Ray Tracing, Part I

Geoffrey Matthews

Department of Computer Science
Western Washington University

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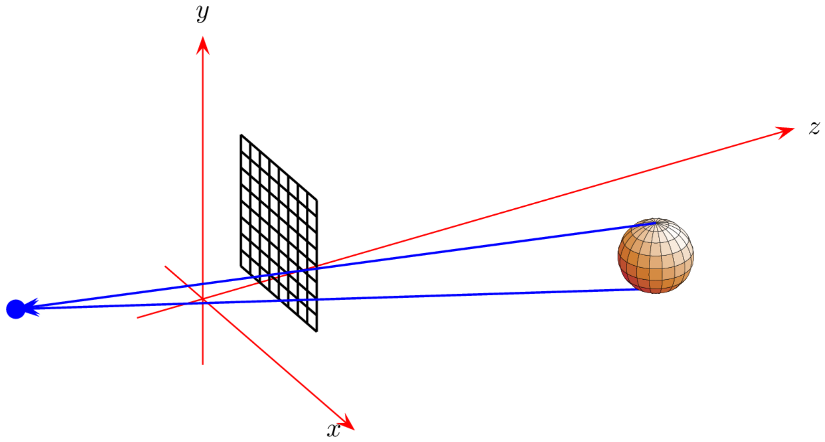
Readings

- https://www.siggraph.org/education/materials/HyperGraph/raytrace/rtrace0.htm
- https://www.cs.unc.edu/~rademach/xroads-RT/RTarticle.html
- http://www.cs.utah.edu/~shirley/books/fcg2/rt.pdf
- http://www.povray.org/
- http://www.pbrt.org/

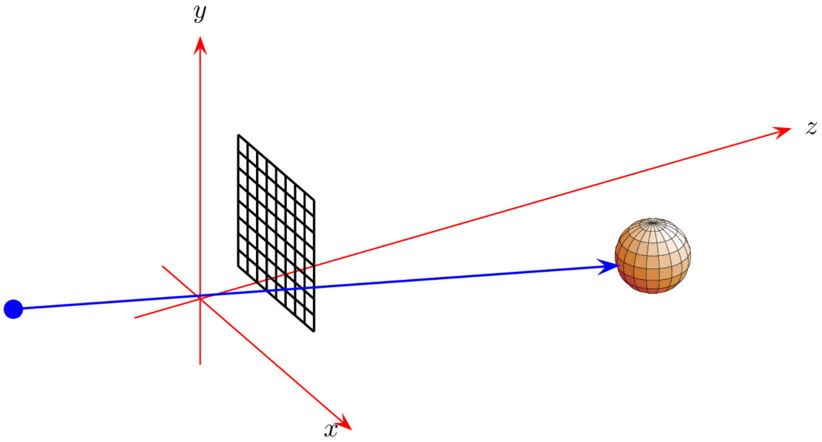
Ray Traced Images Achieve Maximal Realism

Two ways of rendering a picture: object order



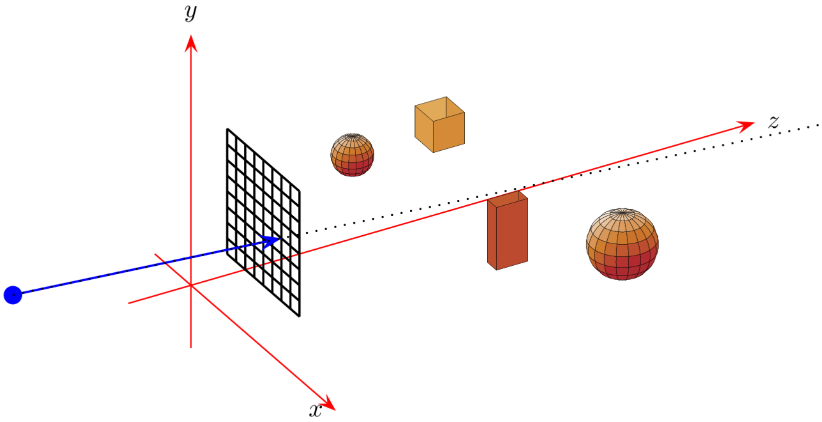
- For each object in the world, find the colors it would put on the screen.

Two ways of rendering a picture: image order



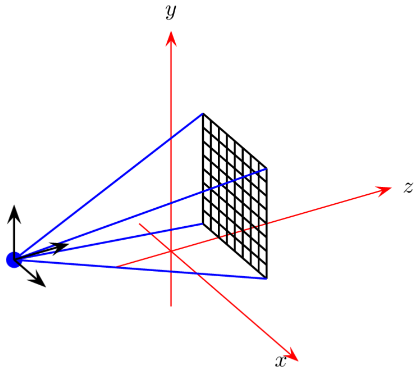
- For each pixel on the screen, find the objects that would color it.

Ray casting, a simplified ray tracing



- Given an eye position and a pixel position, construct a ray.
- Project the ray into the scene and find the closest intersection.
- Use object to compute color.

Camera

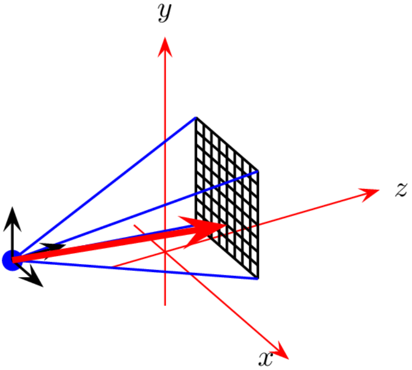


- A frame: **origin** point, and **right**, **up**, **forward** vectors.
- A distance, width and height for the image plane.

$$\langle p, r, u, f, d, w, h \rangle$$

- Would it matter if d, w, h were all doubled?

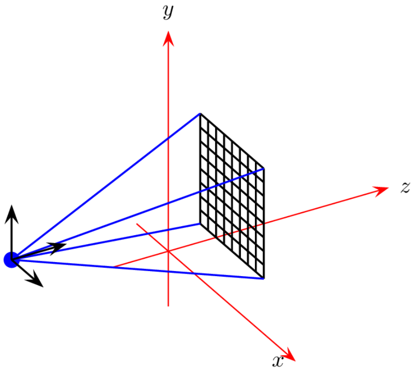
Finding vectors from the eye to the image plane



- Give an expression for the upper left corner.
- Give an expression for the upper right corner.
- Give an expression for a point 30% of the way across the image plane and 10% up from the bottom.

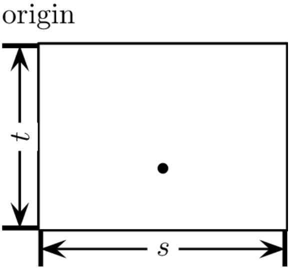
$$\langle p, r, u, f, w, h \rangle$$

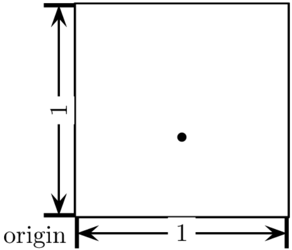
Other camera representations



- Lookat: eye point, lookat point, up vector, width, aspect ratio.
- Eye and four points
- Eye, lower left corner, two vectors
- Eye and four vectors

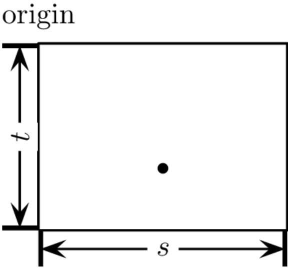
Normalized screen coordinates

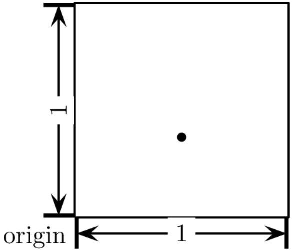




- *Normalized* screen coordinates map the entire surface to the $(0, 1) \times (0, 1)$ square.
- Suppose the screen is 640×480, with origin in the upper left, and we have a point at $(300, 400)$ on the screen.
- What are the point's normalized screen coordinates?

Normalized screen coordinates





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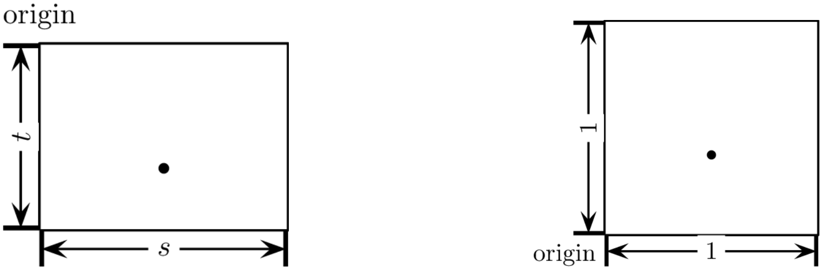
$$\left(\frac{300}{640}, 1 - \frac{400}{480}\right)$$

Normalized screen coordinates



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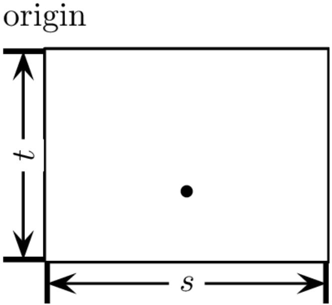
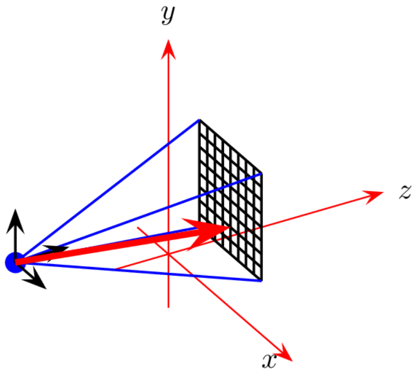
- Why did I use s and t and not w and h?

Mapping from Pixel to Camera Ray



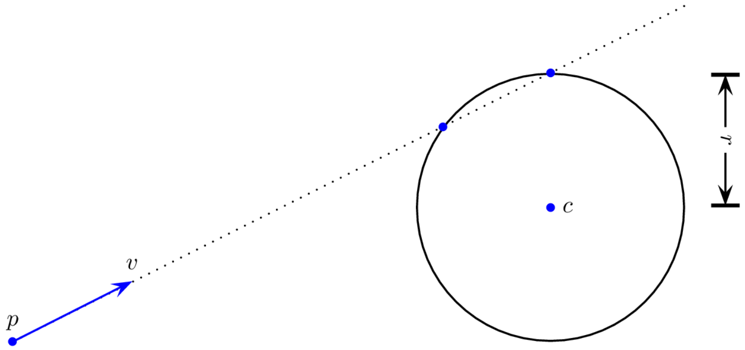


- Given a pixel position on the screen, find the ray in the camera.
- Map screen position to normalized position in $(0, 1) \times (0, 1)$
- Map normalized position in $(0, 1) \times (0, 1)$ to vector in world space.

Ray casting process

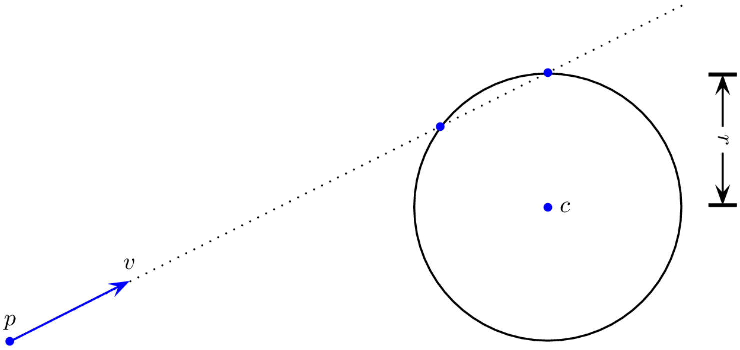
- Input: a camera and a set of objects
- Output: an image
- For each pixel in the image:
  - Find the ray in the camera for that pixel.
  - For all objects in the set:
    - find the closest in front of the camera that intersects the ray
  - Find the color of that object at the intersection point.
  - Color the pixel in the image with that color.

Intersecting a ray and a sphere



- Sphere defined by center and radius.
- Ray defined by point and vector.
- Assume sphere is centered at origin (replace p with $p - c$).
- Equation to solve?

Intersecting a ray and a sphere



- Sphere defined by center and radius.
- Ray defined by point and vector.
- Assume sphere is centered at origin (replace p with $p - c$).
- Equation to solve?
- Solve for t: $|p + tv|^2 = r^2$

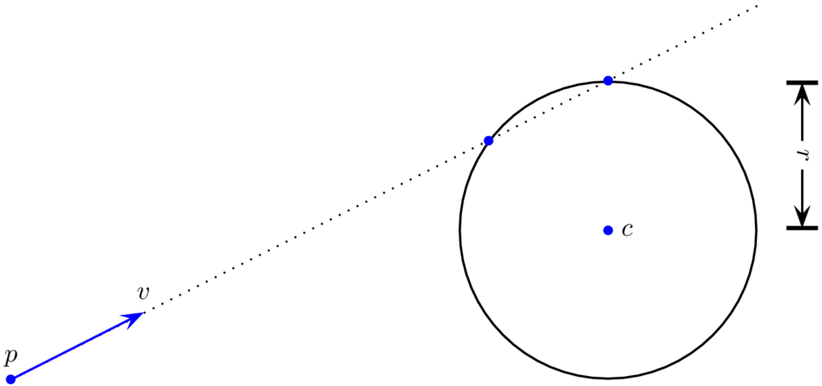
Solving quadratic

$$
\begin{aligned}
|p + tv|^2 &= (p + tv) \cdot (p + tv) \\
&= \sum_i (p_i + tv_i)(p_i + tv_i) \\
&= \sum_i \left(p_i^2 + 2p_i v_i t + v_i^2 t^2\right) \\
&= \sum_i p_i^2 + 2\sum_i p_i v_i t + \sum_i v_i^2 t^2 \\
&= (p \cdot p) + 2(p \cdot v)t + (v \cdot v)t^2
\end{aligned}
$$

So, in the quadratic $at^2 + bt + c = 0$,

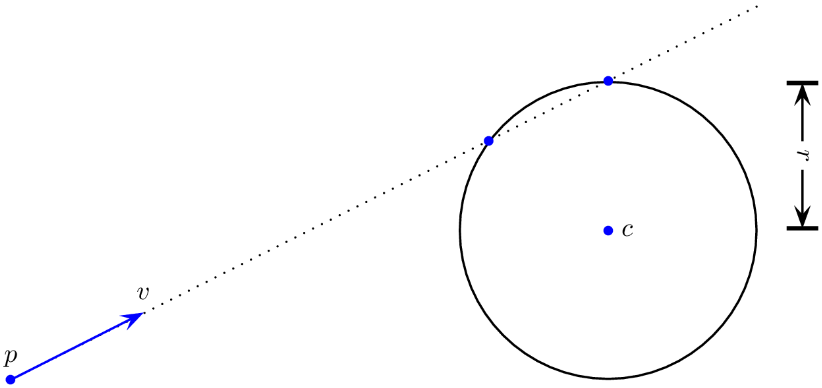
$$
\begin{aligned}
a &= v \cdot v \quad (= 1 \text{ if you normalized your rays}) \\
b &= 2p \cdot v \\
c &= p \cdot p - r^2
\end{aligned}
$$

Intersecting a ray and a sphere



- What equation would we have to solve if we did this in world coordinates?

Intersecting a ray and a sphere



- What equation would we have to solve if we did this in world coordinates?
- $|(p + tv) - c|^2 = r^2$
- Much simpler in object coordinates (replace p with $p - c$).

Still something missing ...