Monotonicity

monotonically increasing:

$$m \leq n \Rightarrow f(m) \leq f(n)$$

monotonically decreasing:

$$m \leq n \Rightarrow f(m) \geq f(n)$$

strictly increasing:

$$m < n \Rightarrow f(m) < f(n)$$

strictly decreasing:

$$m < n \Rightarrow f(m) > f(n)$$

Floors and ceilings

floor: $\lfloor x \rfloor$ largest integer less than or equal to x

ceiling: $\lceil x \rceil$ smallest integer greater than or equal to x

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

$$\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$$

$$\left\lceil \frac{\lceil x/a \rceil}{b} \right\rceil = \left\lceil \frac{x}{ab} \right\rceil$$

$$\left\lfloor \frac{\lfloor x/a \rfloor}{b} \right\rfloor = \left\lfloor \frac{x}{ab} \right\rfloor$$

$$\left\lceil \frac{a}{b} \right\rceil \leq \frac{a + (b-1)}{b}$$

$$\left\lfloor \frac{a}{b} \right\rfloor \leq \frac{a - (b-1)}{b}$$

Exponentials and logarithms

If $a > 1$:

$$
\begin{aligned}
\lim_{n\to\infty} \frac{n^b}{a^n} &= 0 && a > 1 \\
e^x &= \sum_{i=0}^{\infty} \frac{x^i}{i!} \\
e^x &\geq 1 + x \\
1 + x \leq e^x &\leq 1 + x + x^2 && |x| \leq 1 \\
\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n &= e^x \\
\frac{x}{1+x} &\leq \ln(1+x) \leq x && x > -1 \\
\lim_{n\to\infty} \frac{\lg^b n}{(2^a)^{\lg n}} &= \lim_{n\to\infty} \frac{\lg^b n}{n^a} = 0 \\
\lg^b n &= o(n^a)
\end{aligned}
$$

All polynomials grow faster than polylogarithmic functions.

Factorials

$$\begin{aligned}
n! &\leq n^n \\
n! &= \sqrt{2\pi n}\left(\frac{n}{e}\right)^n\left(1+\Theta\left(\frac{1}{n}\right)\right) \\
n! &= o(n^n) \\
n! &= \omega(2^n) \\
\lg(n!) &= \Theta(n \lg n)
\end{aligned}$$

Fibonacci numbers

$$
\begin{aligned}
x^2 &= x + 1 \\
\phi &= \frac{1+\sqrt{5}}{2} \approx 1.618 \\
\hat{\phi} &= \frac{1-\sqrt{5}}{2} \approx -0.618 \\
F_i &= \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}} \\
F_i &= \left\lfloor \frac{\phi^i}{\sqrt{5}} + \frac{1}{2} \right\rfloor
\end{aligned}
$$