evo* 2020

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On the Combined Impact of Population Size and Sub-problem Selection in MOEA/D

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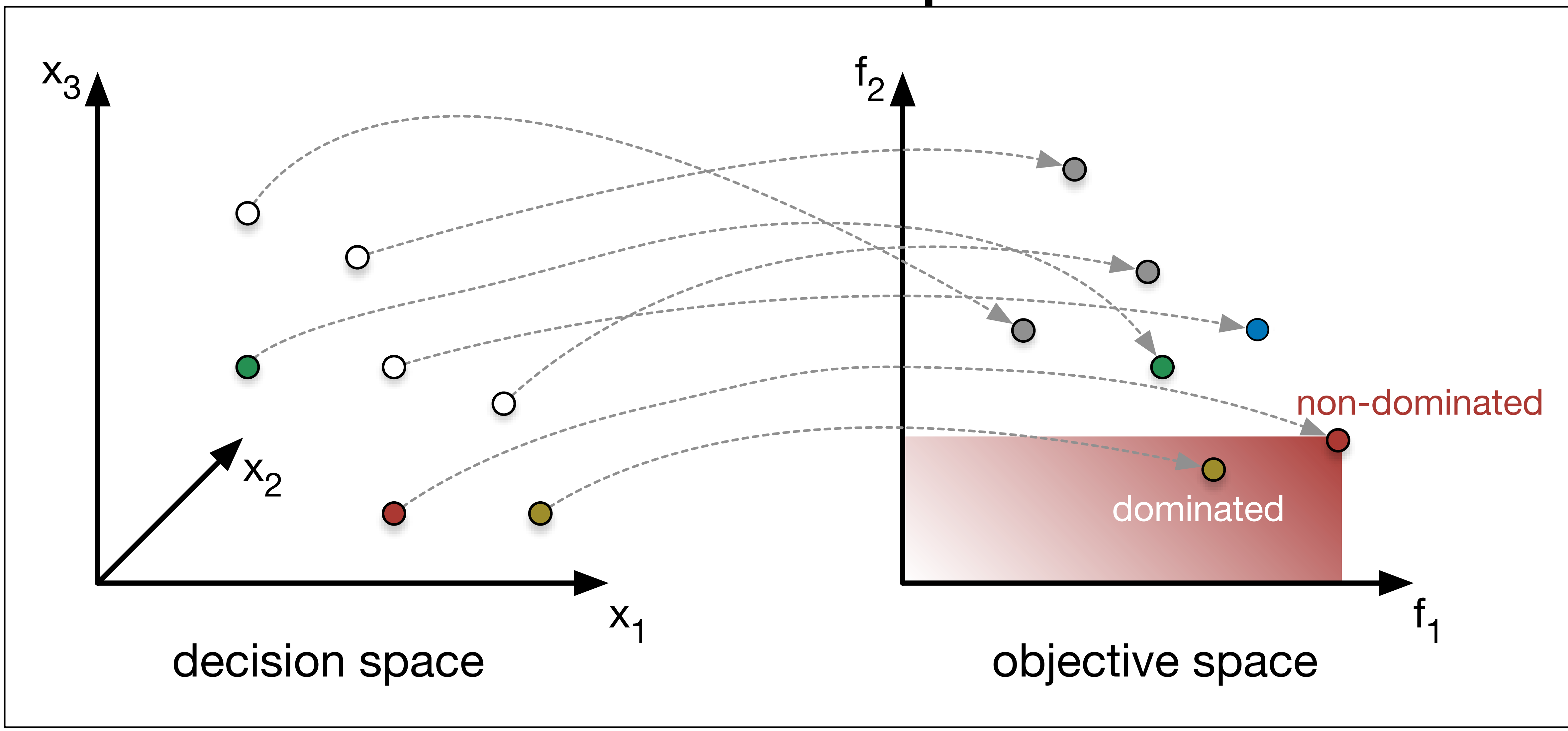
Inria

Outline

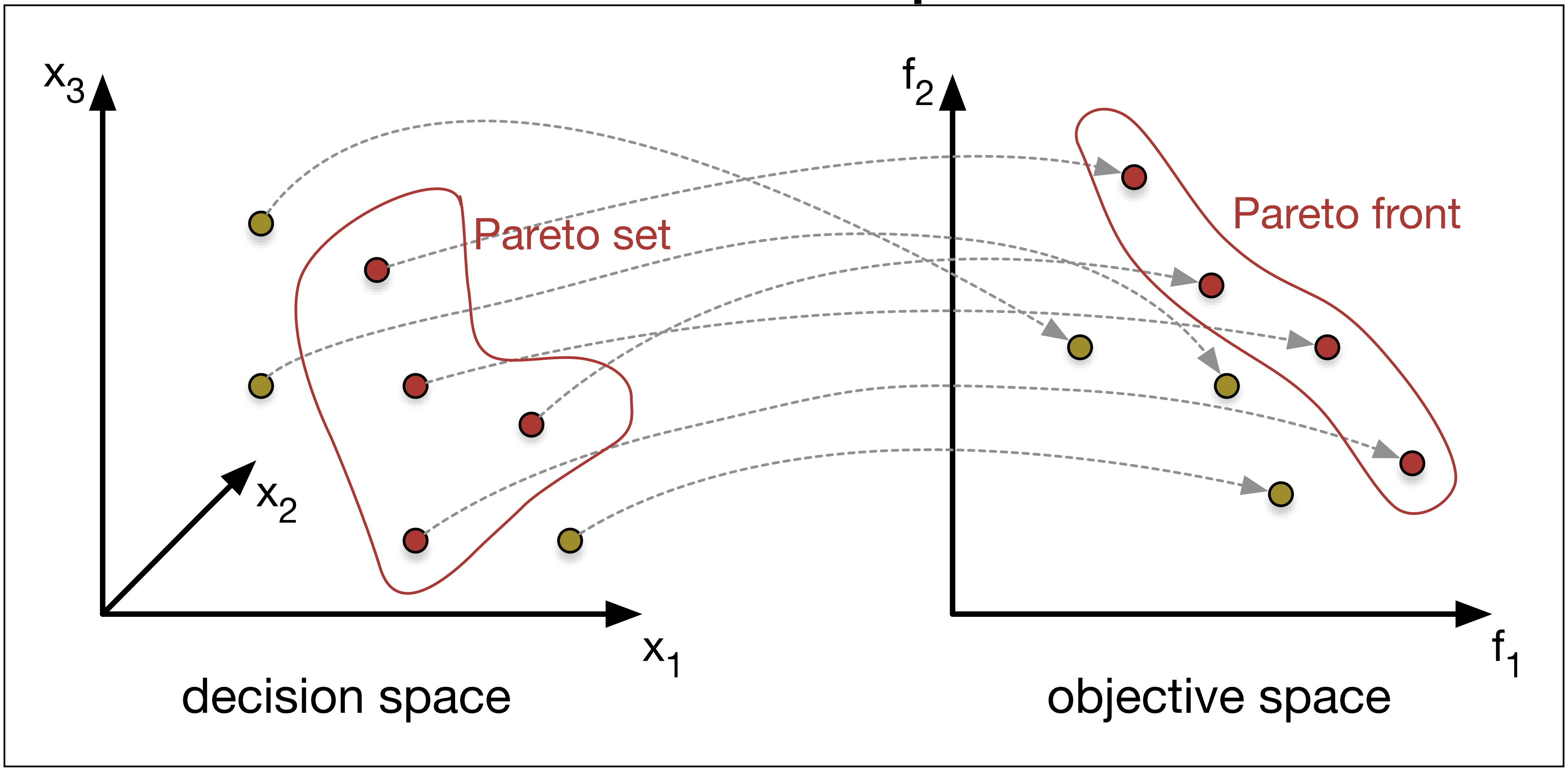
- General Context
  - Multi-Objective Optimization
  - The MOEA/D algorithm
- Review of MOEA/D by dissociating *the Population size* and *Number of sub-problem selected* ?
- Experimental study and main results
- Conclusion

Multi-objective & combinatorial optimization



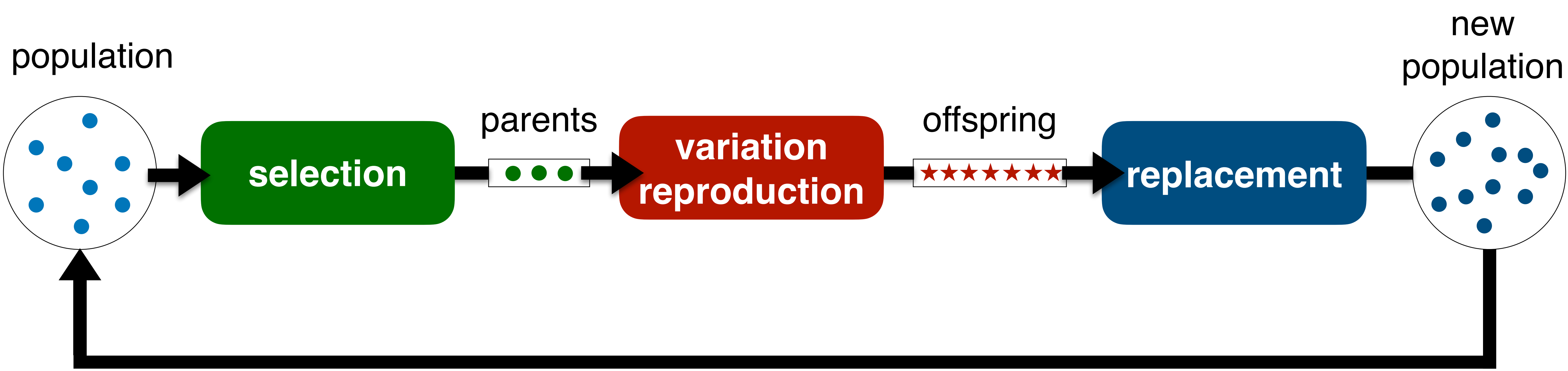
- Conflicting objectives
- **Pareto set/front**

Multi-objective & combinatorial optimization



- Find a good **Pareto set/front** approximation
- **Discrete** variables

Evolutionary Multi-objective Algorithms (EMOA)



The MOEA/D framework

Decompose the original **Multi-Objective Problem** into **multiple (single-objective)** sub-problems solved **cooperatively**

- **Decomposition**
  - Aggregation function
  - weight vectors / subproblems

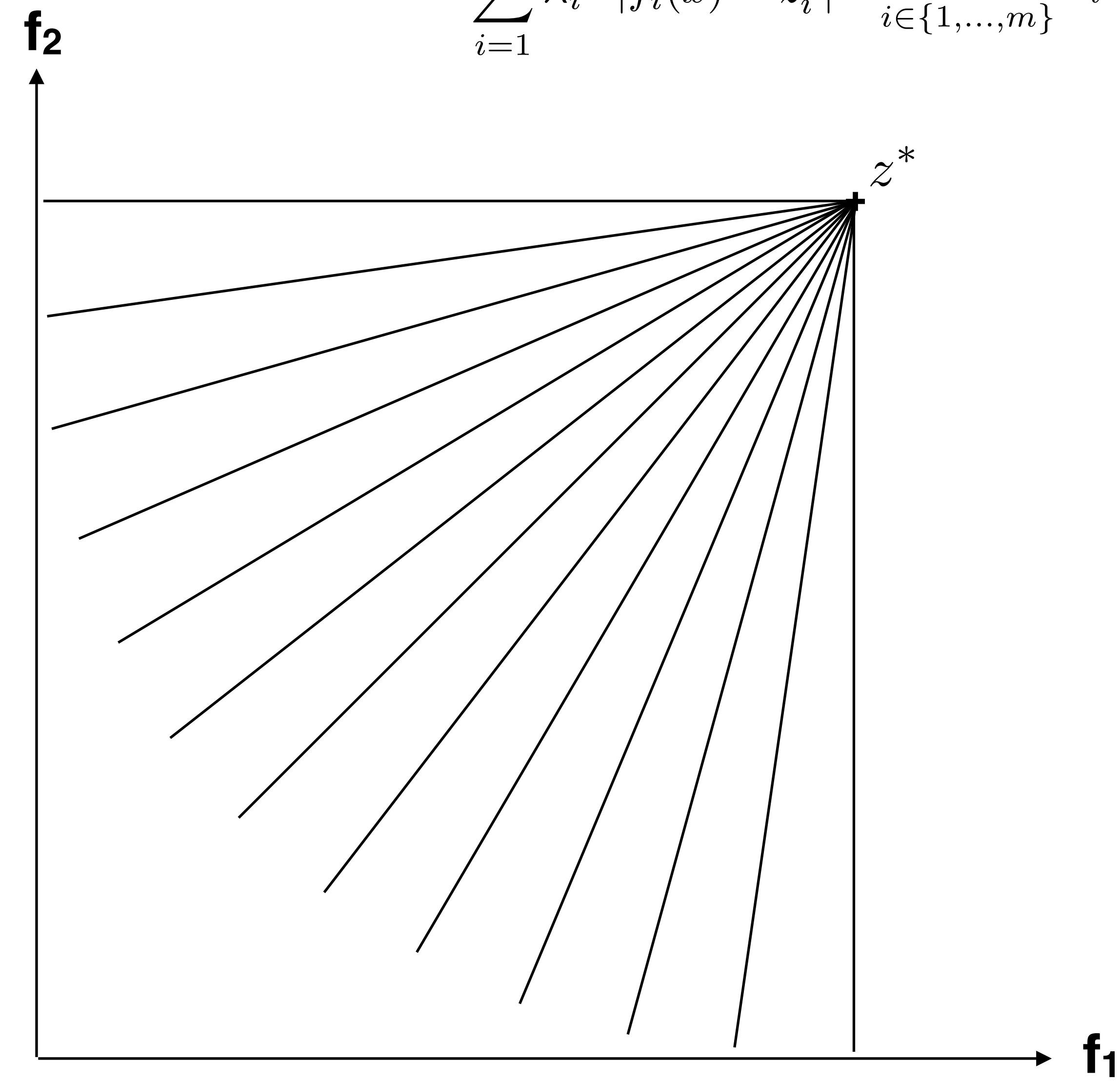
$g(x, \lambda | z^*)$

Weighted sum

$$\sum_{i=1}^{m} \lambda_i \cdot |f_i(x) - z_i^*|$$

Chebychev

$$\max_{i \in \{1,\ldots,m\}} \lambda_i \cdot |f_i(x) - z_i^*|$$



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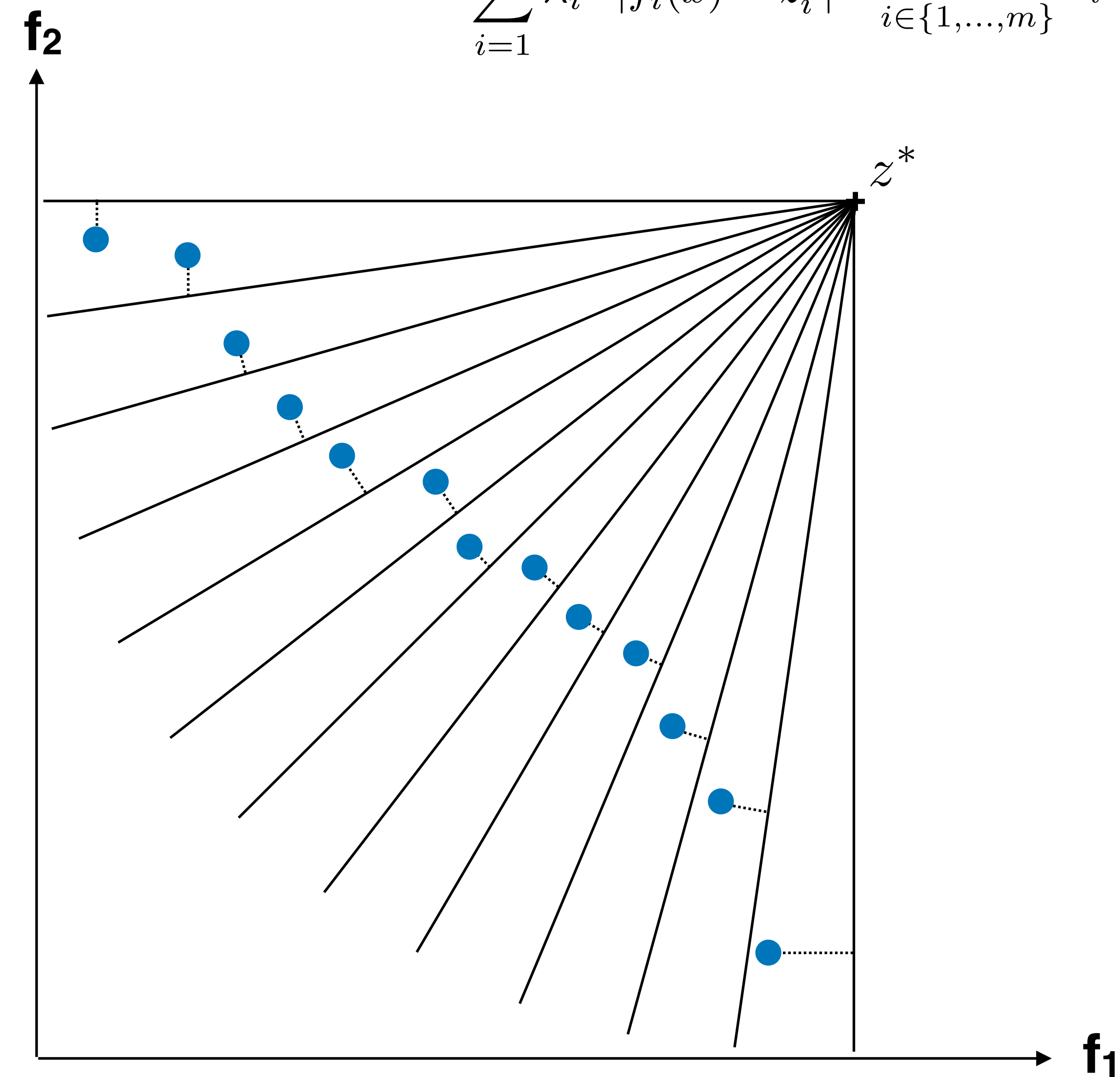

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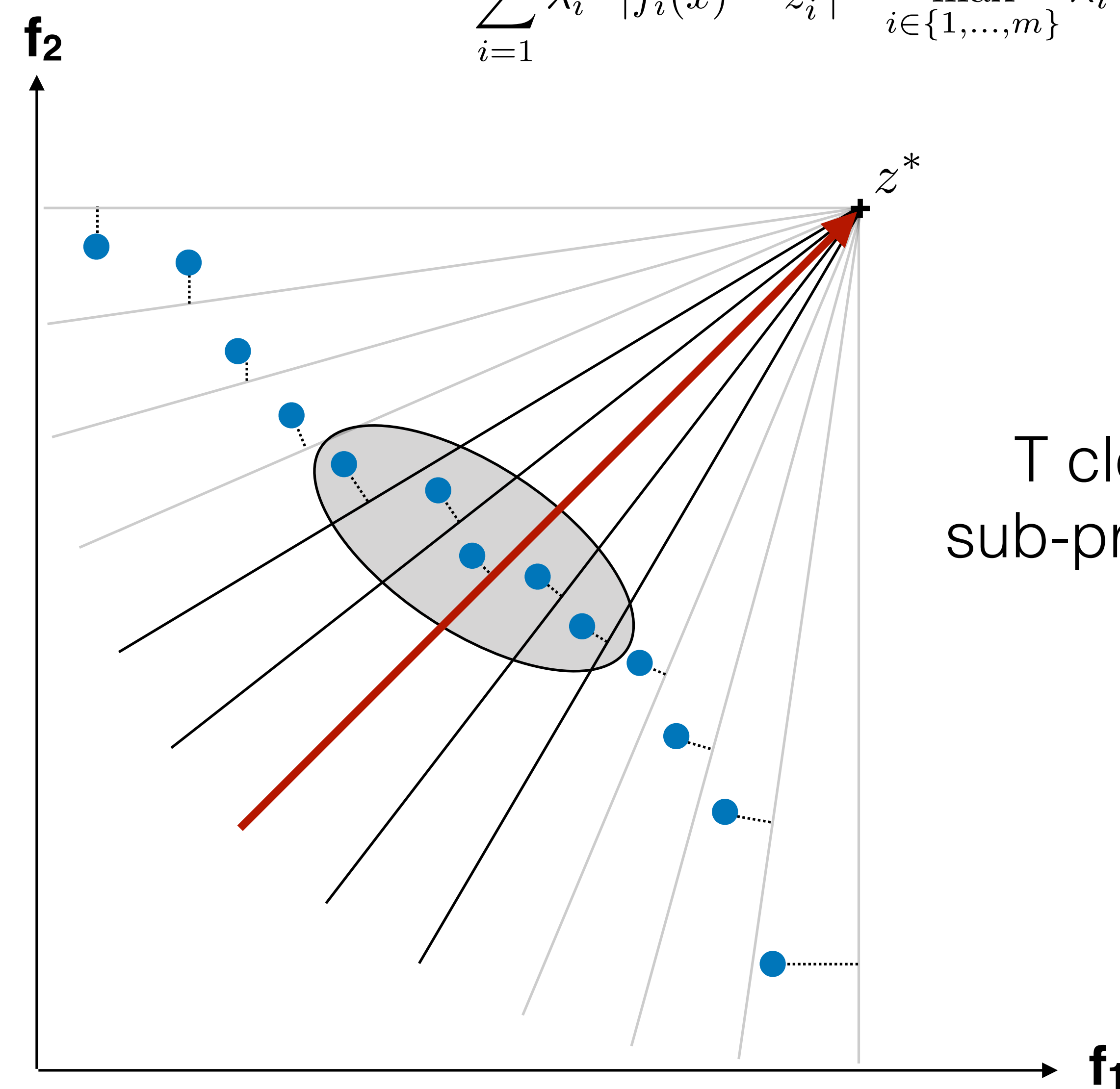

T closest sub-problems

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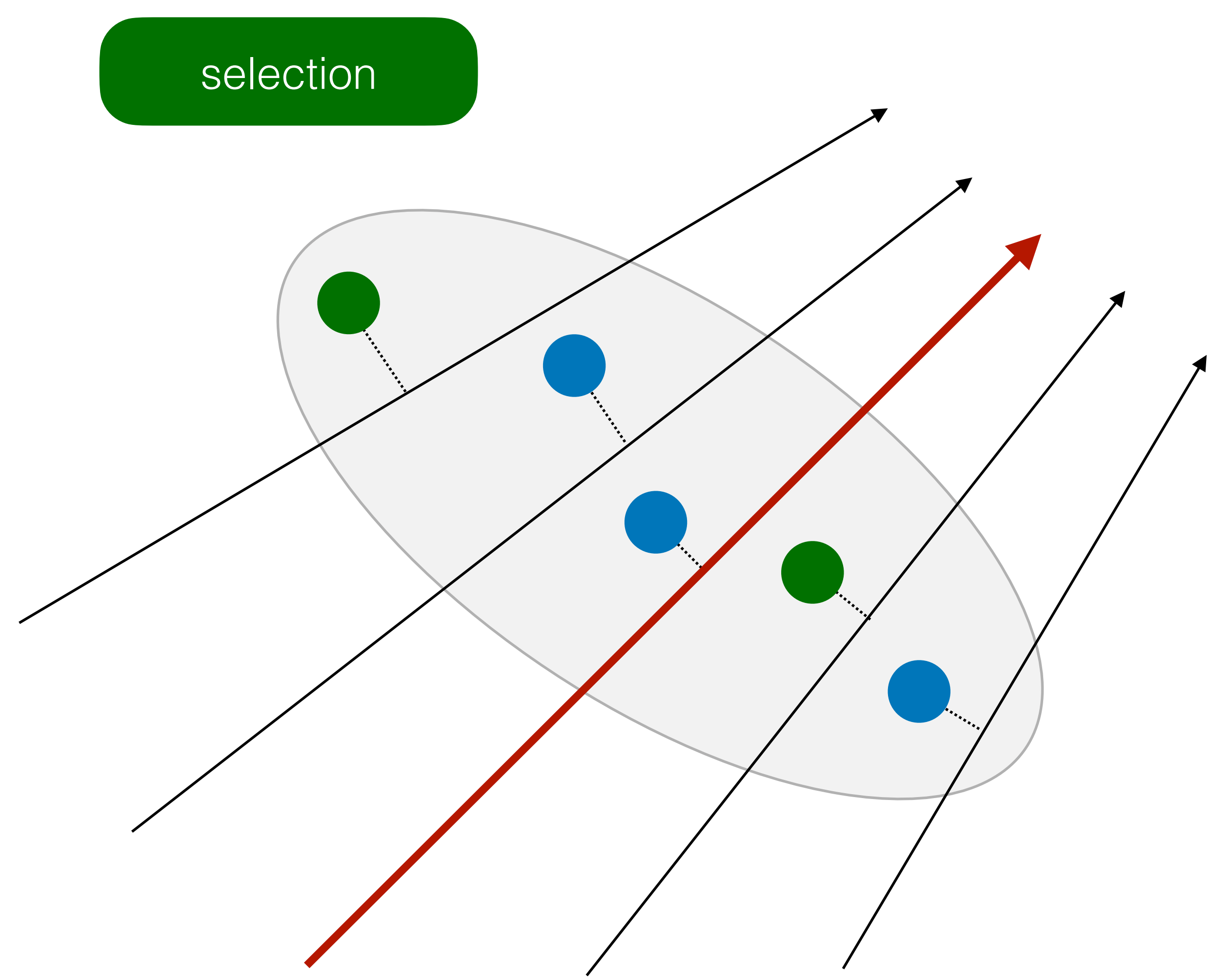

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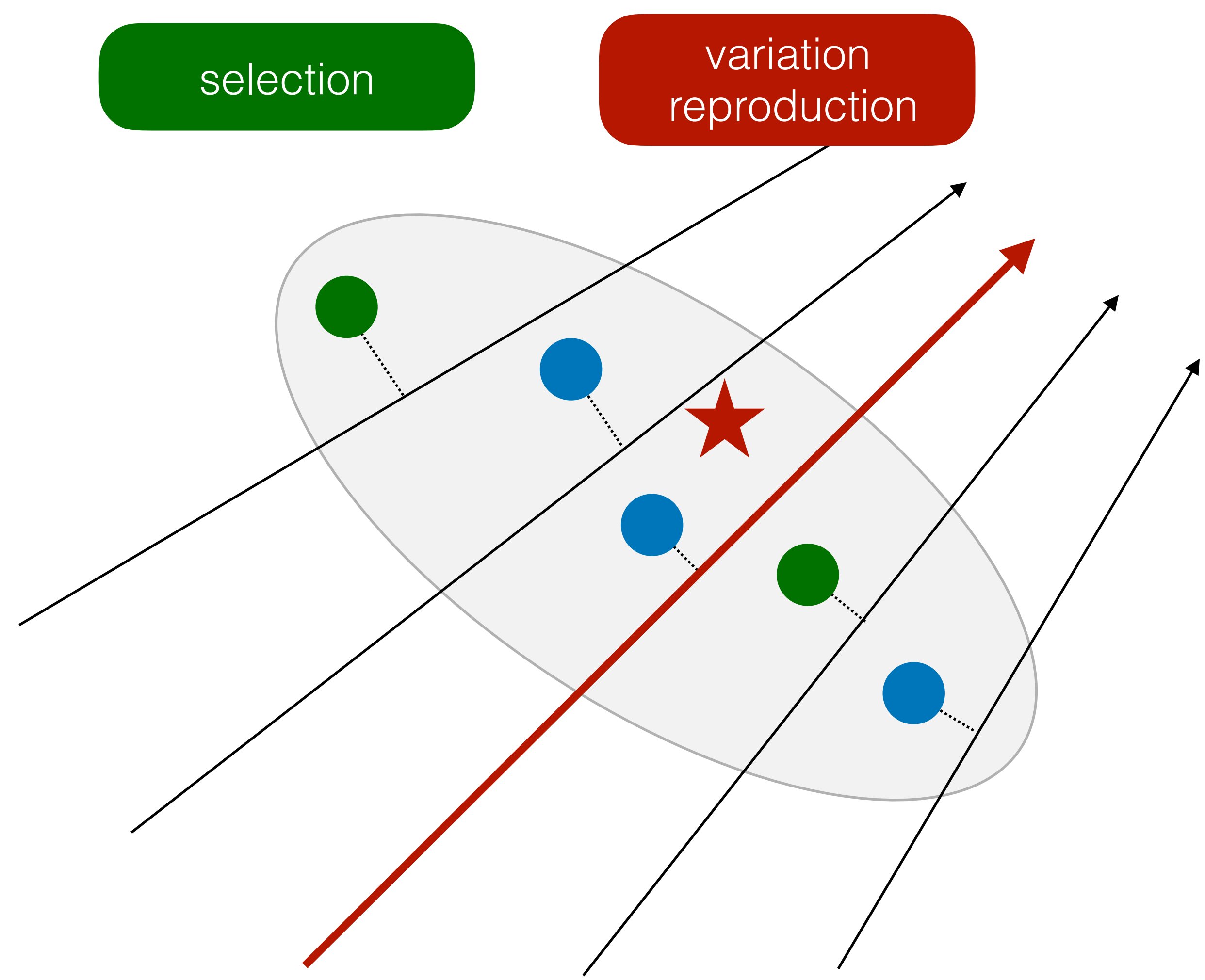

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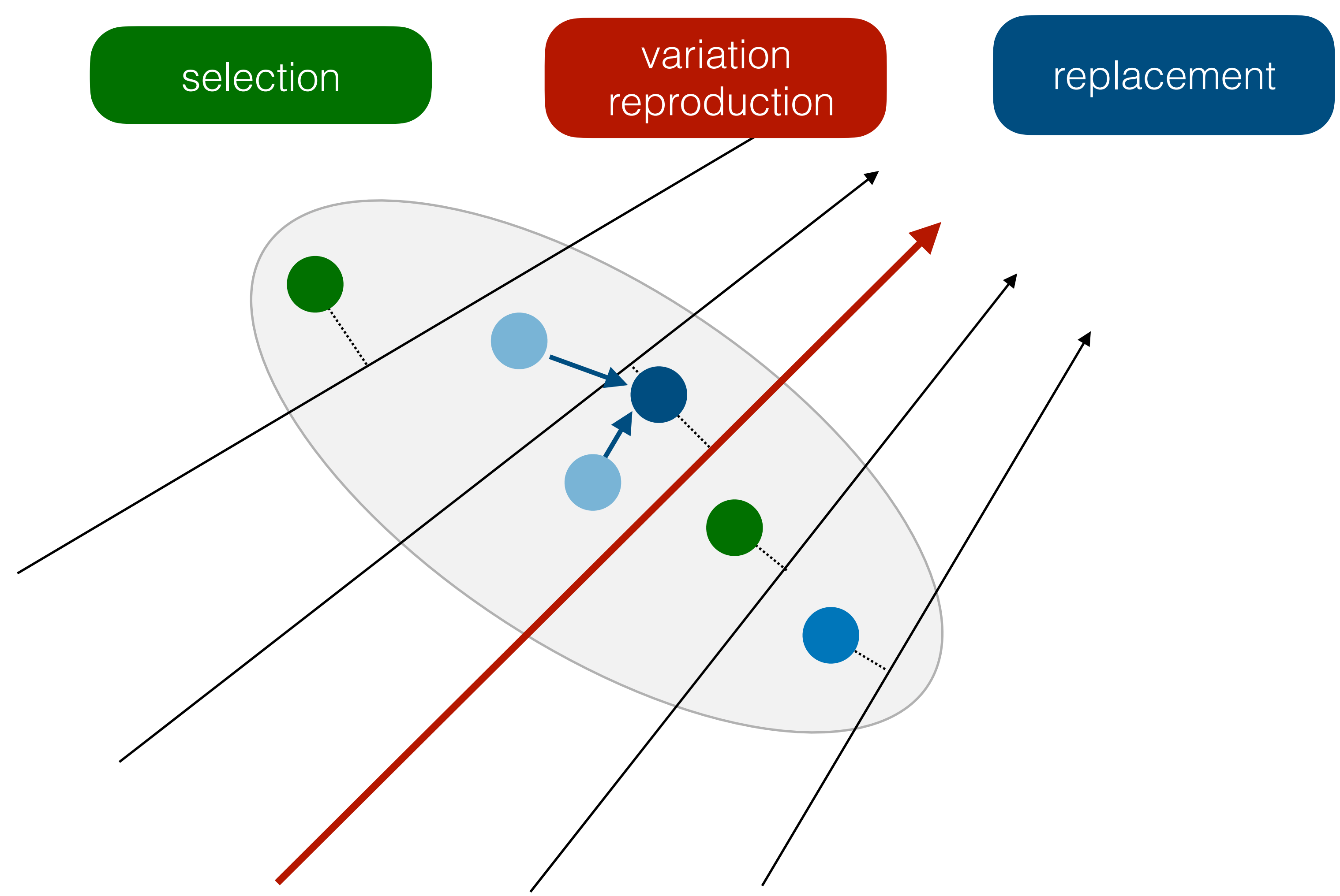

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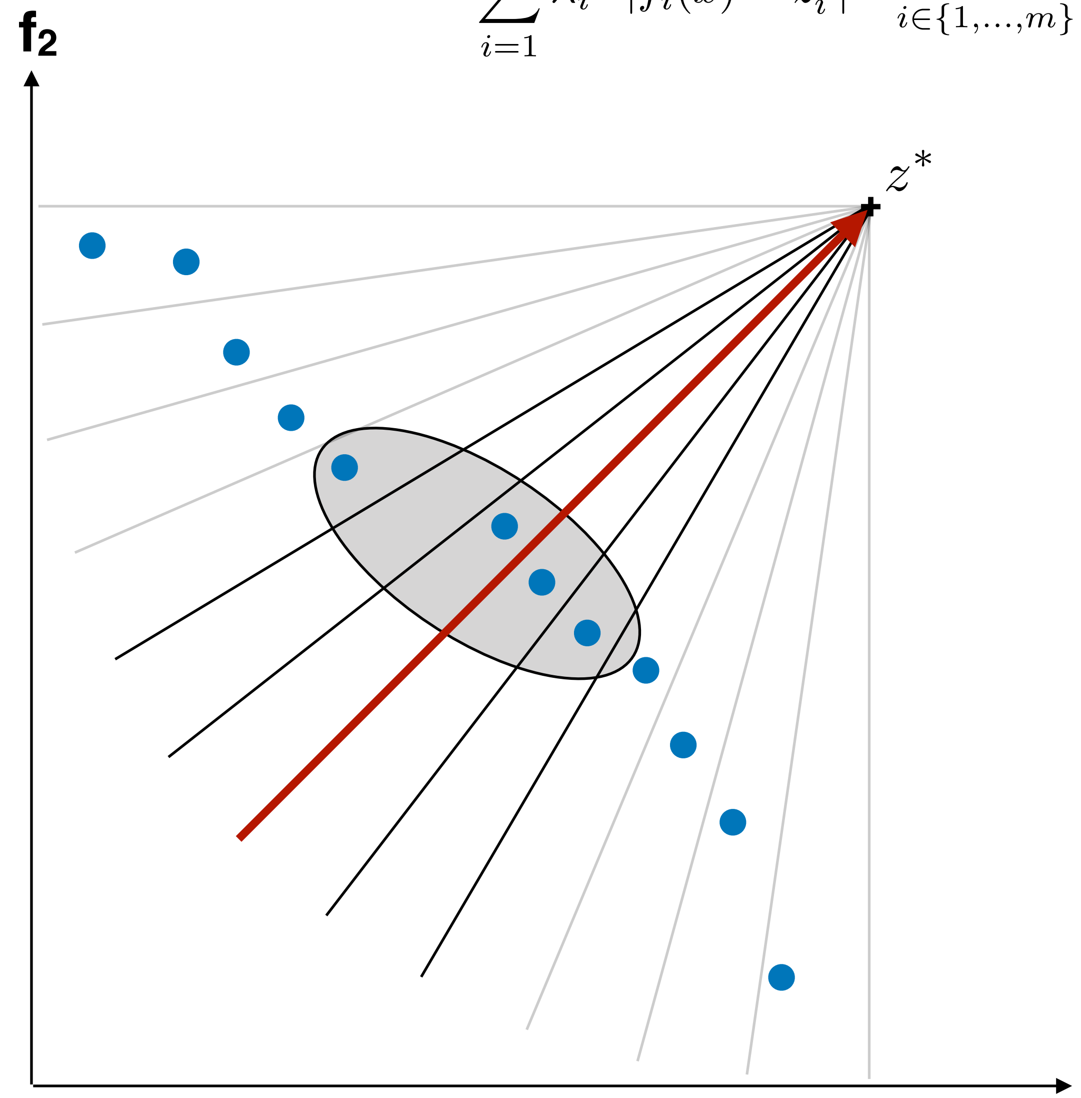

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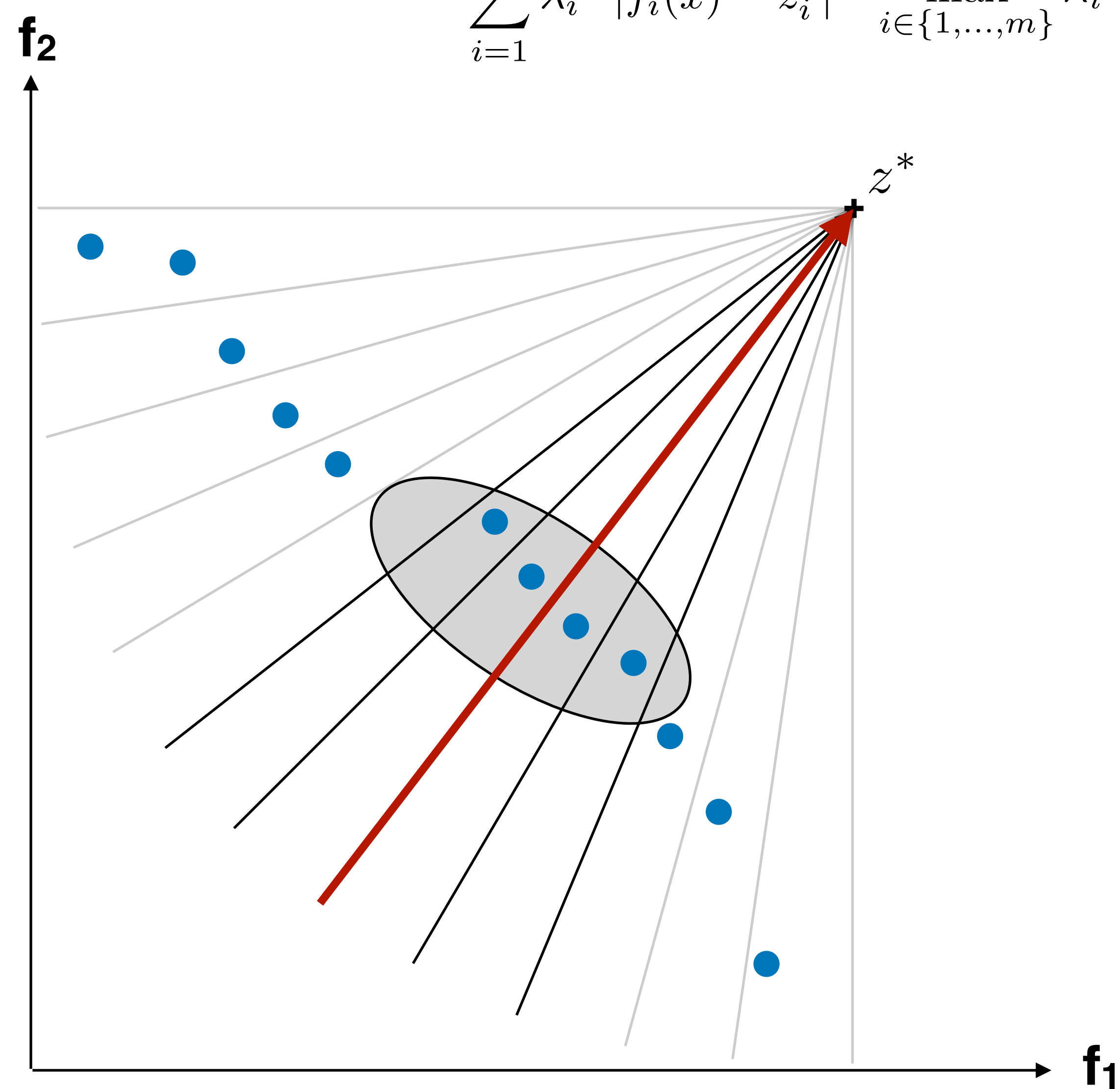

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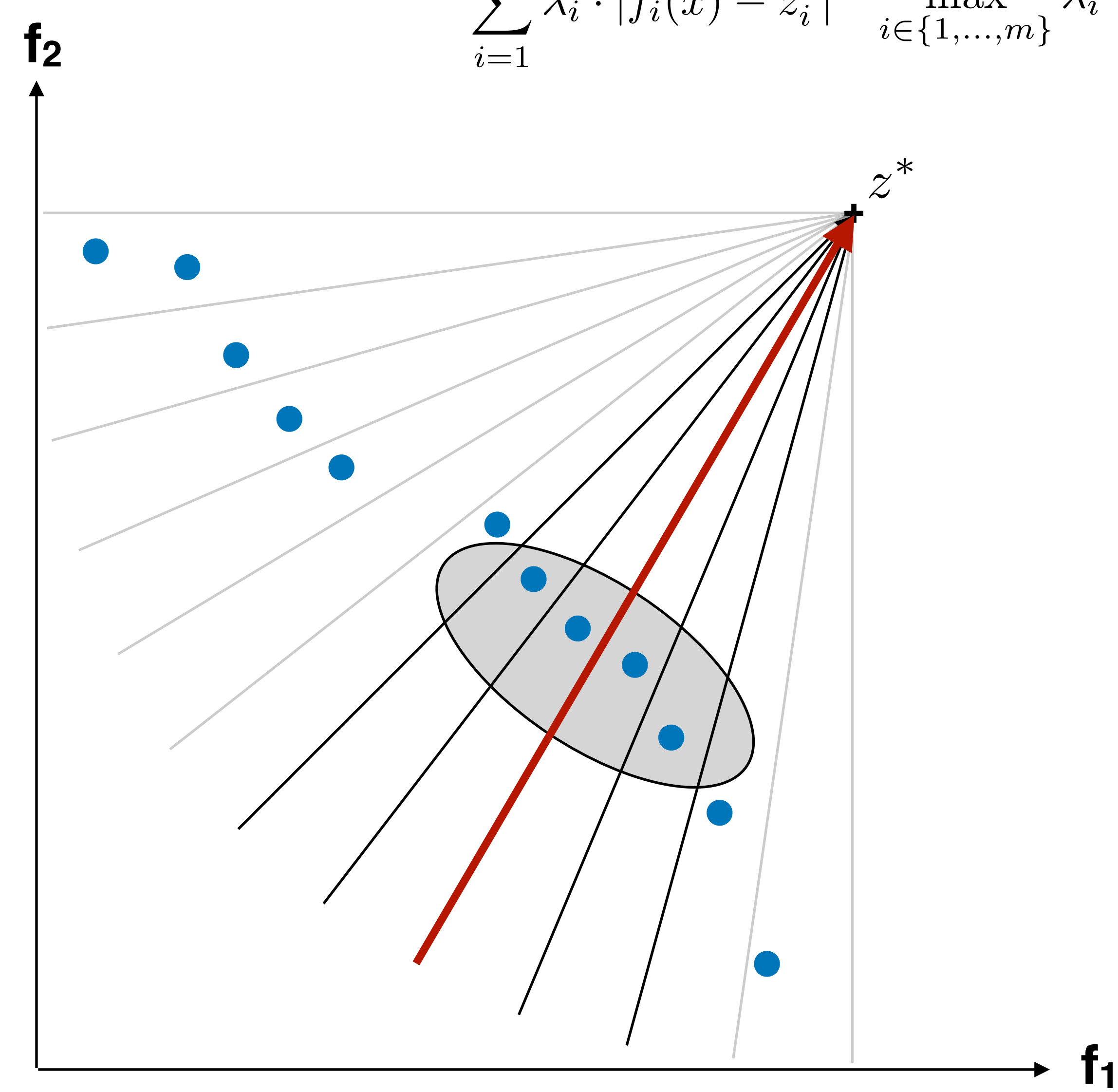

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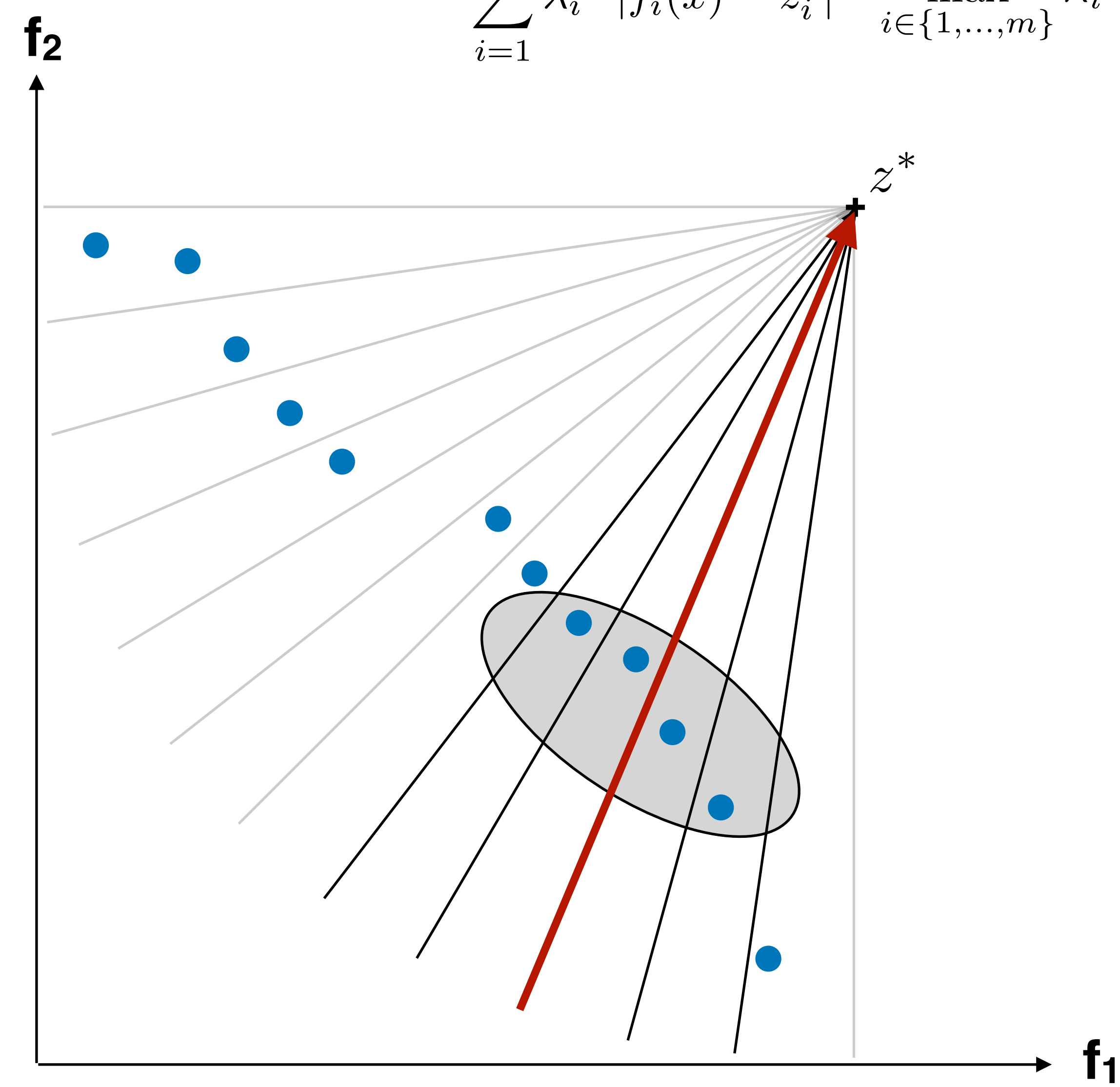

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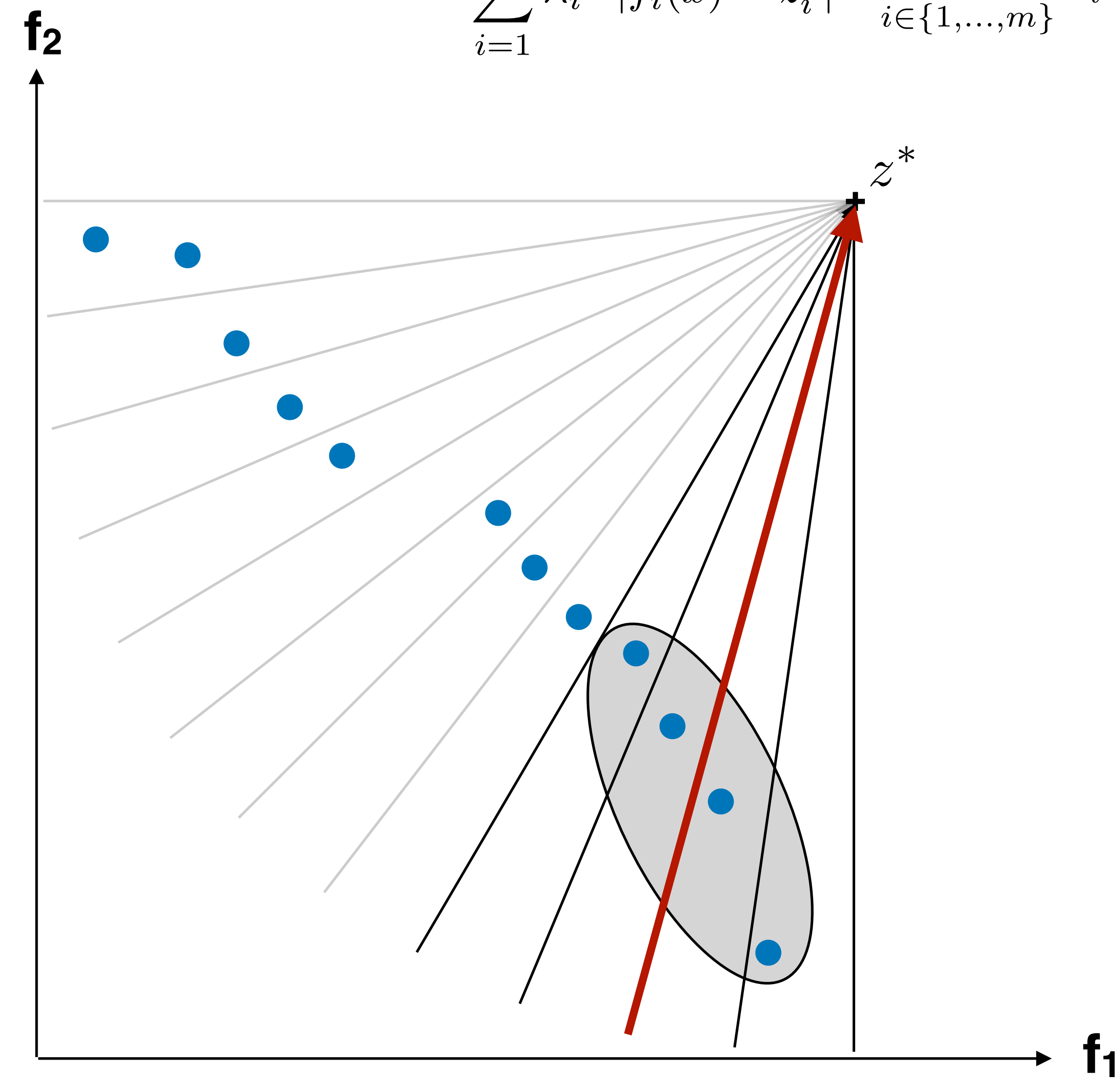
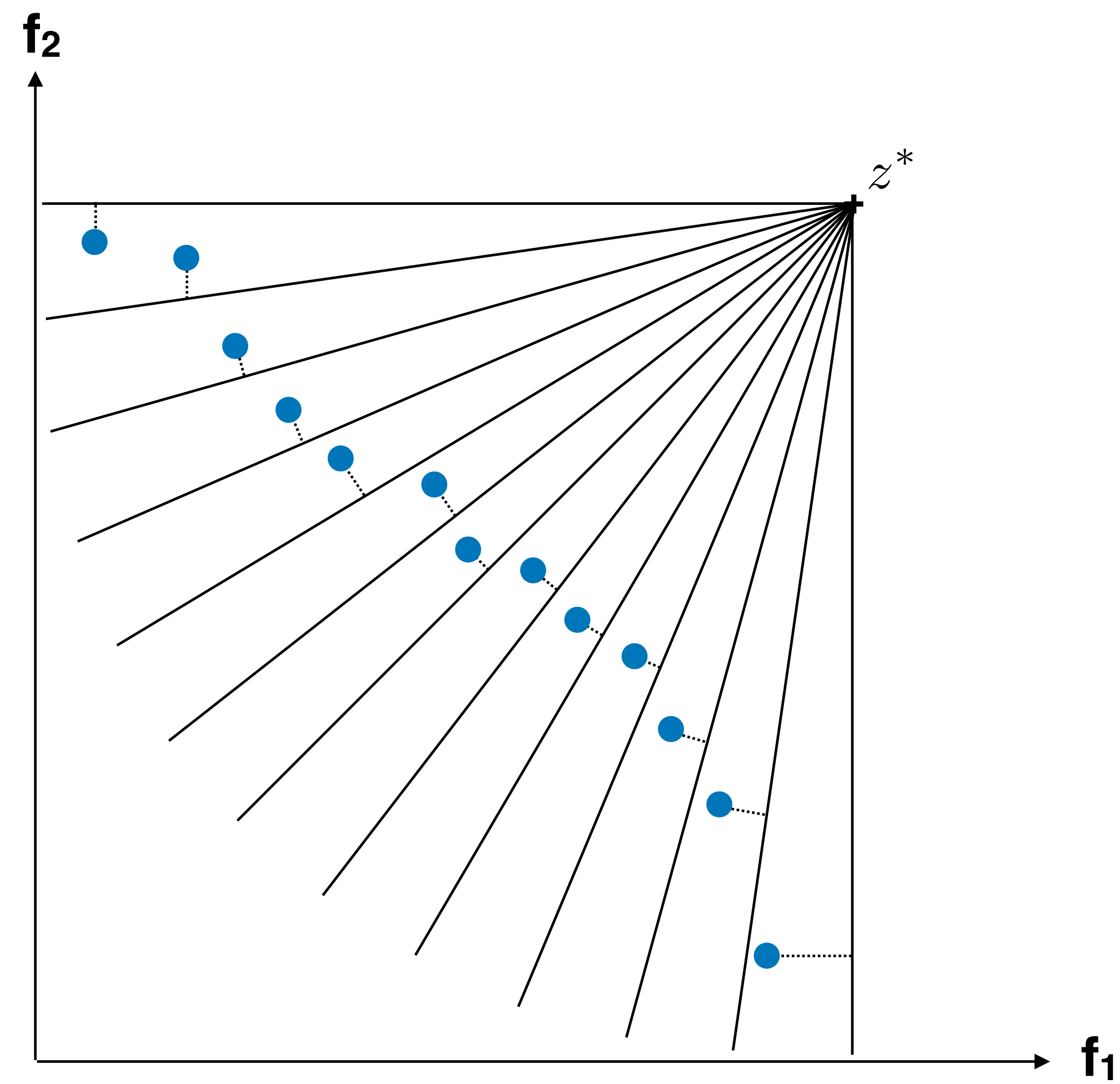


The Population size in MOEA/D

Impact different components of the MOEA/D Framework

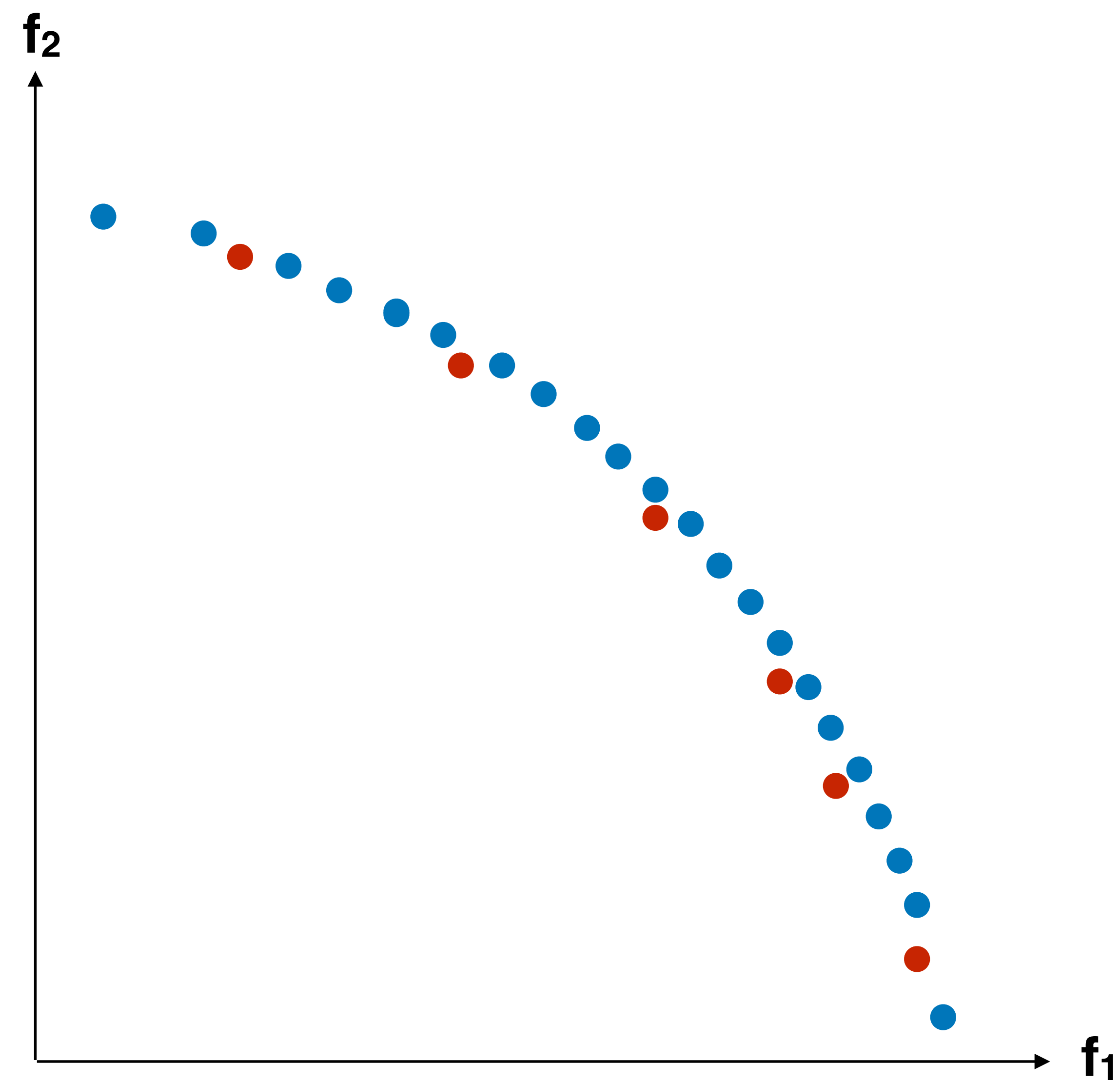
- The **population size** N :
  - The number of solutions in the population
  - The number of sub-problems



The Population size in MOEA/D

Impact different components of the MOEA/D Framework

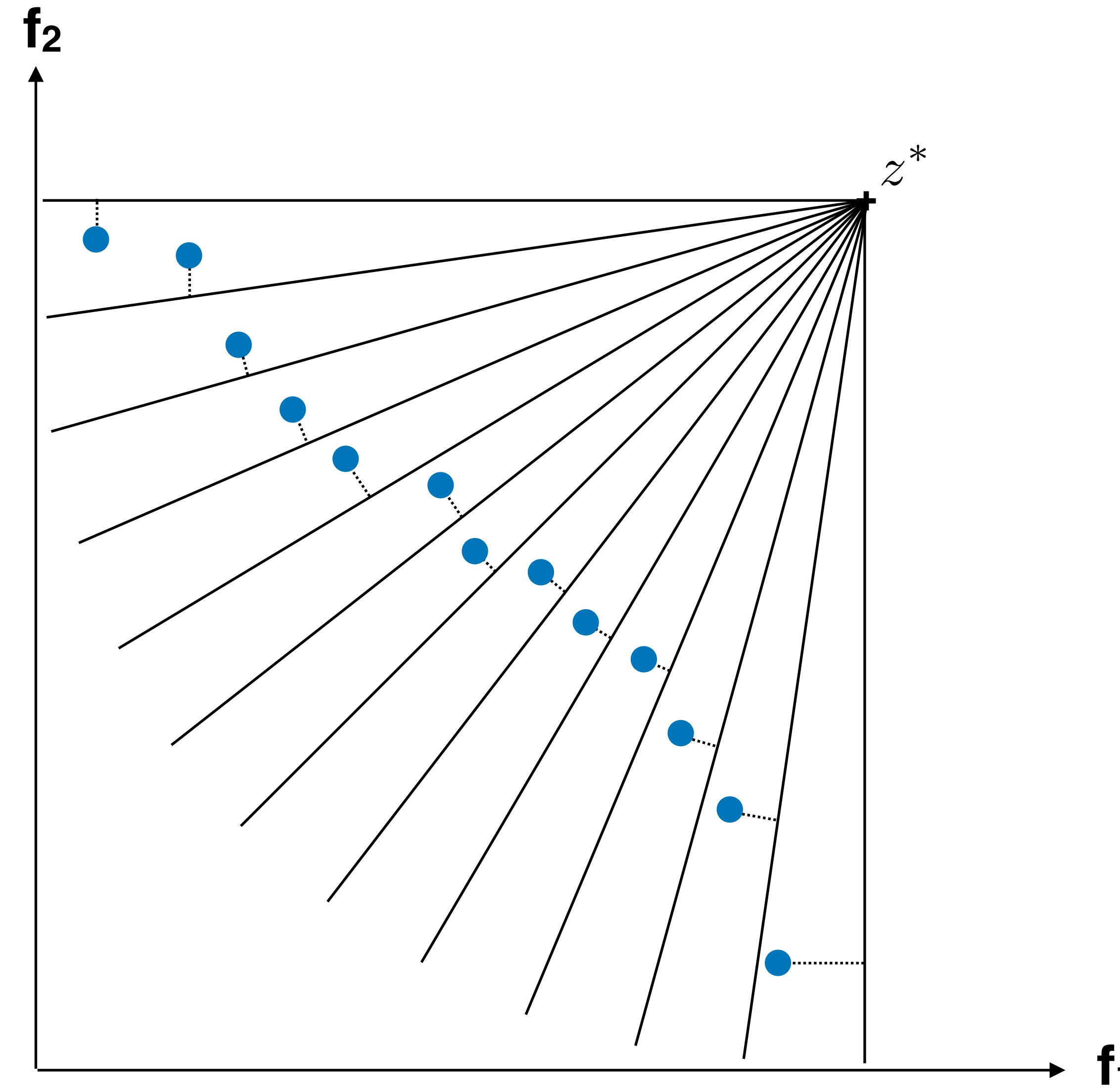
- The **population size** N :
  - The number of solutions in the population
  - **The number of sub-problems**
- Tune with the knowledge from **Evolutionary Algorithms**
  - A lower population :
    - sufficient to approach quickly the PF
    - insufficient to cover well the whole PF
  - A larger population :
    - better cover all the PF
    - waste of resources



The allocation strategy in MOEA/D

Sub-problems of MOEA/D might have different degrees of difficulty

- The progress over some sub-problem can be **unequal**
- Some MOEA/D variants with a ressource allocation :
  - State-of-the-art : MOEA/D-DRA*
    - Utility function
    - Tournament selection (1/5 of the population size)



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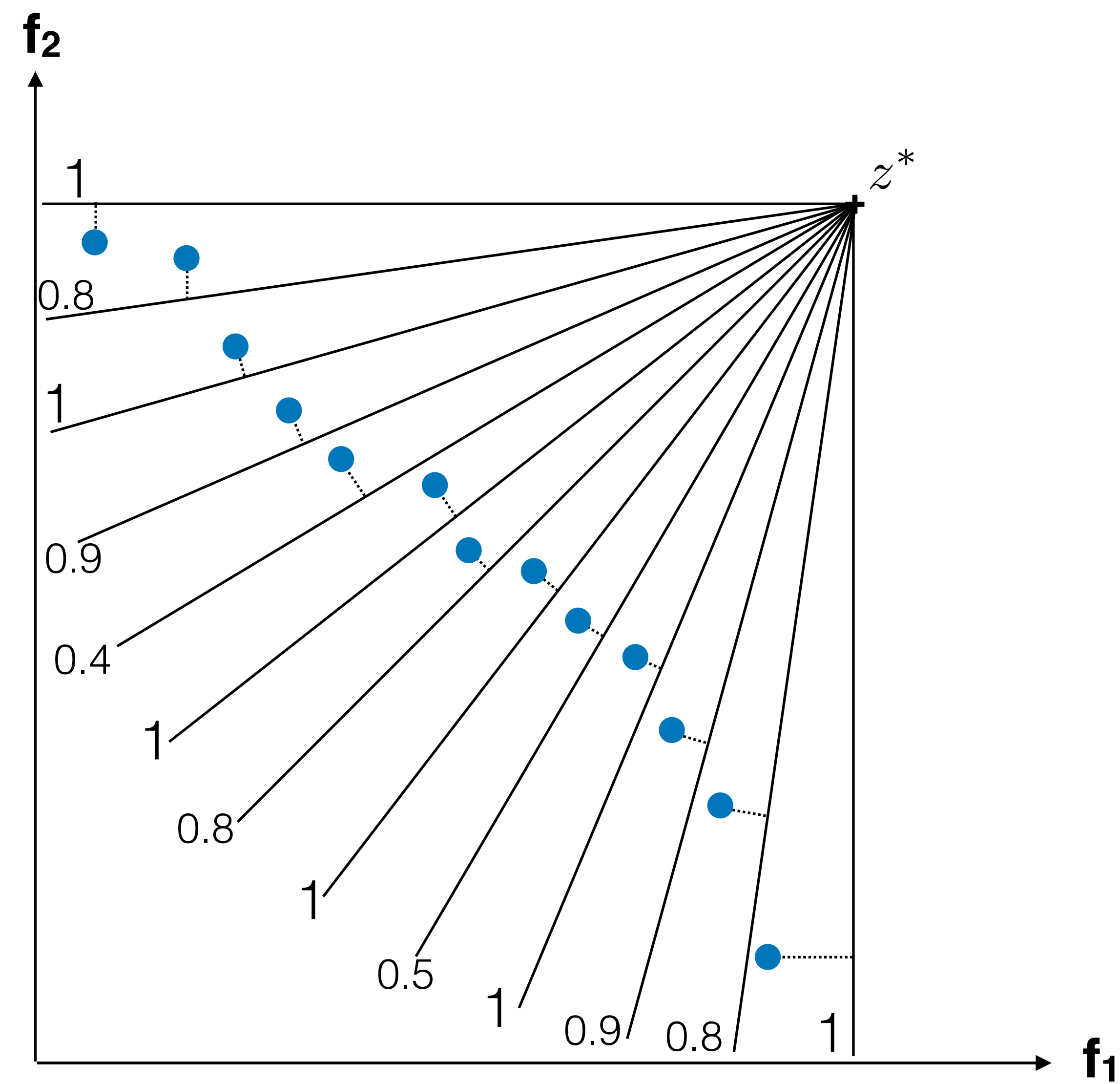

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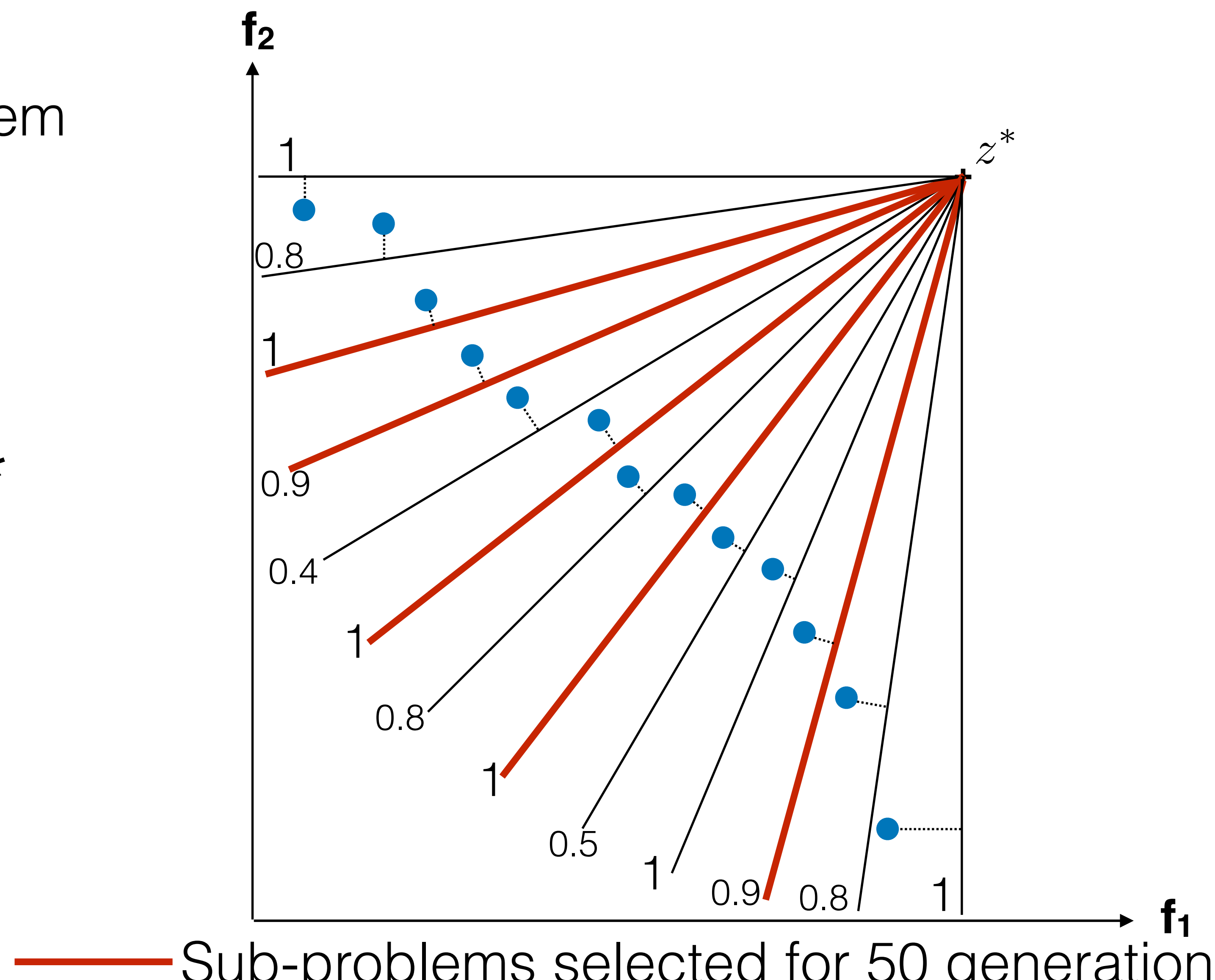

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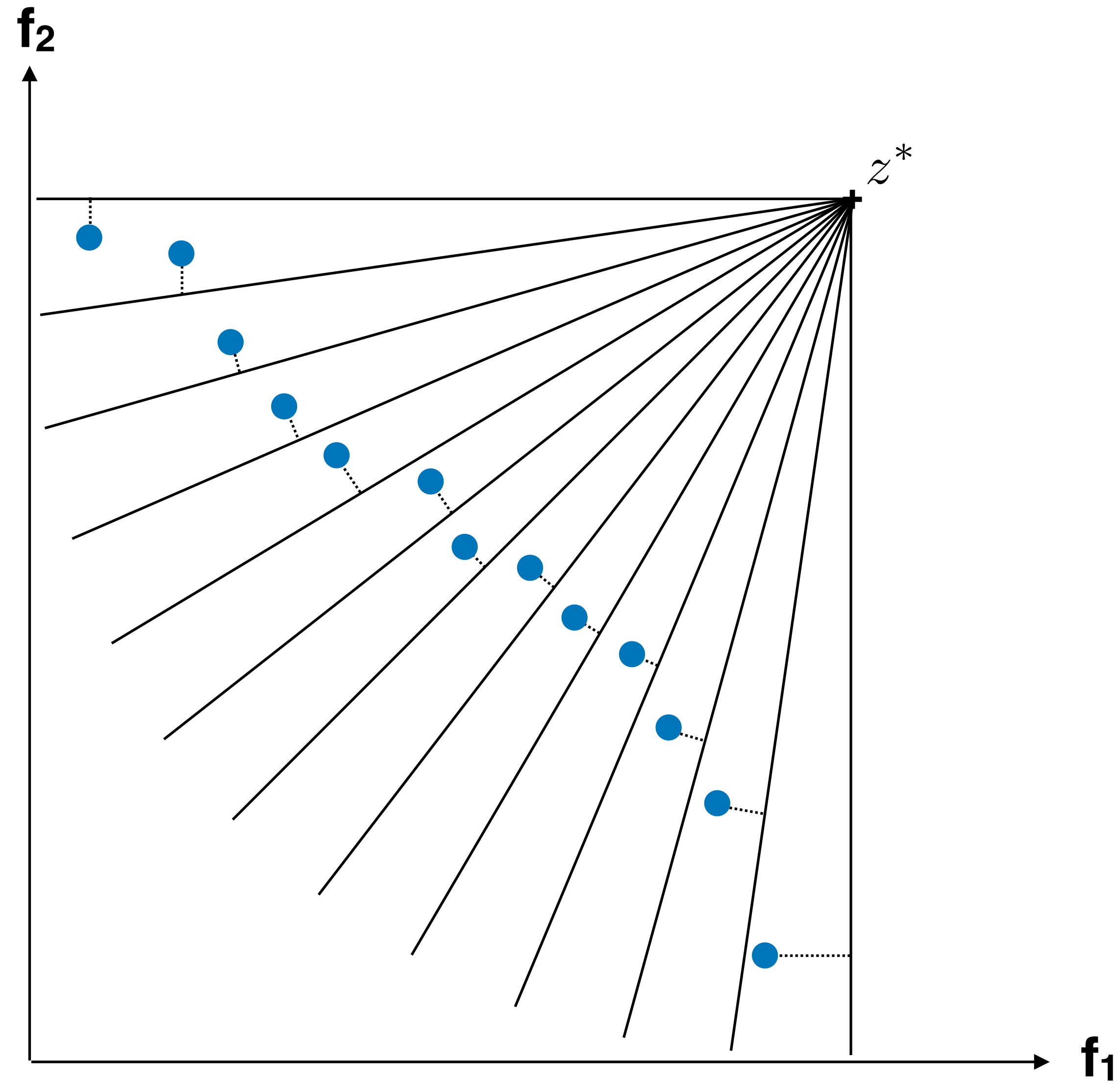
MOEA/D-(μ, λ, sps)

Dissociate the population size of MOEA/D in 3 new components

- **μ :**
- **λ :**
- **Sps :**



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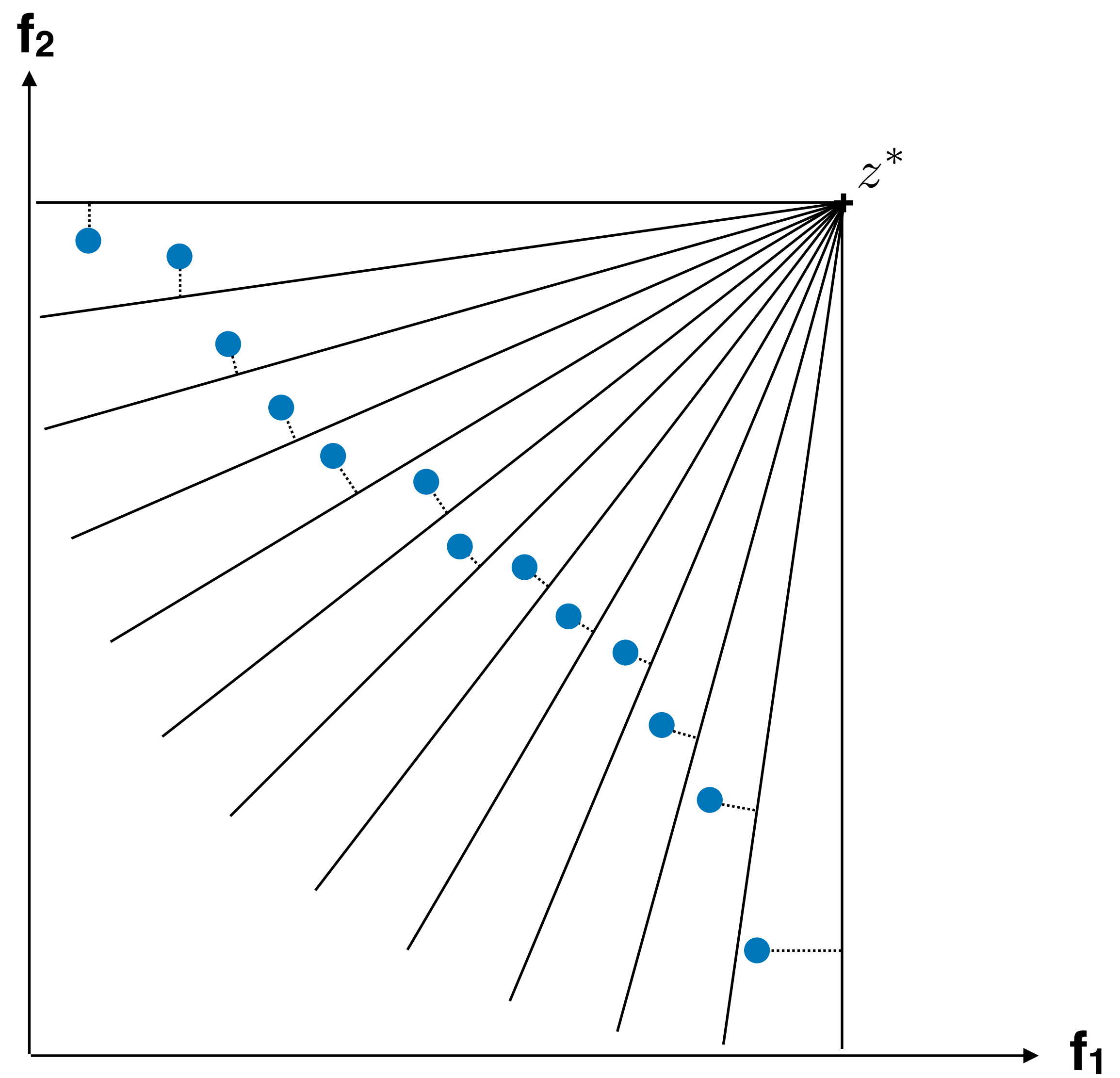

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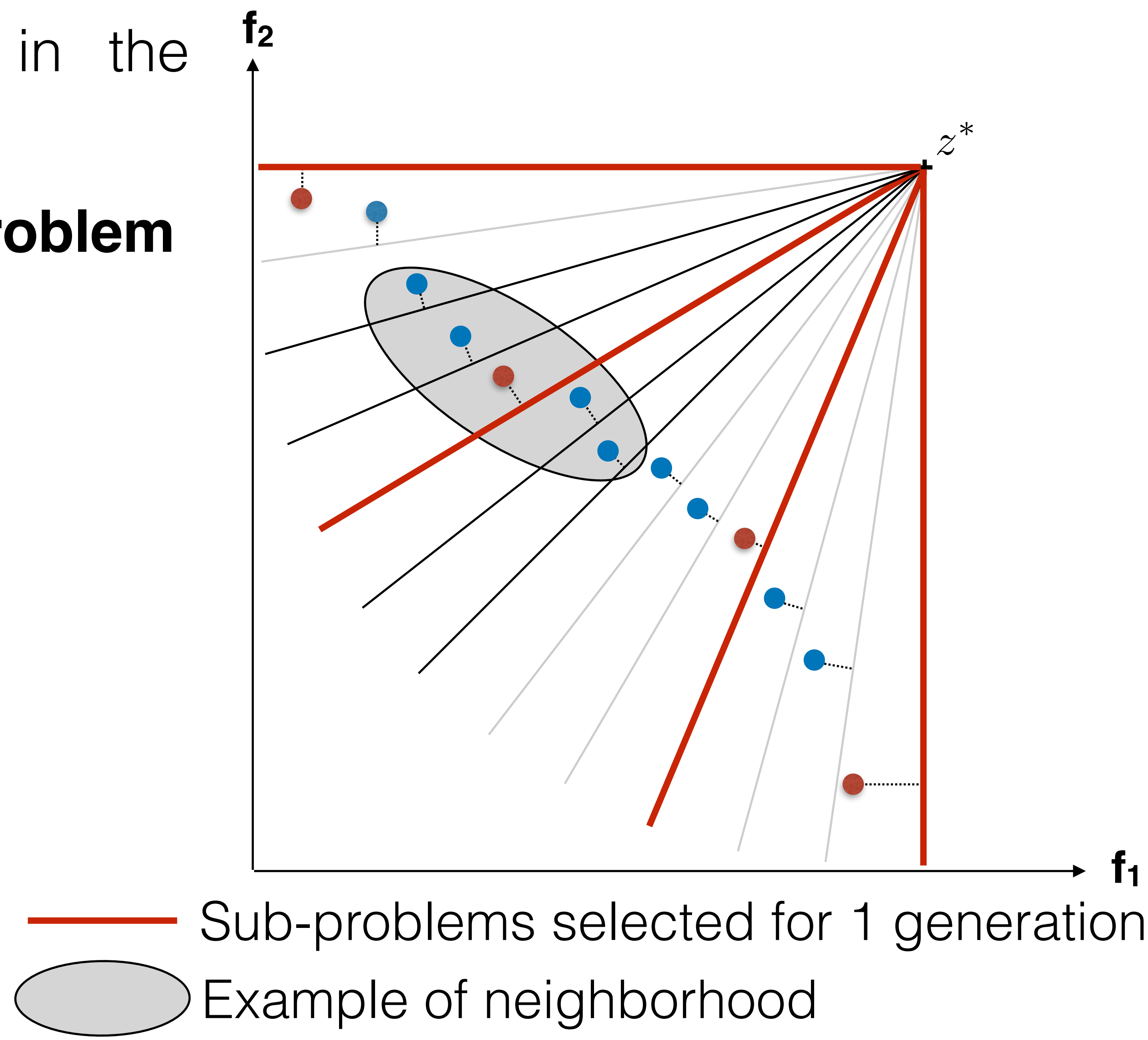

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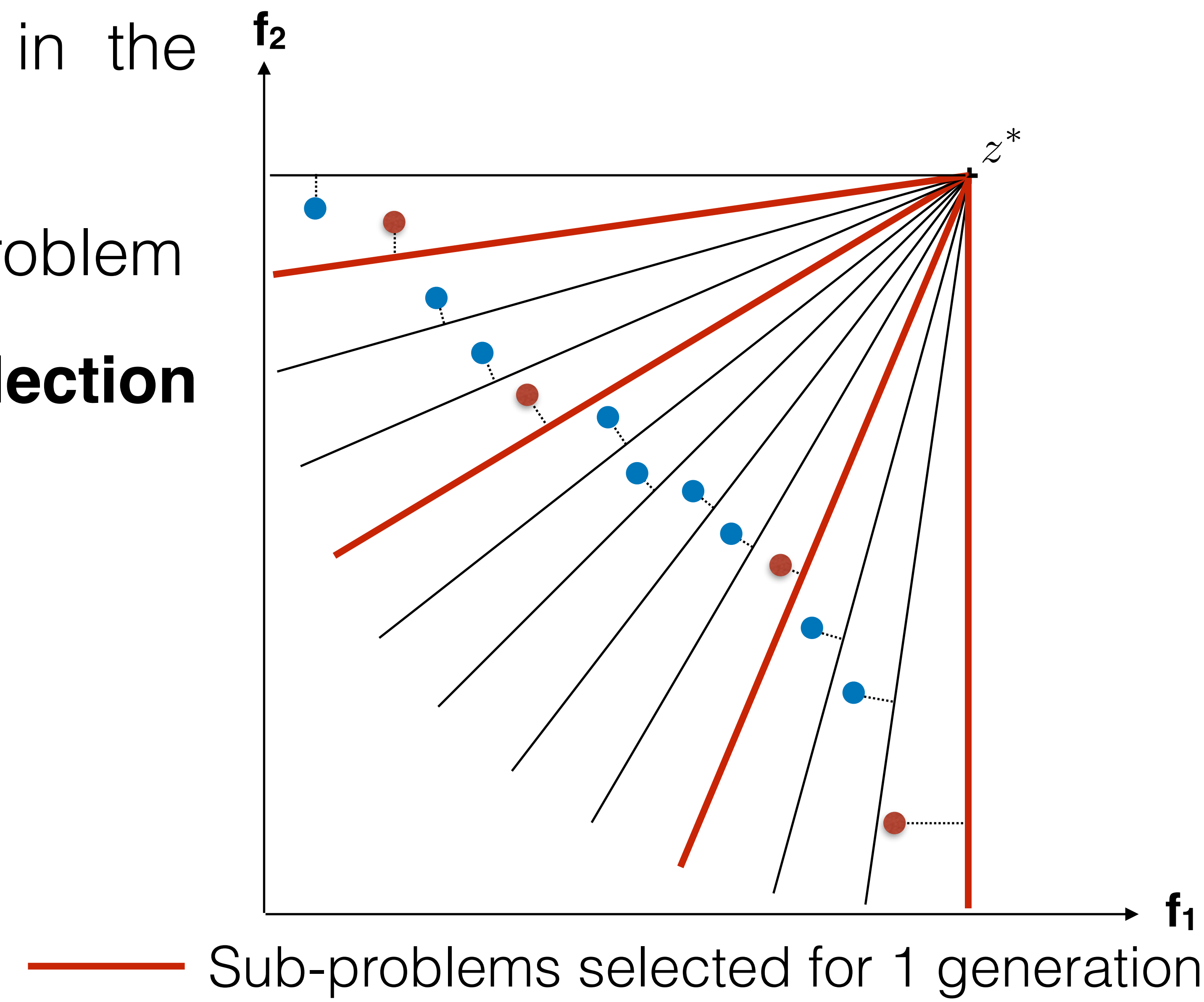
    - Iteratively or ALL
    - DRA
    - Random



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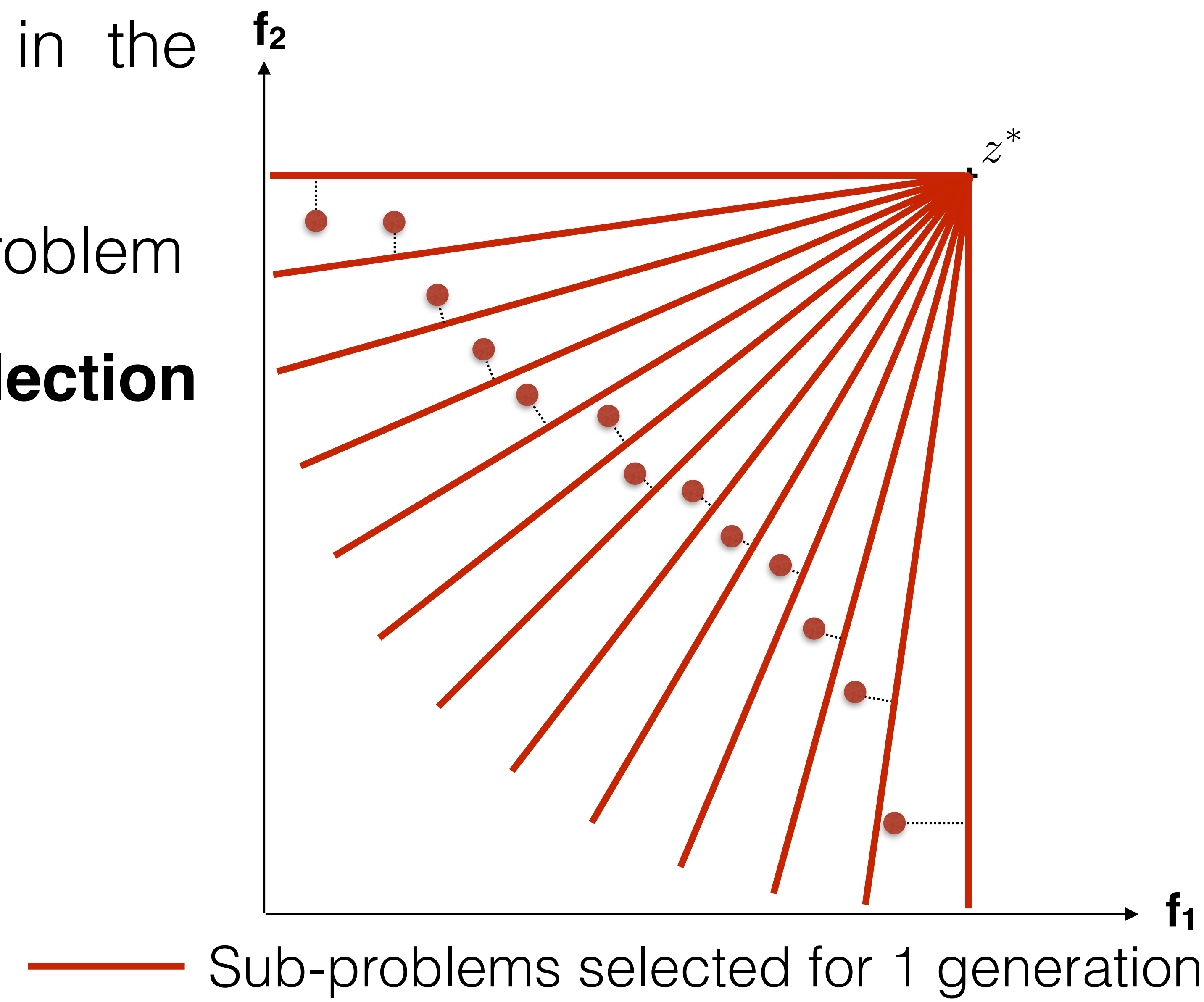

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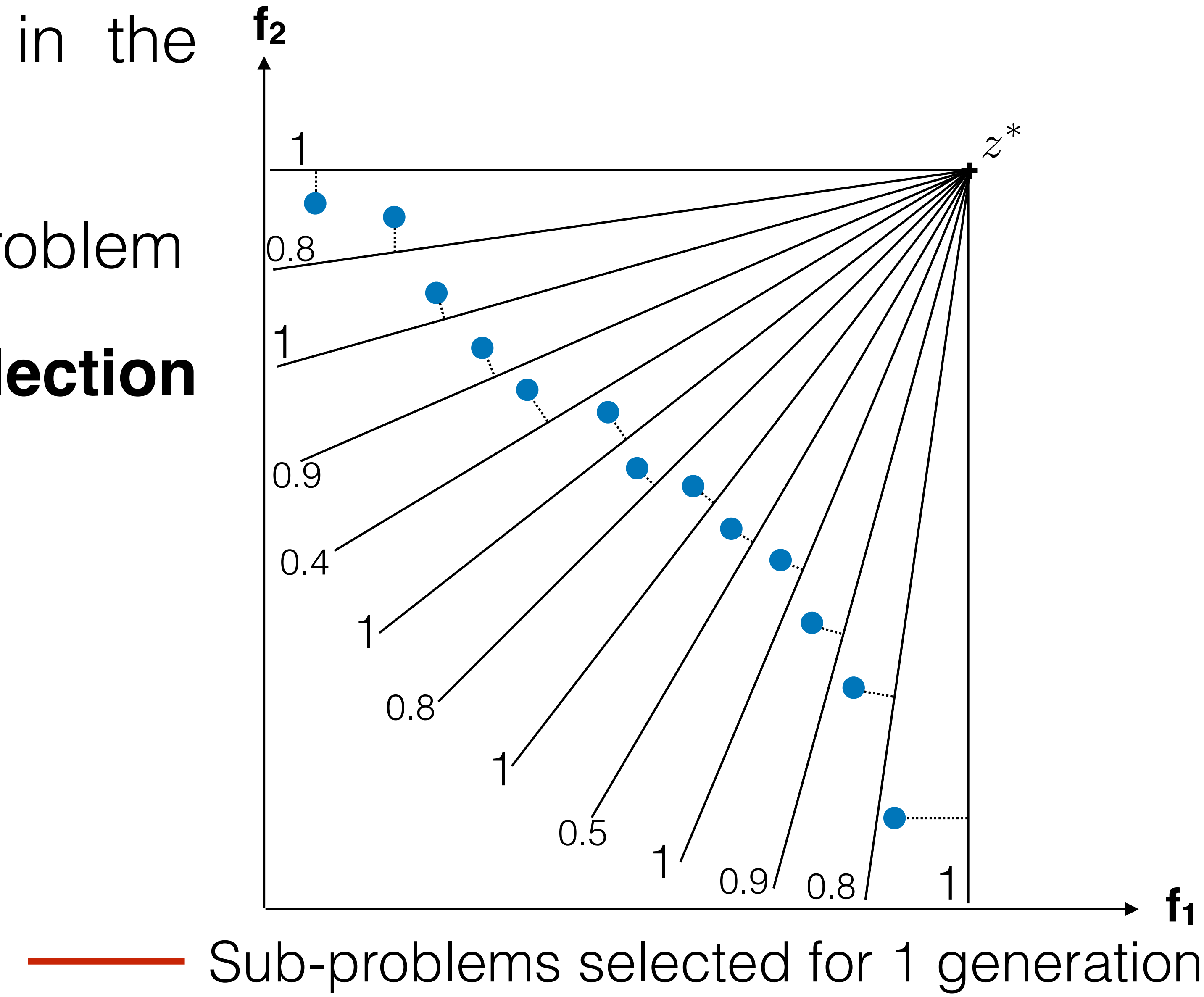

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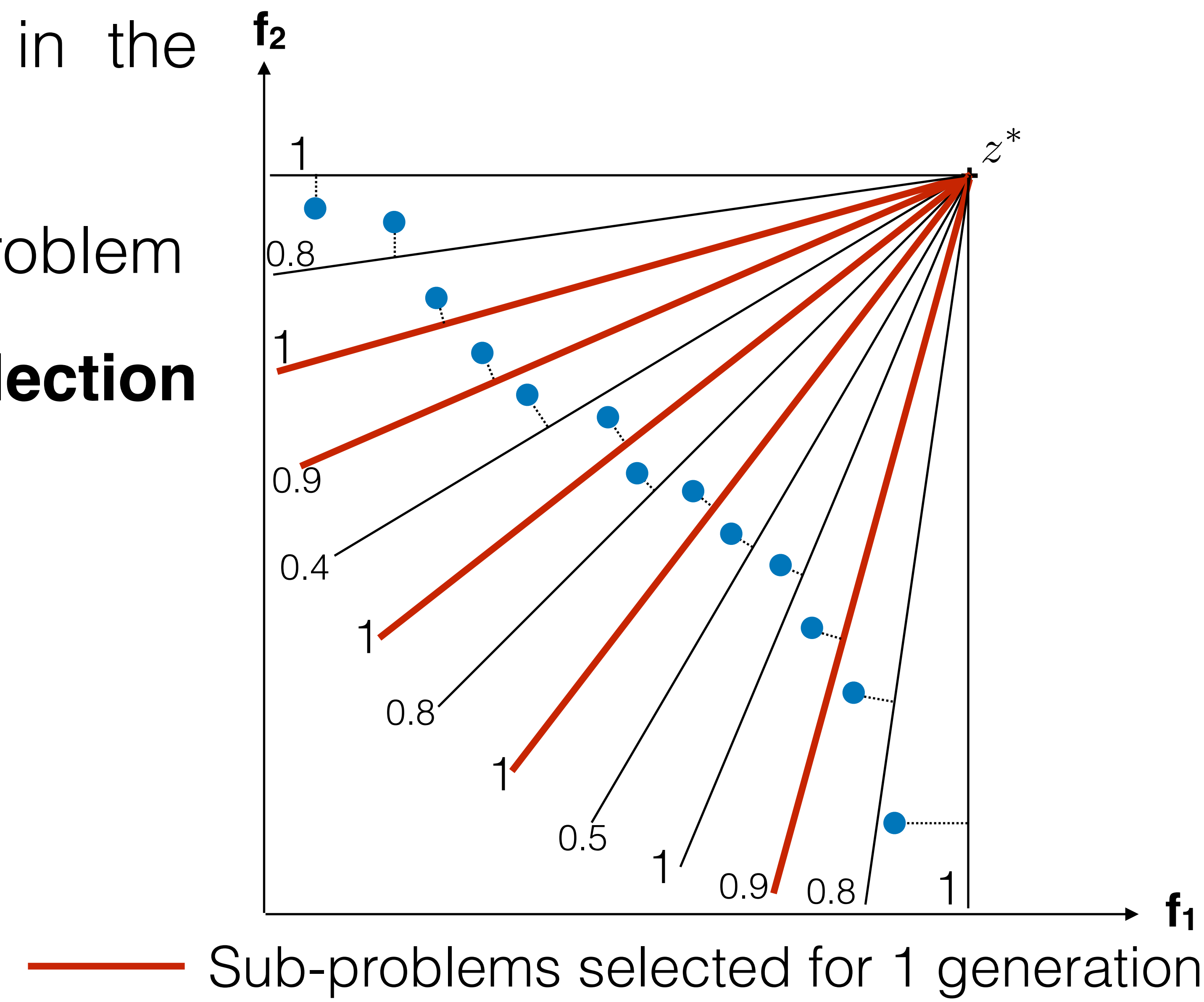

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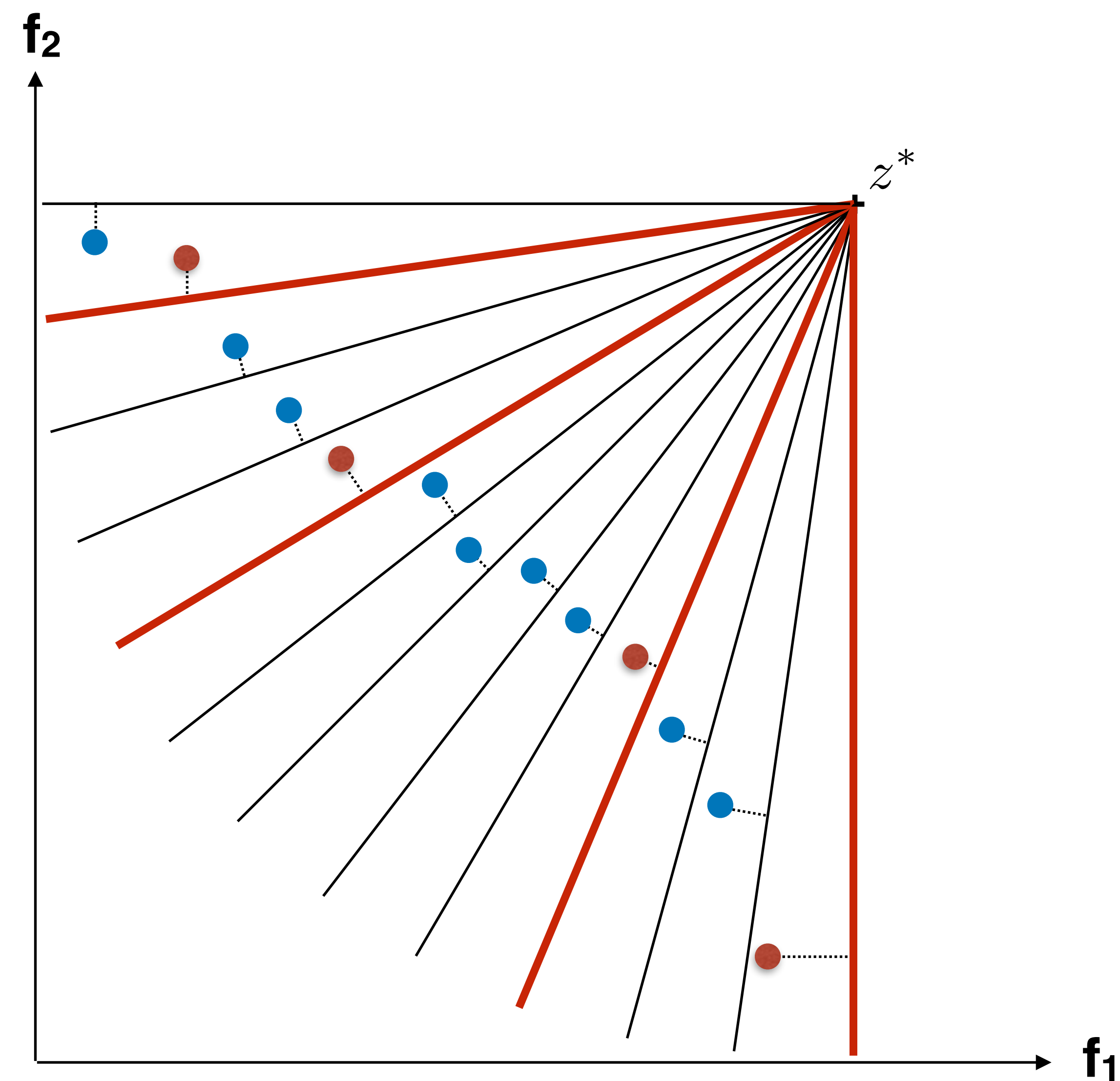
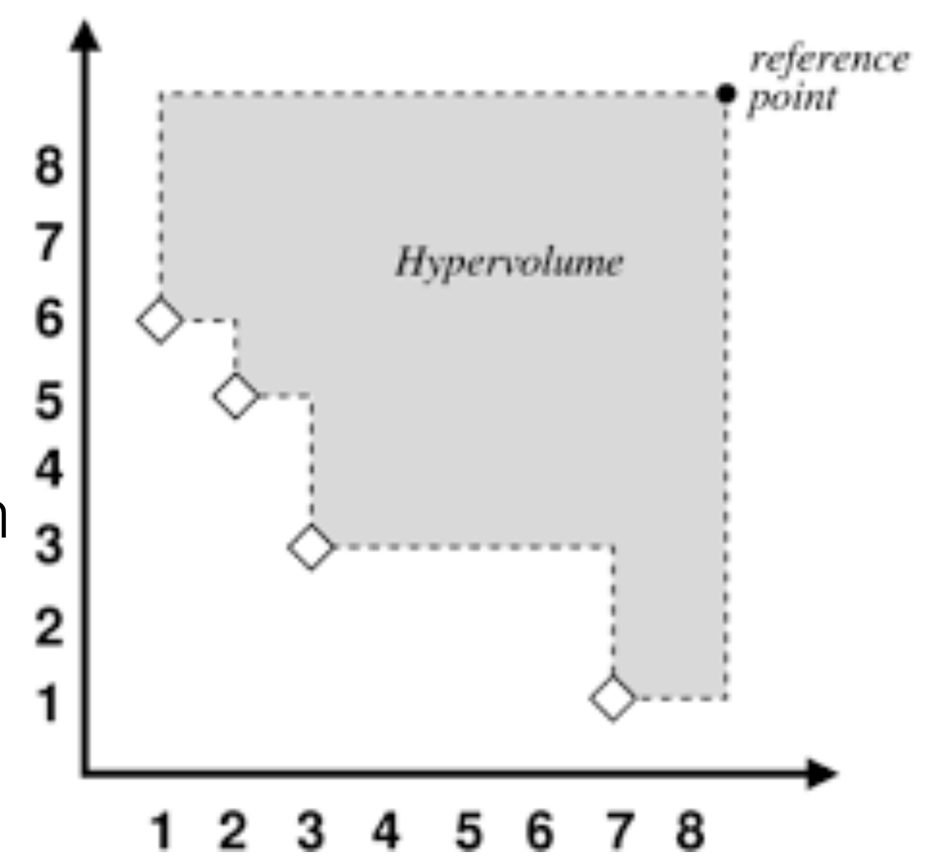


Sub-problems selected for 1 generation

Experimental Setting

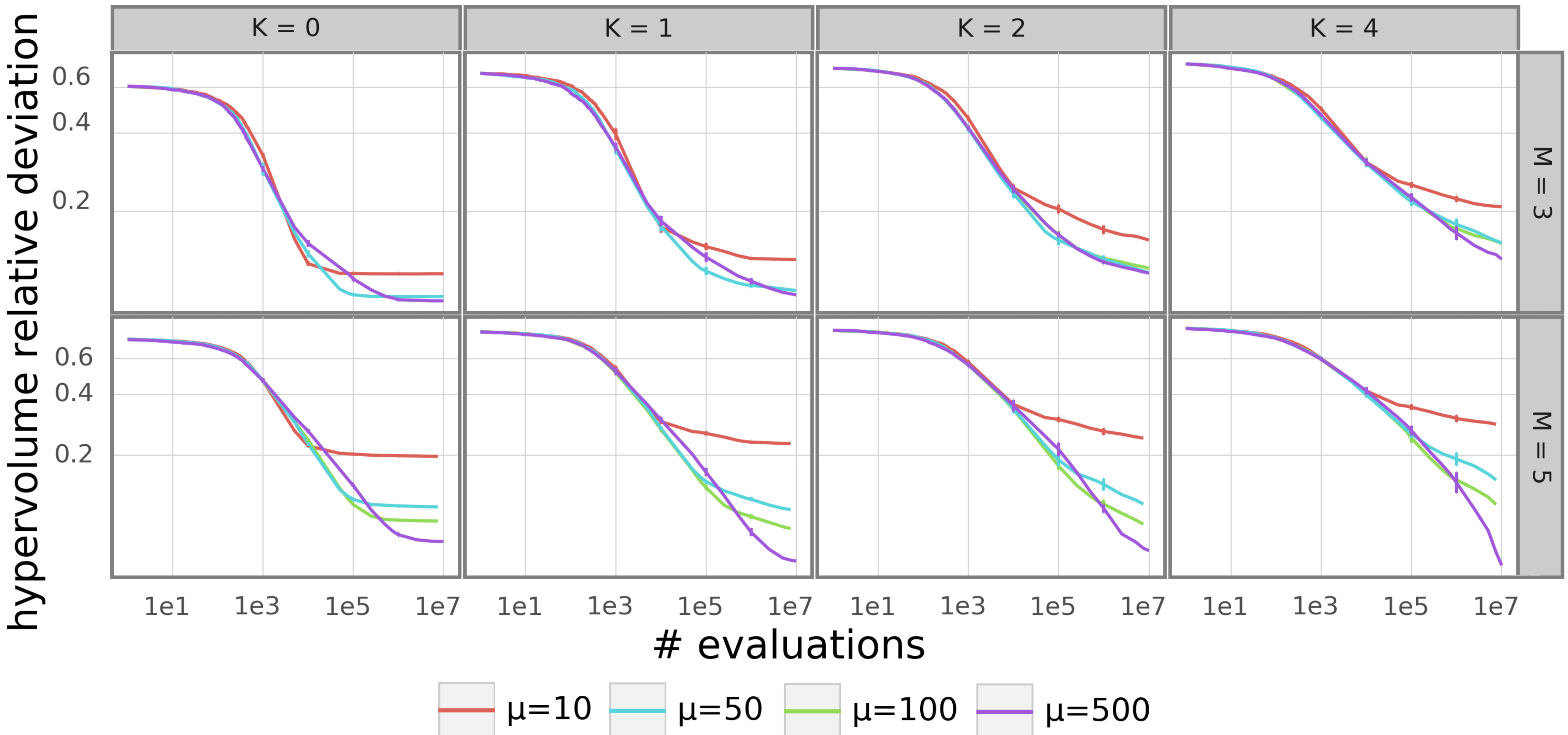
- Benchmark : **multi-objective NK-landscapes**
    - combinatorial problem
    - representation = bit-strings
    - problem size n = 100 (decision space dimension)
    - k={0, 1, 2, 4} epistatic interactions to manage the ruggedness of the problem
    - **m={2,3,4,5} objectives**



- Indicators
    - **Hypervolume**, Hypervolume Relative Deviation
- Number of runs : **10**
- Number of evaluation : **10^0 —> 10^7**

The population size μ

Algorithm : classic MOEA/D with λ=μ and sps = ALL



- With a **small** budget :
  - A small population is equal or better
- With a **larger** budget:
  - A larger population is better

The number of sub-problems selected λ

- The best value is **never** λ = μ for **low budget**
- **Random** strategy :

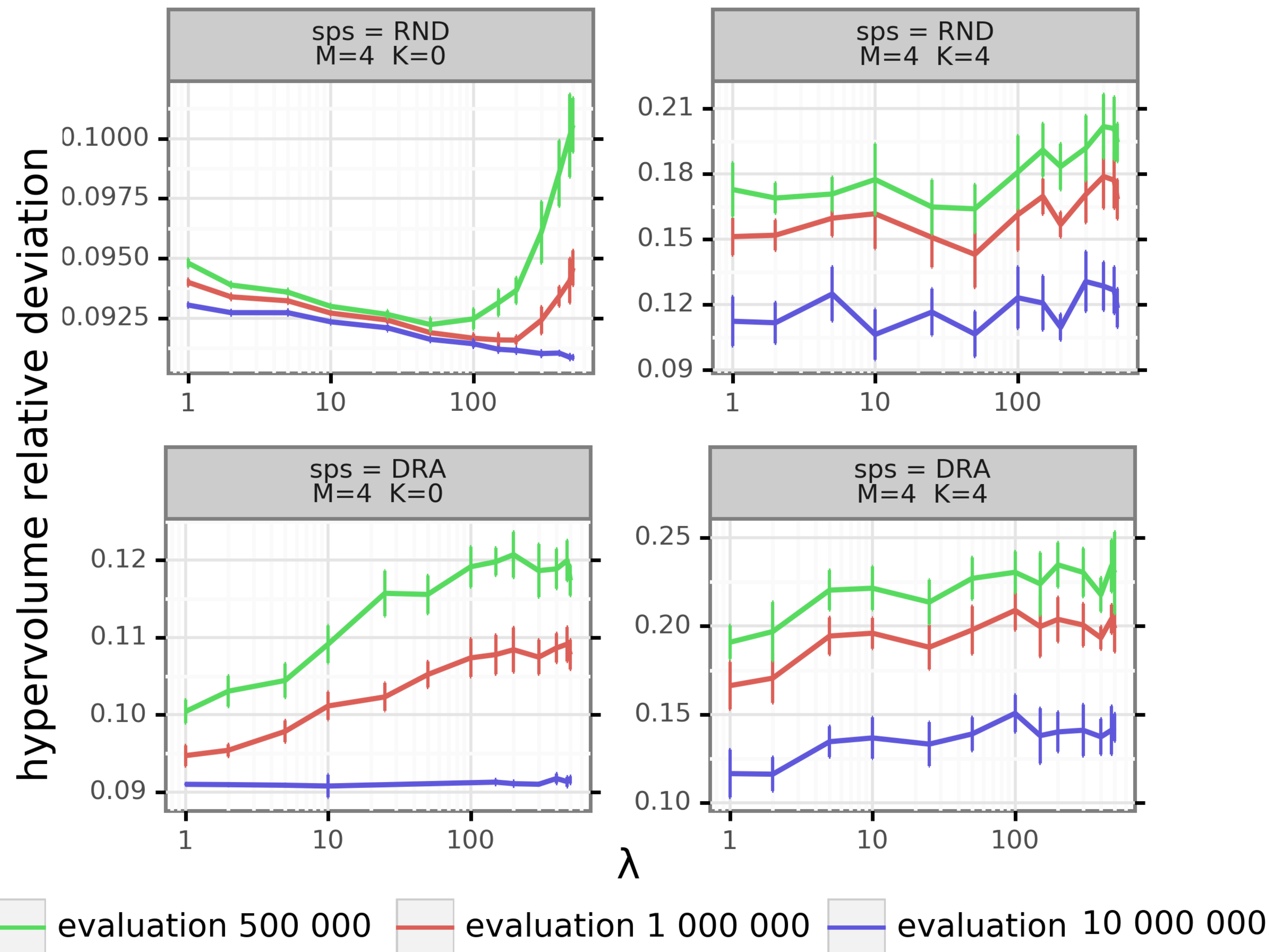
  Depends of budget
  and difficulty of the problem

- **DRA** strategy:

  In original DRA, λ = N/5

  Best value of λ = 1

Algorithm : MOEA/D with μ = 500



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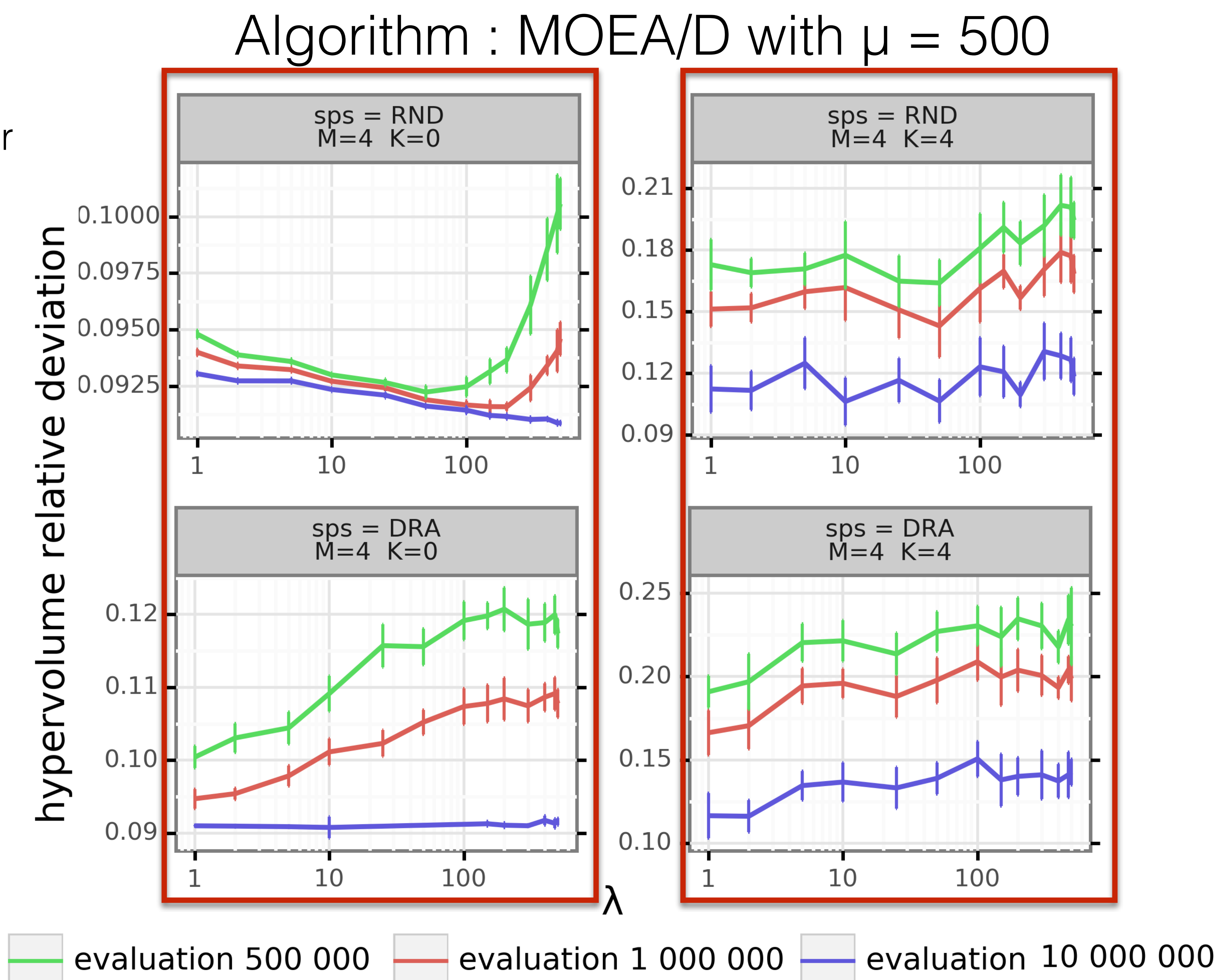

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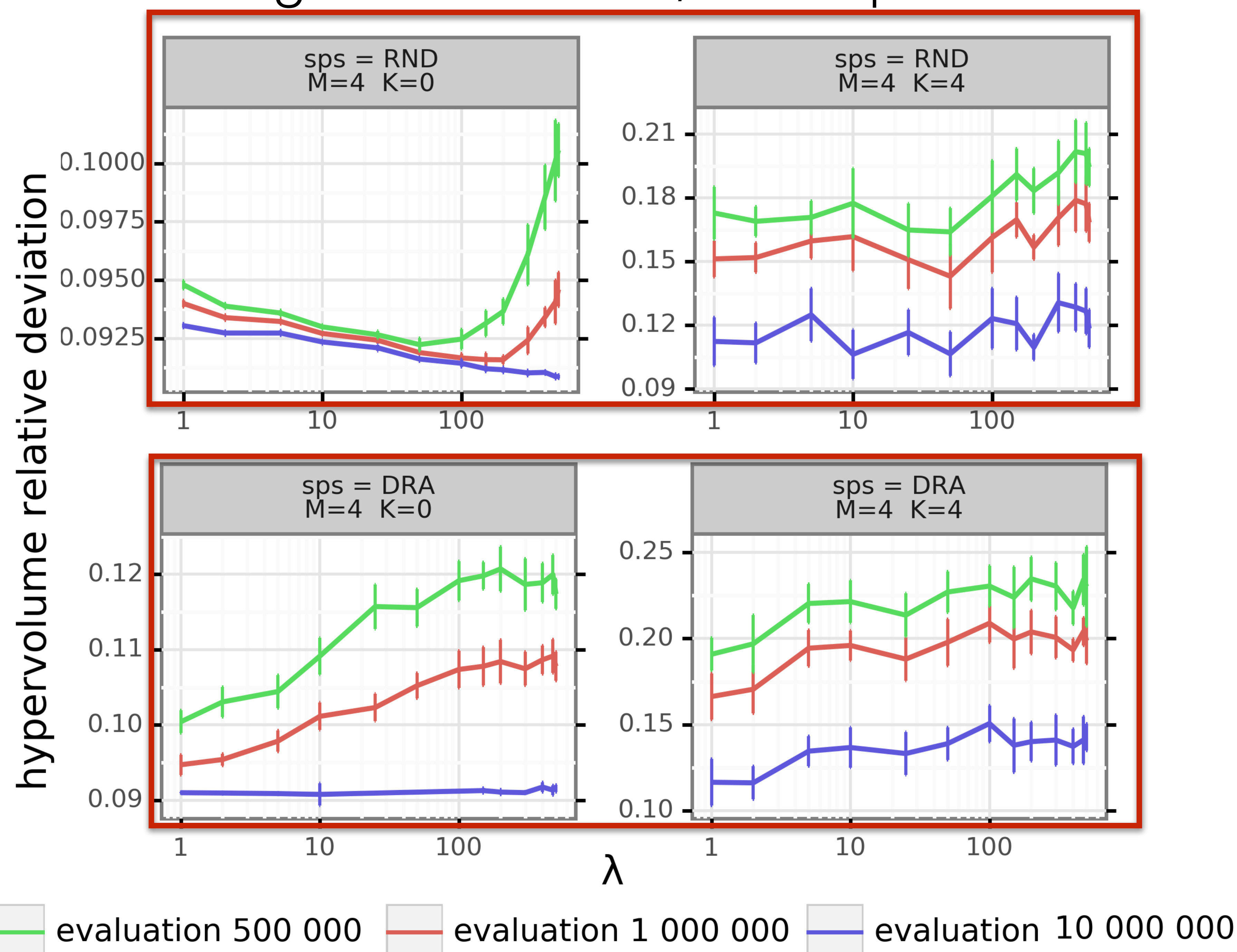

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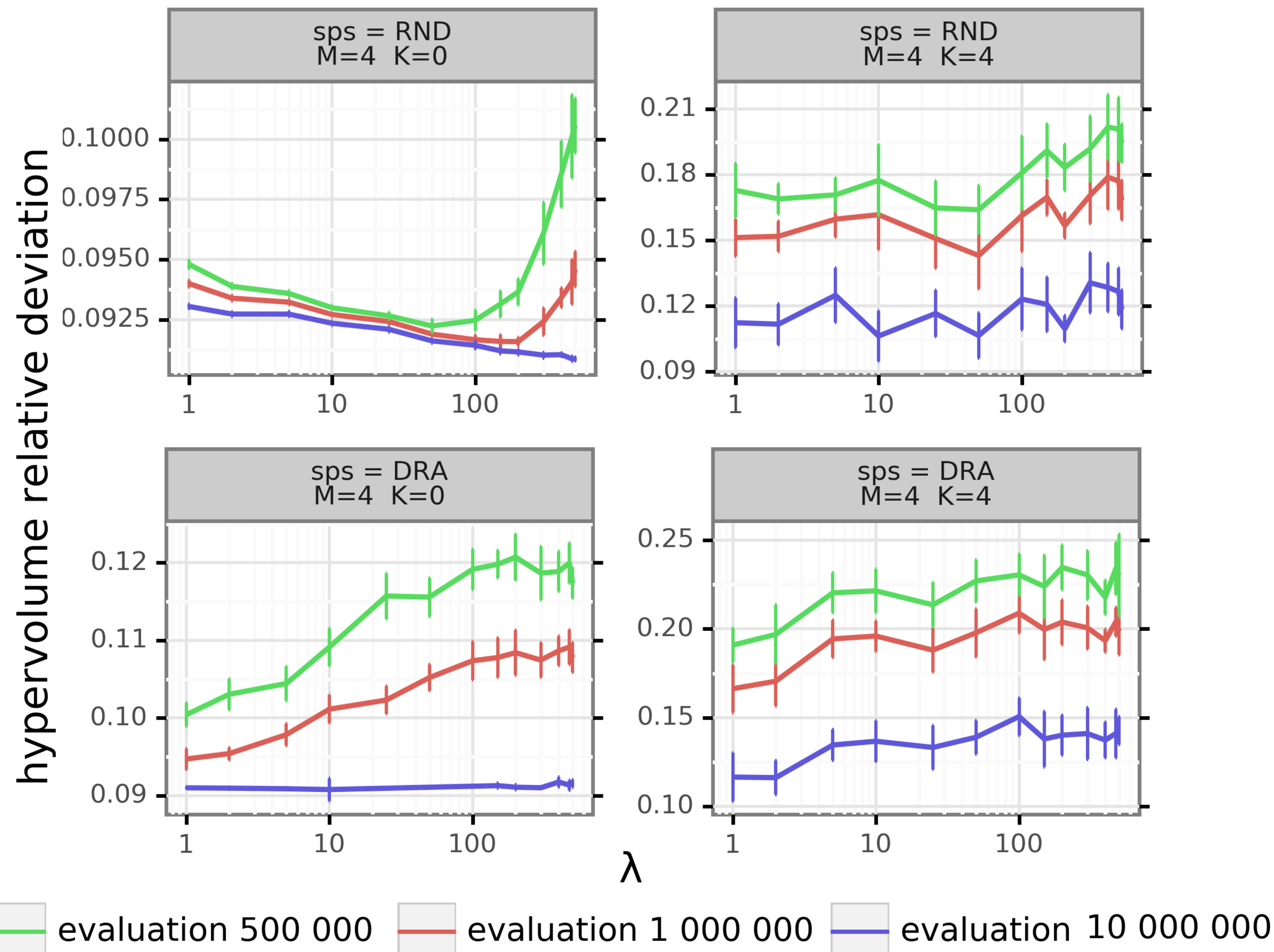
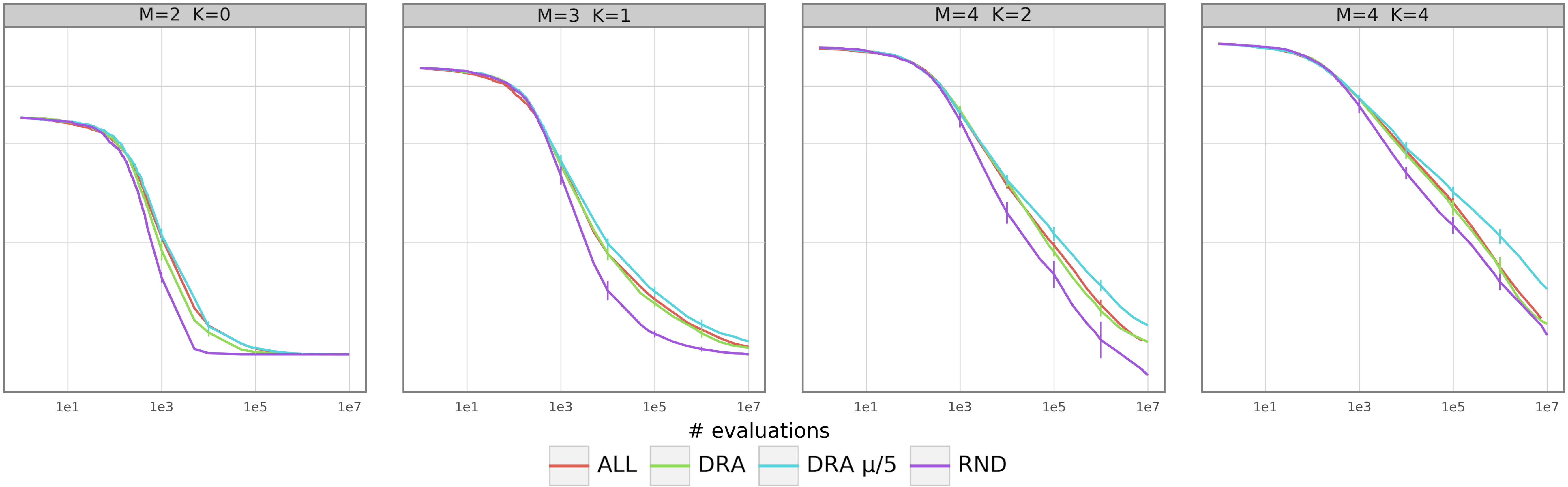


The sps strategy

Algorithm : MOEA/D with μ=500 and λ=1



- **Random (RND)** strategy has a better anytime behavior
- **DRA** and **ALL** strategies are better at the end of the process

The sps strategy

Algorithm : MOEA/D with μ=500 and λ=1

Budget	**10^4**		**10^5**		**10^6**		**10^7**	
SPS Strategy	***RND***	**DRA**	***RND***	**DRA**	***RND***	**DRA**	**RND**	***DRA***
Instance M=4 K=0	**13.2**	20.5	**09.9**	12.8	**09.3**	09.4	09.2	**09.0**

- **Random (RND)** strategy has a better anytime behavior
- **DRA** and **ALL** strategies are better at the end of the process

Conclusion

- We reviewed the design principles of the MOEA/D framework
- Analyse the role of 3 design components
- We are able to derive a parameter setting recommendation
- The perspective would be to extend the analysis to the continuous domain or study the parameter setting (off-line or on-line)

Thank you

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