Linear Algebra Notes

Geoffrey Matthews

Department of Computer Science
Western Washington University

Fall 2011

Online Resources

Readings

- http://chortle.ccsu.edu/vectorlessons/vectorindex.html
- http://mathforum.org/mathimages/index.php/Math_for_Computer_Graphics_and_Computer_Vision
- http://cs229.stanford.edu/section/cs229-linalg.pdf
- http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/, Computer graphics
- http://joshua.smcvt.edu/linearalgebra/

Videos

- http://www.khanacademy.org/math/linear-algebra

Matrices

- A matrix is a set of scalars organized into rows and columns.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Matrix addition, subtraction, multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Multiplication is *not* commutative! $MN \neq NM$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & \dots \\ \dots & \dots \end{bmatrix}$$

$$\begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ea + fc & \dots \\ \dots & \dots \end{bmatrix}$$

Mathematical Vectors

$$
\begin{aligned}
\vec{v} &= \begin{bmatrix} a \\ b \\ c \end{bmatrix} \\
&= [a \ b \ c]^T \\
&= (a, b, c)
\end{aligned}
$$

- A vector is an N row by 1 column matrix.
- We will use mathematical vectors to represent both *points* and *vectors* in space.

Matrices as transforms

- Multiplication of an N-vector by an $N \times N$ matrix *on the left* changes it into another N-vector.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Matrix inverses

- Identity matrix:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $AI = IA = A$
- Some matrices have an inverse: $AA^{-1} = A^{-1}A = I$
- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Inverse of a matrix

$$\begin{bmatrix} a & b & c & 1 & 0 & 0 \\ d & e & f & 0 & 1 & 0 \\ g & h & i & 0 & 0 & 1 \end{bmatrix}$$

- Stick the identity on the right.
- Add multiples of one row to another until the identity is on the left.
- The inverse is now on the right

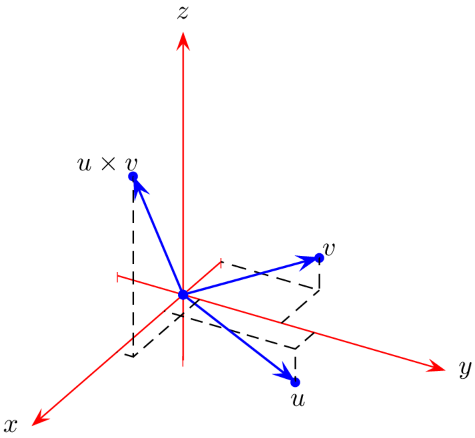
Determinant of a matrix

$$
\begin{aligned}
A &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\
\det(A) &= \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\
&= ad - bc \\
A^{-1} &= \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{\det(A)}
\end{aligned}
$$

Determinant of a matrix

$$\begin{aligned}
\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} &= a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \\
&= (aei - afh) - (bdi - bfg) + (cdh - ceg) \\
&= aei + bfg + cdh - afh - bdi - ceg
\end{aligned}$$

Cross product (vector product)



Application: find the normal to a surface.

- A vector at right angles to u and v.
- Right-hand rule.
- $u \times v = (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)$
- Mnemonic:

$$u \times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

- $|u \times v| = |u||v| \sin(\theta)$

Properties of the cross product

$$
\begin{aligned}
i \times j &= k \\
j \times k &= i \\
k \times i &= j \\
a \times b &= -b \times a \\
a \times (b + c) &= a \times b + a \times c \\
(sa) \times b &= s(a \times b), s \in \mathbb{R} \\
a \times (b \times c) &\neq (a \times b) \times c \\
|a \times b| &= \sin(\theta)|a||b|
\end{aligned}
$$