

Simple Static Typechecking

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Professor Emeritus

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A Little Bit About Me

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- Eight cats

Type Checking

The idea is to *analyze* a program *statically* to find errors that might occur when you run the program. For example:

- Ensure that exp_1 and exp_2 are both type Int in every case of

$$exp_1 + exp_2$$

- Ensure that exp_1 and exp_2 are both type Bool in every case of

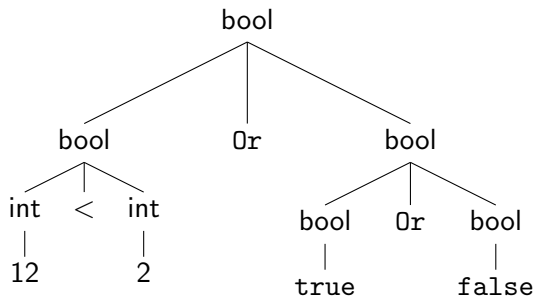
$$exp_1 \text{ Or } exp_2$$

- Ensure that exp_1 is type Bool in every case of

$$\text{If } exp_1 \text{ Then } exp_2 \text{ Else } exp_3$$

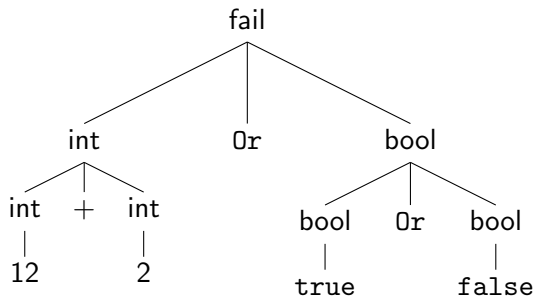
Recursive type checking

`(12 < 2) Or (true Or false)`



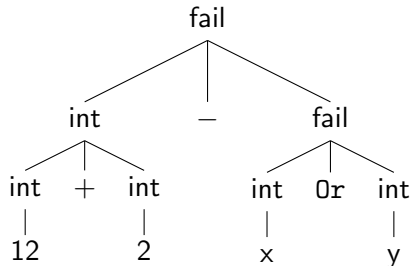
Recursive type checking

`(12 + 2) Or (true Or false)`



Recursive type checking

$(12 + 2) - (x \text{ Or } y)$



The language of Homework 5 has nothing to check!

- The **parser** ensures that only Bool expressions occur in IF and WHILE
- Only Int expressions can occur in assignments to variables
- Only numbers, variables and arithmetic operators can occur on the RHS of assignments
- No program with a type error can even be parsed!
- My original idea was to add boolean variables, but I got a much better idea.

Quick review of Lambda expressions

My syntax:

$$\underbrace{\left[\underbrace{\lambda x \{ x + 1 \}}_{\text{function}} \underbrace{(3 * 5)}_{\text{argument}} \right]}_{\text{application}}$$

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- $[\lambda x \{ x + 1 \} (3 * 5)] \Rightarrow$

Quick review of Lambda expressions

My syntax:

$$\underbrace{[\ \underbrace{\lambda x \{ x + 1 \}}_{\text{function}} \underbrace{(3 * 5)}_{\text{argument}} \]}_{\text{application}}$$

- $[\ \lambda x \{ x + 1 \} (3 * 5) \] \Rightarrow 16$

Quick review of Lambda expressions

My syntax:

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- $[\backslash x \{ x + 1 \} (3 * 5)] \Rightarrow 16$
- $[\backslash f \{ [f \ 3] \} \backslash x \{ 2 * x \}] \Rightarrow$

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- $[\lambda f \{ [f \ 3] \} \lambda x \{ 2 * x \}] \Rightarrow 6$

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- $[\lambda x \{ \lambda y \{ x + y \} \} 3] \Rightarrow$

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- $\lambda x \{ x + 1 \} ::$

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- $\lambda x \{ x + 1 \} :: (\text{int} \rightarrow \text{int})$

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- $\lambda x \{ x + 1 \} :: (\text{int} \rightarrow \text{int})$
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- $\lambda f \{ [f \ 3] \} ::$

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- $\lambda f \{ [f \ 3] \} :: ((\text{int} \rightarrow ??) \rightarrow ??)$

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- $\lambda f \{ [f \ 3] \} :: ((\text{int} \rightarrow ??) \rightarrow ??)$

To do static recursive typechecking, we need **typed lambda expressions**.

Typed Lambda expressions

My syntax:

$$\underbrace{\left[\underbrace{\backslash x:\text{int} \{ x + 1 \}}_{\text{function}} \underbrace{(3 * 5)}_{\text{argument}} \right]}_{\text{application}}$$

Typed Lambda expressions

My syntax:

$$\underbrace{\left[\underbrace{\backslash x:\text{int} \{ x + 1 \}}_{\text{function}} \underbrace{(3 * 5)}_{\text{argument}} \right]}_{\text{application}}$$

- `\f:(int -> int) { [f 3] } ::`

Typed Lambda expressions

My syntax:

$$\underbrace{\left[\underbrace{\backslash x:\text{int} \{ x + 1 \}}_{\text{function}} \underbrace{(3 * 5)}_{\text{argument}} \right]}_{\text{application}}$$

- `\f:(int -> int) { [f 3] } :: ((int -> int) -> int)`
- `\f:(int -> (int -> int)) { [f 3] } ::`

Typed Lambda expressions

My syntax:

$$\underbrace{\left[\underbrace{\backslash x:\text{int} \{ x + 1 \}}_{\text{function}} \underbrace{(3 * 5)}_{\text{argument}} \right]}_{\text{application}}$$

- `\f:(int -> int) { [f 3] } :: ((int -> int) -> int)`
- `\f:(int -> (int -> int)) { [f 3] } ::
((int -> (int -> int)) -> (int -> int))`
- `\f((int -> int) -> int) { [f x] } ::`

Typed Lambda expressions

My syntax:

$$\underbrace{\left[\underbrace{\backslash x:\text{int} \{ x + 1 \}}_{\text{function}} \underbrace{(3 * 5)}_{\text{argument}} \right]}_{\text{application}}$$

- `\f:(int -> int) { [f 3] } :: ((int -> int) -> int)`
- `\f:(int -> (int -> int)) { [f 3] } ::
((int -> (int -> int)) -> (int -> int))`
- `\f((int -> int) -> int) { [f x] } ::
(((int -> int) -> int) -> int)`

Typed lambda expressions and applications in our language

```
f := \x:int {x+1};  x := [f 3]
```

```
g := [\x:int {\y:int {x+y}} 4];  x := [g 5]
```

```
j := \f:(int->int){\y:int{ [f y] };
```

```
k := [j \x:int { 2*x }];
```

```
x := [k 5]
```

```
x := [[\x:int { \y:int {x + y} } 13] 12]
```

```
y := [\x:int { [\y:int {x + y} 13] } 12]
```

Typed lambda expressions and applications in our language

```
f := \x:int {x+1};  x := [f 3]
```

```
g := [\x:int {\y:int {x+y}} 4];  x := [g 5]
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x := [k 5]
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x := [[\x:int { \y:int {x + y} } 13] 12]
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y := [\x:int { [\y:int {x + y} 13] } 12]
```

Are all of these type safe?

Typed lambda expressions and applications in our language

```
f := \x:int {x+1};  x := [f 3]
```

```
g := [\x:int {\y:int {x+y}} 4];  x := [g 5]
```

```
j := \f:(int->int){\y:int{ [f y] };
```

```
k := [j \x:int { 2*x }];
```

```
x := [k 5]
```

```
x := [[\x:int { \y:int {x + y} } 13] 12]
```

```
y := [\x:int { [\y:int {x + y} 13] } 12]
```

Are all of these type safe?

You will learn how to add lambdas to your interpreters very soon.

Today we'll just do a typechecker for them.

Typed recursive functions

What more type information do you need to typecheck recursive functions?

```
REC f = \x { [ f (x+1) ] } IN [f 3] END
```

Typed recursive functions

What more type information do you need to typecheck recursive functions?

```
REC f = \x { [ f (x+1) ] } IN [f 3] END
```

```
REC f = int \x:int { [ f (x+1) ] } IN [f 3] END
```

Two new types of Arithmetic Expressions for Parsing

```
data AExp =  
    Var String  
  | Num Int  
  | Plus AExp AExp  
  | Times AExp AExp  
  | Neg AExp  
  | Div AExp AExp  
  | Lambda String Type AExp --NEW  
  | App AExp AExp           --NEW  
  deriving (Show, Eq)
```

- We won't talk about the parsing today.
- AExp are not always numbers anymore!

Example typechecking

Expressions

$[\lambda x:\text{int} \{99\} \ 22] + 3$	--accept
$[\lambda x:(\text{int} \rightarrow \text{int}) \{99\} \ 22]$	--reject
$[\lambda x:(\text{int} \rightarrow \text{int}) \{[x \ 2]\} \ 9]$	--reject
$[\lambda x:(\text{int} \rightarrow \text{int}) \{[x \ 2]\} \ \lambda x:\text{int}\{9\}]$	--accept
$[\lambda x:(\text{int} \rightarrow \text{int}) \{x + 2\} \ \lambda x:\text{int}\{9\}]$	--reject

Programs

$x := \lambda x:\text{int}\{x+1\};$	
$\text{IF } x < 2 \text{ THEN } y := 3 \text{ ELSE } y := 4;$	--reject
$x := 3;$	
$z := [\lambda y:\text{int} \{x + y\} \ 5];$	
$w := x + y$	--reject

A type for types

```
data Type =  
    BoolType  
  | IntType  
  | LambdaType Type Type  
  | FailureType  
  deriving (Show, Eq)
```

```
type TypeStore = Map.Map VarName Type
```

```
x := 5; y := \x:int {x+x};  z := \x:(int->int) { [x 4] }
```

Typechecking Statements

```
typeCheckStmt :: (Stmt AExp BExp) -> TypeStore
               -> (Bool, TypeStore)

typeCheckStmt program store =
  case program of
    ...
    Seq s1 s2 ->
      let (s1Good, store') = typeCheckStmt s1 store
      in if not s1Good
         then (False, store')
         else let (s2Good, store'') = typeCheckStmt s2 store'
              in (s2Good, store'')
    ...
```

```
x := 5; y := \w:int {x+w}; z := [y x]
```

Typechecking Statements

```
typeCheckStmt :: (Stmt AExp BExp) -> TypeStore
                                   -> (Bool, TypeStore)

typeCheckStmt program store =
  case program of
    ...
    Skip -> (True, store)
    ...
```

Typechecking Statements

```
typeCheckStmt :: (Stmt AExp BExp) -> TypeStore
                                   -> (Bool, TypeStore)

typeCheckStmt program store =
  case program of
    ...
  If b s1 s2 ->
    let (bType, store') = findTypeBExp b store in
    if bType /= BoolType
    then (False, store')
    else let (s1Good, store'') = typeCheckStmt s1 store' in
         if not s1Good
         then (False, store'')
         else let (s2Good, store''') = typeCheckStmt s1 store'''
              in (s2Good, store''')
    -- Do you see a problem here?
```

Typechecking Statements

```
typeCheckStmt :: (Stmt AExp BExp) -> TypeStore
                                -> (Bool, TypeStore)

typeCheckStmt program store =
  case program of
    ...
    If b s1 s2 ->
      let (bType, store') = findTypeBExp b store in
      if bType /= BoolType
      then (False, store')
      else let (s1Good, store'') = typeCheckStmt s1 store' in
           if not s1Good
           then (False, store'')
           else let (s2Good, store''') = typeCheckStmt s1 store''
                in (s2Good, store''')
           -- Do you see a problem here?

IF a < b THEN z := 3 ELSE z := \x:int{x+1} END; w := z + 1
```

Typechecking Statements

```
typeCheckStmt :: (Stmt AExp BExp) -> TypeStore
                                   -> (Bool, TypeStore)

typeCheckStmt program store =
  case program of
    ...
    While b s ->
      let (bType, store') = findTypeBExp b store in
      if bType /= BoolType
      then (False, store')
      else let (sGood, store'') = typeCheckStmt s store' in
           (sGood, store'')
    ...
```

Typechecking Statements

```
typeCheckStmt :: (Stmt AExp BExp) -> TypeStore
                                   -> (Bool, TypeStore)

typeCheckStmt program store =
  case program of
    ...
    Assign x val ->
      let (valType, store') = findTypeAExp val store
      in if valType /= FailureType
         then (True, Map.insert x valType store')
         else (False, store)
    ...
```

Typechecking Boolean Expressions

```
findTypeBExp :: (BExp AExp) -> TypeStore
              -> (Type, TypeStore)
findTypeBExp b store =
  case b of
    ...
    Bool x -> (BoolType, store)
    ...
```


Typechecking Boolean Expressions

```
findTypeBExp :: (BExp AExp) -> TypeStore
              -> (Type, TypeStore)

findTypeBExp b store =
  case b of
    ...
    Or x y ->
      let (t, store') = (findTypeBExp x store) in
      if t /= BoolType
      then (FailureType, store')
      else let (t', store'') = (findTypeBExp y store') in
           if t' /= BoolType
           then (FailureType, store'')
           else (BoolType, store'')
    ...
```

Typechecking Boolean Expressions

```
findTypeBExp :: (BExp AExp) -> TypeStore
              -> (Type, TypeStore)

findTypeBExp b store =
  case b of
    ...
    Lt x y ->
      let (t, store') = (findTypeAExp x store) in
      if t /= IntType
      then (FailureType, store')
      else let (t', store'') = (findTypeAExp y store') in
           if t' /= IntType
           then (FailureType, store'')
           else (BoolType, store'')
    ...
```

Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
  case a of
    ...
    Num x -> (IntType, store)
    ...
```

Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
  case a of
    ...
    Num x -> (IntType, store)
    ...
```

Next: What if it's a variable?

```
data AExp =
  ...
  Var String
```

Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
  case a of
    ...
    Var x -> (Map.findWithDefault FailureType x store, store)
    ...
```

Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
  case a of
    ...
    Var x -> (Map.findWithDefault FailureType x store, store)
    ...
```

Next: What if it's a Plus?

```
data AExp =
  ... Plus AExp AExp
```

Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
  case a of
    ...
    Plus x y ->
      let (t, store') = (findTypeAExp x store) in
        if t /= IntType
        then (FailureType, store')
        else let (t', store'') = (findTypeAExp y store') in
              if t' /= IntType
              then (FailureType, store'')
              else (IntType, store'')
    ...
```

Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
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    ...
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      let (t, store') = (findTypeAExp x store) in
        if t /= IntType
        then (FailureType, store')
        else let (t', store'') = (findTypeAExp y store') in
              if t' /= IntType
              then (FailureType, store'')
              else (IntType, store'')
    ...
```

Next: What if it's a Lambda?

```
data AExp = ... Lambda String Type AExp
```


Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
  case a of
    ...
    Lambda s t a ->
      let (t', store') =
          findTypeAExp a (Map.insert s t store)
          in (LambdaType t t', store)
          -- Why not store' ?
    ...
```

Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
  case a of
    ...
    Lambda s t a ->
      let (t', store') =
          findTypeAExp a (Map.insert s t store)
          in (LambdaType t t', store)
          -- Why not store' ?
    ...
```

Next: What if it's a application of a function?

```
data AExp =
  ...
  App AExp AExp
```

Typechecking Arithmetic Expressions

```
findTypeAExp :: AExp -> TypeStore -> (Type, TypeStore)
findTypeAExp a store =
  case a of
    ...
    App f x ->
      let (t,store') = (findTypeAExp f store) in
        case t of
          LambdaType t1 t2 ->
            let (t',store'') = (findTypeAExp x store) in
              if t1 == t'
              then (t2, store)
              else (FailureType, store)
          _ (FailureType, store)

      -- what about store' or store''?

    ...
```

Your Turn! Write typecheckers for some new expressions.

Let's Do IfExp !

```
IF x < 4 THEN 8 + x ELSE x + 9 END
```

Let's Do Let !

```
LET x = 2 + 2 IN x + x END
```

Let's Do Recursion!!

```
REC f = int \x:int { [f x+1] } IN [f 3] END
```

I've done the parsing for you,
just add clauses to the `findTypeAExp` function.