Multi-Criteria Dimensionality Reduction with Applications to Fairness

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JOINT WORK WITH

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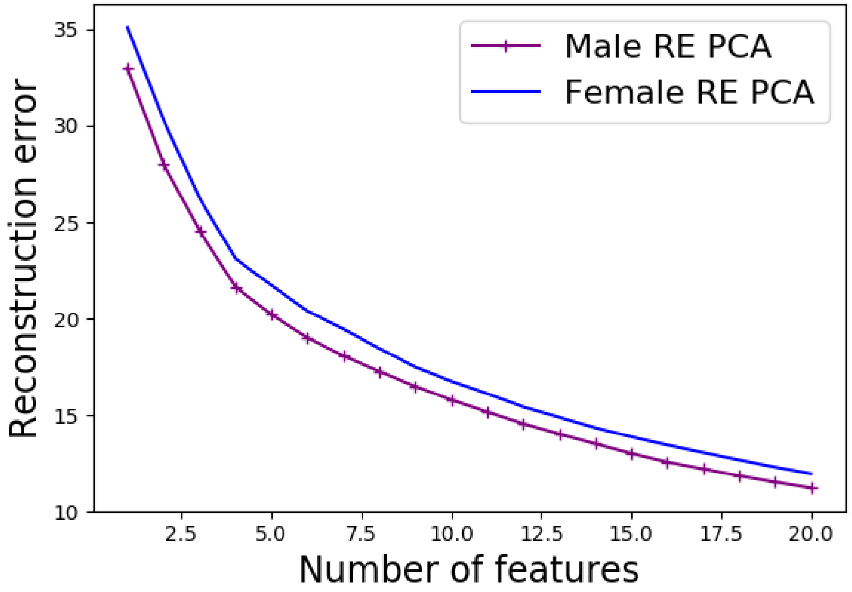
PCA can be unfair!

Standard PCA on face data LFW of male and female

Average reconstruction error (RE) of PCA on LFW



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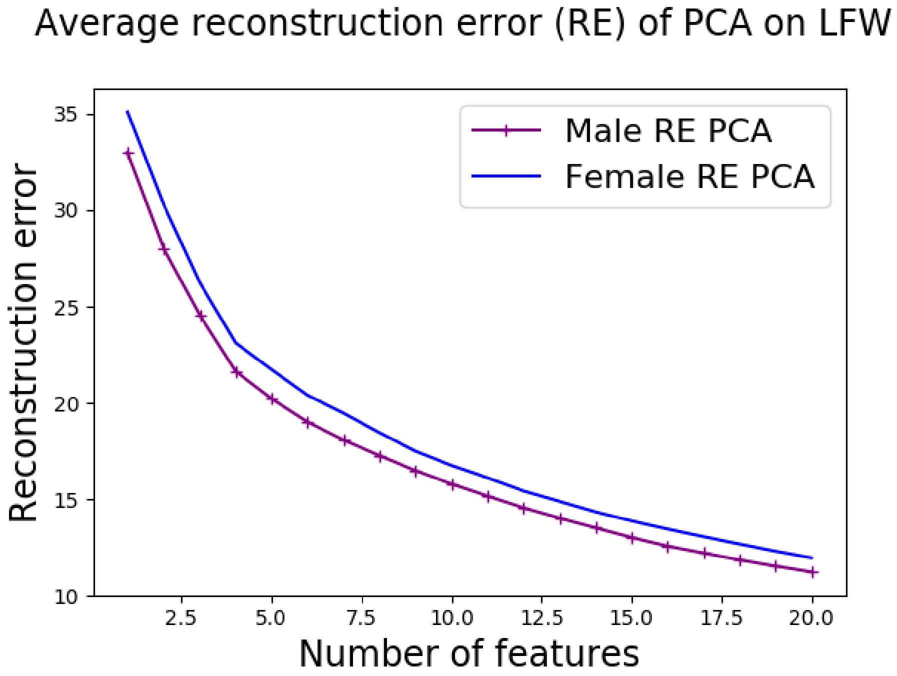
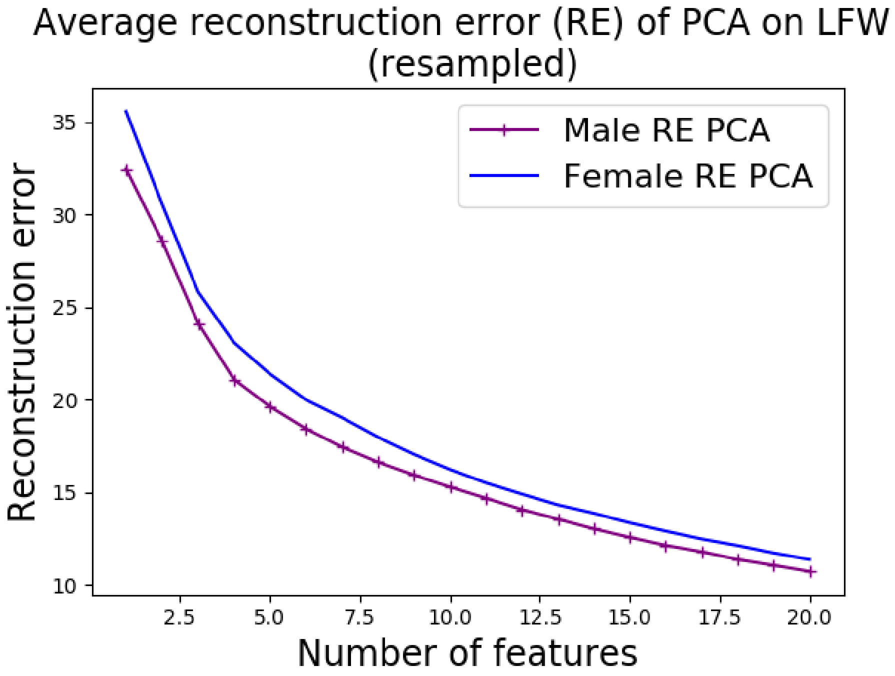


Equalizing male and female weight before PCA



Contribution 1: Problem Formulation

Multi-criteria dimensionality reduction (MCDR):

$$\max_{\text{projection } P} \; g\big(f_1(P), f_2(P), \ldots, f_k(P)\big)$$

Utility criterion f_i's and social welfare g

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- *Mar-Loss*: $\min_P \max_{i\in\{1,\dots,k\}} \left(\max_Q \|A_i Q\|_F^2 - \|A_i P\|_F^2 \right)$
- *NSW*: $\max_P \quad \prod_{i=1}^{k} \|A_i P\|_F^2$

Contribution 2: Algorithms and Guarantees

On linear f_i in PP^{T} and concave g:

- Polynomial-time algorithm for MCDR with optimal utility and small rank violation $s = \sqrt{2k + 1/4} - 3/2$
- Approximation ratio $1 - s/d$ on utility when no rank violation

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- Approximation ratio $1 - s/d$ on utility when no rank violation
- Semidefinite Program (SDP) → Multiplicative Weight (MW) method
  - scalable up to ≈ 1000 dimensions

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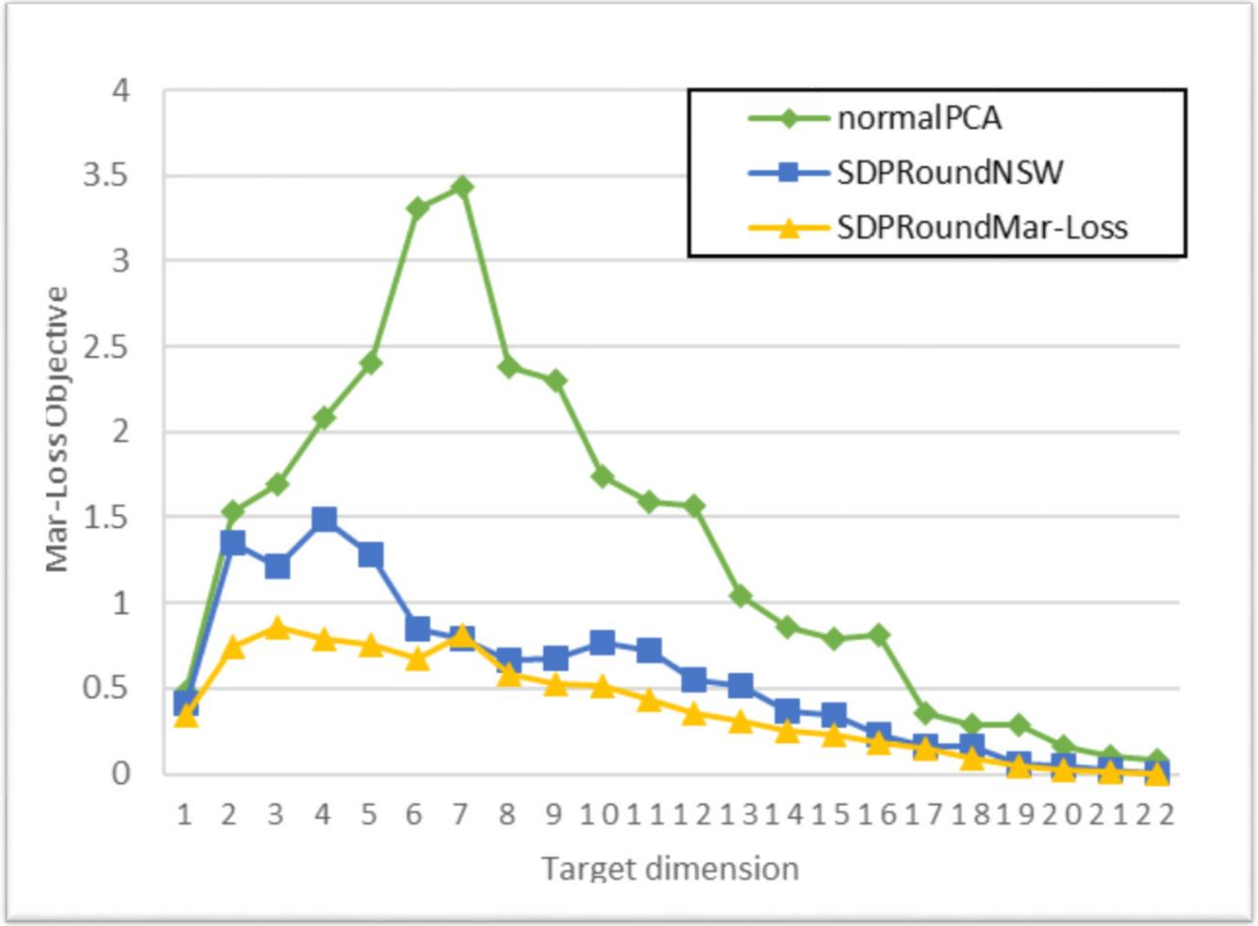

Contribution 3: Optimization Theory

- Every extreme point of the semi-definite program relaxation of MCDR has low rank
  - Generalize work on low-rank property in semi-definite program by Barvinok’95, Pataki’98
- Optimization result + ML application

Contribution 4: Complexity of MCDR

- NP-hard for general k
  - Reduction to MAX-CUT
- Polynomial-time for fixed k
  - Algorithmic theory of quadratic maps.

More details

- Poster: Thursday Dec 12th at 10:45 AM -- 12:45 PM, East Exhibition Hall B + C #80
- Happy to chat!

- Code: github.com/uthaipon/multi-criteria-dimensionality-reduction
- Web: sites.google.com/site/ssamadi/fair-pca-homepage

(searchable links at NeurIPS website or my website Uthaipon Tantipongpipat)