CSCI 510, Fall 2016, Homework # 1

YOUR NAME HERE

Due date: Tuesday, October 4, Midnight

1. Transforming a NFA with n states into a DFA may require as many as 2^n states. Give examples of NFAs with 2 and 3 states that *require* 4 and 8 states, respectively, in their equivalent DFAs. Use the alphabet $\Sigma = \{0, 1\}$. Explain why they require 2^n states and find the DFAs in question.

2. Let
$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$
So Σ contains all columns of 0s and 1s of height two. A string of symbols in Σ give two rows of 0s and 1s. Consider each row to be a binary number and let
$$C = \{w \in \Sigma^* | \text{the bottom row is three times the top row}\}.$$
For example, $\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]\left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right]\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right] \in C$ because the binary number 0110 is three times the binary number 0010, but $\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]\left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right] \notin C$ because the binary number 110 is not three times the binary number 001. Show that C is regular.

   For this problem you may assume the truth of the theorem that says that the reverse of a regular language is regular. The reverse of a language A, written A^R, is the set of all strings $w^R = w_n \dots w_2 w_1$ such that $w = w_1 w_2 \dots w_n \in A$,
$$A^R = \{w^R |\ w^R = w_n \dots w_2 w_1 \text{ where } w = w_1 w_2 \dots w_n \in A\},$$

3. Let Σ be as in the previous problem, and let
$$E = \{w \in \Sigma^* | \text{the bottom row of } w \text{ is the reverse of the top row of } w\}.$$
Show that E is not regular.