Multi-Criteria Dimensionality Reduction with Applications to Fairness

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Joint work with
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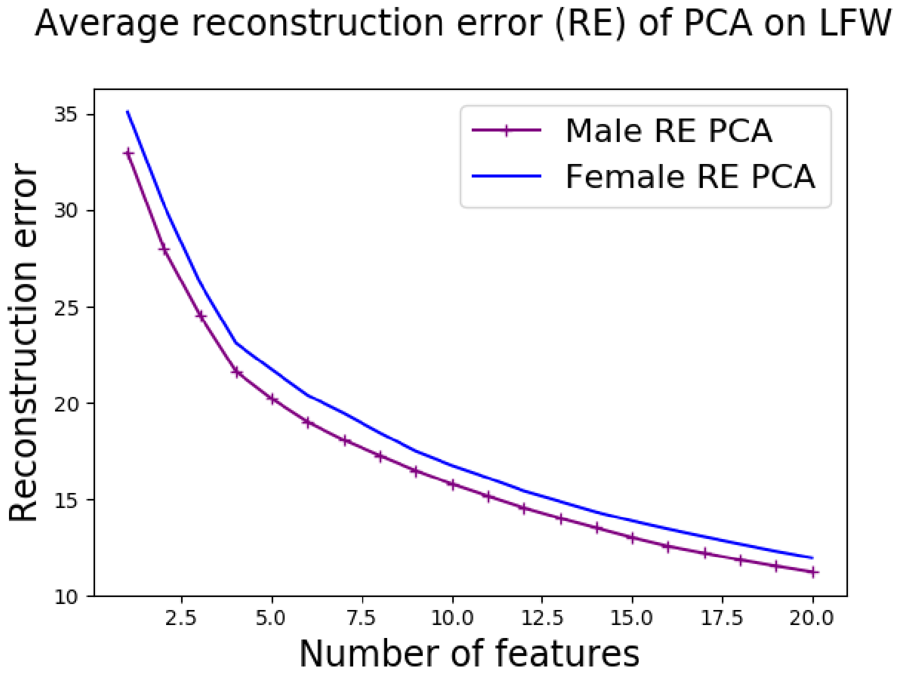
PCA can be unfair!

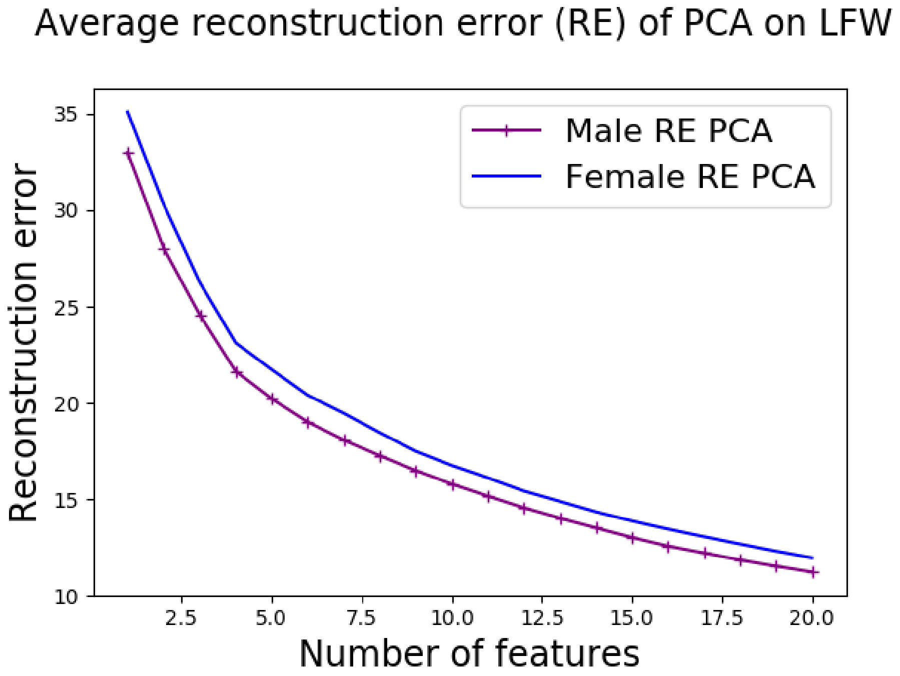
Standard PCA on face data LFW of male and female.

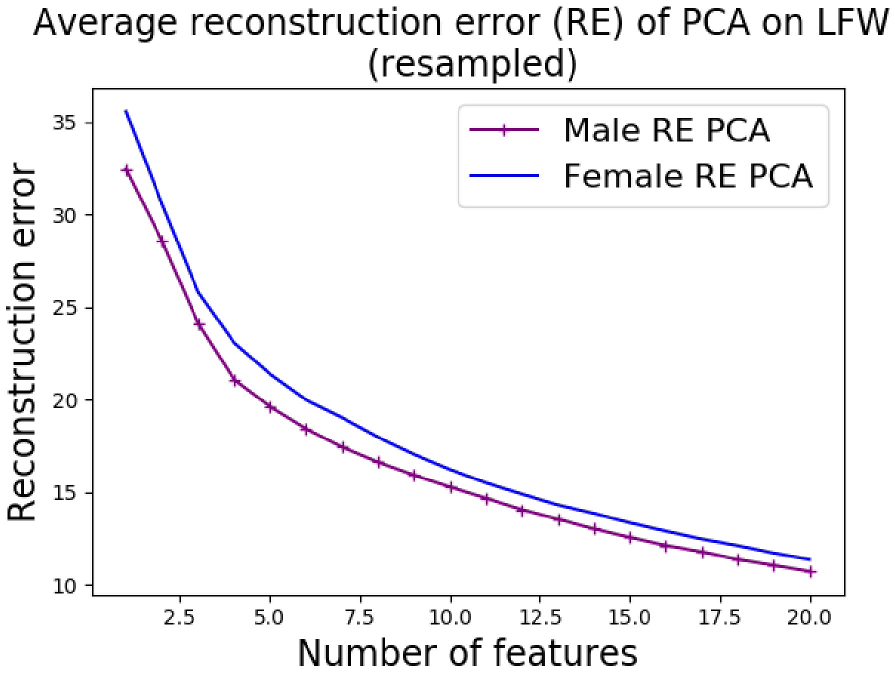


Unfair PCA

Standard PCA on face data LFW of male and female.

Equalizing male and female weight before PCA





Main Contributions

Problem Formulation

Multi-criteria dimensionality reduction (MCDR):

$$\max_{\text{projection } P} \; g\big(f_1(P), f_2(P), \ldots, f_k(P)\big)$$

Utility criterion f_i's and social welfare g

Main Contributions

Mar-Loss: $\min_P \max_{i\in\{1,\dots,k\}} \left(\max_Q \|A_i Q\|_F^2 - \|A_i P\|_F^2 \right)$

NSW: $\max_P \quad \prod_{i=1}^k \|A_i P\|_F^2$

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On linear f_i in PP^{T} and concave g:

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On linear f_i in PP^{T} and concave g:

- Polynomial-time algorithm for MCDR with optimal utility and small rank violation $s = \sqrt{2k + 1/4} - 3/2$
- Approximation ratio $1 - s/d$ on utility when no rank violation

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Multiplicative Weight (MW) method for scalable up to ≈ 1000 dimensions

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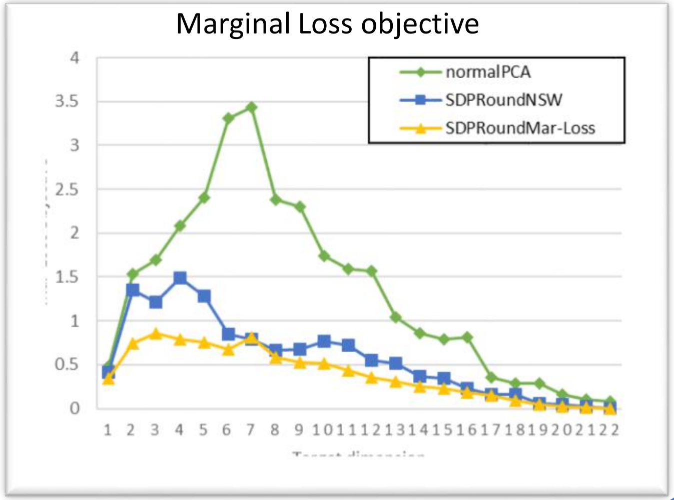

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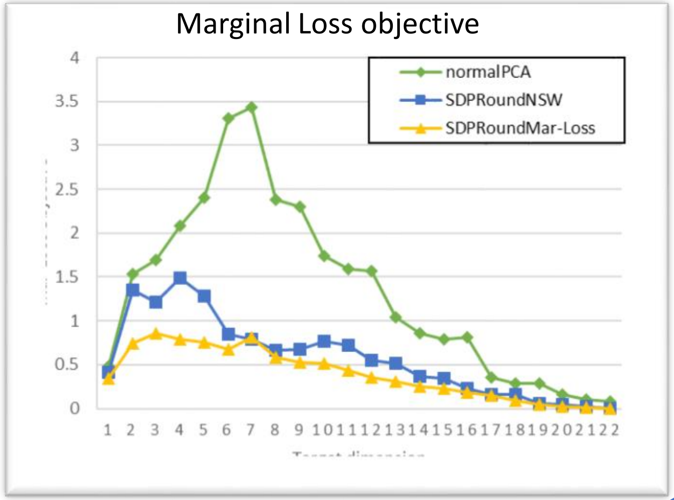

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- Every extreme point of the SDP-Relaxation of MCDR has low rank.
- Optimization result + ML application

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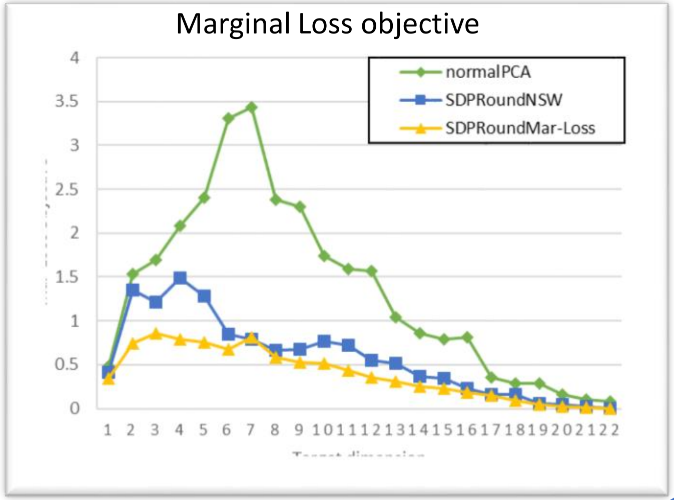

Multiplicative Weight (MW) method for scalable up to $\approx$ 1000 dimensions

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- Algorithmic theory of quadratic maps.

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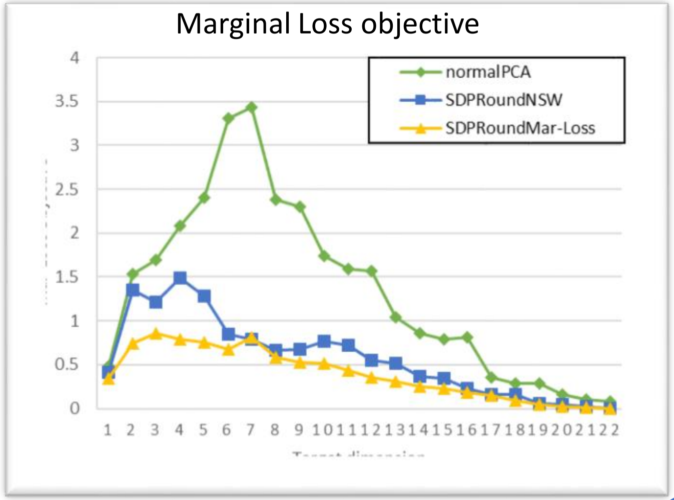

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Code: github.com/SDPforAll/multiCriteriaDimReduction

Web: sites.google.com/site/ssamadi/fair-pca-homepage