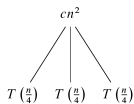


Using trees to guess

- Find a good guess for

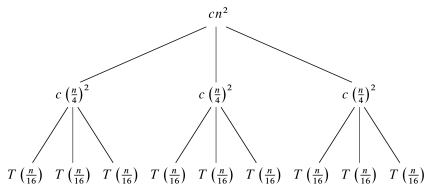
$$T(n) = 3T(n/4) + \Theta(n^2)$$

$T(n)$

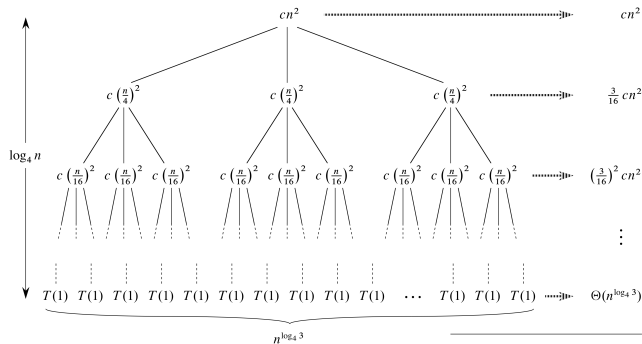


(a)

(b)



(c)



(d)

Total: $O(n^2)$

Bound the summation

- ▶ A geometric series of a number less than 1 can be bounded:

$$\begin{aligned}T(n) &= cn^2 + \frac{3}{16}cn^2 + \left(\frac{3}{16}\right)^2 cn^2 + \dots + \left(\frac{3}{16}\right)^{\log_4 n - 1} cn^2 + \Theta(n^{\log_4 3}) \\&= \sum_{i=0}^{\log_4 n - 1} \left(\frac{3}{16}\right)^i cn^2 + \Theta(n^{\log_4 3}) \\&< \sum_{i=0}^{\infty} \left(\frac{3}{16}\right)^i cn^2 + \Theta(n^{\log_4 3}) \\&= \frac{1}{1 - (3/16)} cn^2 + \Theta(n^{\log_4 3}) \\&= \frac{16}{13} cn^2 + \Theta(n^{\log_4 3}) \\&= O(n^2)\end{aligned}$$

- ▶ Now we can check this guess with substitution (induction).

Explicitly Solving Recursion of p. 89

$$T(n) = \cancel{3T\left(\frac{n}{4}\right)} + cn^2$$

$$\cancel{3T\left(\frac{n}{4}\right)} = \cancel{3^2T\left(\frac{n}{4^2}\right)} + 3c\left(\frac{n}{4}\right)^2$$

$$\cancel{3^2T\left(\frac{n}{4^2}\right)} = \cancel{3^3T\left(\frac{n}{4^3}\right)} + 3^2c\left(\frac{n}{4^2}\right)^2$$

$$\cancel{3^3T\left(\frac{n}{4^3}\right)} = \cancel{3^4T\left(\frac{n}{4^4}\right)} + 3^3c\left(\frac{n}{4^3}\right)^2$$

...

$$\cancel{3^{k-1}T\left(\frac{n}{4^{k-1}}\right)} = \cancel{3^kT\left(\frac{n}{4^k}\right)} + 3^{k-1}c\left(\frac{n}{4^{k-1}}\right)^2$$

...

$$\cancel{3^{\log_4 n - 1}T\left(\frac{n}{4^{\log_4 n - 1}}\right)} = 3^{\log_4 n}T\left(\frac{n}{4^{\log_4 n}}\right) + 3^{\log_4 n - 1}c\left(\frac{n}{4^{\log_4 n - 1}}\right)^2$$

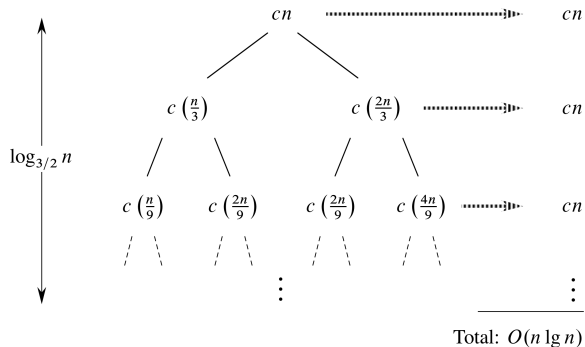
$$T(n) = 3^{\log_4 n}T\left(\frac{n}{4^{\log_4 n}}\right) + \sum_{i=0}^{\log_4 n - 1} 3^i c \left(\frac{n}{4^i}\right)^2$$

Explicitly Solving Recursion of p. 89

$$\begin{aligned}T(n) &= 3^{\log_4 n} T\left(\frac{n}{4^{\log_4 n}}\right) + \sum_{i=0}^{\log_4 n - 1} 3^i c \left(\frac{n}{4^i}\right)^2 \\&= n^{\log_4 3} T(1) + cn^2 \sum_{i=0}^{\log_4 n - 1} \left(\frac{3}{16}\right)^i \\&= n^{\log_4 3} T(1) + cn^2 \left(\frac{(3/16)^{\log_4 n} - 1}{(3/16) - 1}\right) \\&= \Theta(n^2)\end{aligned}$$

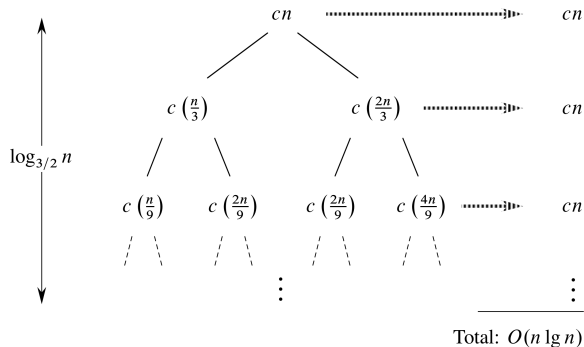
Second recursion tree example

$$T(n) = T(n/3) + T(2n/3) + O(n)$$



Second recursion tree example

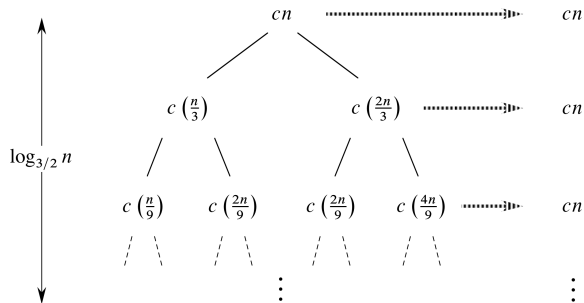
$$T(n) = T(n/3) + T(2n/3) + O(n)$$



- Prove your guess with substitution (induction).

Second recursion tree example

$$T(n) = T(n/3) + T(2n/3) + O(n)$$

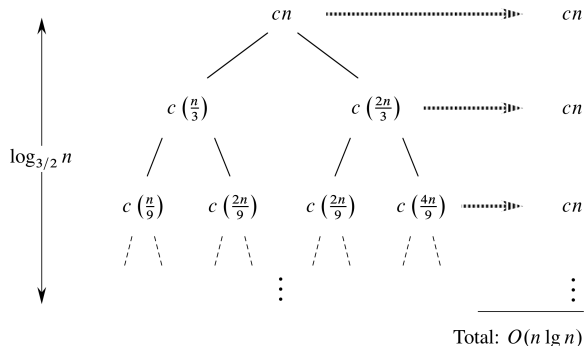


Total: $O(n \lg n)$

- ▶ Prove your guess with substitution (induction).
- ▶ Can you bound the leaves?

Second recursion tree example

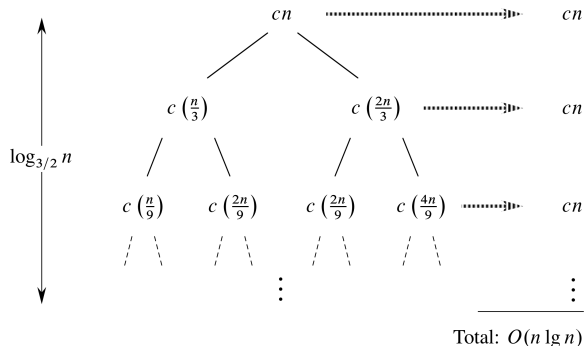
$$T(n) = T(n/3) + T(2n/3) + O(n)$$



- ▶ Prove your guess with substitution (induction).
- ▶ Can you bound the leaves?
- ▶ $n^{\log_{3/2} 2} \approx n^{1.7} = \Omega(n \lg n)$

Second recursion tree example

$$T(n) = T(n/3) + T(2n/3) + O(n)$$



- ▶ Prove your guess with substitution (induction).
- ▶ Can you bound the leaves?
- ▶ $n^{\log_{3/2} 2} \approx n^{1.7} = \Omega(n \lg n)$
- ▶ Substitution will assure you this is not a problem.