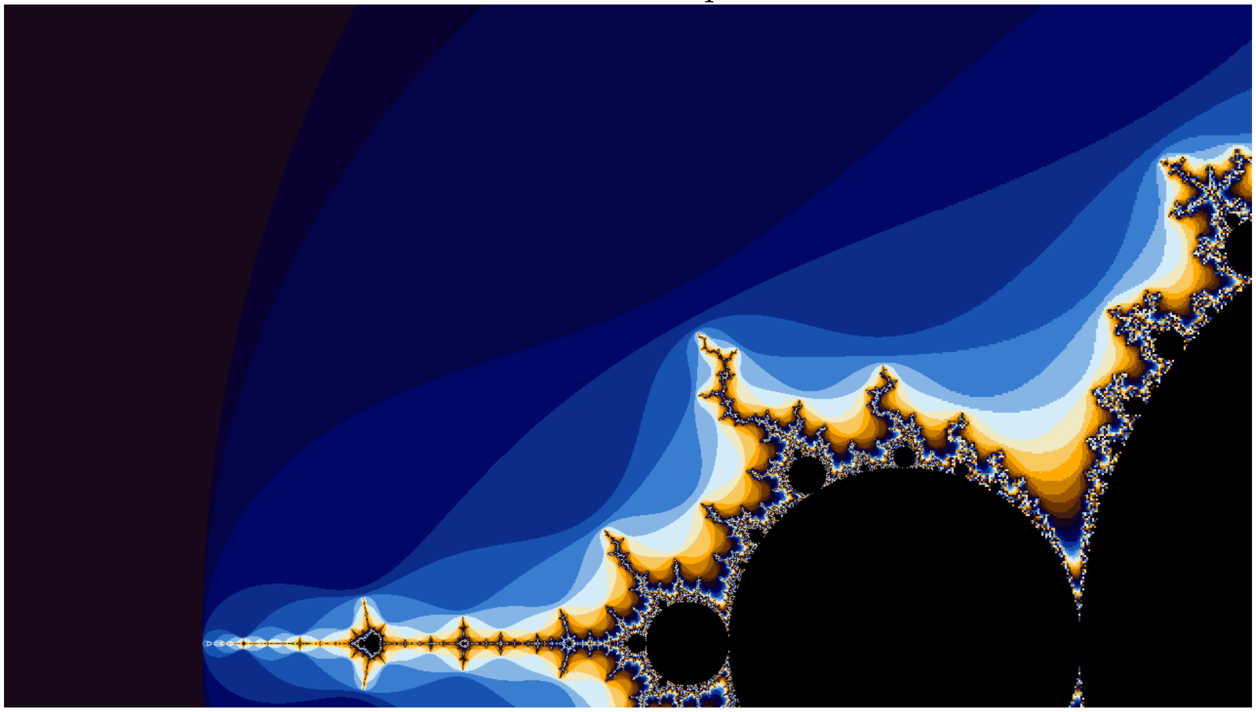
CSCI 111, Lab 9
Mandelbrot Explorer



mandX-1.393Y-0.408R0.5.png

Due date: Midnight, Tuesday, November 15, on Canvas. No late work accepted.

File names: Names of files, functions, and variables, when specified, must be EXACTLY as specified. This includes simple mistakes such as capitalization.

Individual work: All work must be your own. Do not share code with anyone other than the instructor and teaching assistants. This includes looking over shoulders at screens with the code open. You may discuss ideas, algorithms, approaches, *etc.* with other students but NEVER actual code.

The Mandelbrot Set: The **Mandelbrot set** is the set of complex numbers c such that the function $f_c(z) = z^2 + c$ does not diverge to infinity when iterated from $z = 0$. In other words, the sequence $f_c(0), f_c(f_c(0)), f_c(f_c(f_c(0))), \ldots$ remains bounded in absolute value.

You can read a lot about this set online, for example in Wikipedia: `https://en.wikipedia.org/wiki/Mandelbrot_set`, and see many pretty pictures people have made by exploring this set and colorizing it.

Creating an image in pygame: I have provided a program, `pygamecolors.py`, in the lab folder. In that program I create the image of a simple gradient. You will eventually use this framework to display the Mandelbrot set.

Because Mandelbrot pictures can take a long time to generate, this program illustrates how to make a complicated image appear instantly, but roughly, and then gradually refine it. Notice that we start with very large pixels, and reduce them by 1/2 each time we repeat. Eventually the image is as sharp as it can be, but with large sizes it takes quite a bit of time.

The program pauses between each pixelsize, just to give you a chance to see the result before moving on. This pause is entirely unnecessary and can be removed in your versions.

Note that in this program I translated from screen coordinates to ''normalized" coordinates to make it easier to calculate the colors. Instead of (x, y) going from $(0, 0)$ to $(width, height)$, as they do on the screen, they go from $(0, 0)$ to $(1, 1)$. This makes the color computation much easier. You will also transition from screen coordinates to other coordinates in your programs.

Finally, you should note that the program prints out how long each render took, the times for each pixelsize, and the total time for all pixelsizes together. In one of my runs I got 4.4 seconds for the 1×1 pixels, the final render, and 6.6 seconds for all renders together, from 128×128 to 64×64 to ... to 1×1. Thus, all 8 renders only took 2.2 seconds longer than the one final one, or 2.2/6.6 or about 33% longer. This seems a modest price to pay for getting instant feedback about how the image is going to appear. You will notice that I've enabled the user to quit the program before it is finished. If you see an image starting to appear that is not at all what you expected, you don't have to wait until the final, slow image is complete to find out and quit!

Mathematical aside (optional): The fact that 8 renders only takes a bit longer than one render is due to the following interesting theorem (if you're not good at sums you can skip this part):

$$\begin{aligned}
\sum_{i=0}^{n} a^i = \sum_{i=1}^{n} a^i + a^0 &= \sum_{i=1}^{n} a^i + 1 \\
\sum_{i=1}^{n} a^i &= \sum_{i=0}^{n} a^i - 1 \\
a\sum_{i=0}^{n} a^i = \sum_{i=1}^{n+1} a^i &= a^{n+1} + \sum_{i=1}^{n} a^i \\
&= a^{n+1} + \sum_{i=0}^{n} a^i - 1 \\
(a-1)\sum_{i=0}^{n} a^i &= a^{n+1} - 1 \\
\sum_{i=0}^{n} a^i &= \frac{a^{n+1}-1}{a-1}
\end{aligned}$$

Let's check this out. If $a = 2$ then

$$\begin{aligned} 2^0 + 2^1 + 2^3 + 2^4 &= 1 + 2 + 4 + 8 \\ &= 15 = \frac{2^4 - 1}{2 - 1} \end{aligned}$$

How about that.

This is one of my favorite theorems, and really important intuition in computer science. When a is very large, $a - 1 \approx a$ and $a^{n+1} - 1 \approx a^{n+1}$, so, approximately,

$$\sum_{i=0}^{n} a^i \approx a^n$$

This is extraordinary. For large a, then

$$a^0 + a^1 + a^2 + a^3 + a^4 + a^5 + a^6 + a^7 + a^8 + a^9 + a^{10} \approx a^{10}$$

It's as if the smaller exponents don't even count!

Let's see what this has to do with our image viewer. Each time we make the pixels half the width and height of the previous pixels. This means 4 pixels fit into each previous pixel. That means we're computing 4 times as many pixels each time around the pixelsize loop. So, whatever time the first loop took, the next one will take four times longer. And the next one four times longer than that. And so on. So there's a factor of 4^i for the ith trip through the loop. With $a = 4$

$$\sum_{i=0}^{n} 4^i = \frac{4^{n+1} - 1}{4 - 1} \approx \frac{4(4^n)}{3} \approx 1.33(4^n)$$

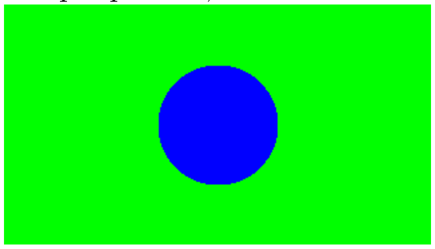
So, before I even wrote the program I expected running all the pixelsize renders would only take about 33% longer than running just the final one.

The data proved me right.

You will see more amazing facts like this in your study of algorithms.

The Dot: OK, now we know why that image rendering program `pygamecolors.py` behaves like it does, and we know how to get an image on the screen. You should notice that if you press the S key at any time the program saves your image to a file, which is convenient.

Before we get to the Mandelbrot set, let's handle a few of the mechanics so we know what we're doing with a simple picture, THE SPOT.



Our world consists, not of the Mandelbrot set, but of the set of points inside a circle of radius one, centered on the origin. We will build a vewer to look at this marvelous set, coloring the set blue and everything else green. We will build a program that will allow us to examine this blue dot in from any perspective we desire.

Screen coordinates and world coordinates: The first challenge in producing the spot is the transition between screen coordinates and world coordinates. The screen coordinates, `(i, j)` range from 0 to `width` and 0 to `height`.

But we want to look at a circle that is centered on the origin, with unit radius, and we want this circle centered in our image as in the figure.

In world coordinates, the **center** of the image is the point $(0, 0)$. The center of the top is two units above the center, $(0, 2)$ and the center of the bottom is a point two units below the center, $(0, -2)$. We call the distance in world coordinates between the center of the image and the top of the screen the **radius** of the image.

Clearly, given the row between 0 and `height`, we need to lerp this value into the range 0 ± 2. Or, more generally, into the range $center_y \pm radius$.

What about the width? Any image has an **aspect ratio**, called R, which is just the width divided by the height. Clearly, if the top is *radius* from the center, in world coordinates, then the left and right sides are, in world coordinates, at $center_x \pm R(radius)$. So, to get from a column between 0 and *width* in screen coordinates, we just have to lerp this number to the interval $center_x \pm R(radius)$.

This should be enough information to write a `screenToWorld` function that translates screen coordinates into world coordinates.

```python
def screenToWorld(i, j, width, height, center, radius):
    ....
    return (x, y)
```

Test data: I ran `screenToWorld` with a screen centered at $(0, 0)$ and radius of 2, and a width and height of 640 and 480, and got these numbers for the input values of i and j:

```
(0,0) => (-2.6666666666666665, 2.0)
(0,50) => (-2.6666666666666665, 1.5833333333333333)
(0,100) => (-2.6666666666666665, 1.1666666666666665)
(0,150) => (-2.6666666666666665, 0.75)
(50,0) => (-2.25, 2.0)
(50,50) => (-2.25, 1.5833333333333333)
(50,100) => (-2.25, 1.1666666666666665)
(50,150) => (-2.25, 0.75)
(100,0) => (-1.8333333333333333, 2.0)
(100,50) => (-1.8333333333333333, 1.5833333333333333)
(100,100) => (-1.8333333333333333, 1.1666666666666665)
(100,150) => (-1.8333333333333333, 0.75)
(150,0) => (-1.4166666666666665, 2.0)
(150,50) => (-1.4166666666666665, 1.5833333333333333)
(150,100) => (-1.4166666666666665, 1.1666666666666665)
(150,150) => (-1.4166666666666665, 0.75)
```

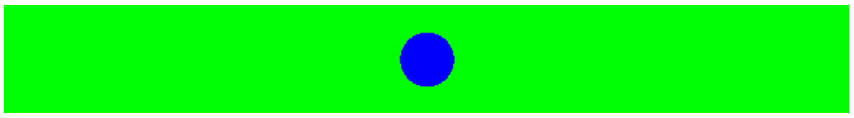
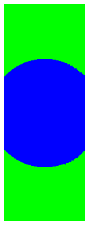
You should be able to get something similar. You should also be able to compute test values by hand to check the correctness of your function.

Colorize: Once you get the world coordinates from the screen coordinates, finding the color is simple! For each pixel on the screen at `(i, j)`, find the world coordinates (x, y) for those screen coordinates. Find the distance from the center, from (c_x, c_y). This is simply $\sqrt{(x - c_x)^2 + (y - c_y)^2}$. If this distance is less than 1, it's blue: (0, 0, 255). Otherwise, it's green: (0, 255, 0).

Now we can finally write an image generating program.

Spot with any size screen: Write a program `spot.py` that will display a blue dot in a green field, as above. You should start with the framework I've given you in `pygamecolors.py` found in the lab's folder.

You should be able to change the height and width of the image by editing the code. No matter what the initial height and width are (try it with several). The spot should be centered horizontally and vertically, and occupy half the distance from top to bottom. It might look like one of these:

Do this before going on!

Resizable screen: Now rewrite the program `spot.py` to handle resizing the window by dragging a corner. Your image should be regenerated at the appropriate size.

To handle resizing a pygame window, you first have to initialize the screen as follows:

```
screen = pygame.display.set_mode((width,height), pygame.RESIZABLE)
```

You also have to handle the `VIDEORESIZE` event, something like this:

```
        elif event.type == VIDEORESIZE:
            width,height = event.dict['size']
            restart(width, height)
```

Restarting should reinitialize the screen to the right size, the background surface we're drawing to, and the pixelsize. Everything you do when you initialize the main loop the first time. Just do it again. If you wrote the program by isolating the intitialization from the main loop, this procedure is probably already written!

When `spot.py` is run now, we see our blue spot right where it should be no matter how we resize the window.

Do this before going on!

Mouse handling: Mouse events are handled just like keyboard events. I have provided a simple program, `testmouse.py`, that shows how to get mouse events.

The mouse event handling should be in the same procedure that handles keyboard events, since all events are put in the same queue.

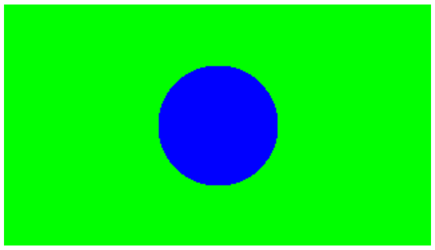
Recentering and zooming: Suppose we want to change the center of the image with the mouse? When we click on a location on the screen, that should be the new center of view in the world, and furthermore we should zoom in on that spot. For instance, if we left click on the upper right side of the spot, the screen should redraw with that spot (in world coordinates) as the new center of the screen, and with the image doubled in size.

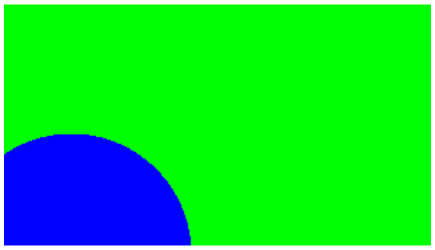
Any mouse click should change the center (in world coordinates) of the image. Remember that when you get the mouse event, it gives you the position of the mouse in **screen** coordinates. You have to change these to world coordinates before resetting the value of `center`.

Clicking the left button should double the size of the image and clicking the right button should halve the size of the image. You can do this wil simple changes to the `radius` of the image. If you change the image world coordinates for the vertical range from $(enter_y \pm 2)$ to $(center_y \pm 4)$ does that make the things in the image look bigger or smaller?

Clicking the mouse will thus also restart the program, just like changing window size did. You should handle mouse events whenever you handle keyboard or resize events, so just add the right cases to the event handler.

For example, starting with the figure on the left, left clicking above and to the right of the spot produces the figure on the right:





We have zoomed in 2$\times$ (the spot is twice as big) and our viewpoint is now above and to the right of the spot.

Finish this before going on! `spot.py` will be the basis for our Mandelbrot explorer, which will be **much** more interesting than a blue spot!

Mandelbrot: The Mandelbrot set pictures are made iterating the Mandelbrot function, described in the Wikipedia article. If the absolute value goes above 2, then the series will diverge. The algorithm simply counts the number of steps until it goes above 2, and colorizes according to the number of steps. If we go for 250 iterations and it doesn't go above 2, we color it black.

Starting with `spot.py`, copy it to `mandelbrot.py` and make the following changes.

There is a pseudocode implementation of the algorithm in the Wikipedia article. Implement this in Python with a function

```
def mandelbrot(x0, y0):
```

that returns the number of iterations.

`max_iterations` will be a global variable. We'll need it elsewhere. A value of 250 is reasonable for moderate zooming, and reasonably quick, but you can try larger values.

The colors I used in my Figures I got from here. They are:

```
    colors = [(66,   30,   15),
              (25,    7,   26),
              (9,     1,   47),
              (4,     4,   73),
              (0,     7,  100),
              (12,   44,  138),
              (24,   82,  177),
              (57,  125,  209),
              (134,  181,  229),
              (211,  236,  248),
              (241,  233,  191),
              (248,  201,   95),
              (255,  170,    0),
              (204,  128,    0),
              (153,   87,    0),
              (106,   52,    3)]
```

If the number of iterations is `n`, we simply take the color at the nth position in `colors` list, modulo the length of the list. Unless, of course, `n == max_iterations`, in which case we color it black.

Adjusting `max_iterations` (optional): If you plan on zooming deeply into the Mandelbrot set you might want to adjust the `max_iterations`. The finer the detail you need, the larger this should be. In my version, whenever I zoomed in (by changing `radius` or out I increased or decreased the `max_iterations` by a percentage. There doesn't seem to be any good guidance on the "right" amount to adjust this, so, maybe 20%?

Alternatively, you could use the plus and minus keys to adjust the `max_iterations`, or the mouse scroll wheel, *etc.*

Black is slow! The black areas inside the Mandelbrot set are the ones where the algorithm had to go to the maximum number of iterations. These are clearly going to be the slowest points. If you select regions without so much black, they'll render faster.

Image names: When the user presses the S key a copy of the image is saved. It is really nice to know the coordinates and radius of the image, so you can find it again! Change the file name to include the x, y, and $radius$ used to generate the image.

Look at the documentation for `format` in Python. A fixed format with three digits is

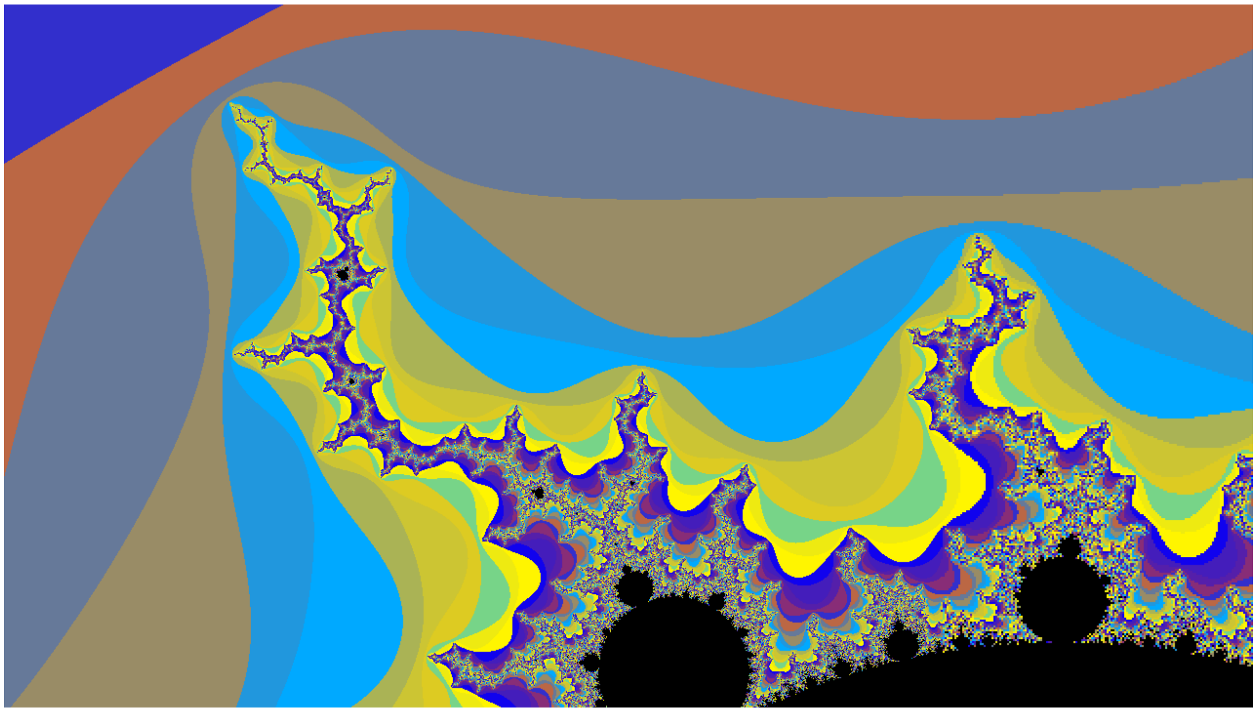
```
'{:.3f}'.format(x)
```

This is what I used for my images.

This will also make it nice to browse and save images, as each image will have a unique name you don't have to worry about overwriting images when you save new ones.

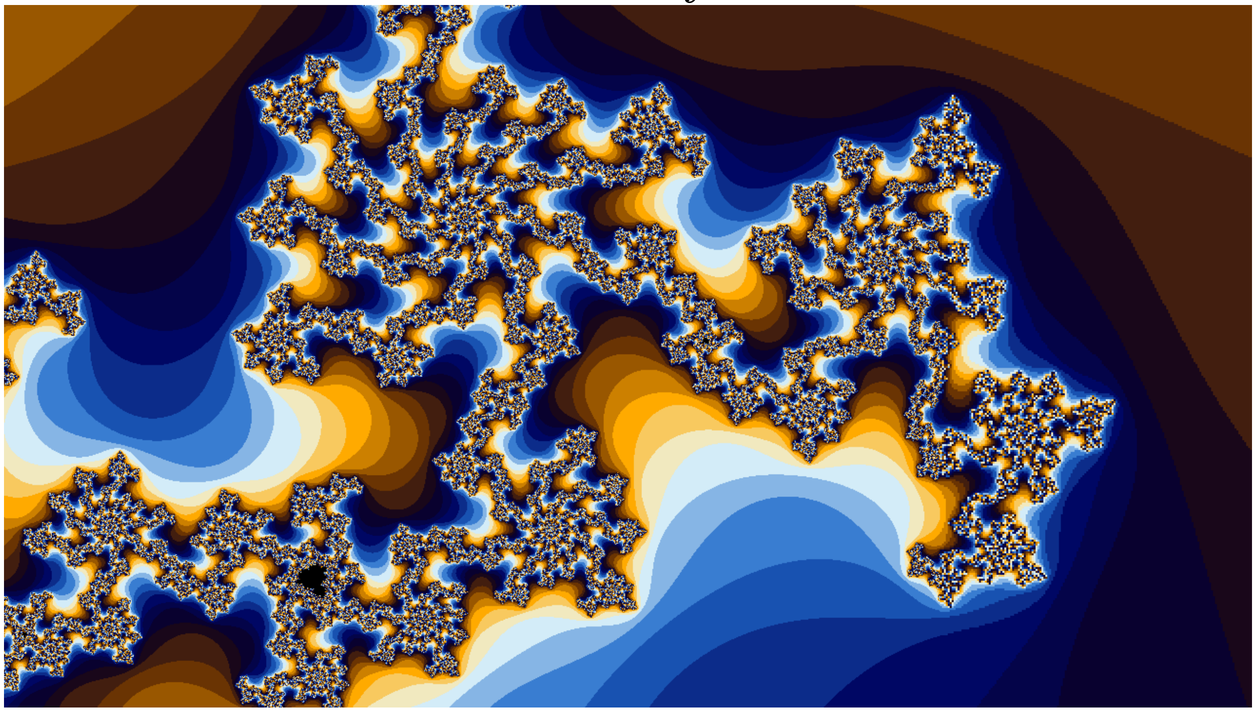
A new gradient: Develop your own color gradient and create at least two nice images of some interesting places in the Mandelbrot set. Include these images in your zipped `lab09` folder.

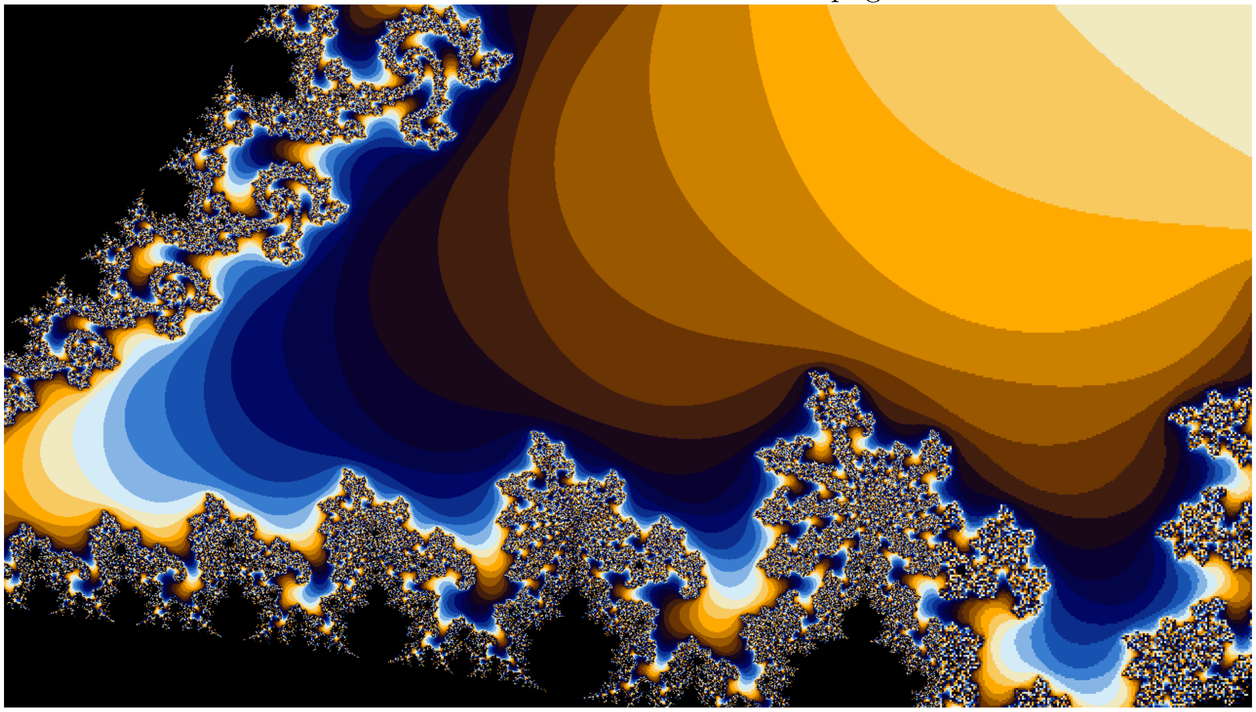
Here is an image I made with a different gradient function:



mandX-1.154Y0.352R0.125.png

Gallery


mandX-0.688Y0.382R0.00390625.png


mandX-0.009Y0.669R0.03125.png

mandX-1.624Y0.002R0.125.png

mandX0.456Y0.111R0.0625.png