Logic and Axiomatic Semantics

Homework #3, CSCI 322, Winter 2016

Due: Monday, February 15, at midnight.

Part I, Formal logic: Provide formal proofs, as illustrated in the class notes, for the following theorems:

1. $((P \Rightarrow R) \wedge (Q \Rightarrow S)) \Rightarrow ((P \wedge Q) \Rightarrow (R \wedge S))$
2. $(P \Rightarrow (Q \Rightarrow R)) \Rightarrow (Q \Rightarrow (P \Rightarrow R))$
3. $(P \Rightarrow (P \Rightarrow Q)) \Rightarrow (P \Rightarrow Q)$
4. $((P \Rightarrow Q) \Rightarrow Q) \Rightarrow ((Q \Rightarrow P) \Rightarrow P)$
5. $(P \Rightarrow Q) \vee (Q \Rightarrow P)$
6. $((P \Rightarrow R) \vee (Q \Rightarrow R)) \Rightarrow (P \wedge Q \Rightarrow R)$
7. $(P \wedge Q \Rightarrow R) \Rightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$

Part II, Axiomatic Semantics: Provide formal proofs of the following triples.

8. $\{x > 5\}$ `x = 2 * x` $\{x > 8\}$
9. $\{y > 0\}$ `if x > y then y = x + y` $\{y > 0\}$

Part III, Concurrency:

10. Provide a formal proof of the following. Show both the annotated code, and proofs of all necessary triples.

    $\{x = 2 \wedge y = 3\}$
    `co x = y; // y = 4; oc`
    $\{x = 3 \vee x = 4\}$

I provide here an example using the natural deduction package for LaTeX. Further examples are in the lectures.

Prove: $(P \Rightarrow Q) \Rightarrow ((P \wedge R) \Rightarrow (Q \wedge R))$

1.	$P \Rightarrow Q$	assumption for conditional proof
2.	$P \wedge R$	assumption for conditional proof
3.	P	2, simplification
4.	R	2, simplification
5.	Q	1, 3, modus ponens
6.	$Q \wedge R$	4, 5, conjunction
7.	$(P \wedge R) \Rightarrow (Q \wedge R)$	conditional proof
8.	$(P \Rightarrow Q) \Rightarrow ((P \wedge R) \Rightarrow (Q \wedge R))$	conditional proof