Master Theorem

Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on nonnegative integers by

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

Then $T(n)$ has the following asymptotic bounds:

$f(n)$	$T(n)$
$O(n^{\log_b a-\epsilon})$	$\Theta(n^{\log_b a})$
$\Theta(n^{\log_b a})$	$\Theta(f(n)\lg n)$
$\Omega(n^{\log_b a+\epsilon})$	$\Theta(f(n))$

The last one only if $af(\frac{n}{b}) \leq cf(n)$ for some $c < 1$ and large enough n.

Master theorem does not apply

$$
\begin{aligned}
T(n) &= 2T(n/2) + n \lg n \\
n^{\log_b a} &= n \\
f(n) &= n \lg n \\
f(n) &= \Omega(n^{\log_b a}) = \Omega(n) \\
f(n) &\neq \Omega(n^{\log_b a + \epsilon}) = \Omega(n^{1+\epsilon}) \\
n \lg n &\neq \Omega(n^{1.0000000000001}) \\
\lg n &\neq \Omega(n^{0.0000000000001})
\end{aligned}
$$

Master theorem does apply

$$
\begin{aligned}
T(n) &= 4T(n/3) + n \lg n \\
n^{\log_b a} &= n^{\log_c 4} = n^{1.26\ldots} \\
f(n) &= n \lg n \\
f(n) &= O(n^{\log_b a}) = O(n^{1.26\ldots}) \\
f(n) &= O(n^{\log_b a - \epsilon}) = O(n^{1.2}) && \epsilon = 0.06\ldots \\
n \lg n &= O(n^{1.2}) \\
\lg n &= O(n^{0.2})
\end{aligned}
$$