CSCI 480, Fall 2015, Midterm Exam, Solutions

Multiple choice. Pick the best answer.

1. **Rigid** transforms include
   (a) Rotation and scale
   (b) Scale and shear
   (c) $\boxed{\text{Translation and rotation}}$
   (d) All linear transforms

2. **Affine** transforms include
   (a) rotation, scale, shear, translation and perspective
   (b) $\boxed{\text{translation, rotation, scale, and shear but not perspective}}$
   (c) translation and rotation, but not scale, shear and perspective
   (d) translation, rotation, and scale, but not shear and perspective

3. **Ray tracing** differs from **direct rendering** in that
   (a) ray tracing is object order while direct rendering is pixel order
   (b) $\boxed{\text{ray tracing is pixel order while direct rendering is object order}}$

4. In **Phong reflection** the specular term is computed by the dot product of
   (a) the light vector and the normal vector
   (b) the reflected light vector and the normal vector
   (c) the normal vector and the eye vector
   (d) $\boxed{\text{the reflected light vector and the eye vector}}$

5. **Homogeneous** coordinates
   (a) add an extra component that is 0 for points and 1 for vectors
   (b) $\boxed{\text{add an extra component that is 1 for points and 0 for vectors}}$
   (c) add an extra component that is arbitrary for both points and vectors, but must be divided out
   (d) do not add an extra component

6. **Texture coordinates** are used to determine
   (a) $\boxed{\text{which 2D image coordinate is used in which 3D position}}$
   (b) which image is attached to which object
   (c) the position of the texture in the world
   (d) the screen position from the world position

7. **Aliasing** refers to
   (a) Using a single texture to color more than one object
   (b) The interference of one texture with another texture at a different frequency
   (c) $\boxed{\text{High frequency noise masquerading as low frequency noise}}$
   (d) Low frequency noise masquerading as high frequency noise

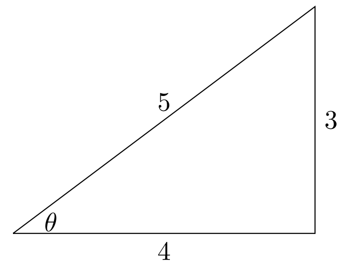
8. The **Gram Schmidt** process refers to

(a) The process of finding texture coordinates for a 3D object

(b) The process of mapping world coordinates to object coordinates

(c) The process of mapping world coordinates to camera coordinates

(d) $\boxed{\text{The process of finding three orthonormal vectors from three arbitrary vectors}}$



Note that the figure at right is a right triangle. Use this fact to find the following matrix inverse:

9.
$$\begin{bmatrix} \frac{3}{5} & 0 & -\frac{4}{5} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{4}{5} & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{3}{5} & 0 & \frac{4}{5} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{4}{5} & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

10. Find the inverse of the following matrix by decomposing it into two simpler matrices representing inverses and then multiplying:

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

11. How many numbers are needed to specify an arbitrary 3D affine transformation? Justify your answer.

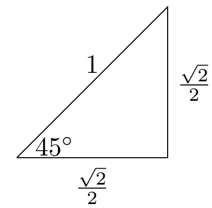
12, because an arbitrary affine transformation is specified by a matrix of the form below.

$$\begin{bmatrix} a & d & g & j \\ b & e & h & k \\ c & f & i & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12. Let $f(x, y, z) = 3xy^2 + 2xz + yz^2$. Find $\nabla f(x, y, z)$

$$\nabla f(x, y, z) = \left(3y^2 + 2z, 6xy + z^2, 2x + 2yz\right)$$

Note: I meant this problem to be much easier than it was. The "easy version" is included after this one



13. The points (in homogeneous coordinates) in the triangle below are multiplied on the left by each of the following homogeneous transform matrices. Draw the two resulting triangles, labelling them A and B. (We start each time with the triangle below; the multiplications are not cumulative.) Give your best approximations for the triangles, the points need not be exact. For your information, the figure at right is a right triangle.

$$A = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & 3 \\ \frac{\sqrt{2}}{2} & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Points of the triangle: $(2, 2, 1)$, $(3, -1, 1)$, and $(1, -1, 1)$. Multiply each by matrix A:

$$\begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1.4 \\ -1.4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -3\frac{\sqrt{2}}{2} \\ 1 \end{bmatrix} \approx \begin{bmatrix} -0.7 \\ -2.1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 1 \end{bmatrix} \approx \begin{bmatrix} -0.7 \\ -0.7 \\ 1 \end{bmatrix}$$

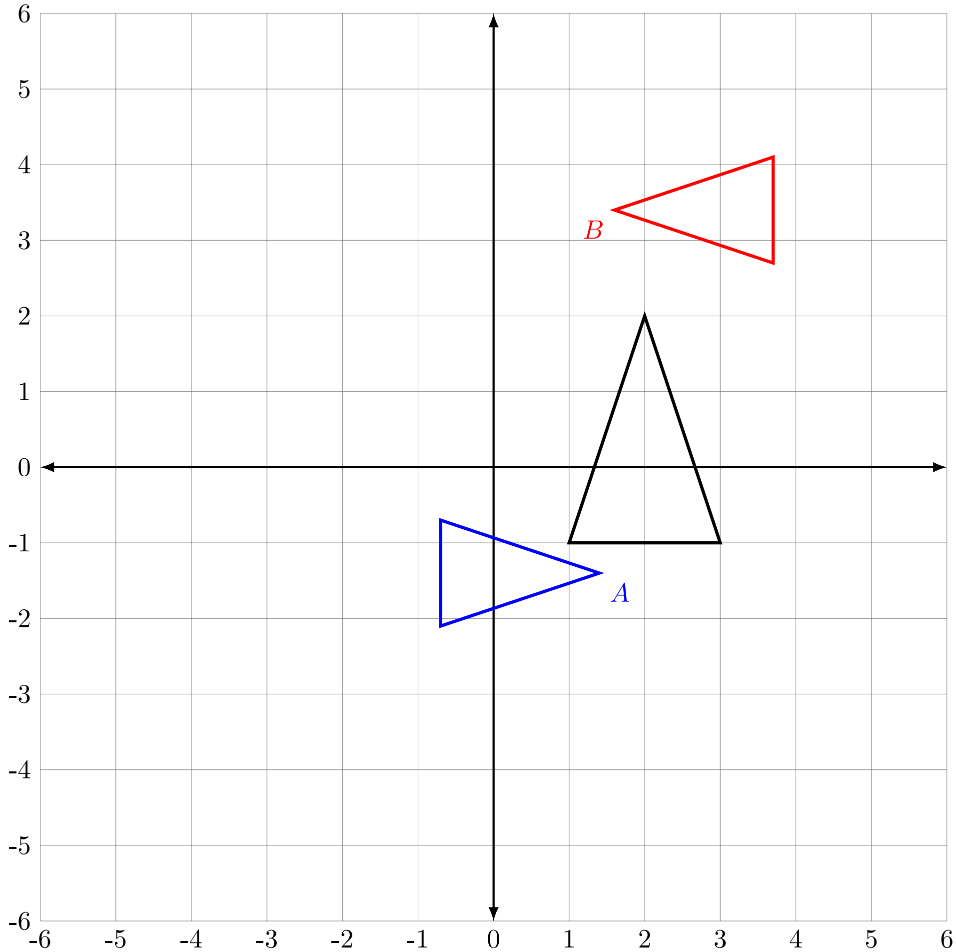
Multiply each by matrix B:

$$\begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & 3 \\ \frac{\sqrt{2}}{2} & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} + 3 \\ \sqrt{2} + 2 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1.6 \\ 3.4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & 3 \\ \frac{\sqrt{2}}{2} & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} + 3 \\ 3\frac{\sqrt{2}}{2} + 2 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 3.7 \\ 4.1 \\ 1 \end{bmatrix}$$

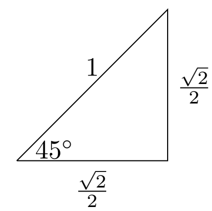
$$\begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & 3 \\ \frac{\sqrt{2}}{2} & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} + 3 \\ \frac{\sqrt{2}}{2} + 2 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 3.7 \\ 2.7 \\ 1 \end{bmatrix}$$

These triangles are drawn in the figure below.



6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
B
A

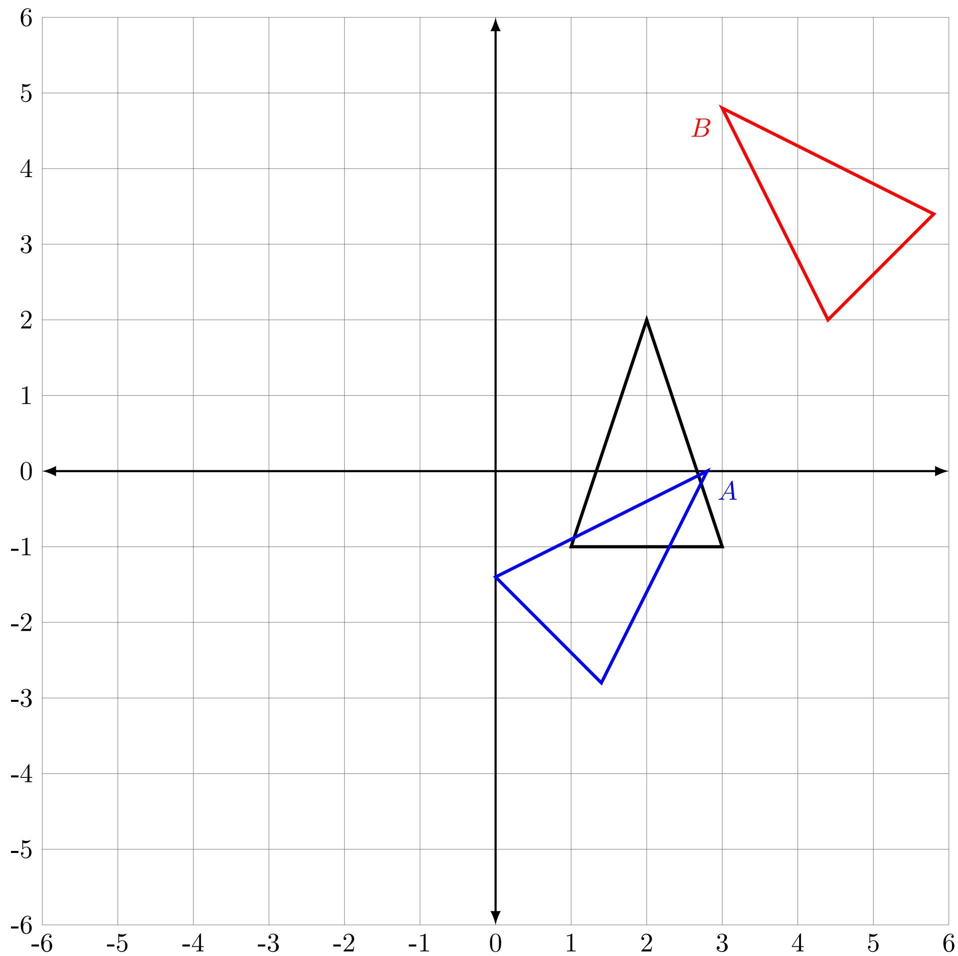
13. Easy version.



The points (in homogeneous coordinates) in the triangle below are multiplied on the left by each of the following homogeneous transform matrices. Draw the two resulting triangles, labelling them A and B. (We start each time with the triangle below; the multiplications are not cumulative.) Give your best approximations for the triangles, the points need not be exact. For your information, the figure at right is a right triangle.

$$A = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 3 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

For this version, the upper left 3×3 matrix is a $45°$ rotation, as can be seen from the “hint” triangle. In A it’s clockwise, in B its counterclockwise, followed by a translation. So we can draw the triangles by inspection:

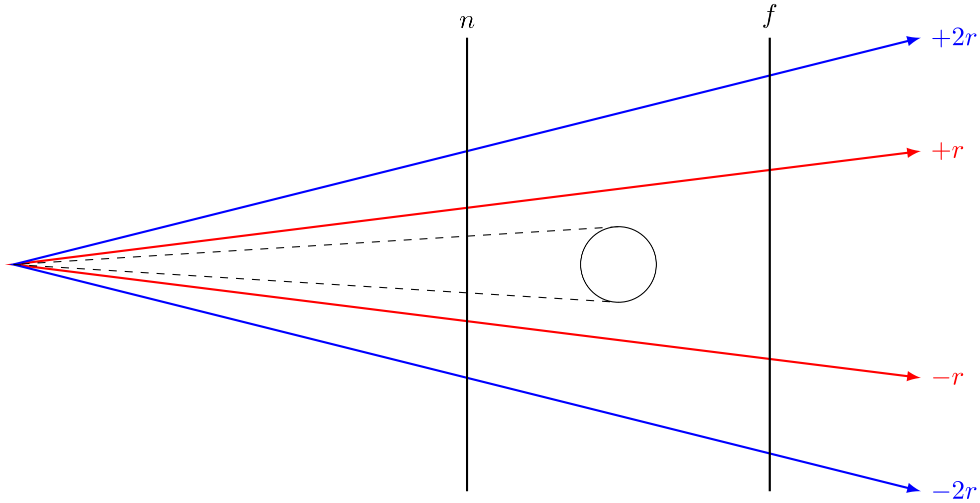


14. Suppose we have a camera frame f and a perspective projection matrix M given by four positive numbers $n = near$, $f = far$, $r = right$, and $t = top$, symmetric around the camera axis so $left = -right$ and $bottom = -top$, as at right and we use this to render an image in OpenGL.

$$M = \begin{bmatrix} \frac{n}{r} & 0 & 0 & 0 \\ 0 & \frac{n}{t} & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

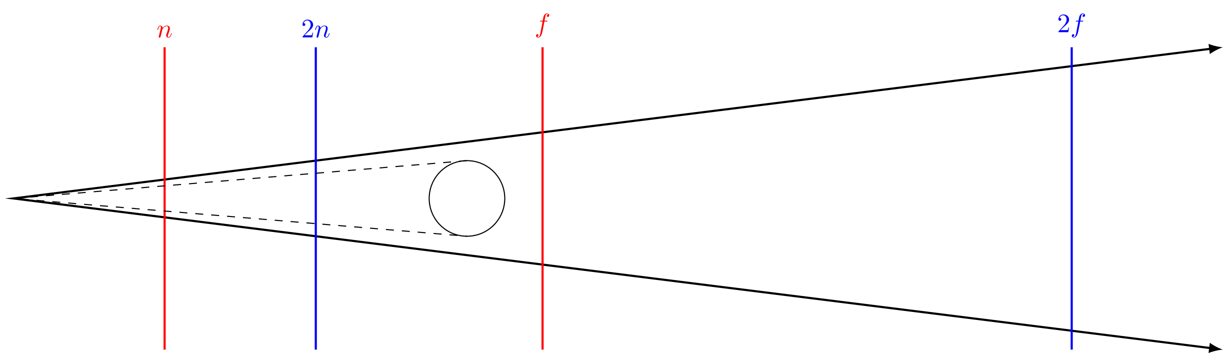
(a) Suppose we now double the values of *right* and *top*. Will this make the objects in our image *bigger* or *smaller*? Draw a picture to illustrate why.

Objects will appear **smaller**. As can be seen in the figure, the circle takes up a smaller part of the image on the near plane when we use $\pm 2r$



(b) Suppose we now double the values of *near* and *far*. Will this make the objects in our image *bigger* or *smaller*? Draw a picture to illustrate why.

This will make **no difference** to the size of the object. As can be seen in the figure, the relative size of the object is not changed if we move the near plane back or forth.



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3/2 & -5/2 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

15. With the projection matrix above, determine if each of the following points ends up inside or outside of normalized device coordinates. Show your work and justify your answer.

    (a) $(-2, 0, -1, 1)$
    Multiply:

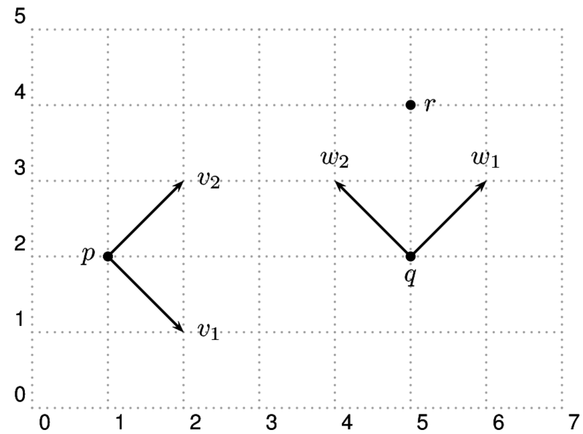
    $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3/2 & -5/2 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

    This point is *not* in the ± 1 cube, so it is *not* in normalized device coordinates.

    (b) $(0, 0, -5, 1)$ Multiply:

    $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3/2 & -5/2 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

    After we divide out the w component, this point *is* in the ± 1 cube, so it *is* in normalized device coordinates.



16. In the above figure, frame $F_1 = (v_1, v_2, p)$ and frame $F_2 = (w_1, w_2, q)$.

    (a) Find the homogeneous coordinates of r in F_2.

    By inspection, $r = (1, 1, 1)_{F_2}$.

    (b) Find the homogeneous coordinates of r in F_1.

    By inspection, $r = (1, 3, 1)_{F_1}$.

    (c) Give the 3×3 homogeneous transform matrix that transforms points in F_2 coordinates into F_1 coordinates.

    There are a couple of ways to approach this problem. By inspection we can see that, moving the *frame* from F_1 to F_2 involves a $90°$ rotation followed by a $(2, 2)$ translation (in F_1 coordinates). The transform for *points* from F_2 to F_1 coordinates (the opposite direction) is therefore the same matrix:

$$\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

    Alternatively, we could have gotten the same matrix by expressing the *frame* F_2 in F_1 coordinates. Giving:

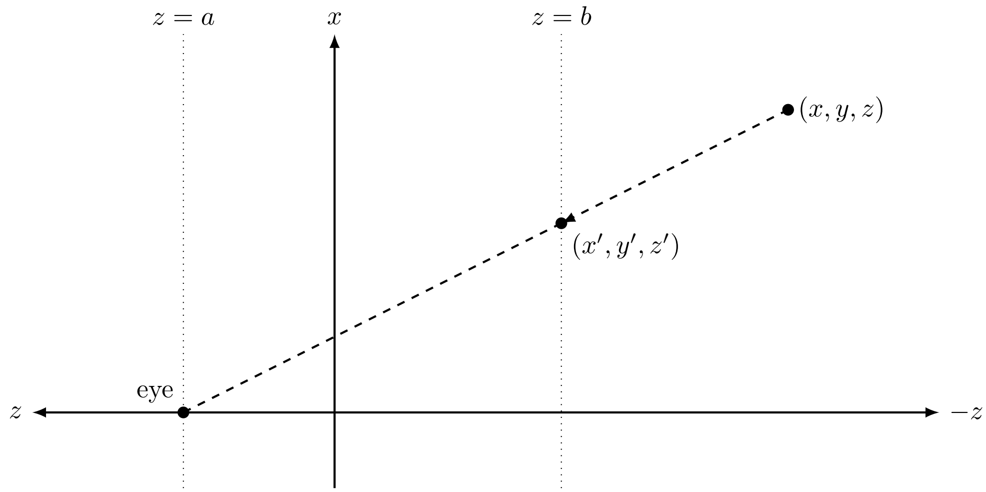
$$\begin{aligned} w_1 &= (0, 1, 0) \\ w_2 &= (-1, 0, 0) \\ q &= (2, 2, 1) \end{aligned}$$

    Using these as the *columns* of a matrix, we get the same result.

    If we test this on our sample point, we get:

$$\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

17. Suppose the eye is at $(0, 0, a)$, and we want to project points onto the $z = b$ plane, as in the figure (the y-z plane is similar). Give the 4×4 matrix that accomplishes this projection.



By similar triangles,

$$\begin{aligned}
\frac{x'}{a-b} &= \frac{x}{(a-z)} \\
\frac{y'}{a-b} &= \frac{y}{(a-z)} \\
x' &= \frac{x(a-b)}{(a-z)} \\
y' &= \frac{y(a-b)}{(a-z)} \\
z' &= b
\end{aligned}$$

Here it looks like it might be nice to divide by $(a - z)$, and in fact that is the "depth" of the point, the distance from the camera, so let's make the last row provide that. To put z' in the same picture, just note:

$$\begin{aligned}
z' &= b \\
&= \frac{b(a-z)}{(a-z)} \\
&= \frac{ba - bz}{(a-z)}
\end{aligned}$$

And so we can construct the matrix:

$$\begin{bmatrix} (a-b) & 0 & 0 & 0 \\ 0 & (a-b) & 0 & 0 \\ 0 & 0 & -b & ba \\ 0 & 0 & -1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} (a-b)x \\ (a-b)y \\ b(a-z) \\ (a-z) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{x(a-b)}{(a-z)} \\ \frac{y(a-b)}{(a-z)} \\ b \\ 1 \end{bmatrix}$$