Divide and Conquer Matrix Multiply

$$T(1) = c$$
$$T(n) = 8T(n/2) + cn^2$$

$$\begin{aligned}
T(n) &= \cancel{8T(n/2)} + c(n)^2 \\
\cancel{8T(n/2)} &= \cancel{8^2T(n/2^2)} + 8c(n/2)^2 \\
\cancel{8^2T(n/2^2)} &= \cancel{8^3T(n/2^3)} + 8^2c(n/2^2)^2 \\
\cancel{8^3T(n/2^3)} &= \cancel{8^4T(n/2^4)} + 8^3c(n/2^3)^2 \\
&\dots \\
\cancel{8^{\lg n-1}T(n/2^{\lg n-1})} &= 8^{\lg n}T(1) + 8^{\lg n-1}c(n/2^{\lg n-1})^2
\end{aligned}$$

Divide and Conquer Matrix Multiply

$$
\begin{aligned}
T(n) &= c8^{\lg n} + \sum_{i=0}^{\lg n - 1} 8^i c(n/2^i)^2 \\
&= cn^{\lg 8} + cn^2 \sum_{i=0}^{\lg n - 1} \frac{8^i}{4^i} \\
&= cn^3 + cn^2 \sum_{i=0}^{\lg n - 1} 2^i \\
&= cn^3 + cn^2 \frac{2^{\lg n} - 1}{2 - 1} \\
&= cn^3 + cn^2(n^{\lg 2} - 1) \\
&= cn^3 + cn^2(n - 1) \\
&= \Theta(n^3)
\end{aligned}
$$

Strassen Matrix Multiply

$$T(1) = c$$
$$T(n) = 7T(n/2) + cn^2$$

$$
\begin{aligned}
T(n) &= \cancel{7T(n/2)} + c(n)^2 \\
\cancel{7T(n/2)} &= \cancel{7^2 T(n/2^2)} + 7c(n/2)^2 \\
\cancel{7^2 T(n/2^2)} &= \cancel{7^3 T(n/2^3)} + 7^2 c(n/2^2)^2 \\
\cancel{7^3 T(n/2^3)} &= \cancel{7^4 T(n/2^4)} + 7^3 c(n/2^3)^2 \\
&\cdots \\
\cancel{7^{\lg n - 1} T(n/2^{\lg n - 1})} &= 7^{\lg n} T(1) + 7^{\lg n - 1} c(n/2^{\lg n - 1})^2
\end{aligned}
$$

$$T(n) = c7^{\lg n} + \sum_{i=0}^{\lg n - 1} 7^i c(n/2^i)^2$$

Solve the summation

$$
\begin{aligned}
T(n) &= c7^{\lg n} + \sum_{i=0}^{\lg n-1} 7^i c(n/2^i)^2 \\
&= cn^{\lg 7} + cn^2 \sum_{i=0}^{\lg n-1} \frac{7^i}{4^i} \\
&= cn^{\lg 7} + cn^2 \sum_{i=0}^{\lg n-1} (7/4)^i
\end{aligned}
$$

Solve the summation

$$
\begin{aligned}
cn^{\lg 7} + cn^2 \sum_{i=0}^{\lg n - 1} (7/4)^i &= cn^{\lg 7} + cn^2 \left(\frac{(7/4)^{\lg n} - 1}{(7/4) - 1} \right) \\
&= cn^{\lg 7} + cn^2 \left(\frac{n^{\lg 7/4} - 1}{3/4} \right) \\
&= cn^{\lg 7} + \frac{4cn^2}{3} \left(n^{\lg 7/4} - 1 \right) \\
&= cn^{\lg 7} + \frac{4cn^2}{3} \left(n^{\lg 7 - \lg 4} - 1 \right) \\
&= cn^{\lg 7} + \frac{4cn^2}{3} \left(\frac{n^{\lg 7}}{n^{\lg 4}} - 1 \right) \\
&= cn^{\lg 7} + \frac{4c}{3} \left(n^{\lg 7} - n^2 \right) \\
&= O(n^{\lg 7}) \\
&= o(n^{2.81}) = o(n^3)
\end{aligned}
$$