Monotonicity

monotonically increasing:

$$m \leq n \Rightarrow f(m) \leq f(n)$$

monotonically decreasing:

$$m \leq n \Rightarrow f(m) \geq f(n)$$

strictly increasing:

$$m < n \Rightarrow f(m) < f(n)$$

strictly decreasing:

$$m < n \Rightarrow f(m) > f(n)$$

Floors and ceilings

floor: $\lfloor x \rfloor$ largest integer less than or equal to x

ceiling: $\lceil x \rceil$ smallest integer greater than or equal to x

$$x - 1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x + 1$$

$$\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$$

$$\left\lceil \frac{\lceil x/a \rceil}{b} \right\rceil = \left\lceil \frac{x}{ab} \right\rceil$$

$$\left\lfloor \frac{\lfloor x/a \rfloor}{b} \right\rfloor = \left\lfloor \frac{x}{ab} \right\rfloor$$

$$\left\lceil \frac{a}{b} \right\rceil \le \frac{a + (b-1)}{b}$$

$$\left\lfloor \frac{a}{b} \right\rfloor \le \frac{a - (b-1)}{b}$$

Logarithms

$$
\begin{aligned}
\lg^k n &= (\lg n)^k \\
\lg \lg n &= \lg(\lg n) \\
b^{\log_b(x)} &= x \\
\log_a(xy) &= \log_a(x) + \log_a(y) \\
\log_a(x/y) &= \log_a(x) - \log_a(y) \\
\log_a(x^r) &= r \log_a(x) \\
\log_a(a^r) &= r \\
\log_a(x) &= (\log_a(b))(\log_b(x)) \\
\log_a(x) &= \frac{\log_b(x)}{\log_b(a)} \\
\log_a(b) &= \frac{1}{\log_b(a)} \\
a^{\log_b(c)} &= c^{\log_b(a)}
\end{aligned}
$$

Factorials

$$
\begin{aligned}
n! &\leq n^n \\
n! &= \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left(1 + \Theta\left(\frac{1}{n}\right)\right) \\
&= \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\alpha_n}
\end{aligned}
\qquad \frac{1}{12n+1} < \alpha_n < \frac{1}{12n}
$$

$$
\begin{aligned}
n! &= o(n^n) \\
n! &= \omega(2^n) \\
\lg(n!) &= \Theta(n \lg n)
\end{aligned}
$$

Iterated log function

$$\lg^*(n) = \min\{i \geq 0 : \lg^{(i)} n \leq 1\}$$

$$\begin{aligned}
\lg^* 2 &= 1 \\
\lg^* 4 &= 2 \\
\lg^* 16 &= 3 \\
\lg^* 65536 &= 4 \\
\lg^* 2^{65536} &= 5
\end{aligned}$$

Fibonacci numbers

$$0 = x^2 - x - 1$$

roots are ϕ and $\hat{\phi}$

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

$$\hat{\phi} = \frac{1-\sqrt{5}}{2} \approx -0.618$$

$$F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$$

$$F_i = \left\lfloor \frac{\phi^i}{\sqrt{5}} + \frac{1}{2} \right\rfloor$$