if and only if part of what it is for it to be the case that snow is white is for it to be the case that grass is green, and part of what

[57]The principles derive their names from category theory. In this framework, $\wedge-$**adjunction** states that conjunction is right adjoint to the "diagonal functor", $\vee$-**adjunctoin** states that disjunction is left adjoint to the diagonal functor (this cannot be formulated explicitly in our type theory since we don't have product types). The $\rightarrow$-**adjunction** states that, for any p, $(\lambda q.p \rightarrow q)$ is right adjoint to $(\lambda r.r \wedge p)$.

it is for it to be the case that snow is white is for it to be the case that grass is green. This principle strikes me as quite plausible given the intended reading of $>$; the $\vee$-adjunction principle is then just a sort of dual to this principle.

The principle $\rightarrow$-**adjunction** may require slightly more comment. Suppose that its being the case that $p \rightarrow q$ is part of its being the case that r. Then when we conjoin r with p, since the conditional $p \rightarrow q$ is already part of its being the case that q, this suggests to me that so too is q; nothing else needs to be added to r, as it were, in order to get p.

These principle provide something like object language descriptions of the standard introduction and elimination rules for $\wedge, \vee$ and $\rightarrow$. On their own they are not all that interesting. But it is somewhat natural to hold that such principles *uniquely* characterize the application of these Boolean operations to propositions. We can force this by adding the following two principles connecting partial identifications with full identification:[58]

Yoneda I: $\forall r(r > p \leftrightarrow r > q) \rightarrow p \equiv q)$

Yoneda II: $\forall r(p > r \leftrightarrow q > r) \rightarrow p \equiv q)$.

The basic idea behind these principles is a broadly structuralist one: the identity of a proposition is determined by those propositions that are "partially identical" to it and those propositions to which it is "partially identical". More precisely if its being the case that r is part of its being the case that p if and only if its being the case that r is part of its being the case that q, then its being the case that p *is* its being the case that q. The package consisting of the Yoneda principles together with the above Boolean adjunction principles is quite strong, since they suffice to uniquely characterize the application of these Boolean operations to propositions.

To see this first consider the case of conjunction. If $\forall r(r > s \leftrightarrow (r > p \wedge r > q))$ then by $\wedge$-**adjunction**, $\forall r(r > s \leftrightarrow r > p \wedge q)$. So by **Yoneda I** $s \equiv p \wedge q$. Using $\vee$-adjunction and **Yoneda II** we observe that disjunctive propositions are also uniquely pinned down. Finally using $\rightarrow$-**adjunction** note that if $\forall r(r > s \leftrightarrow r \wedge p > q)$ it follows

[58]These principles derive their name from the Yoneda lemma in category theory.

that $\forall r(r > s \leftrightarrow r > p \rightarrow q)$. Thus with **Yoneda I** we also uniquely pin down conditional propositions.

So we have some very strong principles with some intuitive plausibility concerning the relationship of partial identification of propositions and full identification of propositions. I now want to show that these principles suffice to give quite a bit of circularity. The simplest example is that the combination of **Yoneda I** and $\wedge$**-adjunction** entail that $(p \equiv p \wedge p)$. This is almost immediate since $\forall p(((r > p) \wedge (r > p)) \leftrightarrow (r > p))$ is a theorem of the background classical logic.

What's more interesting is that these principles also suffice to show that the operation of conjunction *preserves* the operation of disjunction; this gives us a more robust kind of circularity.

The argument goes as follows. Let r be an arbitrary proposition. From $\rightarrow$**-adjunction**:

(1) $\big(((p \vee q) \wedge s) > r\big) \leftrightarrow \big((p \vee q) > (s \rightarrow r)\big)$.

And from $\vee$**-adjunction**:

(2) $\big((p \vee q) > (s \rightarrow r)\big) \leftrightarrow \big(p > (s \rightarrow r) \wedge q > (s \rightarrow r)\big)$

Then, from (1) and (2) we can infer (3):

(3) $\big(((p \vee q) \wedge s) > r\big) \leftrightarrow \big(p > (s \rightarrow r) \wedge q > (s \rightarrow r)\big)$

By $\rightarrow$**-adjunction** again we get (4):

(4) $\big(p > (s \rightarrow r) \wedge q > (s \rightarrow r)\big) \leftrightarrow \big((p \wedge s) > r \wedge (q \wedge s) > r\big)$

And from $\vee$**-adjunction** again we have (5)

(5) $\big((p \wedge s) > r \wedge (q \wedge s) > r\big) \leftrightarrow \big(((p \wedge s) \vee (q \wedge s)) > r\big)$

But (4) and (5) jointly (6):

(6) $\big(((p \vee q) \wedge s) > r\big) \leftrightarrow \big(((p \wedge s) \vee (q \wedge s)) > r\big)$.

Thus, since r was chosen arbitrarily, we can infer using **Yoneda II** that conjunction preserves disjunction:

$$(p \vee q) \wedge s \equiv (p \wedge s) \vee (q \wedge s)$$

So the idea that reality has some circularity to it can be motivated directly by some natural principles governing partial identifications. I don't think an argument like this takes us all the way to a view as strong as **Booleanism**. The sticking point is negation, whose Boolean behavior is difficult to motivate by reflection on partial identifications. I do think it makes more apparent why some amount of circularity in one's theory of propositional fineness of grain is in fact quite natural, at least given a certain "structuralist" perspective on the relation $>$.

This provides us some theoretical reason for endorsing circularity. I also think that there are some rather intuitive cases of circular phenomena. For instance, I find it natural to hold that for a set x to be hereditarily finite is for x to be a finite set all of whose members are hereditarily finite. This is how the notion of "hereditarily finite" is often introduced, afterall. But this seems to involve the sort of circularity ruled out by Dorr's theory. Explicitly, the following principle concerning the property of being hereditarily finite strikes me as intuitively plausible:

$$\mathsf{HF} \equiv_{\langle e\rangle} \lambda x(\mathsf{FinSet}(x) \wedge \forall y(y \in x \rightarrow \mathsf{HF}(y))$$

By substituting β-equivalent constituents on the right-hand side, this identification expands to:

$$\mathsf{HF} \equiv_{\langle e\rangle} \lambda x\big((\lambda X x.\mathsf{FinSet}(x) \wedge \forall y(y \in x \rightarrow X(y))(\mathsf{HF}, x)\big)$$

Thus from **OLC** we can infer

$$\mathsf{Logical}_{\langle\langle e\rangle,e\rangle}\big(\lambda x\big((\lambda X x.\mathsf{FinSet}(x) \wedge \forall y(y \in x \rightarrow X(y))\big)$$

But the idea that *that* property is logical strikes me as very implausible. Thus **OLC** seems to predict that being hereditarily finite is not being a finite set all of whose members are hereditarily finite.[59]

[59]Dorr does discuss cases like this. His response is to accept the consequence but note that in all non-controversial cases of apparently circular definitions like this, there is always available a definition which is not circular. But this doesn't really undercut the point I was intending to making which was merely to point out that there is some evidence against Dorr's theory. Booleanism can accomodate this evidence in a more flatfooted way than **OLC**.

These are by no means knock down arguments. I've included them here just to indicate some metaphysical considerations that might tell in favor of a more coarse grained view like **Booleanism**.

One *could* attempt to motivate a view like **OLC** not on the basis of metaphysical considerations, but rather on the basis that it provides a better theory of the propositional attitudes.[60] In particular, one might attempt to motivate the view by showing that it can more naturally be combined with **Distribution** without, at the same time, running into any sort of conflation type argument. What I want to show now is that even a theory like **OLC** faces a sort of conflation argument, though it is quite a bit less robust than the previous one's considered.

4.4. **A conflation argument for OLC.** As with the previous examples, in order to run an argument like this, we first need to figure out whether there is a plausible algebraic notion of entailment definable in the theory in question. Neither of the defined operations $\leq$ nor $\preceq$ serve as adequate formulations of propositional entailment in the context of **OLC**. Dorr (2016, ft. 16) points out that there is, however, a definition of entailment that is well-behaved in certain extensions of his own theory. Say that propositions p and q are *logically equivalent*, written $p \sim_L q$ if $p \wedge q \equiv p \vee q$. Then we can say that p *entails* q, written $p \implies q$, if $p \vee q \sim_L q$.[61]

In the context of **Agglomerativism**, $p \implies q$ if and only if $p \preceq q$; thus in the context of **Booleanism** $p \leq q$ if and only if $p \implies q$. Neither of these equivalences hold in Dorr's theory.

The relation $\implies$ acts as a natural entailment relation when Dorr's theory is strengthened by adding some of the Boolean identities that are not ruled out by his theory. Which one's are these? Some of the obvious ones are $p \equiv \neg\neg p$ and $p \wedge q \equiv q \wedge p$. But we can also define

[60]It is important to emphasize that this is not at all the kind of things that motivates Dorr since, on his view, the behavior of the belief operator does not act as an independent constraint on identifications.
[61]Goodman (2017b) also puts forward this account of entailment as the proper notion of entailment in Dorr's theory.

which Boolean identities to add in a more systematic way. Let a *complexity assignment*, $\#$, be a map form the set of terms T of the language to the natural numbers $\mathbb{N}$ such that

- $\#(t) = 0$ if t is a logical constant;
- $\#(R(A_1, \ldots A_n) = \#R + \#A_1 + \cdots + \#A_n$;
- $\#(\lambda x_1 \ldots x_n.\varphi) = \#\varphi$.

We write $\varphi\#\psi$ if for any complexity assignment $\#$, $\#(\varphi) = \#(\psi)$. Then, similarly to **Agglomerativism**, we can fill out the theory **OLC** by adding the following schema:

NC: $\varphi \equiv \psi$, whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic and $\varphi\#\psi$.

NC entails that $p \equiv \neg\neg p$ since, for any complexity assignment, $\#(\neg\neg p) = \#\neg + \#\neg + \#p = 0 + 0 + \#p = \#p$. It also allows us to show that, whenever ψ is a classical consequence of φ, the proposition $\varphi \implies \psi$ is a theorem of **NC**. To see this, note that if $\varphi \to \psi$ is a theorem of classical propositional logic, then $\big((\varphi \vee \psi) \wedge \psi\big) \leftrightarrow \big((\varphi \vee \psi) \vee \psi\big)$ is also a theorem. Moreover, it is equally clear that $\big((\varphi \vee \psi) \wedge \psi\big)\#\big((\varphi \vee \psi) \vee \psi\big)$. Thus from **NC**, it follows that $\varphi \vee \psi \sim_L \psi$ (i.e., $\varphi \implies \psi$).

One cannot show that closure follows from **NC** together with **Distribution**.

OLC Closure: $\forall p \forall q(p \implies q \to (Bp \to Bq)$.

But one is able to derive a kind of closure principle from **NC** and **Distribution**.

$\vee$**-Closure:** $\forall p \forall q(p \implies q \to (B((p \vee q) \vee q) \to Bq)$

To see this suppose that $p \implies q$, Bp and $B((p \vee q) \vee q)$. Then, since $p \implies q$, $(p \vee q) \vee q \equiv (p \vee q) \wedge q$. Thus by **LL**, $B((p \vee q) \wedge q)$. So from **Distribution**, Bq.

Intuitively we might think about this closure principle as follows. Suppose that you believe that p and p implies q. Then I can get you to believe q using the following strategy. First, I point out that since p, it's also the case that $p \vee q$. If you are being rational you should accept and thus believe $p \vee q$. I then point out that, since $p \vee q$, it's also the case that $(p \vee q) \vee q$. Again if you a rational, you should accept and so believe $(p \vee q) \vee q$. But given $\vee$-**Closure**, this immediately entails that you believe q, whether or not you are aware that p implies q. Thus, on this theory, we can get an individual to believe the consequence of what they believe merely by getting them to *consider* certain iterated disjunctions of those consequences with what they believe.

This is of course a much more restricted kind of closure. But it strikes me as being equally open to a sort of conflation argument since the principle $\vee$-**Closure** looks like a rather implausible psychological generalization. So it seems to me that, like Boolean and agglomerative views, Dorr's view combined with **Distribution** has odd consequences for one's theory of belief. If one were committed to **Distribution**, one might take this as grounds for rejecting all of these views. I'm inclined to take it to cast doubt on **Distribution**. In order to make this case, I want to now turn to the full structured theory of propositions and investigate how well motivated it might be by **Distribution**

5. The Structured View of Propositions

The above arguments show that theories of propositions that incorporate aspects of the structured view face analogous problems to the Boolean theory of propositions. While they do not validate full closure outright, they do entail somewhat restricted closure principles provided that belief distributes over conjunction. I think that this shows that such theories cannot be justified in the basis of providing us with a better theory of the objects of thought. Either they have to give up **Distribution**, or else accept these restricted closure principles. The problem they face, then, is pretty much analogous to those faced by **Booleanism**. It is thus not clear what the restrictions on **Booleanism** have gotten us. The gains one gets in one's theory of belief is minimal; the costs in the simplicity and strength of one's background theory of propositions seem to me to be no less minimal.[62] The methodology of discounting Booleanism on the basis of its theory of belief is misguided, since it seems that any theory capable of providing a simple, algebraic account of entailment will face some sort of unintuitive consequences once combined with **Distribution**.

[62]The theory **NC** is a restriction on Booleanism. But Dorr's theory **OLC** is not. So this point as stated doesn't quite apply to Dorr. This is part of the reason I spent some extra time responding in particular to Dorr's theory.

The moral that Soames (1987) drew from reflecting on considerations like these is rather different; on his view, there is one view in particular that is motivated on the grounds that it provides a better account of the objects of belief: the *structured* or *Russellian* view. In this section I will formulate the structured theory of propositions and sketch a couple of reasons why the combination of the structured theory of propositions with **Distribution** is less attractive than it might initially appear to be.

5.1. **Basic Structure and Distribution.** Roughly speaking, the structured view of propositions holds that propositions are structured like the sentences that express them. How are sentences structured? Perhaps the key structural property of sentences of a language is that they are *uniquely readable*. The sentence $\ulcorner Fa \urcorner$ is identical to the sentence $\ulcorner Gb \urcorner$ if and only if the predicate F is identical to the predicate G and the constant a is identical to the constant b. The natural thought then is that, if propositions are structured like a language, they should be *uniquely decomposable*. We can start with a restricted structured principle to get a feel for the view:

Basic Structure:

$$(1) \quad R(A_1, \ldots, A_n) \equiv_{\langle\rangle} S(B_1, \ldots, B_n) \rightarrow R \equiv_{\langle \tau_1, \ldots, \tau_n \rangle} S \wedge \bigwedge_{1 \le i \le n} A_i \equiv_{\tau_i} B_i$$

$$(2) \quad F(A_1, \ldots, A_n) \not\equiv_{\langle\rangle} G(B_1, \ldots, B_m)$$

And instance of (1) is obtained by replacing A_i and B_i with terms of type τ_i and replacing R and S with *constants* of type $\langle \tau_1, \ldots, \tau_n \rangle$ (more on this restriction in a moment). An instance of (2) is obtained by replacing F and G with constants of different types and A_i and B_i with terms of the appropriate types. This seems to provide us with an approximation of the idea that propositions are "uniquely readable". This theory predicts, for instance, the following general principle:

$$\forall p \forall q (p \wedge q \not\equiv p \vee q)$$

To see this note that if it were the case that $p \wedge q \equiv p \vee q$ then we could use (1) to infer that $\wedge \equiv_{\langle\langle\rangle,\langle\rangle\rangle} \vee$, which we know to be false. In other words a proposition cannot be *both* the result of applying conjunction to two propositions *and* the result of applying disjunction to those propositions. The view also predicts, for instance, that the "De Morgan law" is universally false:

$$\forall p(\neg(p \wedge q) \not\equiv \neg p \vee \neg q)$$

This follows immediately from (2) since $\neg$ and $\vee$ are constants of different types.

Is **Basic Structure** motivated by conflation type considerations? Here is what Soames says about the "Russellian approach" to propositions, of which **Basic Structure** is plausibly an instance.

> The Russellian approach offers a welcome constrast. Given the intuition that whenever an individual satisfies ⌜x believes that A & B⌝ he also satisfies ⌜x believes that A⌝ and ⌜x believes that B⌝, the Russellian approach supplies a plausible explanation. Since the objects of belief reflect the logical structure of the sentences used to report those beliefs, whenever a belief is correctly reported using a conjunction the agent will believe a conjunctive proposition which includes the propositions expressed by the conjuncts as constituents. Since these constituent propositions are, so to speak, before his mind, no computation is required in order for him to arrive at beliefs in the conjuncts. (1989, 70)

The argument seems to be that (i) **Distribution** is true and (ii) the Russellian approach, which I will for now take to be the view **Basic Structure**, offers the best explanation for why.

It's not quite clear to me why we should accept (ii). For instance if believing a conjunction entails believing its conjuncts, one would have thought the best explanation would be simply that believing a conjunction *just is* believing its conjuncts.

$$B(p \wedge q) \equiv (Bp \wedge Bq)$$

This would provide an immediate explanation for *why* believing a conjunction entails believing its conjuncts. But rather than being predicted by the structured theory, this explanation is in fact inconsistent with it by (2) in **Basic Structure**.

Perhaps the thought is something like the following. The only way **Distribution** could fail is if one believes $p \wedge q$, but believes it under the guise of a sentence that carves up its structure differently. For instance if $p \wedge q \equiv \neg r$ for some proposition r, then one might believe $\neg r$, and so believe $p \wedge q$, but not realize that the proposition one believes is conjunctive, and so what should be an automatic inference from conjunction to conjuncts does not go through. But according to the **Basic Structure** there simply cannot be such a case. So there cannot be any counterexamples to **Distribution**.

While this argument may sound initially somewhat plausible, I think it ultimately rests on a conflation between the structure of propositions and the structure of the things we use to represent propositions (whether it be sentences, utterances, actions etc.) The theory **Basic Structure** is a theory about the structure of propositions, not a theory about what sorts of devices can be used to represent propositions. And there doesn't seem to be any theory that could rule out the basic worry of the kind raised above. That is, even if propositions are structured, there are plenty of ways to represent them that don't make evident what their structure *is*. So if we can exploit misleading representations to get individuals to believe, e.g., $p \wedge q$, without making evident what the structure of this proposition is, then perhaps there are counterexamples to **Distribution**. **Basic Structure** says nothing that could rule this out.

5.2. **Basic Structure and Entailment.** Sometimes the structured view is presented as one that is "fine-grained enough" to avoid the conflation argument. But presenting things this way I think misses a bit of the interworkings of the argument. As we saw above, in each case the argument only goes through given a certain analysis of propositional entailment. These analyses are all hopelessly implausible given the full structured view. But this isn't clearly a virtue. In fact, I'm inclined to view it as a weakness of the view. **Booleanism** and the other quasi-structured views considered, were able to give simple algebraic analyses of entailment for propositions, properties and relations. These analyses made use only of extensional resources: conjunction, disjunction, identity etc. The structured theory is not capable of providing account of entailment like this.

Of course, one might reply that the package consisting of **Basic Structure** and **Distribution** is still nevertheless a better overall theory than, for instance, the package consisting of **Booleanism** and the negation of **Distribution**. I will argue that it is not. My argument for this has several parts. First, I'll argue that the theory **Basic Structure** is both objectionably weak and objectionably arbitrary. However, strengthening the theory in a nonarbitrary way proves to be difficult. I will then argue that accepting the negation of **Distribution** is not at all as bizarre that its proponents often make out. I will also provide a couple of *direct* arguments against **Distribution** for good measure.

5.3. **Structured Propositions and Russell-Myhill.** The principle **Basic Structure** restricts its instances to only those in which the main connective is replaced by a constant. The result is that the theory fails to predict a lot of what a theory of structured propositions *should* predict. For instance, a structured proposition theorist will presumably want to reject theses like

$$p \equiv (\lambda p.p)p$$

and

$$p \wedge q \equiv (\lambda p.p \wedge q)p$$

Basic Structure does not predict this. There is another, related worry. When the principle is restricted to *just* to the constants, the view seems arbitrary. What we have in effect done is chosen some class of properties arbitrarily and said that the principle governs these properties. But if that is true then we should either expect the principle to govern all properties, or else we should be in a position to describe in a nonarbitrary manner *which* properties it governs and which ones it does not. Let's consider both of these options in turn.

Suppose that one thought that every instance of **Basic Structure** was true even when R and S are replaced by arbitrary terms. Then the following generalization would hold:[63]

Full Structure: $\forall p \forall q \forall X \forall Y (Xp \equiv Yp \rightarrow (X \equiv Y \wedge p \equiv q))$.

According to full structure, for any proposition p and property X, the proposition that Xp is uniquely decomposable into the property X and the proposition p.

The problem with **Full Structure** is that it can shown to be inconsistent given very minimal principles. This problem has been well documented in the recent literature in higher-order metaphysics.[64] But I think it is worth some further discussion for several reasons. First, it is important to emphasize this problem in the overall abductive comparison of the theories we have been discussing. Second, the proofs that have been given in recent literature showing that **Full Structure** is inconsistent have all relied on *classical* principles. I think it is worth emphasizing, however, that the proof of inconsistency can be given *constructively*. This shows the problem is somewhat more robust than has been noted. Finally, I just want to emphasize that the problem is not one that is somehow implicitly having to do with cardinality issues. It's not exactly clear to me whether anyone thinks this. But sometimes the result is informally presented as a cardinality issue. I think to do this slightly obscures what is actually happening in the argument.

Starting with the first task, let's first show that **Full Structure** can be shown to be inconsistent when we restrict our background logic to intuitionistically acceptable principles. We will need one further principle, a principle that I have been implicitly using up to this point but is good to state explicitly:

Extensional β-equivalence: $(\lambda v_1 \ldots v_n.\varphi)(A_1, \ldots, A_n) \leftrightarrow \varphi[A_i/v_i]$.[65]

Thus according to this principle, $p \leftrightarrow (\lambda p.p)p$. This principle does *not* assert that these propositions are identical; just that they are materially equivalent.

[63]Recall that the instances of a schema are closed under generalization.
[64]The problem can be traced back to Russell (1937) and Myhill (1958). There has been renewed attention to the problem by metaphysicians however. See for instance Dorr(2016) and Goodman (2017).
[65]This formulation is from Dorr (2016).

With this in place we can give a constructive proof of the following claim:

$$\forall p \neg \forall X \forall Y (Xp \equiv Yp \rightarrow X \equiv Y)$$

The basic proof is similar to the classical one's that have appeared in the literature. The difference is that the constructive proof is given a reductio. Let r be an arbitrary term of type $\langle\rangle$ and let

$$O = \lambda p \exists X (Xr = p \wedge \neg Xp)$$

Thus O is the property of being a proposition that predicates a property of r that it does not instantiate. Then the following is an instance of **Extensional β-equivalence**:

(1) $O(Or) \leftrightarrow \exists X (Xr \equiv Or \wedge \neg X(Or))$

From **Full Structure** and **LL**:

(2) $\exists X (Xr \equiv Or \wedge \neg X(Or)) \leftrightarrow \exists X (X \equiv O \wedge \neg X(Or))$

From **LL** and existential generalization:

(3) $\exists X (X \equiv O \wedge \neg X(Oq)) \leftrightarrow \neg O(Oq))$

And (1), (2) and (3) entail (4):

(4) $O(Or) \leftrightarrow \neg O(Or)$

Since (4) is inconsistent in intuitionistic logic we can infer (5) by reductio:

(5) $\neg \forall X \forall Y (Xr \equiv Yr \rightarrow X \equiv Y)$

And so since r was chosen arbitrarily, this establishes our claim.

The proof is constructive. And the assumptions needed to make it go through in addition to intuitionistic propositional logic are quite weak. This seems to me to be strong evidence against **Full Structure**.

It is tempting to think that the proof is at bottom a cardinality type argument. When presenting the argument, Dorr (2016) sums up the conclusion:

> The argument is essentially Cantorian: one can think of the conclusion as saying that the domain of properties of propositions is larger than the domain

> of propositions, so that there can be no one-one correspondence between the two domains. (p. 64)

It is also the case that the *proof* directly mirrors the constructive proof of the proposition that there is no injection from $\mathcal{P}(X)$ to X.[66] However I think it can nevertheless be a bit misleading to present the view as having anything to do with size. The *explanation* for why the two theorems are so similar is that they are both instances of diagonalization arguments; however as Bruno Whittle (2018) has argued, the fact that Cantor's theorem is an instance of such an argument shows, if anything, that Cantor's theorem itself has nothing *intrinsically* to do with size. This might sound like a somewhat radical conclusion to draw, but it is actually one that I think *most* mathematicians would accept. William Lawvere (1969) showed that Cantor's theorem was a straightforward corollary of a simple algebraic theorem; a theorem that holds in contexts much broader than set theory.[67] Indeed it can be shown to hold in contexts in which no notion of *size* can even be defined. In the case of sets, there happens to be a connection between Cantor's theorem and size; but this is *only* because, in the case of sets, the Schroeder-Bernstein theorem holds: if there are injections from X to Y and Y to X then there is a bijection from X to Y. It is this fact that allows us to get some well defined notion of "larger than" and "smaller than" in set theory. Without some analogue of this theorem, the connection between these sorts of diagonalization arguments and size becomes obscure.

In order to avoid the inconsistency, the structured proposition theorist would have to embrace much more radical weakenings of their background logic. It's not clear what would motivate this in the present context. The argument doesn't rely on any general principles concerning truth, or any other factive operators. This suggests to me that the costs of **Full Structure** are quite high; if the structured theorists wants to maintain **Basic Structure**, they will need some principled way of telling us which *instances* of **Full Structure** are

[66] See for instance Bell (2004).

[67] In particular, it is a corollary of a theorem that holds in any cartesian closed category, of which the category of sets is an example.

acceptable and which instances are not. While there has been some work that is suggestive in this area, I do not know of any worked out answers to this question.

I want to briefly sum up where I think we are in this argument. First, we've shown that **Distribution** combined with a variety of strong and simple theories of propositional fineness of grain predicts implausible results. And while **Distribution** conjoined with the principle **Basic Structure** does not predict implausible results, the view suffers from both weakness in terms of what it can prove, and arbitrariness in terms of what it does prove. I then argued that the structured proposition theorist faces an uphill battle when attempting to correct those defects. What I now want to do is address head on the question of how bad it really is to deny **Distribution** in addition to addressing the question of whether some *other* theory we have not yet considered will be able to accomodate **Distribution** without any collateral damage. I will argue that the denial of **Distribution** is much less implausible than it is sometimes thought to be; I will also argue that in a certain precise sense, we shouldn't expect any theory to be able to accommodate **Distribution** unscathed.

6. The Denial of Distribution

This section assesses the costs of denying **Distribution**. I'll do this by looking at some of the motivations for accepting the principle, and seeing how well those motivations support it.

6.1. **Distribution and Belief Ascriptions.** Perhaps the main motivation for accepting **Distribution** comes from the philosophy of language. Our intuitive reaction to belief ascriptions like 'Jones believes that grass is green and snow is white' is that it can only be true if 'Jones believes that grass is green' and 'Jones believes that snow is white' are also true. As Scott Soames puts the point:

> For many propositional attitude verbs distribution over conjunction is a fact whereas closure under necessary consequence is not. (Soames 1989, 49)

This suggests that many of us, at least implicitly, accept the following principle as valid in English:

> **Linguistic Distribution:** For any sentences φ and ψ and individual x, if x satisfies $\ulcorner x$ believes that φ and $\psi\urcorner$ then x satisfies $\ulcorner x$ believes that $\varphi\urcorner$ and x satisfies $\ulcorner x$ believes that $\psi\urcorner$.

But it is hard to see why **Linguistic Distribution** would be true if **Distribution** were false. If some individual could believe a conjunction without believing the conjuncts, then surely we should be able to report that this is so in English. The idea that there are failures of **Distribution** that are nevertheless inexpressible does not seem credible.

I think the argument from **Linguistic Distribution** to **Distribution** is convincing. The **Distribution** denying Boolean should thus reject **Linguistic Distribution**. This might seem like bad news for the Boolean since most of the instances of **Linguistic Distribution**that immediately come to mind seem obviously true. But if we look closer at the Boolean view it becomes clearer what the kinds of instances that turn out false will look like. I'll go over one sort of example.

Plausibly, the proposition foxes run entails the proposition that vixens run. So suppose that one believes that foxes run but disbelieves that vixens run, due to some sort of confusion. Since the proposition that foxes run entails the proposition that vixens run, the Boolean view entails that the proposition that foxes run and vixens run *is* the proposition that foxes run. Hence they believe that foxes run and vixens run. But, by stipulation, they do not believe that vixens run. So in this example the individual in question satisfies 'x believes that foxes run and vixens run' but does not satisfy 'x believes that vixens run'. **Linguistic Distribution** thus has a false instance.

There is something distinctive about this case that bears mentioning. The false instance of **Linguistic Distribution** that we get from **Booleanism** is an instance where intuitively the individual in question doesn't satisfy the antecedent. That is to say, the inference that looks unintuitive here is not that one can believe the proposition that foxes run without believing that vixens run, which given **Booleanism** generates a false instance of **Linguistic Distribution**, but rather the inference from believing that foxes run to believing that foxes run *and* vixens run. Thus the sorts of counterexamples that **Booleanism** provides are not unintuitive because we think the individual actually does believe both conjuncts. They are unituitive because we think the individual doesn't believe the conjunction. This turns out to be actually good news for the Boolean.

The reason why I think this is good news is that we can appeal to general heuristics we use in assessing belief ascriptions to explain this sort of data, without appealing to the truth of **Distribution**. For instance, the following rule is plausibly one that speakers apply in attempting to figure out the truth values of belief ascriptions:

> **ND:** Do not assert 'S believes that φ and ψ' if S fails to believe that φ or S fails to believe that ψ.

An instance of **ND** is obtained by substituting a name N for 'S', declarative sentences S and S' for φ and ψ respectively, and replacing 'S believes that φ and ψ' with $\ulcorner N$ believes that S and $S' \urcorner$.

One reason we might expect there to be a norm like this is the following. If I use some sentence in specifying one's belief, and that individual fails to believe that the sentence is true, then they are likely to reject my characterization of their belief. So if I can specify their belief using only sentences that they believe to be true, I should do so if what I want is to facilitate communication. Moreover, if one were to use the sentence 'foxes run and vixens run' where the sentence 'foxes run' would do, this raises the question as to why one went through the extra effort. If one merely believed that foxes run without believing that vixens run, describing their belief using the conjunctive sentence is likely to cause confusion. Consequently **ND** can be motivated independently of whether belief actually does distribute.[68]

[68] These rules of cooperation are of course somewhat rough and are violated often; but I am suggesting them more as heuristics we use to quickly assess the truth values of belief ascriptions rather than hard norms of discourse. I do suspect that **ND** is something like a full fledged norm. But I think this norm is plausibly explained by our background heuristics like those mentioned above.

One might worry that if **Linguistic Distribution** really were false, we would have no explanation as to why it is difficult for English speakers to discriminate 'John believes that it is raining and it is Tuesday' from 'John believes that it is raining and John believes that it is Tuesday'. These sentences will likely strike ordinary speakers as equivalent; one good explanation of this fact is that they *are* equivalent. But notice that were this explanation correct, it would also support a principle which many theorists are happy to deny:

Agglomeration: $\forall p \forall q(Bp \wedge Bq \to B(p \wedge q))$

It is generally thought that this principle is false: an individual might be "fragmented" and so fail to put together some of their beliefs. I suspect the reason for the linguistic indiscriminability of the the two cases is better explained by **ND**: there would just never be an occasion in day to day life to utter the one without uttering the other.

6.2. **Distribution and Assertion.** Another argument one might give for **Distribution** appeals to analogues between assertion and belief. If belief is the inner analogue of assertion, one might claim, then assertion distributes only if belief does. But certainly, the proponent says, assertion *does* distribute. So belief distributes.

We needn't inquire into whether belief really is the inner analogue of assertion nor ask after what exactly that claim amounts to: assertion does not distribute over conjunction. Indeed, the Boolean must reject the thesis that assertion distributes over conjunction for the same reasons that they must reject the thesis that belief does. If assertion distributes over conjunction, it is closed under logical consequence. But since assertion is not closed under logical consequence, it must not distribute.

This might seem like very bad news for the Boolean. How could one assert a conjunction but fail to assert both conjuncts? In addressing this question we need to be very careful to distinguish between *assertions* and *assertive utterances*. The objects of assertions are propositions. The objects of assertive utterances are sentences. The reason it might seem unassailable that assertion distributes is that one is prone to confuse it with a principle that *is* unassailable:

(SD): For any sentences S and S', if one assertively utters $\ulcorner S$ and $S'\urcorner$ then one assertively utters S and one assertively utters S'.

An assertive utterance of 'snow is white and grass is green' *just is* an assertive utterance of 'snow is white' closely followed by an utterance of 'and' closely followed by an assertive utterance of 'grass is green'. As far as I can tell, there is no reason whatsoever to reject **(SD)**. Once we have distinguished assertions from assertive utterances it is natural to ask after principles relating the two notions. One principle (schema to be exact) is particularly useful for our purposes:

(SA): If one assertively utters 'φ and ψ' then one asserts that φ and one asserts that ψ.

An instance of this schema results from replacing φ and ψ with declarative sentences S and S' respectively and replacing 'φ and ψ' with $\ulcorner S$ and $S'\urcorner$. Nothing in this paper challenges **(SA)**; it is perfectly consistent with the negation of **Distribution** and **Booleanism**. Any instance of **(SA)** can be derived from **(SD)** together with a corresponding instance of the schema **(A)**:

(A): If one assertively utters 'φ' then one asserts that φ.

instances of which are obtained by replacing φ with a declarative sentence S and 'φ' with $\ulcorner S\urcorner$. Since both **(SD)** and **(A)** look pretty plausible, this provides a strong case for **(SA)**.

6.3. **Distribution and the Language of Thought.** When it comes to the distribution of assertion over conjunction, it is easy to come up with replacements that nevertheless capture much of the original intuition. Are there analogous replacements for the case of belief? The obvious strategy is to appeal to mental representations in place of sentences. To implement this strategy symmetrically first note that we can redescribe the schema **(SA)** as a first order generalization over sentences:

(SA^{+}): For any sentences S and S' if one assertively utters $\ulcorner S$ and $S'\urcorner$ then for any propositions p and q such that S means p and S' means q, one asserts p and one asserts q.

Now suppose that individuals have a language of thought (a certain system of mental representations) and that beliefs are had via internal tokenings (the analogue of utterances) of these sentences. More precisely, there is some relation, *acceptance*, such that an individual believes that p if and only if they accept a mental representation that means that p. One's system of mental representations contains logical connectives such as conjunction. I'll write $\ulcorner m \dot{\wedge} m' \urcorner$ for the conjunction of mental representations m and m'. We can then formulate the analogue of **(SA$^+$)** for belief:

> **(SB):** For any mental representations m and m' if one accepts $\ulcorner m \dot{\wedge} m' \urcorner$ then for any propositions p and q such that m means that p and m' means that q, one believes that p and one believes that q.

Provided that there is a language of thought, **(SB)** is a natural replacement for **Distribution**. When we try to imagine counterexamples to **Distribution**, for instance, we presumably do so by way of trying to imagine an individual with a conjunctive *mental representation* that nevertheless fails to believe the conjuncts. **(SB)** explains why such cases cannot be found.

As in the case of assertion, one can accept **(SB)** without accepting **Distribution**. One might initially worry that **(SB)** itself provides strong evidence for **Distribution**. Why should **(SB)** be true were **Distribution** false? Given that one can believe a conjunctive proposition without believing its conjuncts, why couldn't one do so by way of a conjunctive mental representation?

But in the case at hand I just don't think these rhetorical questions carry much weight. It seems like a perfectly plausible and acceptable psychological principles governing mental representations that when conjunctive mental representations get in the belief box, so do their conjuncts. It may be for instance that conjunctive mental representations get in the belief box *only* by way of getting their conjuncts in the belief box. If that's right, then whether one believes $p \wedge q$ via a conjunctive mental representation is obviously relevant as to whether one's belief distributes.

I think that this shows that the failure of **Distribution** is not as bad as we might have initially thought. On the one hand, every counterexample to the principle according to the Boolean is a proposition that, according to the structured theorist, isn't a counterexample. That is, whenever the Boolean say that one believes that $p \wedge q$ without believing that p, the structured theorist will just say that one doesn't actually believe that $p \wedge q$. The problem thus reduces to a more familiar problem: explaining why certain belief ascriptions, in our case certain conjunctive belief ascriptions, are true despite appearing to be false. This is the sort of problem that structured theories themselves face. For instance Soames' (1987) view entails that one believes that Hesperus is Hesperus if and only if one believes that Hesperus is Phosphorus. And in general I think there are a lot of responses available to these sorts of worries. In our case, I have sketched two routes: one via a norm and the other via a principle concerning mental representations.

In the concluding section of this appear, I want to address one final worry one might have with the sort of argument I have been pushing in this paper. The worry is that, while it may be true that given our present state of knowledge, **Booleanism** without **Distribution** seems to offer a compelling theory, what if the structured theorist is able to answer the arbitrariness worries raised above? Then there might be a strong and simple theory that can be combined with **Distribution** without any collateral damage. Since I haven't shown that this isn't the case, shouldn't I be agnostic moving forward? I'm not quite sure if this is a convincing argument. Nevertheless, I want to respond by sketching an argument that shows that in a certain sense no theory can accept **Distribution** without at least a little bit of unintuitive consequences.

7. Distribution and Arbitrary Conjunctions

The principle of **Distribution** can be iteratively applied. Thus it entails each instance of the following schema:

n-Distribution: $\forall p_1 \ldots \forall p_n(B(p_1 \wedge \cdots \wedge p_n) \rightarrow (Bp_1 \wedge \cdots \wedge Bp_n))$[69]

One might take this as grounds itself for rejecting **Distribution**: while the inference from $p \wedge q$ to p seems obvious, perhaps the inference from $p_1 \wedge \cdots \wedge p_{1,000,000}$ to p_{500} will seem less obvious. In this concluding section, I want to press a different sort of argument against **Distribution**. The argument basically runs as follows. First, we should accept n-**Distribution** only if we accept that belief distributes over *arbitrary* conjunctions. But no one, not even structured views, should accept that belief distributes over arbitrary conjunctions. Thus we shouldn't accept $n-$**Distribution**. Since **Distribution** entails n-**Distribution**, we shoulndn't accept **Distribution**.

One of the main premises of this argument is that no one should accept that belief distributes over arbitrary conjunctions. In order to make this case we first need to provide some way to *express* this idea. Here is one way to do this. Let us for set our typed language aside and work in a simple two-sorted higher-order language that has both propositional variables $p, q, r, \ldots$ as well as plural propositional variables $pp, qq, rr, \ldots$ with quantifiers to bind them. We suppose that there are still the Boolean operators, the belief operator B and the propositions identity operator $\equiv$. We also add two novel resources: an operator $p \prec pp$ which informally states that p is one of pp and an operator $\bigwedge pp$ which represents the conjunction of the propositions pp.[70] In this language, arbitrary distribution can be formulated as follows:

Arbitrary Distribution: $\forall pp(B \bigwedge pp \rightarrow (p \prec pp \rightarrow Bp))$

I will sketch an argument against this principle. The argument will proceed informally, but I suspect we should be able to provide a fully formal version of the argument as well. For reasons of space I will only briefly indicate how this is to be done.

[69] In the Boolean setting $p \wedge \cdots \wedge p_n$ can be read unambiguously. But in a structured setting there may be many propositions we are intending to pick out depending on how we bracket the terms of the conjunction. On the intended reading of this schema, there is, for each way of bracketing the formulas, and instance in which the formulas are bracketed that way.

[70] For a much more leisurely description of this language see Hall (forthcoming) as well as Fritz (unpublished).

Peter Fritz (unpublished) argues that recent theories of grounding are committed to inconsistent principles. In particular, he argues that they are commited to the following two, jointly inconsistent principles:

$S \bigwedge$: $\forall pp \forall qq(Tpp \wedge Tqq \to (\bigwedge pp \equiv \bigwedge qq \to pp \equiv qq)$.

$T \bigwedge$: $\forall pp(Tpp \to \bigwedge pp)$

Here Tpp abbreviate the claim that all of the proposition among pp are true (i.e. $\forall p(p \prec pp \to p)$ and $pp \equiv qq$ abbreviates the something is in pp iff it is in qq (i.e., $\forall p(p \prec pp \leftrightarrow p \prec qq)$. The argument is just the Russell-myhill argument in plural form: if both of these were true, then $\bigwedge$ would define an "injection" from pluralities of truths to truths. But by a familiar argument, we know that there cannot be any such injection. As Fritz notes, turning this argument into a fully deductive one is just a matter of translating Russell's (1903, Appendix B) argument into plural form.

Now consider a variant of this argument. Let's say that a proposition p is *non-conjunctive* if there are no propositions pp such that $p \equiv \bigwedge pp$ (i.e., $\neg\exists pp(p \equiv \bigwedge pp)$. We write this as $\mathsf{NC}(p)$. Some propositions pp are nonconjunctive if every one of them is nonconjunctive (i.e. $\forall p(p \prec pp \to \mathsf{NC}(p))$. We write this as NC. Now consider the following two principles:

Non-Conjunctive Structure: $\forall pp \forall qq(\mathsf{NC}(pp) \wedge \mathsf{NC}(qq) \to (B \bigwedge pp \equiv B \bigwedge qq \to pp \equiv qq))$

Non-Conjunctive Belief: $\forall pp(\mathsf{NC}(pp) \to B \bigwedge pp)$

By an argument anologous to Fritz's, these principles are plausibly inconsistent. If both were true, then the complex operator $B \bigwedge$ would define an injection from pluralities of non-conjunctive propositions to non-conjunctive propositions. Baut by general diagonal type arguments, we should expect this to fail.

Okay so suppose that is correct. Any structured view should accept **Non-Conjuntive Belief**. Thus a structured theorist is commited to rejecting **Non-Conjunctive Structure**. So for two distinct collections of *non-conjunctive* propositions pp and qq, believing $\bigwedge pp$ just is believing $\bigwedge qq$. But this fact, together with *arbitrary distribution* looks to entail implausible closure conditions on belief. For suppose that one believes $\bigwedge pp$ by explicitly considering each of the conconjunctive propositions in pp and inferring $\bigwedge pp$. Then it follows that one believes $\bigwedge qq$. So by **Arbitrary Distribution**, one believes each $q \prec qq$. But there is some proposition $q \prec qq$ that one did not explicitly consider yet, since it is not among pp. Moreover, since none of the $p \prec pp$ were conjunctive, one cannot have aquired the belief in q by believing some conjunctive proposition in pp of which q was a conjunct. So it looks like the principle of **Arbitrary Distribution**, together **Non-Conjunctive Belief** entail undesired closure condition on belief *all on their own*. This casts doubt on **Arbitrary Distribution** and so, it seems to me, cast doubt on **Distribution**.

8. Conclusion

There are various principles the Boolean can appeal to as replacements for Distribution. Furthermore, there is a general worry about the principle of Distribution. If a given individual believes a conjunctive proposition $p \wedge q$ under the guise of a sentence that is not conjunctive, then we might expect them to fail to believe p and q by failing to notice that they follow from $p \wedge q$. To insist that such cases are impossible just seems to be a reflection of the intuitions that lead to the structured view, which has been shown to be inconsistent. I conclude that one ought to reject Distribution and accept one of the various possible restrictions.

9. References

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