CSCI 510, Fall 2016, Homework # 5

YOUR NAME HERE

Due date: Wednesday, November 30, Midnight

1. Show that A is Turing-recognizable iff $A \leq_m A_{TM}$.

2. Let $J = \{w \mid \text{either } w = 0x \text{ for some } x \in A_{TM}, \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$. Show that neither J nor $\overline{J}$ is Turing-recognizable.

3. Let
$$f(x) = \begin{cases} 3x+1 & \text{for odd } x \\ x/2 & \text{for even } x \end{cases}$$
for any natural number x. If you start with a natural number x and iterate f, you obtain a sequence, $x, f(x), f(f(x)), \ldots$ Stop if you ever hit 1. For example, if $x = 17$, you ge the sequence $17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1$. Extensive computer tests have shown that every starting point between 1 and a very large positive integer gives a sequence that ends in 1. But the question of whether all positive starting points end up at 1 is unsolved; it is called the $3x + 1$ problem.

   Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that is guaranteed to state the answer to the $3x + 1$ problem.

4. Let $T = \{\langle M\rangle | M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that T is undecidable.

5. Show that the Post Correspondence Problem is decidable over the unary alphabet $\Sigma = \{1\}$.

6. Prove that there exists an undecidable subset of $\{1\}^*$.