Parametric Surfaces

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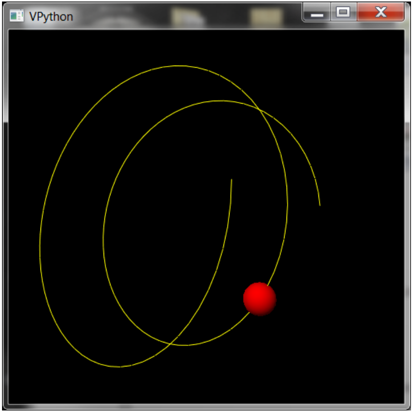
October 22, 2015

Online Resources

Readings

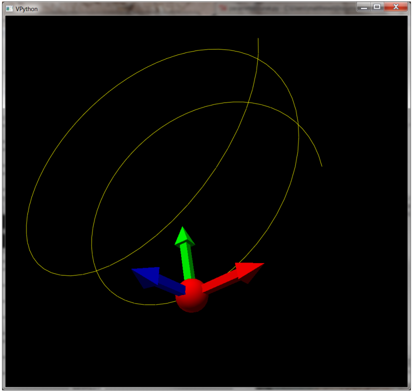
- https://www.math.duke.edu//education/ccp/materials/mvcalc/parasurfs/para1.html
- http://en.wikipedia.org/wiki/Parametric_surface
- http://www.math.uri.edu/~bkaskosz/flashmo/tools/parsur/
- http://people.cs.clemson.edu/~dhouse/courses/405/notes/implicit-parametric.pdf
- http://msenux.redwoods.edu/Math4Textbook/Plotting/ParametricSurfaces.pdf

Parametric Curves



$$f(t) = (10\cos(t), 10\sin(t), t)$$

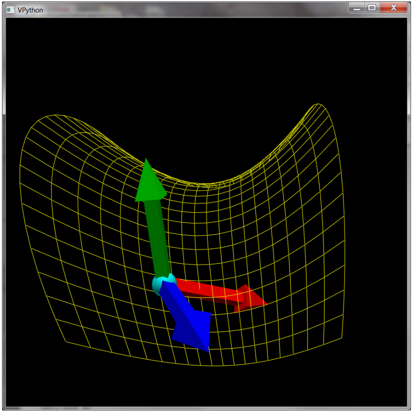
Parametric Curves: Tangent, Normal, Binormal



$$
\begin{aligned}
f(t) &= (10\cos(t), 10\sin(t), t) \\
\frac{df}{dt} &= (-10\sin(t), 10\cos(t), 1) \\
\frac{d^2 f}{dt^2} &= (-10\cos(t), -10\sin(t), 0)
\end{aligned}
$$

$$
\frac{df}{dt} \times \frac{d^2 f}{dt^2} \propto (\sin(t), \cos(t), 10)
$$

Parametric Surfaces



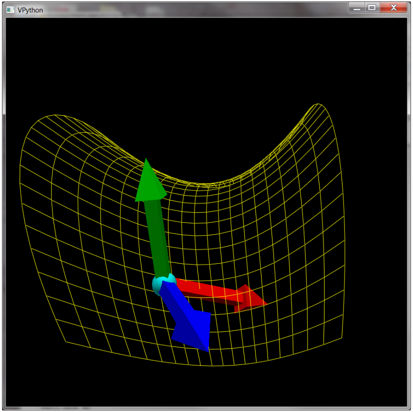
Explicit:

$$f(s,t) = (s, t, s^2 - t^2)$$

Implicit:

$$F(x,y,z) = 0$$

Parametric Surfaces



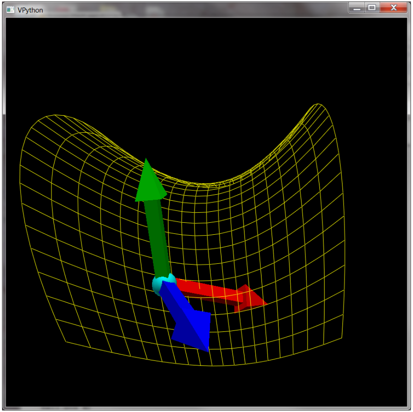
Explicit:

$$f(s,t) = (s, t, s^2 - t^2)$$

Implicit:

$$\begin{aligned} F(x,y,z) &= 0 \\ x^2 - y^2 &= z \end{aligned}$$

Parametric Surfaces



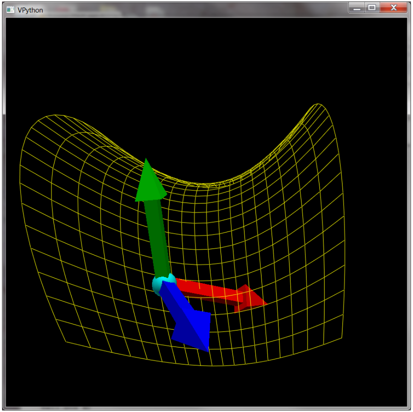
Explicit:

$$f(s,t) = (s, t, s^2 - t^2)$$

Implicit:

$$\begin{aligned} F(x,y,z) &= 0 \\ x^2 - y^2 &= z \\ x^2 - y^2 - z &= 0 \end{aligned}$$

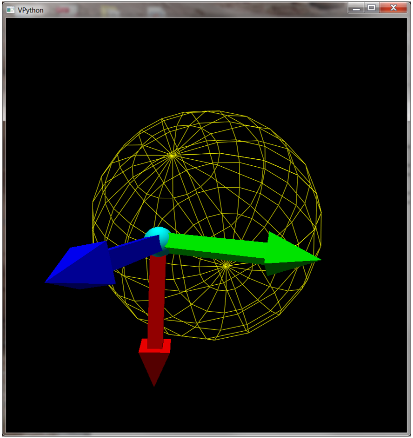
Parametric Surfaces: Normal, Tangent, Binormal



$$
\begin{aligned}
f(s,t) &= (s, t, s^2 - t^2) \\
\frac{\partial f}{\partial s} &= (1, 0, 2s) \\
\frac{\partial f}{\partial t} &= (0, 1, -2t) \\
\frac{\partial f}{\partial s} \times \frac{\partial f}{\partial t} &= (-2s, 2t, 1)
\end{aligned}
$$

$$
\begin{aligned}
F(x,y,z) &= x^2 - y^2 - z \\
\nabla F(x,y,z) &= (2x, -2y, -1)
\end{aligned}
$$

Parametric Surfaces: Sphere



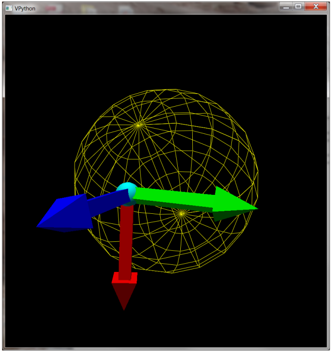
Explicit:

$$\begin{aligned} f(s,t) &= (\cos(t)\cos(s), \\ &\quad \cos(t)\sin(s), \\ &\quad \sin(t)) \end{aligned}$$

Implicit:

$$\begin{aligned} x^2 + y^2 + z^2 &= 1 \\ x^2 + y^2 + z^2 - 1 &= 0 \end{aligned}$$

Parametric Surfaces: Normal, Tangent, Binormal



$$
\begin{aligned}
F(x, y, z) &= x^2 + y^2 + z^2 - 1 \\
\nabla F(x, y, z) &= (2x, 2y, 2z)
\end{aligned}
$$

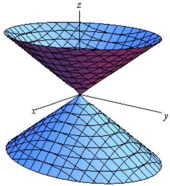
$$
\begin{aligned}
f(s, t) &= (\cos(t)\cos(s), \cos(t)\sin(s), \sin(t)) \\
\frac{\partial f}{\partial s} &= (-\cos(t)\sin(s), \cos(t)\cos(s), 0) \\
\frac{\partial f}{\partial t} &= (-\sin(t)\cos(s), -\sin(t)\sin(s), \cos(t))
\end{aligned}
$$

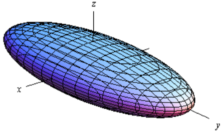
Approximations to Tangent, Normal, Binormal

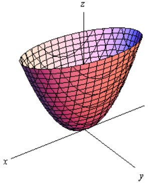
$$
\begin{aligned}
\frac{\partial f}{\partial s} &\approx\propto \frac{f(s+\epsilon,t)-f(s,t)}{|f(s+\epsilon,t)-f(s,t)|} \\
\frac{\partial f}{\partial t} &\approx\propto \frac{f(s,t+\epsilon)-f(s,t)}{|f(s,t+\epsilon)-f(s,t)|} \\
Normal &= \frac{\partial f}{\partial s} \times \frac{\partial f}{\partial g}
\end{aligned}
$$

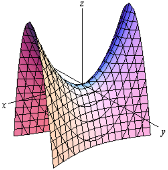
Quadrics

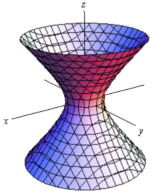
http://tutorial.math.lamar.edu/Classes/CalcIII/QuadricSurfaces.aspx

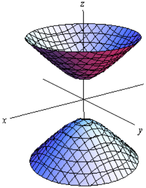








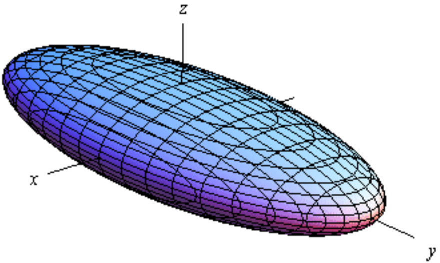




Ellipsoid

$x^2 + y^2 + z^2 - 1$

$$(\cos(v)\cos(u), \cos(v)\sin(u), \sin(v))$$
$$v \in [-\pi/2, \pi/2], u \in [-\pi, \pi]$$

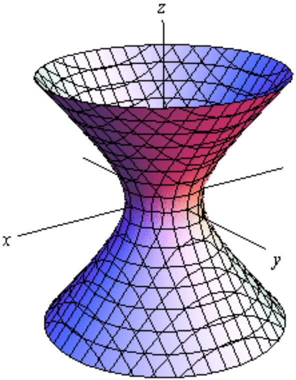


Equations from *Computer Graphics Using Open GL, 2E*

Hyperboloid of one sheet

$x^2 + y^2 - z^2 - 1$

$$(\sec(v)\cos(u), \sec(v)\sin(u), \tan(v))$$
$$v \in [-\pi/2, \pi/2], u \in [-\pi, \pi]$$

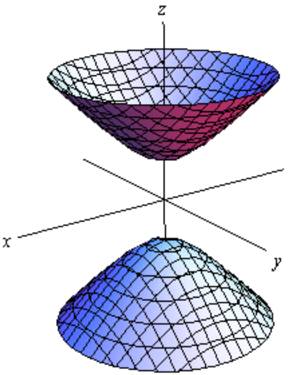


Hyperboloid of two sheets

$x^2 - y^2 - z^2 - 1$

$$(\sec(v)\cos(u), \sec(v)\tan(u), \tan(v))$$
$$v \in [-\pi/2, \pi/2], [\pi/2, 3\pi/2], u \in [-\pi, \pi]$$

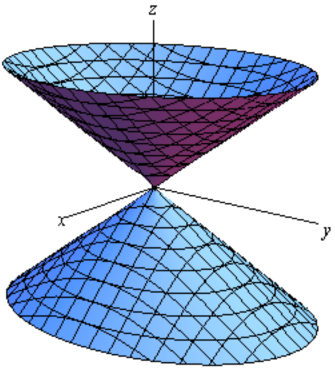
?



Elliptic cone

$$x^2 + y^2 - z^2$$

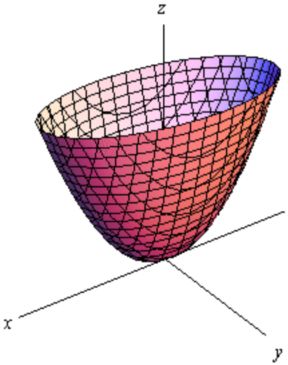
$$(v\cos(u), v\sin(u), v)$$
$$v \in [-\infty, +\infty], u \in [-\pi, \pi]$$



Elliptic paraboloid

$$x^2 + y^2 - z$$

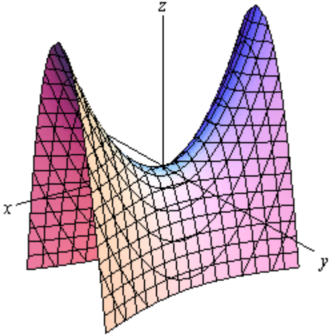
$$(v\cos(u), v\sin(u), v^2)$$
$$v \in [0, +\infty], u \in [-\pi, \pi]$$



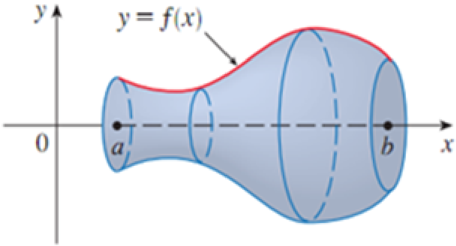
Hyperbolic paraboloid

$$-x^2 + y^2 - z$$

$$(v\tan(u), v\sec(u), v^2)$$
$$v \in [0, +\infty], u \in [-\pi, \pi]$$



Surfaces of Revolution

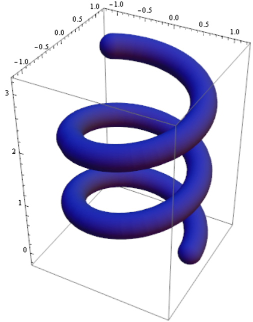


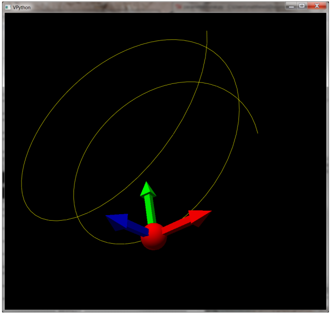
(a) Surface of revolution

If $y = f(x)$ is determined by a series of points $(x_0, y_0), \ldots (x_n, y_n)$, then let $s \in [0, \ldots, n]$ and $t \in [0, 2\pi]$, and the parametric surface is

$$(x_s, y_s \cos(t), y_s \sin(t))$$

Tubes along 3D curves





- Assume the curve is parameterized by $f(s)$, $s \in [a, b]$
- Find the tangent, normal, and binormal to the curve: t, n, b
- The parametric surface, parameterized by $s \in [a, b]$, $t \in [0, 2\pi]$:

$$f(s) + \cos(t)n + \sin(t)b$$

- Could you extrude an arbitrary shape instead of a circle?