

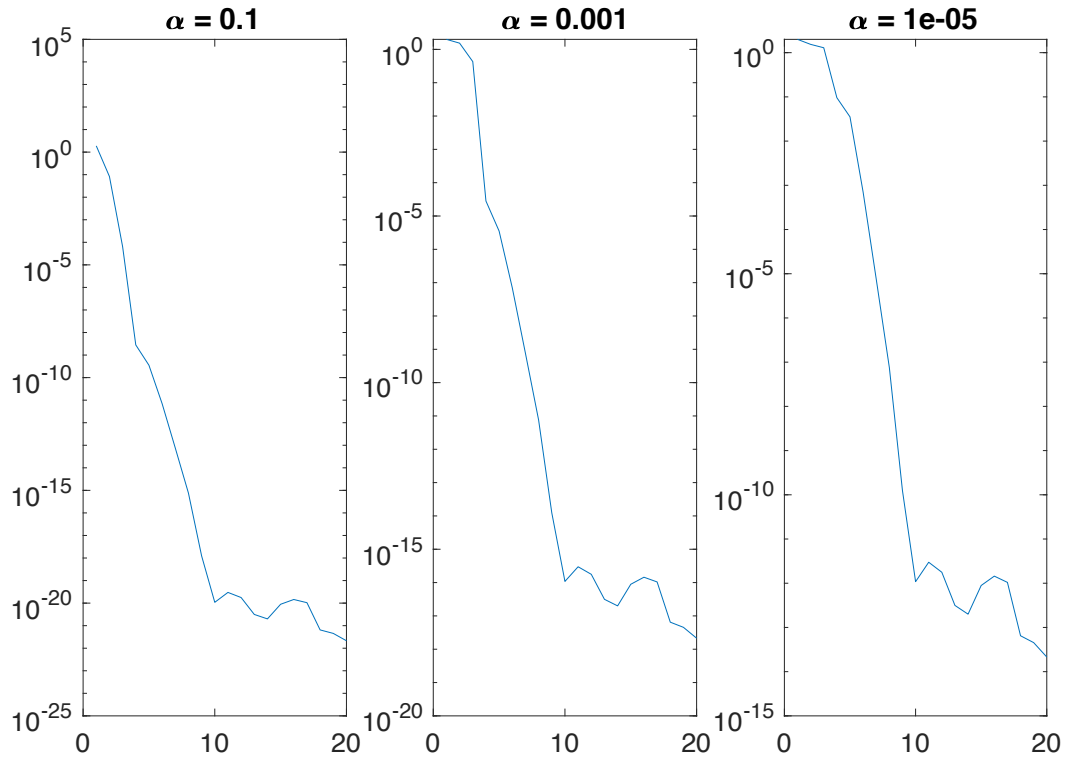
Question 1

I select truncated value $p = 2$ and 4 and the χ^2 values are 336.6076 and 3.7732 . The χ^2 value for the true model is 89.693 . The expected value of the χ^2 value is 0 that the predicted data and analytical solution fit exactly. From the χ^2 value, the $p = 4$ is close to expected value most.

But the analytical solution looks bad from the χ^2 value. I think it may be due to our sampling rate to the analytical solution. Since our analytical solution is a continuous function, and we discrete it by sampling with $1/20$ interval, after discrete operation it will lose some information and cannot be as precise as the original analytical solution. If I use $1/40$ interval, the χ^2 value become 8.3 and become much smaller. Due to the limited data size, I cannot make the interval smaller, in which the interpolation will make observed data distortion.

Question 2

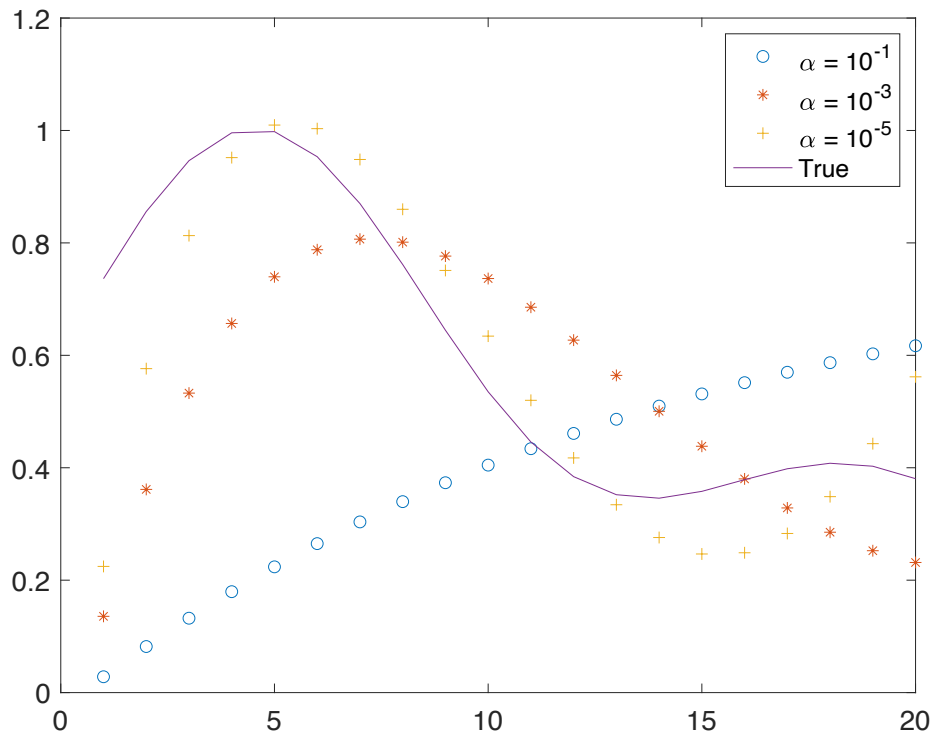
The log plot of the magnitude of the filtered Picard ratios are below. Compared to the unfiltered Picard ratios, the magnitude in filtered Picard ratios is larger (except for $\alpha = 0.1$). When $\alpha = 1e-5$, the Picard ratios has largest magnitude.



Question 3

a)

The model estimated from the different alpha and true model are plotted at figure below. We can find that $\alpha = 1e-5$ the estimated model is closest to the true model. It has direct relation to the magnitude of the Picard ratios that the larger the magnitude of the Picard ratios, the perfect model estimation.



(b)

The chi2 value for the $\alpha = 0.1, 0.001$ and $1e-5$ are $1.3e6, 153.67$ and 3.7733 .

The chi2 value from $\alpha = 1e-5$ is similar to the TSVD with $p=4$ which also has the estimated model close to true model.