Question 1

From the figure below, we find the singular value is close to the O(1/j4) and thus it is moderately ill-posed.



Question 2

From the log plot, I will choose *i* = 2, after that, the magnitude drops fast and I will consider they as the zero.



Question 3

From the log plot, I will choose *i* = 4, after that, the magnitude increases fast.



Question 4

I plot the m with p = from 1 to 6 and the observed and predicted data in following two figures. I found that when p is larger than 4, then the magnitude of model will be larger quickly. The observed and predicted data fit well after p = 2. Therefore, I prefer p =4, which is close to Question 3.





Question 5

The estimated model can reflect the magnitude and trend of the true model in most part. The beginning and ending part of the model cannot fit well.



Codes

clear

clc

%% question 1

load('ifk.mat')

delta = 1/20;

% median points

x = [0:delta:1-delta]+delta/2;

y = x;%x;

[x1,y1] = meshgrid(x,y);

G(:,:) = x1.\*exp(-x1.\*y1)\*delta;

[U,S,V] = svd(G);

j = 1:length(diag(S));

c = S(1,1);

figure(1)

plot(diag(S),'lineWidth',1.5)

hold on

plot(c./j.^1,'lineWidth',1.5)

hold on

plot(c./j.^2,'lineWidth',1.5)

hold on

plot(c./j.^3,'lineWidth',1.5)

hold on

plot(c./j.^4,'lineWidth',1.5)

legend('Singular value','O(1/j^1)','O(1/j^2)','O(1/j^3)','O(1/j^4)')

set(gca,'FontSize',14)

%% question 2

semilogy(diag(S),'lineWidth',1.5)

xlabel('i')

ylabel('Magnitude')

set(gca,'fontsize',14)

%% question 3

i = 20;

ratio = U(:,1:i)'\*d./diag(S(1:i,1:i));

figure

semilogy(abs(ratio),'lineWidth',1.5)

xlabel('i')

ylabel('Magnitude')

set(gca,'fontsize',14)

%% question 4

for p = 1:6

Vp=V(:,1:p);

Sp = S(1:p,1:p);

Up=U(:,1:p);

m = Vp\*Sp^(-1)\*Up'\*d;

subplot(2,3,p)

plot(m)

title(['p=',num2str(p)])

set(gca,'fontsize',14)

end

figure

for p = 1:6

Vp=V(:,1:p);

Sp = S(1:p,1:p);

Up=U(:,1:p);

m = Vp\*Sp^(-1)\*Up'\*d;

subplot(2,3,p)

plot(G\*m)

hold on

plot(d)

title(['p=',num2str(p)])

set(gca,'fontsize',14)

end

legend('Predicted','Observed')

%% question 5

p=4;

Vp=V(:,1:p);

Sp = S(1:p,1:p);

Up=U(:,1:p);

m = Vp\*Sp^(-1)\*Up'\*d;

mt = exp(-10\*(x-0.2).^2) + 0.4\*exp(-10\*(x-0.9).^2);

figure;

plot(m,'o')

hold on

plot(mt)

legend('Estimated','True')

set(gca,'Fontsize',14)