

# CSCI 305, Homework # 6

YOUR NAME HERE

Due date: Midnight, May 14

1. Alternative quicksort analysis (problem 7-3 in the text).

An alternative analysis of the running time of randomized quicksort focuses on the expected running time of each individual recursive call to RANDOMIZED-QUICKSORT, rather than on the number of comparisons performed.

- (a) Argue that, given an array of size  $n$ , the probability that any particular element is chosen as the pivot is  $1/n$ . Use this to define indicator random variables

$$X_i = I\{\textit{ith smallest element is chosen as the pivot}\}$$

What is  $E[X_i]$ ?

- (b) Let  $T(n)$  be a random variable denoting the running time of quicksort on an array of size  $n$ . Argue that

$$E[T(n)] = E \left[ \sum_{q=1}^n X_q (T(q-1) + T(n-q) + \Theta(n)) \right] \quad (1)$$

- (c) Show that we can rewrite equation 1 as

$$E[T(n)] = \frac{2}{n} \sum_{q=2}^{n-1} E[T(q)] + \Theta(n) \quad (2)$$

- (d) Show that

$$\sum_{k=2}^{n-1} k \lg k \leq \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2 \quad (3)$$

(Hint: Split the summation into two parts, one for  $k = 2, 3, \dots, \lceil n/2 \rceil - 1$  and one for  $k = \lceil n/2 \rceil, \dots, n-1$ )

- (e) Using the bound from equation 3, show that the recurrence in equation 2 has the solution  $E[T(n)] = \Theta(n \lg n)$ . (Hint: Show, by substitution, that  $E[T(n)] \leq an \lg n$  for sufficiently large  $n$  and for some positive constant  $a$ .)