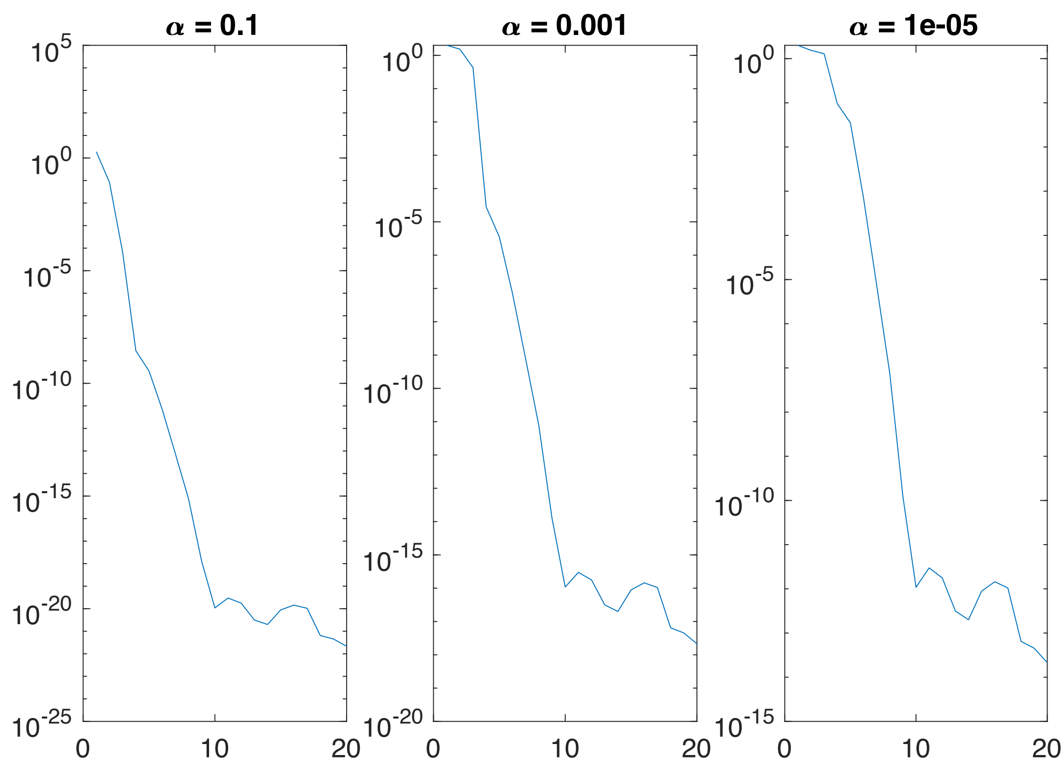
Question 1

I select truncated value p =2 and 4 and the chi2 values are 336.6076 and 3.7732. The chi2 value for the true model is 89.693. The expected value of the chi2 value is 0 that the predicted data and analytical solution fit exactly. From the chi2 value, the p = 4 is close to expected value most.

But the analytical solution looks bad from the chi2 value. I think it may be due to our sampling rate to the analytical solution. Since our analytical solution is a continuous function, and we discrete it by sampling with 1/20 interval, after discrete operation it will lose some information and cannot be as precise as the original analytical solution. If I use 1/40 interval, the chi2 value become 8.3 and become much smaller. Due to the limited d data size, I cannot make the interval smaller, in which the interpolation will make observed data distortion.

Question 2

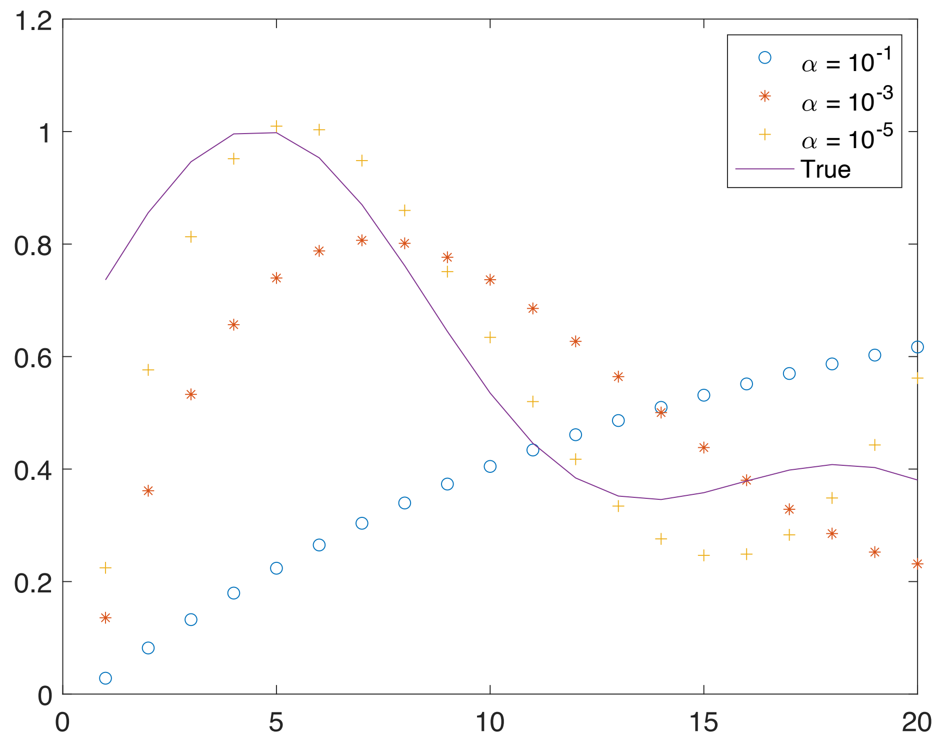
The log plot of the magnitude of the filtered Picard ratios are below. Compared to the unfiltered Picard ratios, the magnitude in filtered Picard ratios is larger (except for alpha = 0.1). When alpha = 1e-5, the Picard ratios has largest magnitude.



Question 3

a)

The model estimated from the different alpha and true model are plotted at figure below. We can find that alpha = 1e-5 the estimated model is closest to the true model. It has direct relation to the magnitude of the Picard ratios that the larger the magnitude of the Picard ratios, the perfect model estimation.



(b)

The chi2 value for the alpha = 0.1, 0.001 and 1e-5 are 1.3e6, 153.67 and 3.7733. The chi2 value from alpha = 1e-5 is similar to the TSVD with p =4 which also has the estimated model close to true model.

```
clear
clc
%% question 1
load('ifk.mat')

delta = 1/20;

% median points
x = [0:delta:1-delta]+delta/2;
y = x;%x;

% samping the analytic solution with higher frequency
% delta = 1/40;
% % median points
% x = [0:delta:1-delta]+delta/2;
% y = x;%x;
% d2 = interp1(y2,d,y,'linear','extrap');
% d = d2.';

[x1,y1] = meshgrid(x,y);

G(:,:) = x1.*exp(-x1.*y1)*delta;
[U,S,V] = svd(G);

sigma = 5e-5;
% p = 2
p=2;
Vp=V(:,1:p);
Sp = S(1:p,1:p);
Up=U(:,1:p);
m = Vp*Sp^(-1)*Up'*d;

chi2_1 = (d - G*m)'*(d - G*m)/sigma^2;

% p = 4
p = 4;
Vp=V(:,1:p);
Sp = S(1:p,1:p);
Up=U(:,1:p);
m = Vp*Sp^(-1)*Up'*d;

chi2_2 = (d - G*m)'*(d - G*m)/sigma^2;

% true model
mt = exp(-10*(x-0.2).^2) + 0.4*exp(-10*(x-0.9).^2);
chi2_3 = (d - G*mt')'*(d - G*mt')/sigma^2;



%% question 2

alpha = [1e-1,1e-3,1e-5];
figure
```

```
for i =1:3
    si = diag(S);
    fi = si.^2./(si.^2+alpha(i)^2);
    Pr = fi.*U'*d./si;
    subplot(1,3,i)
    semilogy(abs(Pr))
    title(['\alpha = ',num2str(alpha(i))])
    set(gca,'Fontsize',14)
end

%% question 3

m_1 = inv(G'*G + alpha(1).^2*eye(size(G'*G)))*G'*d;

m_2 = inv(G'*G + alpha(2).^2*eye(size(G'*G)))*G'*d;

m_3 = inv(G'*G + alpha(3).^2*eye(size(G'*G)))*G'*d;

figure;
plot(m_1,'o')
hold on
plot(m_2,'*')
hold on
plot(m_3,'+')
hold on
plot(mt)
legend('\alpha = 10^{-1}','\alpha = 10^{-3}','\alpha = 10^{-5}','True')
set(gca,'Fontsize',14)

% b

chi2_new1 = (d - G*m_1)'*(d - G*m_1)/sigma^2
chi2_new2 = (d - G*m_2)'*(d - G*m_2)/sigma^2
chi2_new3 = (d - G*m_3)'*(d - G*m_3)/sigma^2

%% samping with higher frequency

delta = 1/20;
% median points
x2 = [0:delta:1-delta]+delta/2;
y2 = x2;%x;

% samping the analytic solution with higher frequency
delta = 1/40;
% median points
x = [0:delta:1-delta]+delta/2;
y = x;%x;
d2 = interp1(y2,d,y,'linear','extrap');
d = d2.';

[x1,y1] = meshgrid(x,y);

G2(:,:) = x1.*exp(-x1.*y1)*delta;

mt = exp(-10*(x-0.2).^2) + 0.4*exp(-10*(x-0.9).^2);
```

```
chi2_3_new = (d - G2*mt')'*(d - G2*mt')/sigma^2
```