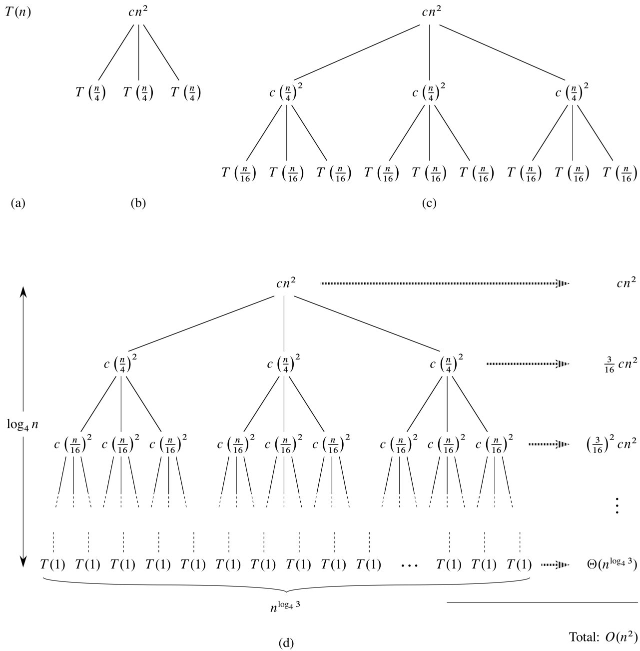
Using trees to guess

- Find a good guess for

$$T(n) = 3T(n/4) + \Theta(n^2)$$



$T(n)$
cn^2
$T\left(\frac{n}{4}\right)$ $T\left(\frac{n}{4}\right)$ $T\left(\frac{n}{4}\right)$
$c\left(\frac{n}{4}\right)^2$
$T\left(\frac{n}{16}\right)$
(a) (b) (c)
$\log_4 n$
$c\left(\frac{n}{16}\right)^2$
$T(1)$ … $T(1)$
$n^{\log_4 3}$
cn^2
$\frac{3}{16}cn^2$
$\left(\frac{3}{16}\right)^2 cn^2$
$\Theta(n^{\log_4 3})$
Total: $O(n^2)$
(d)

Bound the summation

- A geometric series of a number less than 1 can be bounded:

$$
\begin{aligned}
T(n) &= cn^2 + \frac{3}{16}cn^2 + \left(\frac{3}{16}\right)^2 cn^2 + \ldots + \left(\frac{3}{16}\right)^{\log_4 n - 1} cn^2 + \Theta(n^{\log_4 3}) \\
&= \sum_{i=0}^{\log_4 n - 1} \left(\frac{3}{16}\right)^i cn^2 + \Theta(n^{\log_4 3}) \\
&< \sum_{i=0}^{\infty} \left(\frac{3}{16}\right)^i cn^2 + \Theta(n^{\log_4 3}) \\
&= \frac{1}{1 - (3/16)} cn^2 + \Theta(n^{\log_4 3}) \\
&= \frac{16}{13} cn^2 + \Theta(n^{\log_4 3}) \\
&= O(n^2)
\end{aligned}
$$

- Now we can check this guess with substitution (induction).

Explicitly Solving Recursion of p. 89

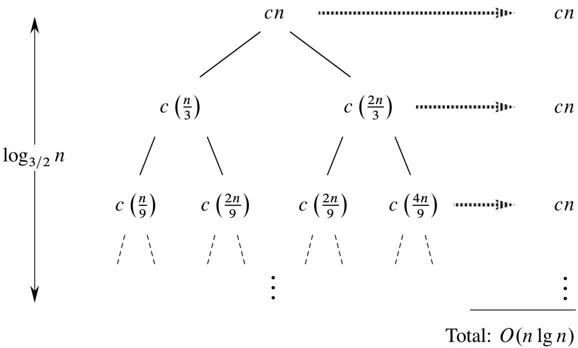
$$
\begin{aligned}
T(n) &= \cancel{3T\left(\frac{n}{4}\right)} + cn^2 \\
\cancel{3T\left(\frac{n}{4}\right)} &= \cancel{3^2T\left(\frac{n}{4^2}\right)} + 3c\left(\frac{n}{4}\right)^2 \\
\cancel{3^2T\left(\frac{n}{4^2}\right)} &= \cancel{3^3T\left(\frac{n}{4^3}\right)} + 3^2c\left(\frac{n}{4^2}\right)^2 \\
\cancel{3^3T\left(\frac{n}{4^3}\right)} &= \cancel{3^4T\left(\frac{n}{4^4}\right)} + 3^3c\left(\frac{n}{4^3}\right)^2 \\
&\cdots \\
\cancel{3^{k-1}T\left(\frac{n}{4^{k-1}}\right)} &= \cancel{3^kT\left(\frac{n}{4^k}\right)} + 3^{k-1}c\left(\frac{n}{4^{k-1}}\right)^2 \\
&\cdots \\
\cancel{3^{\log_4 n-1}T\left(\frac{n}{4^{\log_4 n-1}}\right)} &= 3^{\log_4 n}T\left(\frac{n}{4^{\log_4 n}}\right) + 3^{\log_4 n-1}c\left(\frac{n}{4^{\log_4 n-1}}\right)^2 \\
T(n) &= 3^{\log_4 n}T\left(\frac{n}{4^{\log_4 n}}\right) + \sum_{i=0}^{\log_4 n-1} 3^i c\left(\frac{n}{4^i}\right)^2
\end{aligned}
$$

Explicitly Solving Recursion of p. 89

$$
\begin{aligned}
T(n) &= 3^{\log_4 n} T\left(\frac{n}{4^{\log_4 n}}\right) + \sum_{i=0}^{\log_4 n - 1} 3^i c \left(\frac{n}{4^i}\right)^2 \\
&= n^{\log_4 3} T(1) + cn^2 \sum_{i=0}^{\log_4 n - 1} \left(\frac{3}{16}\right)^i \\
&= n^{\log_4 3} T(1) + cn^2 \left(\frac{(3/16)^{\log_4 n} - 1}{(3/16) - 1}\right) \\
&= \Theta(n^2)
\end{aligned}
$$

Second recursion tree example

$$T(n) = T(n/3) + T(2n/3) + O(n)$$



Second recursion tree example

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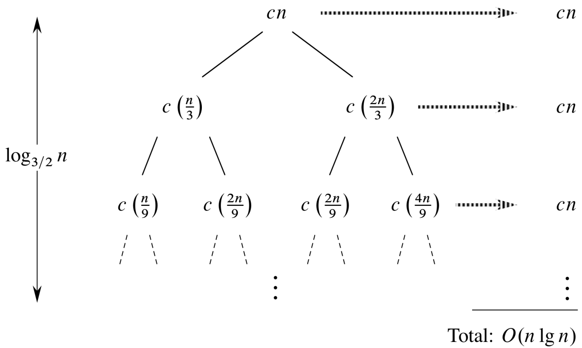

- Prove your guess with substitution (induction).

Second recursion tree example

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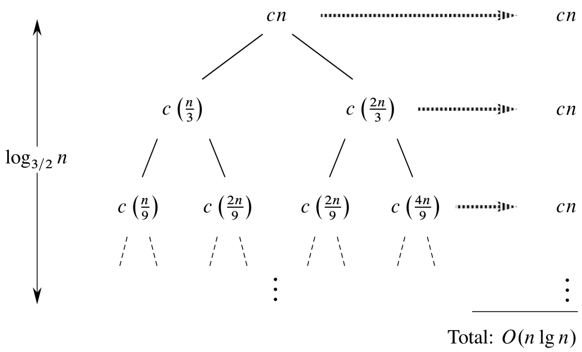

- Prove your guess with substitution (induction).
- Can you bound the leaves?

Second recursion tree example

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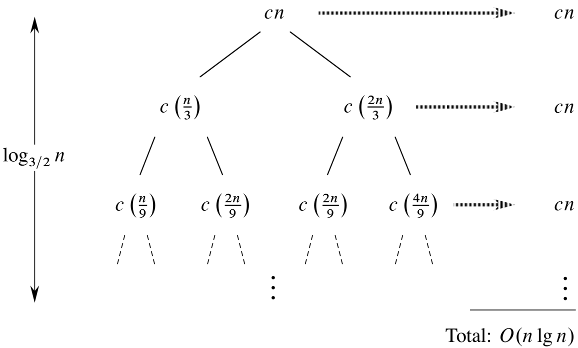

- Prove your guess with substitution (induction).
- Can you bound the leaves?
- $n^{\log_{3/2} 2} \approx n^{1.7} = \Omega(n \lg n)$ $\qquad$ $n^{\log_3 2} \approx n^{0.6} = O(n \lg n)$

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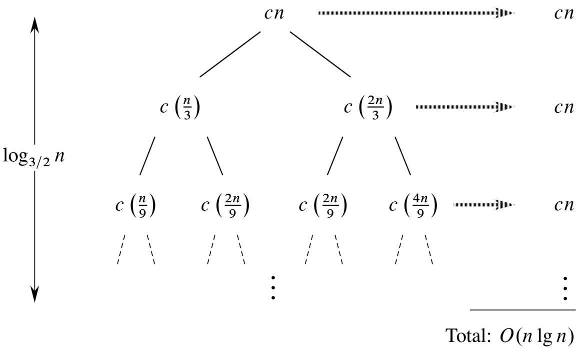

- Prove your guess with substitution (induction).
- Can you bound the leaves?
- $n^{\log_{3/2} 2} \approx n^{1.7} = \Omega(n \lg n)$ $\quad\quad$ $n^{\log_3 2} \approx n^{0.6} = O(n \lg n)$
- Substitution will assure you this is not a problem.