Ray Tracing, Part IV

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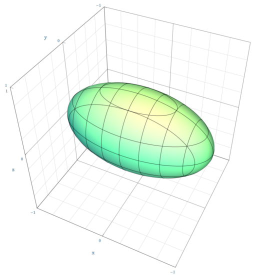
Fall 2015

Ellipsoids



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- How can we intersect a ray with this surface?
- How can we find the normal?
- How can we rotate this shape?

Intersect a ray with an ellipsoid

Constraint on the surface:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Points on the line generated by:

$$p + tv = (p_0, p_1, p_2) + t(v_0, v_1, v_2)$$

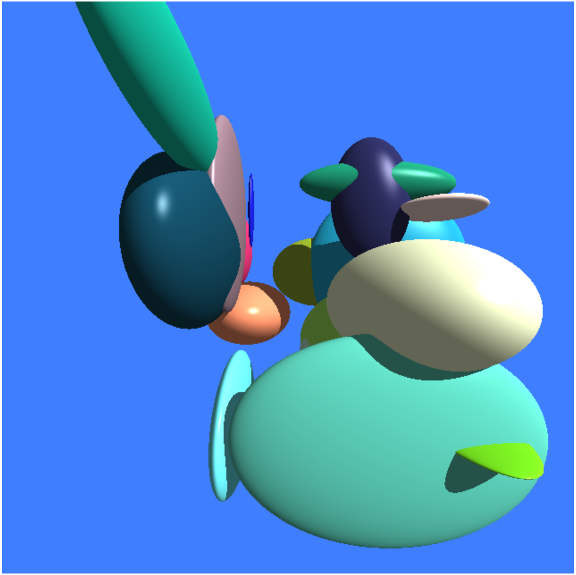
Plug one into the other to satisfy both:

$$\frac{(p_0 + tv_0)^2}{a^2} + \frac{(p_1 + tv_1)^2}{b^2} + \frac{(p_2 + tv_2)^2}{c^2} = 1$$

Collect terms to get our quadratic to solve:

$$\left(\frac{v_0^2}{a^2} + \frac{v_1^2}{b^2} + \frac{v_2^2}{c^2}\right) t^2 + \left(\frac{2p_0 v_0}{a^2} + \frac{2p_1 v_1}{b^2} + \frac{2p_2 v_2}{c^2}\right) t + \left(\frac{p_0^2}{a^2} + \frac{p_1^2}{b^2} + \frac{p_2^2}{c^2}\right)$$

What about the normal for an ellipsoid?



Calculus to the rescue!

https://en.wikipedia.org/wiki/Gradient

Calculus to the rescue!

- Ellipsoids satisfy the constraint:

$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- This means that the function f is *constant* across the surface. If we take the *gradient* of f, it will be a vector pointing in the direction of maximum change of f, which will be perpendicular to the directions in which it is not changing at all.
- The gradient is defined as

$$\nabla f(x, y, z) = \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \right)$$

- The partial derivative of a function, for example, $\frac{\partial f}{\partial x}$, is simply the derivative of f treating everything except x as a constant.

Normals for the ellipsoid

- For the ellipsoid, the function is

$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- For this function, the gradient is easy to compute:

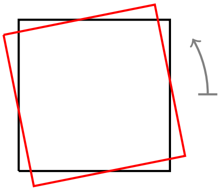
$$\nabla f(x, y, z) = \frac{2x}{a^2} + \frac{2y}{b^2} + \frac{2z}{c^2}$$

- Since we want to normalize this anyway, we can drop the factor of 2.
- Note that this is in coordinates where the ellipsoid has not been translated. After we find our translated intersection point, we have to translate it back by subtracting the ellipsoid's center before calculating the normal.

Other quadrics

- Using the tips above for the ellipsoid, you should be able to render any of the quadrics from this page: `https://en.wikipedia.org/wiki/Quadric`

What about rotating quadrics?



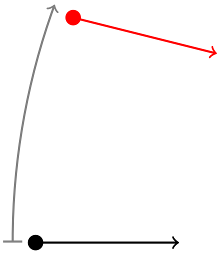
What about rotating quadrics?

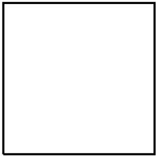
How do we find the intersection point?

How do we find the intersection point?

- Translate the ray point (subtract the object center).
- Rotate the ray point by the *inverse* rotation of the object.
- Rotate the ray vector by the same rotation.
- Find t, the distance along the ray to the intersection.
- Use t with the untransformed ray to find the point in world coordinates.

How do we find the normal?





How do we find the normal?

- Find the point on the *untransformed* object (use *transformed* ray).
- Find the normal at that point using the gradient, as before.
- Rotate this normal using the original rotation matrix.