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Indefinite Extensibility and the Principle of Sufficient Reason

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Abstract The principle of sufficient reason threatens modal collapse. Some have suggested that by appeal to the indefinite extensibility of contingent truth, the threat is neutralized. This paper argues that this is not so. If the indefinite extensibility of contingent truth is developed in an analogous fashion to the most promising models of the indefinite extensibility of the concept set, plausible principles permit the derivation of modal collapse.

1 Introduction

The principle of sufficient reason (PSR) is the thesis that every contingent truth has a sufficient reason.[1] Peter van Inwagen (1983, pp. 202-204) has shown that the PSR collapses modal distinctions against a backdrop of plausible principles governing explanation, necessity, and their interaction. (Jonathan Bennett (1984, p. 115) presents a similar argument.) If the PSR is true, then, it would appear, there are no contingent truths and so all truths are necessary (i.e., *necessitarianism* is true).[2]

Recently, Samuel Levey has proposed an *extensibilist* response to van Inwagen's argument.[3] Just as the indefinite extensibility of the concept set would, according to the extensibilist, resolve the set theoretic antimonies, so too the indefinite extensibility of the concept of contingent truth would, according to the extensibilist, block van Inwagen's argument. If correct, extensibilism offers

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[1] Some authors state a stronger version of the PSR which says that all truths have sufficient reasons. In Appendix 2 I show that the stronger version not only guarantees modal collapse, but is in fact inconsistent with commonly accepted principles.

[2] Given some further assumptions, one can then conclude that the true, the possible and the necessary coincide.

[3] See Levey 2016 and for some criticisms see McDaniel Forthcoming.

the rationalist a novel way to maintain their commitment to the PSR while recognizing genuine contingency in reality.

While extensibilist responses to the set theoretic paradoxes are familiar, apart from Levey 2016, there has not been much discussion concerning the viability of an extensibilist response to van Inwagen's argument. This paper argues that there is an important asymmetry between extensibilist responses to set theoretic antimonies and extensibilist responses to van Inwagen's argument. Extensibilist responses to the set theoretic antimonies can be given a modal characterization that does not involve changing background logical principles; extensibilist responses to van Inwagen's argument, I will argue, cannot. Thus whereas extensibilism in the set theoretic case can be developed by a classically minded philosopher, extensibilism in the present case requires a more constructivist outlook.

The following provides a rough overview of the paper. In this first couple of sections we work towards a rigorous statement of van Inwagen's argument. We will see that an important property of this argument is that it contains a comprehension principle (in this case a sort of higher-order analogue of plural comprehension) and a collapse premise (the principle of sufficient reason). In this way the argument is analogous to plural formulations of Russell's paradox. In response to Russell's paradox, extensibilists will either restrict plural comprehension or restrict the collapse premise. Many philosophers have thought that restricting plural comprehension is the less plausible of these two implementations. I agree with this sentiment and will provide some arguments that the relevant higher order analogue of comprehension should be preserved. I then outline the extensibilist response to van Inwagen's argument that involves restricting the PSR (analogous to the way extensibilists restrict the principle that every plurality of things forms a set). I will argue that given classical principles, this extensibilist restriction fails to avoid the consequence of necessitarianism.

2 The Argument

Since my main opponent is Levey I will work with his formulation of van Inwagen's argument:

> Let C be the conjunction of all contingent truths. Then C itself is a contingent truth, for no necessary truth can have a contingent truth as a conjunct. By PSR, there is an explanatory ground G that is a sufficient reason for C. G entails C and explains C. Is G itself a contingent truth? If so, then G is in C. But then in explaining C, G would explain itself, and no contingent truth can explain itself. If G is not a contingent truth but a necessary truth, then because G entails C, it follows that C is a necessary truth, contrary to hypothesis. So, given PSR, there can be no conjunction C of all contingent truths. If there is no conjunction C of all contingent truths, then it must be that there are no contingent truths.

Therefore, PSR entails that there are no contingent truths. (2016, 399-400)

The goal in what follows will be to assess the bearing indefinite extensibility has on this argument. In order to do so, it will prove useful to provide a more precise formulation. Only by doing so, I suggest, will we be able to properly assess whether the extensibilist response can be carried out. Below I offer one such formalization. The formalization will have another benefit worth mentioning. Several authors have claimed that ultimately the paradox rests on some controversial claim about the structure of a certain kind of abstract object: propositions, states of affairs or facts perhaps.[4] The language in which I formalize the paradox will not make use of *any* first order quantifiers and so will be strictly speaking neutral as to whether there are any of these objects. In this sense, the formulation that I offer can be seen as a "pure" version of the paradox of sufficient reason.

2.1 Formalizing the Argument

One natural setting in which to formalize the paradox is a higher order language that permits, in addition to quantification into sentence position, a kind of quantification that stands to quantification into sentence position as plural quantification stands to ordinary first order quantification. With this language in place we will be able to formally simulate both the singular and plural quantification over truths involved in the paradox of sufficient reason.

I'll suppose we have a countable collection of propositional variables, p_1, p_2, $\ldots$, Boolean operators $\neg$ for negation and $\wedge$ for conjunction, a unary modal operator $\Box$ for metaphysical necessity, and a universal quantifier $\forall$ binding propositional variables. Each propositional variable is a well formed formula, and in addition to the standard clauses for building complex well formed formulas we stipulate that if p_i is a propositional variable and φ a well formed formula, $\forall p_i \varphi$ is a well formed formula. Other Boolean and modal operators are treated as abbreviations.

In order to formalize the PSR, I'll add a further connective $<$ to the language so that, intuitively, $\varphi < \psi$ formalizes that the proposition that φ is a sufficient reason for the proposition ψ. For a given formula φ and propositional variable q not free in φ, $C\varphi$ abbreviates $\varphi \wedge \Diamond\neg\varphi$ (it is contingent that φ) and $E\varphi$ abbreviates $\exists q(q < \varphi)$ (φ has a sufficient reason). The PSR can now be stated as follows:

(PSR) $\quad \forall p\big(Cp \to Ep\big)$.

While the language so defined has the expressive resources to *state* the PSR, it is not yet expressive enough to state the argument against the PSR. The argument against the PSR involves forming the conjunction of *all* contingent truths. For all we know, there are infinitely many contingent truths. Thus we

4 See for instance Ross (2013).

will need some way to talk about arbitrary conjunctions in our language.[5] While there may be several ways to do this, the approach I take will facilitate comparison of the paradox of sufficient reason with more familiar set theoretic paradoxes such as Russell's paradox.[6]

To this end let us extend the language with a countable collection of new *plural* propositional variables $pp_1, pp_2, \ldots$ together with a universal quantifier—also written $\forall$—to bind them. By analogy with plural logic, I will also suppose that we have a binary logical operator with propositional variables going in the first place and plural propositional variables going in the second, $p_i \prec pp_i$, which may be read "the proposition that p_i is one of the propositions that pp_i." Lastly, to make generalizations about arbitrary conjunctions I'll suppose that we have a monadic operator that combines with plural propositional variables, $\bigwedge pp_i$, which may be read as "the conjunction of the propositions that pp_i."

The definition of well formed formula is then extended as follows: $p_i \prec pp_i$ and $\bigwedge pp_i$ are well formed formulas whenever p_i is a propositional variable and pp_i is a plural propositional variable. Additionally when pp_i is a plural propositional variable and φ a well formed formula, $\forall pp_i \varphi$ is a formula. And we say that t is a *term* if t is either a formula or a plural propositional variable. Call the resulting language $\mathcal{L}_{HP}$.

How should this language be interpreted? I have suggested informal pronunciations for $\forall p \ldots$ and $\forall pp \ldots$ as "for any proposition $p \ldots$ and "for any propositions $pp \ldots$'. Arguably, however, we needn't rely on any first order theory of propositions to make sense of this sort of quantification. Rather, the higher order quantifiers can be taken as new fundamental ideology. So interpreted, even a nominalist, one who denies that there are any propositions, can make use of the language in order to understand the paradox of sufficient reason.[7]

There is an additional interpretive issue. Should $\exists pp \ldots$ be informally understood as "there are one or more propositions..." or "there are two or more propositions ..." or what? Since it is technically simpler, I will make use of a different reading on which it is to be read as 'there are zero or more propositions ...".[8] The universal quantifier $\forall pp$ can then be read as asserting

[5] We could introduce the conjunction of contingent truths as the unique truth that necessitates each contingent truth and is necessitated by everything that necessitates every contingent truth. The problem is that this builds in the assumption that there *is* such a unique truth. Without supposing that necessarily equivalent propositions are identical, this assumptions looks unjustified.

[6] An alternative route is to formulate the argument in a simply typed λ-calculus and treat the arbitrary conjunction operator as an operator that combines with sentential operators to form sentences. So for instance given the truth operator λpp we form the conjunction of all truths as $\wedge\lambda pp$. In a functionally typed setting, $\wedge$ is of type $(t \to t) \to t$. For further discussion see section 3.

[7] This is of course a controversial stance to take on higher order quantification and the ideas presented in this paper do not require such an interpretation. For defenses of the idea of taking higher order quantification as fundamental see Prior 1971 and Williamson 2003.

[8] Arguably this has become the standard reading of the plural quantifiers. See for instance Burgess 2004, Uzquiano 2015 and Linnebo 2010 among others.

"however some propositions pp may be ...". I do not mean to commit myself to any controversial metaphysical thesis in doing so. The purpose of permitting "empty pluralities" is mere technical convenience. Everything that follows could be suitably tweaked to account for a reading which did not permit such pluralities.[9]

Turning now from grammar to logic, I will suppose that we have all of classical logic in addition to the analogies of classical axioms and rules of inference governing propositional and plural propositional quantification. Where x is either a propositional variable or a plural propositional variable:

A1 Any substitution instance of a propositional tautology.
A2 $\forall x\varphi \to \varphi[t/x]$ (where t is a term of the appropriate sort free for x in φ).
A3 $\forall x(\varphi \to \psi) \to (\forall x\varphi \to \forall x\psi)$.
R1 $\varphi, \varphi \to \psi/\psi$.
R2 $(\varphi \to \psi)/(\varphi \to \forall x\psi)$ if x is not free in φ.

Since $\Box$ is meant to formalize metaphysical necessity it would be natural to suppose that it obeyed the logic of S5. For our purposes, nothing so strong is required. The argument goes through even in the weakest normal modal logic K:

A4 $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$.
R3 $\varphi/\Box\varphi$.

This covers the logical principles for the quantifiers, Boolean connectives and modal operators. The only other logical constant is $\prec$. In the first order case, the standard principles governing this connective are *plural comprehension* and *extensionality.* I will suppose for now that we can appeal to the analogue of plural comprehension:

A5 $\exists qq\forall p(p \prec qq \leftrightarrow \varphi(p))$ (provided that p does, and qq does not, have free occurrence in $\varphi(p)$).

Let the *background logic*, BL, be the set comprised of all instances of A1-A5 and closed under R1-R3. I'll write $\vdash_{\mathsf{BL}} \varphi$ for $\varphi \in \mathsf{BL}$. We can extend $\vdash$ to a finitary consequence relation by letting $\varphi_1, \ldots, \varphi_n \vdash \psi$ abbreviate $\vdash \varphi_1, \ldots, \varphi_n \to \psi$. I will now move to formalizing the argument. I will attempt to stay as close as possible to the actual principles that Levey appeals to in his formulation of the argument; some of these principles will plausibly follow from more fundamental principles concerning sufficient reasons, entailment, and conjunction.

2.2 The Paradox of Sufficient Reason

Let's start at the beginning:

[9] For discussion see Burgess and Rosen (1997, 154-156).

> Let C be the conjunction of all contingent truths. Then C itself is a contingent truth, for no necessary truth can have a contingent truth as a conjunct.

There are two principles implicitly being appealed to here. First if the conjunction of some propositions is necessary, then each of those propositions is necessary:

(Inheritance) $\quad \forall pp\big(\Box \bigwedge pp \to \forall p(p \prec pp \to \Box p)\big)$.

The principle Inheritance does not entail that the conjunction of contingent truths is contingent since it does not entail that the conjunction of some truths is *true*. Since this principle is completely uncontroversial it is easy to skip over:[10]

(Truth) $\quad \forall pp\big(\forall p(p \prec pp \to p) \to \bigwedge pp\big)$.

Next:

> By PSR, there is an explanatory ground G that is a sufficient reason for C. G entails C and explains C. Is G itself a contingent truth? If so, then G is in C. But then in explaining C, G would explain itself, and no contingent truth can explain itself.

I have already formulated the PSR. Two other principles are appealed to here. First, nothing (or at least no contingent truth) explains itself:

(Irreflexivity) $\forall p(p \not< p)$.[11]

And second, if p is a sufficient reason for q then p *entails* q:

(Sufficiency) $\forall p \forall q\big(p < q \to \Box(p \to q)\big)$.[12]

(Sufficiency is *not* being proposed as a principle that governs explanation more generally. Rather, it is supposed to govern that specific sort of explanation that authors within this literature are concerned with when they say that one proposition is a sufficient reason for another. Sufficiency is a standard assumption in the literature concerning sufficient reason.)

These are not the only principles appealed to in the passage; Levey also assumes that if G explains C then "G would explain itself." Why should this be so? The reasoning seems to be twofold: that if G explains and entails C then G is contingent, and that if G explains a conjunction, it explains each conjunct. The latter principle is:

[10] Strictly speaking, as we will see, these two principles are also not jointly sufficient to derive that the conjunction of contingent truths is contingent since it may turn out, indeed it does turn out if the argument is correct, that there are no contingent truths. The difference between my formulation of the argument and Levey's is that I am allowing empty pluralities whereas he does not appear to be. The differences are not deep however.

[11] Recently several philosophers, such as Jenkins (2011), Wilson (2014) and Wood (2018), have argued against this principle (or against analogous principles).

[12] Some philosophers reject this, for instance, Amijee (2017). Moreover, if $<$ is interpreted as grounding, the principle becomes quite a bit more controversial.

(Distribution) $\forall p \forall qq(p < \bigwedge qq \to \forall q(q \prec qq \to p < q))$.[13]

As for the former, suppose that q is contingent and that $p < q$. Then from S it follows that $\Box(p \to q)$. And so if $\Box p$ we know that $\Box q$. Thus S guarantees that if p explains a contingent truth then p is not necessary. Thus in order to guarantee that p is contingent, we need only add that it is *true*. I will suppose the stronger principle that whenever p explains q both p and q are true. Thus $<$ is *factive*:

(Factivity) $\forall p \forall q(p < q \to (p \wedge q))$.[14]

Let the *theory of sufficient reason*, TSR, be the theory (set) comprised of PSR, $\mathsf{Iheritance}$, $\mathsf{Irreflexivity}$, $\mathsf{Sufficiency}$, $\mathsf{Distribution}$, Truth and $\mathsf{Factivity}$. Define *necessitarianism* to be the following principle:

(Necessitarianism) $\forall p(p \to \Box p)$.

We can then show the following:

Theorem 1 $\mathsf{TSR} \vdash_{\mathsf{BL}} \mathsf{Necessitarianism}$.

The proof of theorem 1 is almost immediate given the following lemma:

Lemma 1 $\mathsf{TSR} \vdash_{\mathsf{BL}} \forall pp\big((\forall p(p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)) \to \Box \bigwedge pp\big))$

The proof of lemma 1 is given in the appendix. One might think this Lemma a bit odd. Didn't we explicitly set up the principles in TSR so that they would entail that the conjunction of contingent truths is *contingent*? Not exactly. Here is how to think of the situation: The conjunction of some truths entails each of those truths and is true if each of them are true. So indeed the conjunction of the contingent truths must be contingent *provided there are contingent truths*. But if there are no contingent truths, the conjunction of the contingent truths is the conjunction of the "empty plurality." Informally, think of the conjunction of some things as the greatest lower bound under entailment of those things. This means that the conjunction of the empty plurality must (i) entail every member of that plurality and (ii) be entailed by everything that entails every member of that plurality. But since the plurality has no members *every* truth entails every member of the plurality. And so the conjunction of the empty plurality must be entailed by everything; this is just another way of saying that it must be necessary.

Once this lemma is in place the proof of Theorem 3.1 is straightforward. Since some details of the proof will become philosophically relevant in the next

[13] Not all formulations of the puzzle make use of this principle.
The formulation of the puzzle in Van Inwagen 1986 does not make use of D but rather of the principle that if $\Box(p \leftrightarrow q)$ then neither $p < q$ nor $q < p$. But it has not been standard practice to follow van Inwagen in this respect since many authors would like to allow the possibility of explanations that holds between necessarily materially equivalent propositions. Thus most formulations of the argument contain D or something like D (see for instance Pruss 2006). Nevertheless D is certainly one of the more controversial principles in the derivation. If $<$ is read as grounding, many authors will reject it. See Dasgupta 2016.

[14] Note that F could be derived from $\forall p \forall q(p < q \to p)$, S and the principle that $\forall p(\Box p \to p)$.

section I will state it here (what follows is more of an informal explanation of how to produce the proof. The main purpose of including it is to highlight the role that A5 plays in the derivation):

Proof (of Theorem) The principle

$$\forall pp\big(\forall p(p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)) \to \Box \bigwedge pp\big)$$

together with the principle Inheritance immediately implies (in BL) the principle

$$\forall pp\big(\forall p(p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)) \to \forall p(p \to \Box p)\big)$$

And so given Lemma 3.1 Necessitarianism then follows in BL from the following instance of plural comprehension A5:

$$\exists pp\forall p\big((p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)\big)$$

Necessitarianism is extremely implausible.[15] For instance, the following application of universal instantiation delivers an absurd conclusion: If for any P, P only if it is metaphysically necessary that P then if I raise my hand it is metaphysically necessary that I raise my hand. Since that instance of necessitarianism is clearly false, the principle itself should be rejected. Thus the above puzzle strikes me as genuine in the following sense: plausible principles can be precisely formulated so as to entail an absurd conclusion. In order to avoid the absurd conclusion one must either reject some part of the theory of sufficient reason or weaken the background logic. What is the extensibilist response?

3 Extensibilism and Plural Comprehension

A concept is *indefinitely extensible* if no matter what some instances of that concept may be, we are in a position to define, by reference to them, a new instance of that concept that is not one of them. Recently Samuel Levey (2016) has argued that by recognizing the indefinite extensibility of the concept of contingent truth, we are in a position to respond to the above argument. In his words:

> [I]f *contingent truth* is indefinitely extensible, then Completeness [the thesis that if there are any contingent truths, there is such a thing as all contingent truths] is incorrect: there can be contingent truths and yet no such thing as all contingent truths and so no conjunction C of all contingent truths. And thus - on the extensibilist interpretation - the van Inwagen-Bennet argument falls through. (2016, 403)

[15] The higher order version of necessitarianism should be sharply distinguished from views sometimes called ‘necessitarianism’ that are formulated in terms of first order quantification over facts or propositions or states of affairs (see for instance Schaffer 2012). Perhaps all facts conceived of as first order entities of a given sort are necessary. But this would just mean that a view that attempted to provide an analysis of higher order quantification in terms of first-order quantification over facts would face an uphill battle. The negation of Necessitarianism seems to me to be a datum.

How should we interpret Levey's response?

The following strikes me as a plausible reading. Levey is advocating the following two theses: first, indefinitely extensible concepts provide untrue instances of plural comprehension. And second the condition '$p \wedge \Diamond\neg p$' is indefinitely extensible. These two theses jointly entail that the following instance of plural comprehension is untrue:

(IE) $\exists pp\forall p\big((p \prec pp \leftrightarrow (p \wedge \Diamond\neg p)\big)$

And this suffices to defuse van Inwagen's argument since IE plays an ineliminable role in the proof of Theorem 3.1.

There are difficult interpretative issues surrounding the concept of indefinite extensibility. But I am willing to grant for the sake of argument that *contingent truth* is indefinitely extensible in whatever sense of that term that applies to concepts such as *set* and *ordinal* (according to the extensibilist). Why should we think that indefinitely extensible concepts provide untrue instances of plural comprehension?

Levey says surprisingly little about this. And without saying more this solution is not completely satisfying. Consider the following analogy. Suppose that you have just presented your friend with the liar paradox for the first time, and suppose that they respond as follows:

> There is no paradox here. The false premise is simply that the sentence $L =$ 'L is not true' satisfies the T-schema. It is just not the case that 'L is not true' is true if and only if L is not true. And since this premise is false, the reasoning that leads to a contradiction in the liar paradox is blocked.

This would be a rather unsatisfying response, I think, since we are told nothing about what the underlying difference is between sentences that obey disquotational reasoning and sentences that do not. Any viable classical solution to the liar paradox should contain such an explanation.

Similarly, to merely *assert* that '$p \wedge \Diamond\neg p$' provides an untrue instance of plural comprehension is no solution to the paradox of sufficient reason since it does not give us any explanation of the underlying difference between the conditions that provide true instances and those that provide false instances of plural comprehension. As Gabriel Uzquiano says in a slightly different setting:

> [T]o claim that some conditions fail to determine some objects as all and only the objects that satisfy the condition is no better than "to wield the big stick" without offering an explanation. (2015, 149)

What might a principled restriction of plural comprehension look like? One possible route to restrict plural comprehension is to restrict it to all and only those truths that have a common explanation:

$\mathsf{A5}^{<}$ $\forall p\exists qq\forall q\big(q \prec qq \leftrightarrow (p < q \wedge \varphi(q))\big)$

There are several reasons why this is a natural restriction. First it is analogous to a restriction one might place on first-order plural comprehension in light of the set theoretic antimonies. There, the restriction is that for any condition, there are all and only those things that satisfy that condition provided they are all contained within some set. And second, it is designed to block the derivation of Necessitarianism. In order to conjoin some truths there must be those truths. But given $\mathsf{A5}^{<}$ there are those truths only provided there is some single truth that explains each of them. This truth in turn is naturally taken to be a sufficient reason for the conjunction of those truths.

There are serious costs in rejecting A5 however. First let us note that $\mathsf{A5}^{<}$ is a radically weak principle. For instance, given the factivity of $<$, the only pluralities of propositions guaranteed to exist will be true propositions. But given some truths *pp* it is natural to suppose that there are all of the negations of propositions in *pp*, which would then be comprised of falsities. Even if some propositions lack a common explanation, it might have been that they had a common explanation. Cases like this might arise when p is true and q is false but were q true there would have been some r that explained both p and q.

There is a second worry that is perhaps slightly more difficult to pin down. In order to know whether there are some truths that are the truths that satisfy some condition, we need to know whether those truths have a common explanation. In the finite case, this might just be a matter of taking note that r explains p and r explains q. But if we are wondering whether some things satisfying some condition have a common explanation when infinitely many things satisfy that condition, it would appear that we *first* need to know whether there are those things and then consider the explanation of their conjunction (provided the conjunction is contingent). This is at least one plausible way to come to know that there is a common explanation. There is thus a kind of epistemic circularity. In order to know whether there are some propositions meeting a certain condition we must first know that everything meeting that condition has a common explanation. In order to know that everything meeting that condition has a common explanation we must know that there are all and only those propositions meeting the condition (so that we can conjoin them and infer by PSR that they have a common explanation).

Putting this restriction aside, how credible is any restriction of plural comprehension? An initial motivation for plural comprehension is that it is a plausible, strong and simple principle. That is a reason to think that it is true. But the proponent of PSR might respond that the PSR too has such virtues. While that may be true, I don't think the virtues of PSR will be enough to convert a philosopher who starts out with more "extensionalist" sympathies. For instance, suppose that we are not reasoning about metaphysical necessity, or sufficient reasons or any other such metaphysical exotica, but are rather simply reasoning about what is the case, and Boolean combinations of these states of affairs. We will inevitably find ourselves appealing to principles like the following:

$$\exists qq \forall p(p \prec qq \leftrightarrow p)$$

and

$$\exists qq \forall p(p \prec qq \leftrightarrow \neg\neg p)$$

and

$$\exists qq \forall p\big((p \prec qq \leftrightarrow \exists q \exists r(p \leftrightarrow q \wedge r)\big)$$

The worry is that once we give up the instance of plural comprehension for '$p \wedge \Diamond\neg p$' we will have to give up these instances as well. For instance, if there are all and only the truths, surely there are also all and only the contingent truths, since the contingent truths are among the truths. And if there are all and only those propositions whose double negations are true, then presumably there are all those propositions whose double negations are contingently true. And if there are all and only those propositions materially equivalent to conjunctions then there are all and only those propositions materially equivalent to conjunctions of the form '$p \wedge \Diamond\neg p$'. By giving up plural comprehension, not only is our theory of modal metaphysics weakened, but also principles in extensional metaphysics are weakened.

Perhaps there is a plausible theory that allows us to appeal to comprehension for extensionally definable pluralities but not intensionally definable pluralities. Levey's proposed solution does not provide this. This seems to be significant since without a criterion for distinguishing true and untrue instances of plural comprehension, it is hard to evaluate how radical the extensibilist position is.

There is a further consequence of the rejection of plural comprehension that that is worth pointing out. I have formulated the argument using quantification into sentence position together with plural quantification into sentence position. On the current interpretation of Levey, he is suggesting that while there are contingent truths, there are no things that are *the* contingent truths. But he is not suggesting that there is no such thing as *being contingently true*. But just as ordinary plural quantification can be traded out for second order quantification, so too plural quantification into sentence position can be traded out for third order quantification. Thus if we can make sense of the infinitary conjunction of the condition of being contingently true independently of the instances of that property, then the argument can be reformulated without quantifying over pluralities of propositions but instead by quantifying over properties of propositions. Once this is done, we see that a rejection of plural comprehension at the level of plural sentential variable inevitably leads to a rejection of comprehension at the level of variables occupying the position of a monadic sentential operator. In other words, there is an inherent tension between there being some property of being contingently true while there being no things that are the contingent truths.

A final potentially worrying consequence of the rejection of plural comprehension is that it looks to be at odds with some of the motivations behind the PSR itself. One historically use of the principle was to infer that there was some explanation of every member of some descending chain of contingent explanations $\ldots p_2 < p_1 < p_0$. But without plural comprehension, there isn't a guarantee that there are those proposition that are the propositions in the chain. Here again what the extensibilist needs is either some further argument that indefinitely extensible concepts must provide untrue instances of plural comprehension or else some other formulation of what the indefinite extensibility of contingent truth amounts to. In the next section, I will consider one such argument suggested by Øystein Linnebo (2010) and Stephen Yablo (2006) in the context of the iterative conception of set. I will argue that Linnebo's and Yablo's arguments do not generalize to the context of the paradox of sufficient reason. Rather, the considerations that they mention motivate a restriction on the PSR.

4 Set Theory and Indefinite Extensibility

The concept of indefinite extensibility is often appealed to as a response to Russell's paradox. One standard way of formulating the paradox is in *plural-first order logic*. In this setting, the paradox can be seen as an inconsistency between (first order) plural comprehension and the principle that any things form a set, where some things are said to form a set if and only if there is a set whose members are precisely those things. The first order principle of plural comprehension can be written in a two sorted first order language with a predicate $\prec$:

(Comprehension) $\exists xx \forall u(u \prec xx \leftrightarrow \varphi(u))$

An instance of Comprehension results from replacing $\varphi(u)$ with a formula in which u occurs free and the plural variable xx does not occur free. The principle can be shown to be inconsistent with a naive conception of set formation:

(Collapse) $\forall xx \exists y \forall u(u \prec xx \leftrightarrow u \in y)$

By replacing $\varphi(u)$ with $u \notin u$ in Comprehension one obtains:

1. $\exists xx \forall u(u \prec xx \leftrightarrow u \notin u)$

Then by Collapse and uncontroversial reasoning one can infer

(2) $\exists y \forall u(u \in y \leftrightarrow u \notin u)$

But (2) is inconsistent in classical first order logic.

Yablo has suggested that we reject Comprehension in light of the inconsistency:

> The condition $\phi(u)$ that (I say) fails to define a plurality can be a perfectly determinate one; for any object x, it is a determinate question whether x satisfies $\phi(u)$ or not. How then can it fail to be a determinate matter what are *all* the things that satisfy $\phi(u)$? I see only one answer to this. Determinacy of the ϕ's follows from
> (i) determinacy of $\phi(u)$ in connection with particular candidates,
> (ii) determinacy of the pool of candidates.
> If the difficulty is not with (i), it must be with (ii). (Yablo , 151-152)[16]

[16] The quotation has been altered to fit the notation of this paper.

Yablo goes on to suggests that in the case of sets, there is indeterminacy in the pool of candidates. The indeterminacy has to do with the iterative conception of sets. The universe of sets is built up in stages. At each stage all of the sets are formed from objects at previous stages. Nevertheless, at no stage is this process complete. At any stage, it is possible to go on and form new sets.

Linnebo (2010) develops this response in more detail. On Linnebo's reading of Yablo, the failure of plural comprehension in the context of set theory is attributed to the implicit modal character of the set theorist's quantifiers. When the set theorist quantifies over, for instance, all sets, they are to be read as quantifying over all of the sets *no matter what sets one goes on to form.* To capture this "implicitly modal character" of the set theorist's quantifiers, he introduces a new primitive modal operator $\blacksquare$ together with its dual $\blacklozenge$. Informally, $\blacksquare\phi$ is interpreted as "no matter what sets we go on to form it will remain the case that ϕ" and $\blacklozenge\phi$ as "it is possible to go on to form sets so as to make it the case that ϕ." The set theorist's quantifiers, $\forall$ and $\exists$, are then read as $\blacksquare\forall$ and $\blacklozenge\exists$. So interpreted Comprehension becomes

(Potential Comprehension) $\blacklozenge\exists xx\blacksquare\forall u(u \prec xx \leftrightarrow \varphi(u))$

and Collapse becomes:

(Potential Collapse) $\blacksquare\forall xx\blacklozenge\exists y\forall u(u \prec xx \leftrightarrow u \in y)$

But Potential Comprehension, Linnebo contends, is false:

> This scheme translates as the claim that, given any formula $\phi(u)$ it is possible for there to be some objects xx such that *no matter what sets we go on to form* xx are all and only the ϕ's. It is not hard to see that this claim is problematic. (2010, 157)

But what if we insist on reading the quantifiers in their normal, "non-potentialized" manner? On that reading, Linnebo is clear, Comprehension is true and Collapse is false: it is only on the suitably "potentialized" reading of Collapse that we get a truth. For notice that if we insist on the implicitly potential reading of the quantifiers, all that this shows is the tenability of Potential Collapse; it does not show the tenability of collapse on its flat footed reading.

One could read Linnebo's account as a defense of rejecting Comprehension. Alternatively, we can read it as a way of rejecting Collapse and replacing it with Potential Collapse. The principle Potential Collapse provides one with a way of preserving the intuition that any things whatsoever form a set: what our intuitions are tracking are merely that wherever we are in the set theoretic hierarchy, the things at that level will go on to form sets at some further level (indeed they will go on to form sets as soon as possible in the hierarchy, as it were). What I do want to do is look at whether we can pursue a similar strategy in the context of the paradox of sufficient reason.

5 PSR and Indefinite Extensibility

Just as the set theoretic antimonies can be conceived as a tension between Comprehension and Collapse, the paradox of sufficient reason can be conceived as a tension between the PSR and the higher order version of plural comprehension (the axiom A5 of the BL). The principle of Collapse says that for each plurality there is a set. The PSR says that for each contingent truth, there is a sufficient reason. In the one case, pluralities are collapsed into sets; in the other, contingent truths are collapsed into explanans. We might thus try to model failures of comprehension in the context of the paradox of sufficient reason on the failures of comprehension in the set theoretic case.[17]

One might object that PSR and Collapse are not analogous. Whereas Collapse truly collapses *some* things to sets, PSR just pairs each contingent truth with an explanation. The analogy can be made stronger by either looking at a plural corollary of PSR or by looking at a non-plural basis for Collapse. Let Cp abbreviate $p \wedge \Diamond\neg p$. The following principle is derivable from TSR:

$$\forall pp(\forall p(p \prec pp \rightarrow Cp) \rightarrow \exists q \forall p(p \prec pp \rightarrow q < p)$$

In prose, if some propositions are all contingent, then there is some single sufficient reason for each of them.[18] Thus pluralities of contingent truths are collapsed into their grounds. We might call that this the *principle of explanatory ground*. It is simpler to conceive of the tension as being between PSR and A5 than between the principle of explanatory ground and A5. If one does set up the tension between A5 and the principle of explanatory ground, one quickly realizes that what one says about the tension will ultimately rest on what one says about PSR.

Alternatively note that Collapse itself stands to the non-plural principle that anything is a member of some set as the principle of explanatory ground stands to P:

$$\forall x \exists y x \in y$$

If this principle is true, then any things xx will form the set y that is the union of the singletons of each xx. (Since for each $x \prec xx$ there is some z such

[17] There is one important disanology that merits further discussion. The set theoretic paradoxes can be recast as a tension between second-order comprehension and the principle that any property has an extension. But in the presence of second-order comprehension, not only is the principle that any property has an extension inconsistent, but so too is the appropriate modalized version of it. It matters quite a bit to the contours of the problem whether it is given plural or second-order formulations. As mentioned in section 3, an alternative formulation of van Inwagen's argument eschews plurals and with it plural comprehension in terms of higher-order comprehension (or some form of λ-conversion if lambdas are used). The tension is then conceived as between there being the property of being a contingent truth and the principle of sufficient reason. What's interesting is that it does not seem to matter as much to van Inwagen's argument whether it is formulated in terms of plurals or using higher-order resources.

[18] To construct the proof take some contingent truths pp. If they are non-empty, then $\bigwedge pp$ is contingent by Truth and Sufficiency and so by PSR there is a sufficient reason q for $\bigwedge pp$. Then by Distribution q is a sufficient reason for each p. If pp are empty then the consequent of the conditional holds vacuously.

that $x \in z$ we can separate x's singleton from z by considering $\{z' \in z \mid z' = x\}$, which is a set by the separation axiom.)

Thus, assuming that the extensibilist will maintain all of the standard set theoretic principles, the replacement of Collapse with Potential Collapse requires a corresponding replacement:

$$\blacksquare\forall x \blacklozenge \exists y y \in x$$

This suggests that the analogy between the two paradoxes is actually quite robust.

There is another motivation for modeling a solution to the paradox of sufficient reason off a solution to the set theoretic paradoxes. Both the relation of set formation and the relation of *being a sufficient reason for* are what Karen Bennet (2017) calls "building relations." The exact definition of 'building relation' needn't concern us here. What matters is that they fall into a natural class of relations. Both the paradox of sufficient reason and the set theoretic paradoxes might then be viewed as arguments to the effect that building relations, or building relations of a certain sort, must meet certain substantive constraints: given plausible principles governing some sort of entities over which the building relation is defined, it cannot be that everything is built up by something according to that building relation. Perhaps, though, it can be that *potentially* everything is built by something according to that building relation. If similar principles governing similar relations lead to similar problems, we should expect similar solutions.

How then should we model a response to the paradox of sufficient reason off of the response given to the set theoretic paradox? Suppose we start by trying to mirror Yablo's response:

> The condition 'Cp' fails to define a plurality despite being a perfectly determinate condition; for any proposition p, it is a determinate question whether p satisfies 'Cp' or not. How then can it fail to be a determinate matter what *all* the things that satisy 'Cp'? I see only one answer to this. Determinacy of 'Cp' follows from:
> (i) determinacy of 'Cp' in connection with particular candidates,
> (ii) determinacy of the pool of candidates.
> If the difficulty is not with (i), it must be with (ii).

But what are the pool of candidates? In setting up the language, I introduced the higher order quantifiers as irreducible and absolutely unrestricted. In the case of set theory, it is clear that one's quantifiers are restricted (implicitly or explicitly) by the non-logical predicate 'set'. In the case of the paradox of sufficient reason, there does not appear to be any analogous restriction.

The proponent of the PSR might take up a particularly strong stance: there is indeterminacy in reality itself, as it were. But how should this response be understood and developed? Here is one suggestion. Let us for the moment conceive of the higher order quantifiers as ranging over some special abstract objects, *propositions*. Talk of reality is to be understood as talk of everything that is the case, or alternatively, as talk of the collection of all true propositions. The proponent of the PSR sees indeterminacy in this pool of candidates by seeing reality as a hierarchy with no bottom level: given any collection of true propositions, there is some more fundamental level at which those propositions are jointly grounded. In the set theoretic case, the sets at higher levels are generated by collecting together sets at lower levels. In the present case, propositions at lower levels are "generated" by taking sufficient reasons for conjunctions of propositions at higher levels. The concept of contingent truth is indefinitely extensible because the contingent truths are indefinitely *explicable*.

We needn't take this talk of levels of reality as ontologically serious. Just as in the case of the extensibilist response to the set theoretic antimonies we can introduce a new modal operator $\blacksquare$ as before together with its dual $\blacklozenge$. Here $\blacksquare\varphi$ is to be read, intuitively, as "no matter what contingent truths get explained it will remain the case that φ" and $\blacklozenge\varphi$ as "it is possible to go on to explain some contingent truths so that φ.' If we then insist on reading our initial quantifiers on their "unpotentialized readings" the PSR is rejected in favor of the principle of *potential* sufficient reason:

$(\mathsf{PSR}^{\blacksquare})$ $\blacksquare\forall p(Cp \to \blacklozenge\exists q(q < p))$

We of course want to ensure that no matter what stage of explanation we are at, all of the principles in TSR apply. Thus in addition to replacing the P with the $\mathsf{P}^{\blacksquare}$ we ought also to replace the other principles TSR with the result of prefixing $\blacksquare$ to them. Let then the *theory of potential sufficient reason*, TPSR, be the result of prefixing $\blacksquare$ to each principle in TSR and then replacing the P with the $\mathsf{P}^{\blacksquare}$.

Alternatively, if a potentialized reading of the quantifiers is in play, the principle $\mathsf{A5}$ is dropped and replaced with:

$\mathsf{A5}^{\blacksquare}$ $\blacklozenge\exists qq\blacksquare\forall p(p \prec qq \leftrightarrow \varphi(p))$

The principle $\mathsf{A5}^{\blacksquare}$ might be rejected on the grounds that there is no level of reality, or stage of explanation, at which all contingent truths from all stages are collected together: at every stage, one generates a new contingent truth by taking the sufficient reason for the conjunction of contingent truths at that stage. Notice though that this does not give us a reason to reject the original comprehension principle, $\mathsf{A5}$, since that principle is now being read as asserting that at each level, one can collect together the instances of a given condition at that level.

Similarly, $\mathsf{PSR}^{\blacksquare}$ can be maintained since at each level, every truth can go on to be explained at some more fundamental level. But P on its flatfooted reading should be rejected: the conjunction of all contingent truths is not explained at a given level but only at some more fundamental level.

It is important here to not get confused by talk of "levels of explanation." This is just a way of informally stating claims fronted by $\blacklozenge$. But it is crucial for the success of this model of indefinite extensibility that $\blacklozenge$ be a prophylactic operator. Thus an assertion of $\blacklozenge\exists x\varphi$ should not in general commit one to what an assertion of $\exists x\varphi$ commits them to. In particular our understanding of $\blacklozenge$ should guarantee that there are failures of the inference from $\blacklozenge\exists x\varphi$ to $\exists x\varphi$. The unpotentialized quantifier $\exists$ is supposed to be absolutely unrestricted. Nevertheless one could go on to explain things so that there would be propositions doing things that they are not actually doing.

As I mentioned above, I prefer a reading of Linnebo's response on which it is $\mathsf{Collapse}$ that is being rejected while $\mathsf{Comprehension}$ is maintained. I think that in the present case, the analogous move is a natural one to make. The background logic should be suitable for many metaphysical contexts, not just special purpose generalizations about the hierarchy of explanation. If we thus insist that the unrestricted flatfooted reading is in play, the theory TSR should be replaced by TPSR.

What are the appropriate logical principles for the newly introduced operator $\blacksquare$? While I do not make any claims of completeness, I suggest that *at least* $\blacksquare$ should have the modal logic T. Thus we should assume the following principles:

$\mathsf{A6}$ $\blacksquare(\varphi \to \psi) \to (\blacksquare\varphi \to \blacksquare\psi)$
$\mathsf{A7}$ $\blacksquare\varphi \to \varphi$
$\mathsf{R4}$ $\varphi/\blacksquare\varphi$

In the contexts of set theory, Linnebo (2010) supposes that $\blacksquare$ has the logic of $\mathsf{S4.2}$; the above principles will do for my purposes. It will also not matter for present purposes what principles govern the interaction of $\blacksquare$ and $\Box$. Let us then extend BL with the axioms $\mathsf{A6}$ and $\mathsf{A7}$ together with the rule of inference $\mathsf{R4}$. Call the result the *extended background logic*, BL^+.

The official response from indefinite extensibility is then this: in order to reflect the indefinite extensibility of the concept of contingent truth, we ought to expand our language with $\blacksquare$ in order to reason explicitly about the potential stages of explanation. The proper theory of sufficient reason is TPSR and the proper background logic at least *includes* BL^+.

Does the proposed response avoid modal collapse? Surprisingly no. Or perhaps it is better to say, *almost* no. We need to make one assumption that has not been explicitly made at this point. And this is that the finitary version of distribution holds:

(Finite Distribution) $\blacksquare\forall p\forall q\forall r(r < p \wedge q \to (r < p \wedge r < q))$.

Note that Levey himself appeals to the more general version of this principle, and so he is hardly in a position to reject it. Moreover if we add an identity operator to our language, then there is no need to appeal to $\mathsf{Finite\ Distribution}$ since we can mimic its effect with $\mathsf{Distribution}$ by defining pluralities comprised of the conjuncts of some finite conjunction, e.g., $r \in rr \leftrightarrow r = p \vee r = q$. The principle $\mathsf{Distribution}$ then guarantees that something explains $\bigwedge rr$ only if it explains both p and q.

We can then show the following:

Theorem 2 $\mathsf{TPSR} \cup \{\mathsf{Finite\ Distribution}\} \vdash_{\mathsf{BL}^+} \mathsf{Necessitarianism}$

A proof of theorem 2 is given in the appendix. Informally the idea is just that if p is an unexplained contingent fact then it is an inexplicable, but contingent, fact that p is an unexplained fact. And so given the sort of classical principles we have in the background logic, if all contingent truths are potentially explicable, all contingent truths are explicable. This combined with other easily verifiable facts shows that the theory of potential sufficient reasons entails every member of the theory of sufficient reason. And so since the extended background logic includes all of the background logic, the theory of potential sufficient reason entails Necessitarianism if the theory of sufficient reason does.

With a bit of reflection on the proposed response, it is not all that surprising that it fails. The main part of the proof is exactly analogous to the the proof that "weak-verificationism", the principle that all truths are knowable, entails "strong verificationism", the principle that all truths are known.[19] If we let Ep abbreviate $\exists q(q < p)$ then one can show that the following two facts hold:

$$Ep \to p$$
$$E(p \wedge q) \to (Ep \wedge Eq)$$

The proof of the first principle makes use of the principle F, the proof of the second makes use of Finite Distribution. Thus E is what we might call a "Fitch operator." But with respect to such Fitch operators, together with classical principles, the following two principles are equivalent:

1. $\forall p(p \to Op)$
2. $\forall p(p \to \Diamond Op)$

Where O is a Fitch operator and $\Diamond$ is a normal modal operator. That weak verificationism entails strong verificationism is an instance of this fact. That the principle of potential sufficient reason entails the principle of sufficient reason is also an instance of this fact.

Given this fact, potential responses to the argument can be developed by analogy with responses to Fitch's argument. For instance, Williamson (1982) argues that the derivation of strong verificationism from weak verificationism does not go through if one's background logic is intuitionistic. As Williamson (1998) states the conclusion, Fitch's argument provides the weak-verificationist with a "reason to revise logic." A similar conclusion is suggested here concerning the extensibilist. Theorem 6.1. shows that when one models the putative indefinite extensibility of contingent truth on standard models of the indefinite extensibility of the concept of set, one is still able to deduce necessitarianism. This combined with the points made in §4 shows that the extensibilist proponent of the PSR has a reason to revise logic: either by rejecting plural comprehension, or else by rejecting classical propositional logic in favor of intuitionisitic logic.

[19] For discussion see Williamson 2000. For the original presentation of the proof see Fitch 1963.

6 Conclusion

The paradox of sufficient reason, as Levey calls it, can be formalized precisely in a two sorted higher order language that permits in addition to quantification into sentence position, plural quantification into sentence position. When the paradox is precisely set out in this language, one sees that it can naturally be described as a tension between a higher order version of plural comprehension and the principle of sufficient reason; the tension is very much analogous to the tension between a crucial set collapse principle and first order plural comprehension. It is thus natural to wonder whether an extensibilist solution to the one extends to the other. The main take away of this paper is that extensibilist solutions may require constructivist commitments over and above those required by the extensibilist solutions in the set theoretic case.

7 Appendix I

The following appendix contains proofs of lemma 1 and theorem 2.

For convenience the principles of TSR and BL are listed again:

A1 Any substitution instance of a propositional tautology.
A2 $\forall x\varphi \to \varphi[t/x]$ (where t is a term of the appropriate sort free for x in φ).
A3 $\forall x(\varphi \to \psi) \to (\forall x\varphi \to \forall x\psi)$.
A4 $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$.
R1 $\varphi, \varphi \to \psi/\psi$.
R2 $(\varphi \to \psi)/(\varphi \to \forall x\psi)$ if x is not free in φ.
R3 $\varphi/\Box\varphi$.

(I) $\forall pp\big(\Box \bigwedge pp \to \forall p(p \prec pp \to \Box p)\big)$.
(T) $\forall pp\big(\forall p(p \prec pp \to p) \to \bigwedge pp\big)$.
(S) $\forall p\forall q\big(p < q \to \Box(p \to q)\big)$.
(Ir) $\forall p(p \not< p)$.
(D) $\forall p\forall qq(p < \bigwedge qq \to \forall q(q \prec qq \to p < q))$.
(F) $\forall p\forall q(p < q \to (p \wedge q))$.
(P) $\forall p\big(Cp \to Ep\big)$.

Let $C(pp)$abbreviate $\forall p(p \prec pp \leftrightarrow (p \wedge \Diamond\neg p))$. Then we want to show that $\mathsf{TSR} \vdash_{\mathsf{BL}} \forall pp(C(pp) \to \Box \bigwedge pp)$. The following derivation provides a sketch of how the proof goes:

(1)	$(\mathsf{D}) \vdash p < \bigwedge pp \to (p \prec pp \to p < p)$	A1, A2, R1
(2)	$(\mathsf{Ir}) \vdash p \not< p$	A2
(3)	$(\mathsf{D}), (\mathsf{Ir}) \vdash (p < \bigwedge pp \to p \not\prec pp)$	(1, 2)
(4)	$(\mathsf{D}), (\mathsf{Ir}) \vdash (C(pp) \to (p < \bigwedge pp \to (p \to \Box p)))$	(3)
(5)	$(\mathsf{F}) \vdash p < \bigwedge pp \to p$	A1, A2, R1

(6)	$(\mathsf{D}), (\mathsf{Ir}), (\mathsf{F}) \vdash (C(pp) \to \forall p(p < \bigwedge pp \to \Box p))$	$(4, 5)$
(7)	$(\mathsf{S}) \vdash \forall p(p < \bigwedge pp \to \Box p) \to (E \bigwedge pp \to \Box \bigwedge pp)$	A1, A2, A3, A4, R1
(8)	$(\mathsf{P}) \vdash \neg E \bigwedge pp \to (\bigwedge pp \to \Box \bigwedge pp)$	A1, A2, R1
(9)	$(\mathsf{T}) \vdash C(pp) \to \bigwedge pp$	A1, A2, R1
(10)	$(\mathsf{D}), (\mathsf{Ir}), (\mathsf{F}), (\mathsf{S}) \vdash C(pp) \to (E \bigwedge pp \to \Box \bigwedge pp)$	$(6, 7)$
(11)	$(\mathsf{P}), (\mathsf{T}) \vdash C(pp) \to (\neg E \bigwedge pp \to \Box \bigwedge pp)$	$(8, 9)$
(12)	$(\mathsf{D}), (\mathsf{Ir}), (\mathsf{F}), (\mathsf{T}), (\mathsf{P}), (\mathsf{S}) \vdash \forall pp(C(pp) \to \Box \bigwedge pp))$	$(10, 11)$

Turning now to theorem 2 recall that the axioms and rules of inference of BL^+ are those of BL plus

A6 $\blacksquare(\varphi \to \psi) \to (\blacksquare\varphi \to \blacksquare\psi)$
A7 $\blacksquare\varphi \to \varphi$
R4 $\varphi / \blacksquare\varphi$

We have already shown that $\mathsf{TSR} \vdash_{\mathsf{BL}} \mathsf{N}$. And so since $\mathsf{BL}^+ \supset \mathsf{BL}$ it suffices to show that for each $\varphi \in \mathsf{TSR}$, $\mathsf{TPSR} \vdash_{\mathsf{BL}^+} \varphi$. For most $\varphi \in \mathsf{TSR}$, that sequent has a one line proof that sites A7. The only difficult case is showing that $\mathsf{TPSR} \vdash_{\mathsf{BL}^+} \mathsf{P}^\blacksquare$.

For readability and convenience I'll introduce the following abbreviations:

$(\mathsf{P}^\blacksquare)$ $\blacksquare\forall p(Cp \to \blacklozenge\exists q(q < p))$
$(\mathsf{F}^\blacksquare)$ $\blacksquare\forall p\forall q\forall q(p < q \to (p \wedge q))$
$(\mathsf{D}^\blacksquare)$ $\blacksquare\forall p\forall q\forall r(p < q \wedge r \to (p < q \wedge p < r))$

Then the following lemma suffices to prove the theorem:

Lemma 2 $(\mathsf{P}^\blacksquare), (\mathsf{F}^\blacksquare), (\mathsf{D}^\blacksquare) \vdash_{\mathsf{BL}^+} (\mathsf{P})$

Proof

(1)	$(\mathsf{P}^\blacksquare) \vdash_{\mathsf{BL}^+} C(p \wedge \neg Ep) \to \blacklozenge E(p \wedge \neg Ep))$	A2, A7
(2)	Finite Distribution $\vdash_{\mathsf{BL}^+} q < (p \wedge \neg Ep) \to (q < p \wedge q < \neg Ep)$	A2
(3)	$\mathsf{F} \vdash_{\mathsf{BL}^+} (q < \neg Ep) \to \neg Ep$	A2
(4)	Finite Distribution, $\mathsf{F} \vdash_{\mathsf{BL}^+} q < (p \wedge \neg Ep) \to (q < p \wedge \neg Ep)$	$(2, 3)$
(5)	$\vdash_{\mathsf{BL}^+} \neg(q < p \wedge \neg Ep)$	A2
(6)	Finite Distribution, $\mathsf{F} \vdash_{\mathsf{BL}^+} \neg(q < (p \wedge \neg Ep))$	$(4, 5)$
(7)	Finite Distribution, $\mathsf{F} \vdash_{\mathsf{BL}^+} \neg E(p \wedge \neg Ep))$	(6), R2
(8)	$(\mathsf{D}^\blacksquare), (\mathsf{F}^\blacksquare) \vdash_{\mathsf{BL}^+} \blacksquare\neg E(p \wedge \neg Ep)$	(7), A6, R4
(9)	$(\mathsf{P}^\blacksquare), (\mathsf{D}^\blacksquare), (\mathsf{F}^\blacksquare) \vdash_{\mathsf{BL}^+} \neg C(p \wedge \neg Ep)$	$(1, 8)$
(10)	$(\mathsf{P}^\blacksquare), (\mathsf{D}^\blacksquare), (\mathsf{F}^\blacksquare) \vdash_{\mathsf{BL}^+} Cp \to \neg C(p \wedge \neg Ep)$	(9)

(11) $\vdash_{\mathsf{BL}^+} (Cp \wedge \neg C(p \wedge \neg Ep)) \rightarrow Ep$ A1, A4, R1, R3

(12) $(\mathsf{P}^{\blacksquare}), (\mathsf{D}^{\blacksquare}), (\mathsf{F}^{\blacksquare}) \vdash_{\mathsf{BL}^+} Cp \rightarrow Ep$ (10, 11)

(13) $(\mathsf{P}^{\blacksquare}), (\mathsf{D}^{\blacksquare}), (\mathsf{F}^{\blacksquare}) \vdash_{\mathsf{BL}^+} (\mathsf{P})$ (12), R2

8 Appendix II

When formulating van Inwagen's argument Levey (2016) makes use of a somewhat stronger version of the the PSR:

(Full PSR) $\forall p(p \rightarrow Ep)$

Let Full TSR be TSR $\cup$ {Full PSR}. Then we can show the following:

Proposition 1 Full TSR $\vdash_{\mathsf{BL}^+} \bot$

Proof The conjunction of all truths is true and so has a sufficient reason t. By Factivity t is also a truth. Applying Distribution gives us that $t < t$ violating Irreflexivity.

Interestingly, Full TSR without Distribution, however, is consistent. In fact we can show something a bit stronger. First we require that explanation be transitive:

(Transitivity) $\forall p \forall q \forall r(((p < q) \wedge (q < r)) \rightarrow p < r)$

Let the *basic theory of sufficient reason* BTSR be the set comprised of Full PSR, Irreflexivity, Transitivity, Inheritance, Truth and Sufficiency. Then we can show the following theorem:

Theorem 3 BTSR $\nvdash$ Necessitarianism

As an immediate corollary we also have that BTSR is consistent. We might take this as evidence for the thesis that it is Distribution that is to blame rather the indefinite extensibility of contingent truth.

To prove the theorem we will define a class of structures with the following properties: (i) BL is true in every member of this class, (ii) if $\varphi \rightarrow \psi$ and φ are true in a structure, ψ is true in that structure and (iii) BTSR is true in a structure that Necessitarianism is not. Exhibiting such a class suffices to prove theorem 3.

Let a *relational structure* be a triple $\langle W, R, E, \alpha \rangle$ such that W is a set, $\alpha \in W$, $R \subset W \times W$ and $E \subset \mathcal{P}(W) \times \mathcal{P}(W)$. An *assignment* function v maps propositional variables into $\mathcal{P}(W)$ and plural propositional variables into $\mathcal{P}(\mathcal{P}(W)$. A function $[\![\cdot]\!]^v$ from formulas of $\mathcal{L}_{HP}$ to subsets of W is defined as follows:

$[\![p_i]\!]^v = v(p_i)$
$[\![p_i \prec pp_j]\!]^v = \{w \in W \mid v(p_i) \in v(pp_i)\}$
$[\![\bigwedge pp_i]\!]^v = \bigcap v(pp_i)$
$[\![\neg\varphi]\!]^v = W \setminus [\![\varphi]\!]^v$

$[\![\varphi < \psi]\!]^v = \{w \in W \mid [\![\varphi]\!]^v E [\![\psi]\!]^v\}$
$[\![\varphi \wedge \psi]\!]^v = [\![\varphi]\!]^v \cap [\![\psi]\!]^v$
$[\![\Box\varphi]\!]^v = \{w \in W \mid \forall w'(wRw' \to w' \in [\![\varphi]\!]^v)\}$
$[\![\forall p_i \varphi]\!]^v = \{w \in W \mid w \in [\![\varphi]\!]^{v[p_i/X]}, \forall X \subset W\}$
$[\![\forall pp_i \varphi]\!]^v = \{w \in W \mid w \in [\![\varphi]\!]^{v[pp_i/X]}, \forall X \subset \mathcal{P}W\}$

Here $v[x/X]$ is the function like v except that it maps x to X. A formula φ is true on an assignment v in $\langle W, R, E, \alpha\rangle$ if $\alpha \in [\![\varphi]\!]^v$. A closed sentence is true in a relational structure if and only if it is true on some assignment. A theory is true in a relational structure if and only if each member of that theory is true in that relational structure.

It is not hard to verify that the class of relational structures has properties (i) and (ii) described above. Thus we need only show that it has property (iii).

Proposition 2 *It's not the case that* Necessitarianism *is true in every relational structure that* BTSR *is true in.*

Proof Let ω be the first infinite von Nuemann ordinal and let α and β be any distinct objects that are not in ω nor in $\mathcal{P}(\omega)$. We define a relational structure $\langle W, \sim, E, \alpha\rangle$ such that $W = \{\alpha, \beta\} \cup \omega$, $\sim$ is the equivalence relation corresponding to the partition $\{\{\alpha, \beta\}, \omega\}$ and E is the transitive closure of the smallest relation E^- satisfying the following principles:

1. For any $x \subset \omega$, $\langle\{\alpha\}, \{\alpha, \beta\} \cup x\rangle \in E^-$.
2. If $x \subset \omega$ is infinite then $\langle\{\alpha\}, \{\alpha\} \cup x\rangle \in E^-$.
3. If $x \subset \omega$ is finite, then $\langle\{\alpha\} \cup n, \{\alpha\} \cup x\rangle \in E^-$, where $n = \sup x + 1$.

Then BTSR is true in $\langle W, \sim, E, \alpha\rangle$ but Necessitarianism is not[20]

Let me end briefly with some philosophical remarks. One way of thinking about the above relational structure is this. Here α is the actual world and β a possible world. The finite ordinals represent the space of impossible worlds. Thus the model requires that propositions be individuated more finely than necessary equivalence. Nevertheless, it is still a *Boolean* model of propositional granularity since the higher order quantifiers are interpreted over (complete and atomic) Boolean algebras. Thus the above construction shows that the theory of sufficient reasons without distribution is consistent even when interpreted over a relatively coarse grained conception of propositions.

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[20] One should be careful in thinking of this model. The finite ordinals $n < \omega$ are worlds in the model. But since each finite ordinal n is the set of its predecessors, each finite ordinal is also a proposition of the model; thus each finite ordinal is also a plurality of propositions in the model.

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