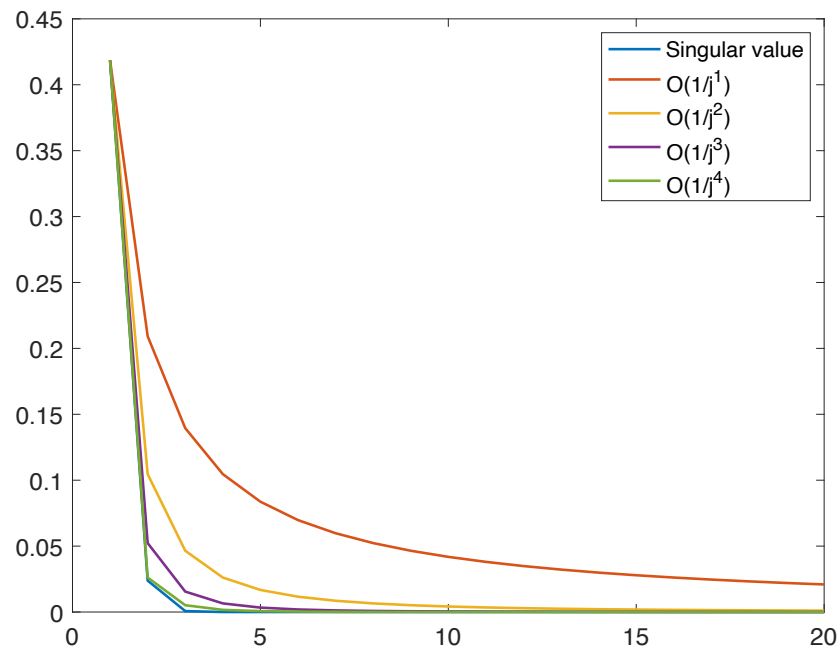


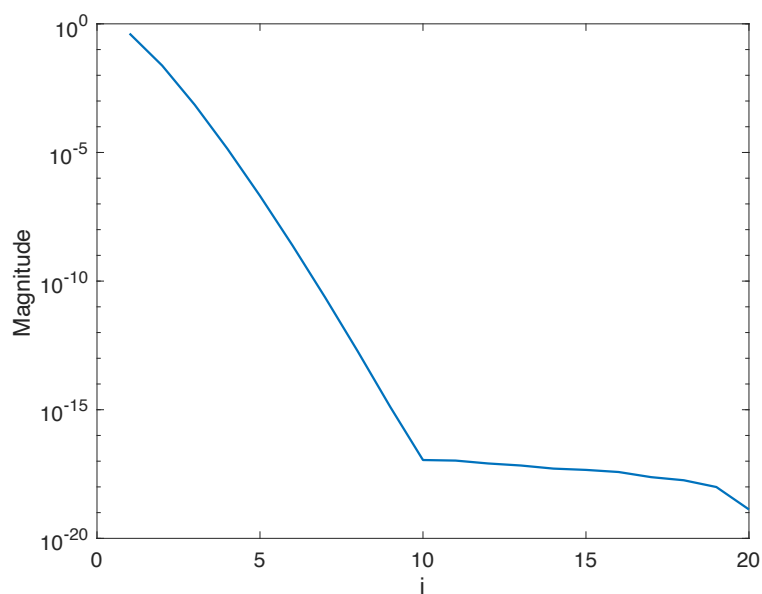
Question 1

From the figure below, we find the singular value is close to the $O(1/j^4)$ and thus it is moderately ill-posed.



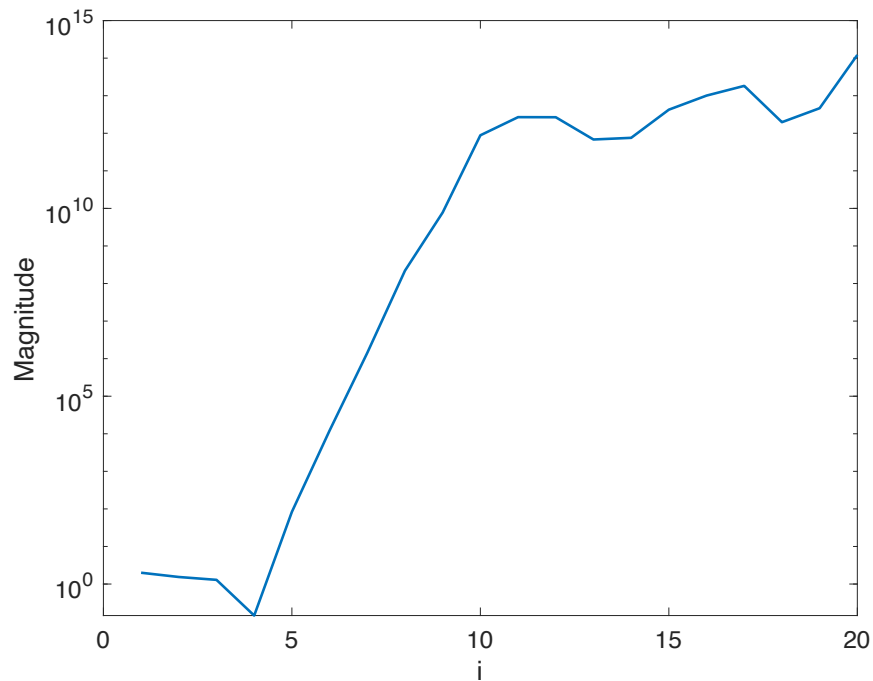
Question 2

From the log plot, I will choose $i = 2$, after that, the magnitude drops fast and I will consider they as the zero.



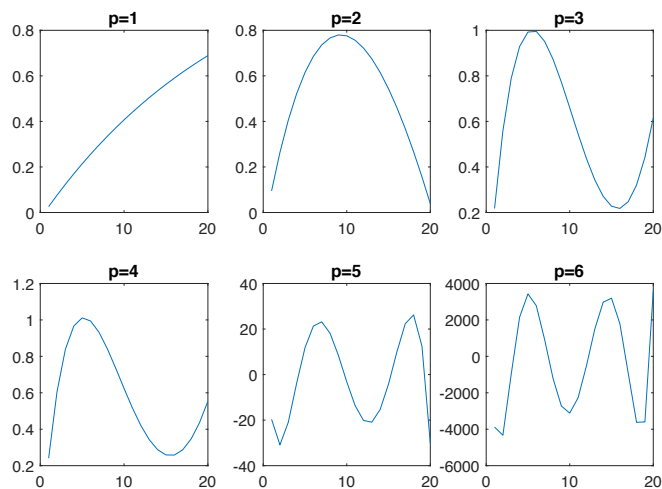
Question 3

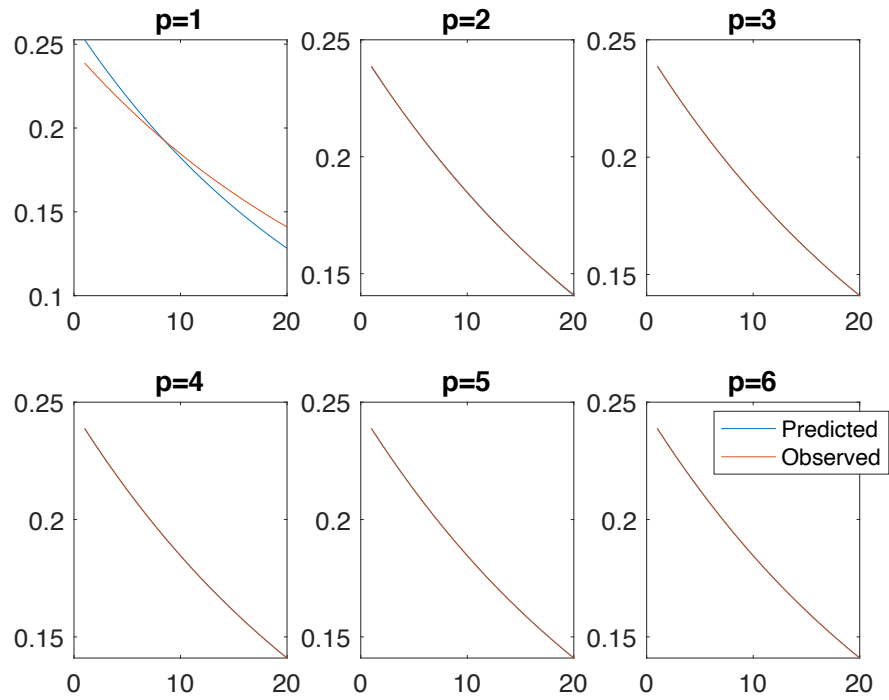
From the log plot, I will choose $i = 4$, after that, the magnitude increases fast.



Question 4

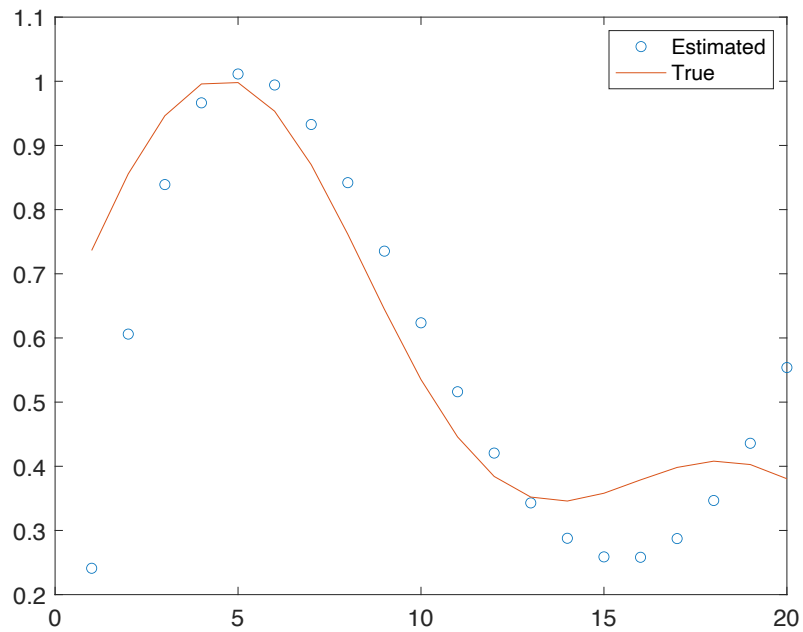
I plot the m with $p =$ from 1 to 6 and the observed and predicted data in following two figures. I found that when p is larger than 4, then the magnitude of model will be larger quickly. The observed and predicted data fit well after $p = 2$. Therefore, I prefer $p = 4$, which is close to Question 3.





Question 5

The estimated model can reflect the magnitude and trend of the true model in most part. The beginning and ending part of the model cannot fit well.



Codes

```
clear
clc
%% question 1
load('ifk.mat')

delta = 1/20;

% median points
x = [0:delta:1-delta]+delta/2;

y = x;%x;

[x1,y1] = meshgrid(x,y);

G(:, :) = x1.*exp(-x1.*y1)*delta;

[U,S,V] = svd(G);

j = 1:length(diag(S));
c = S(1,1);

figure(1)
plot(diag(S), 'lineWidth',1.5)
hold on
plot(c./j.^1, 'lineWidth',1.5)
hold on
plot(c./j.^2, 'lineWidth',1.5)
hold on
plot(c./j.^3, 'lineWidth',1.5)
hold on
plot(c./j.^4, 'lineWidth',1.5)

legend('Singular value', '0(1/j^1)', '0(1/j^2)', '0(1/j^3)', '0(1/j^4)')
set(gca, 'FontSize',14)
%% question 2

semilogy(diag(S), 'lineWidth',1.5)
xlabel('i')
ylabel('Magnitude')
set(gca, 'fontsize',14)

%% question 3

i = 20;

ratio = U(:,1:i)'*d./diag(S(1:i,1:i));

figure
semilogy(abs(ratio), 'lineWidth',1.5)
xlabel('i')
ylabel('Magnitude')
set(gca, 'fontsize',14)
```

```
%% question 4
```

```
for p = 1:6
    Vp=V(:,1:p);
    Sp = S(1:p,1:p);
    Up=U(:,1:p);
    m = Vp*Sp^(-1)*Up'*d;
    subplot(2,3,p)
    plot(m)
    title(['p=',num2str(p)])
    set(gca,'fontsize',14)
end
figure
for p = 1:6
    Vp=V(:,1:p);
    Sp = S(1:p,1:p);
    Up=U(:,1:p);
    m = Vp*Sp^(-1)*Up'*d;
    subplot(2,3,p)
    plot(G*m)
    hold on
    plot(d)
    title(['p=',num2str(p)])
    set(gca,'fontsize',14)
end
legend('Predicted','Observed')
```

```
%% question 5
```

```
p=4;
Vp=V(:,1:p);
Sp = S(1:p,1:p);
Up=U(:,1:p);
m = Vp*Sp^(-1)*Up'*d;

mt = exp(-10*(x-0.2).^2) + 0.4*exp(-10*(x-0.9).^2);
figure;
plot(m,'o')
hold on
plot(mt)
legend('Estimated','True')
set(gca,'FontSize',14)
```