In addition to proper scaling, the signal score should be *neutralized* so that the alphas do not include biases or undesirable bets on the benchmark or on risk factors. As we illustrate in the exercises at the end of the chapter, neutralization can be achieved in various ways, as there are multiple portfolios that hedge out a bet on the benchmark or on other risk factors.

6.7 Performance Analysis

How can the performance of a portfolio manager be evaluated? Are the *ex post* results due to skill or luck? The goal of performance analysis is to answer these questions. The efficient market hypothesis suggests that skillful active management is impossible. However, there is considerable evidence against the efficient market hypothesis (Shleifer, 2000).

Empirical results also suggest that an *average* active fund manager underperforms their benchmark on a risk-adjusted basis. Furthermore, empirical evidence also shows that good performance does not persist: The winners this year are almost as likely to be winners or losers next year. These are bleak conclusions about asset management. So how could we tell which asset managers are the good ones?

The fundamental goal of performance analysis is to separate skill from luck. The simplest type of performance analysis is a cross-sectional comparison of returns over some time period. This would distinguish winners from losers. However, these kinds of comparisons have several drawbacks. First, they typically do not represent the complete universe of investment managers but only those in existence during a specific time period. They generally contain survivorship bias. Perhaps worst of all, cross-sectional comparisons do not adjust for risk. By contrast, time-series analysis of returns can do a better job at separating skill from luck by measuring both return and risk. An even more complete picture can be obtained via time-series analysis of returns and portfolio holdings.

Return-Based Performance Analysis (Basic)

The development of the CAPM and the notion of market efficiency in the 1960s encouraged academics to tackle the problem of performance analysis. According to the CAPM, consistent exceptional returns are unlikely. Academics devised tests to check if the theory was correct. As a byproduct the first performance analysis techniques emerged. One approach, proposed by Jensen, consists of regressing the time series of *realized* portfolio excess returns against benchmark excess return:

$$r_P(t) = \alpha_P + \beta_P r_B(t) + \epsilon_P(t).$$

Jensen's alpha is simply the intercept α_P of this regression. According to the CAPM, this intercept is zero. The regression yields not only alpha and beta, but t-statistics that give information about their statistical significance. The *t-statistic* for α_P is

$$t\text{-stat} = \frac{\alpha_P}{\mathrm{SE}(\alpha_P)}.$$

As a rule of thumb, a t-statistic of 2 or more indicates that the performance of the portfolio is due to skill rather than luck. Assuming normality, the probability of observing such a large t-statistic purely by chance is smaller than 5%.

The t-statistic and the information ratio are closely related. The main difference between them is that the information ratio is annualized. By contrast, the t-statistic scales with the number of years of data. If we observe returns over a period of T years, the information ratio is approximately the t-statistic divided by the square root of the number of years of observation:

$$IR \approx \frac{t\text{-stat}}{\sqrt{T}}.$$

The standard error of the information ratio is approximately

$$\mathrm{SE}(IR) \approx \frac{1}{\sqrt{T}}.$$

A simple alternative to Jensen's approach is to compare Sharpe ratios for the portfolio and the benchmark. A portfolio with

$$\frac{\bar{r}_P}{\sigma_P} > \frac{\bar{r}_B}{\sigma_B},$$

where $\bar{r}$ denotes mean excess return over the period, has demonstrated positive performance. Once again, the statistical significance of this relationship is relevant for distinguishing luck from skill. If we assume that the standard errors of the portfolio and benchmark volatilities are fairly small compared to $\bar{r}$ standard errors, then the standard error of the Sharpe ratio is approximately $1/\sqrt{N}$, where N is the number of observations. Hence a statistically significant demonstration of skill occurs when

$$\frac{\bar{r}_P}{\sigma_P} - \frac{\bar{r}_B}{\sigma_B} > 2\sqrt{\frac{2}{N}}.$$

Return-Based Style Analysis

Style analysis was developed by Nobel laureate William Sharpe (1992). The popularity of this concept was aided by a study (Brinson et al., 1991) concluding that 91.5% of the variation in returns of 82 mutual funds could be explained by the allocation to bills, stocks, and bonds. Later studies considering asset allocation across a broader range of asset classes have shown that as much as 97% of fund returns can be explained by asset allocation alone.

Style analysis attempts to determine the effective asset mix of a fund using only the time series of returns for the fund and for a number of carefully chosen asset classes. Like a factor model approach, style analysis assumes that portfolio returns have the form

$$r_P(t) = \sum_{j=1}^{m} w_j f_j(t) + u_P(t),$$

where the $f_j(t)$ are the returns of m benchmark asset classes. The holdings $w_j,\ j = 1, \ldots, m$, represent the *style* of the portfolio. That is, the effective allocation to the m asset classes that could be replicated via a passive portfolio. The term $u_P(t)$ represents the *selection return*; that is, the portion of the portfolio return that style cannot explain. The effective holdings can be estimated via the quadratic program

$$\begin{array}{ll} \min\limits_{\mathbf{w}} & \operatorname{var}(u_P(t)) \\ \text{s.t.} & \sum\limits_{j=1}^{m} w_j = 1 \\ & w_j \geq 0,\ j = 1, \ldots, m. \end{array} \tag{6.21}$$

Notice that there are two key differences between this model and conventional multiple regression. First, the weights are constrained to be non-negative and to add up to 1. Second, instead of minimizing the sum of squared errors $\sum_{t=1}^{T} u_P(t)^2$, we minimize the variance of these quantities. The reason for the first restriction is that the w_j are to be interpreted as an effective asset allocation representing the style of the fund. In essence, they create a fund-specific benchmark. The reason for the second restriction is that we want to allow for a non-zero selection effect by the fund manager. The model finds the style that minimizes the variance of this effect. Once the optimal weights are determined, the average value of $u_P(t)$ gives the value added by the manager's selection skills, which can be negative or positive.

Assume the data available for style analysis are the return time series $r_P(t)$, $f_1(t), \ldots, f_m(t)$ for $t = 1, \ldots, T$. For ease of notation, put

$$\mathbf{r} := \begin{bmatrix} r_P(1) \\ \vdots \\ r_P(T) \end{bmatrix}, \quad \mathbf{F} := \begin{bmatrix} f_1(1) & \cdots & f_m(1) \\ \vdots & \ddots & \vdots \\ f_1(T) & \cdots & f_m(T) \end{bmatrix}, \quad \mathbf{1} := \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}.$$

Then the objective function in (6.21) can be written as

$$\begin{aligned} \operatorname{var}(\mathbf{r} - \mathbf{F}\mathbf{w}) &= \frac{1}{T}\|\mathbf{r} - \mathbf{F}\mathbf{w}\|^2 - \frac{1}{T^2}(\mathbf{1}^\mathsf{T}(\mathbf{r} - \mathbf{F}\mathbf{w}))^2 \\ &= \left(\frac{\|\mathbf{r}\|^2}{T} - \frac{(\mathbf{1}^\mathsf{T}\mathbf{r})^2}{T^2}\right) - 2\left(\frac{\mathbf{r}^\mathsf{T}\mathbf{F}}{T} - \frac{\mathbf{1}^\mathsf{T}\mathbf{r}}{T^2}\mathbf{1}^\mathsf{T}\mathbf{F}\right)\mathbf{w} \\ &\quad + \mathbf{w}^\mathsf{T}\left(\frac{1}{T}\mathbf{F}^\mathsf{T}\left(I - \frac{1}{T}\mathbf{1}\mathbf{1}^\mathsf{T}\right)\mathbf{F}\right)\mathbf{w}. \end{aligned}$$

Style analysis provides an improvement tool for measuring performance. The constructed style usually tracks the performance of the fund more accurately than a predefined benchmark. Style analysis has also some limitations. For instance, the weights may not necessarily match the style disclosed by the fund manager. However, as Sharpe puts it: "If it acts like a duck, it is ok to assume it is a duck." Style analysis also makes the simplifying assumptions that the weights are constant. This is clearly not the case in actively managed funds, even without active trading. There exist some variations of style analysis that allow for weights to change. The model gets a bit more technical because it needs to incorporate some "regularization" term that prevents the weights from changing too much too often.

6.8 Notes

The mean–variance model was introduced in the seminal article of Markowitz (1952). The CAPM was developed by Treynor[1], Sharpe (1964), Lintner (1965), and Mossin (1966), by building on the mean–variance approach of Markowitz. In recognition of their work on portfolio choice and the CAPM, Sharpe and Markowitz were jointly awarded the 1990 Nobel Prize in Economics. Both Lintner and Mossin passed away before 1990 and Treynor's manuscript was never published.

The textbook by Grinold and Kahn (1999) is a classical reference in active portfolio management. In their textbook, Grinold and Kahn developed and relied extensively on characteristic portfolios.

6.9 Exercises

Exercise 6.1 The purpose of this exercise is to prove the two-fund theorem (Theorem 6.1).

(a) Find the Lagrangian function $L(\mathbf{x}, \theta)$ for (6.1).
(b) Solve the optimality conditions $\nabla L(\mathbf{x}, \theta) = \mathbf{0}$ to conclude that the optimal solution to (6.1) is

$$\mathbf{x}^* = \lambda \cdot \frac{1}{\mathbf{1}^\mathsf{T}\mathbf{V}^{-1}\boldsymbol{\mu}}\mathbf{V}^{-1}\boldsymbol{\mu} + (1-\lambda) \cdot \frac{1}{\mathbf{1}^\mathsf{T}\mathbf{V}^{-1}\mathbf{1}}\mathbf{V}^{-1}\mathbf{1}$$

where $\lambda = \mathbf{1}^\mathsf{T}\mathbf{V}^{-1}\boldsymbol{\mu}/\gamma$.

Exercise 6.2 Assume $\boldsymbol{\mu}$ and $\mathbf{V}$ are respectively the vector of expected returns and covariance matrix of n risky assets. Assume V is non-singular and $\bar{\mu} > \boldsymbol{\mu}^\mathsf{T}\mathbf{V}^{-1}\mathbf{1}/\mathbf{1}^\mathsf{T}\mathbf{V}^{-1}\mathbf{1}$. Consider the mean–variance optimization problem

[1] "Toward a theory of market value of risky assets". Unpublished manuscript, 1961.