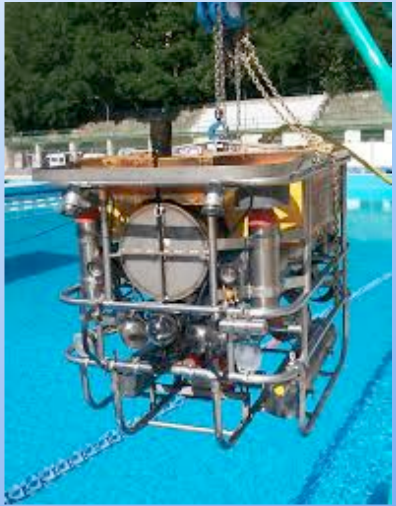

Modelling and identification of UMVs

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Thruster model

- Propeller-Hull interactions
  - for the vessels equipped with stern thrusters, the thrust available for the propulsion is reduced by the so-called thrust deduction number (typical values are 0.05-0.2), that takes into account the interaction between the propeller and the hull. This available thrust may be considered independent of the vessel speed.
- Thrust exerted by a propeller

$$\boxed{T_a = (1 - t)T}$$

$$T = T_{n|n|}n|n| + T_{v_a|n|}v_a|n| \longrightarrow \boxed{T_a = (1 - t)T_{n|n|}n|n||}$$

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Rudder model

- : rudder angle
- δ_a : angle of attack, relative angle between the rudder and the flow
- δ_f : angle of the flow in the vehicle fixed reference frame
- δ_s : rudder stall angle

$$F = \begin{cases} c_F v_{av}^2 \sin\left(\frac{\pi}{2}\frac{\delta_a}{\delta_s}\right) \ , if \ \left|\delta_a\right| < \delta_s \\ c_F v_{av}^2 sign\left(\delta_a\right) \ , if \ \left|\delta_a\right| \geq \delta_s \end{cases}$$

$$\delta_a = \delta - \delta_f = \delta - atan\frac{v + Lr}{u}$$

If the sway speed is negligible and the yaw rate is small, the angle of attack can be approximated by the rudder angle and the sine by its argument

$$\left|\delta_a\right| \approx \delta < \delta_s$$

$$F = c_F v_{av}^2 \sin\left(\frac{\pi}{2\delta_s}\delta_a\right) \approx c_F v_{av}^2 \frac{\pi}{2\delta_s}\delta = k_F v_{av}^2 \delta$$

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Marine vehicle modelling

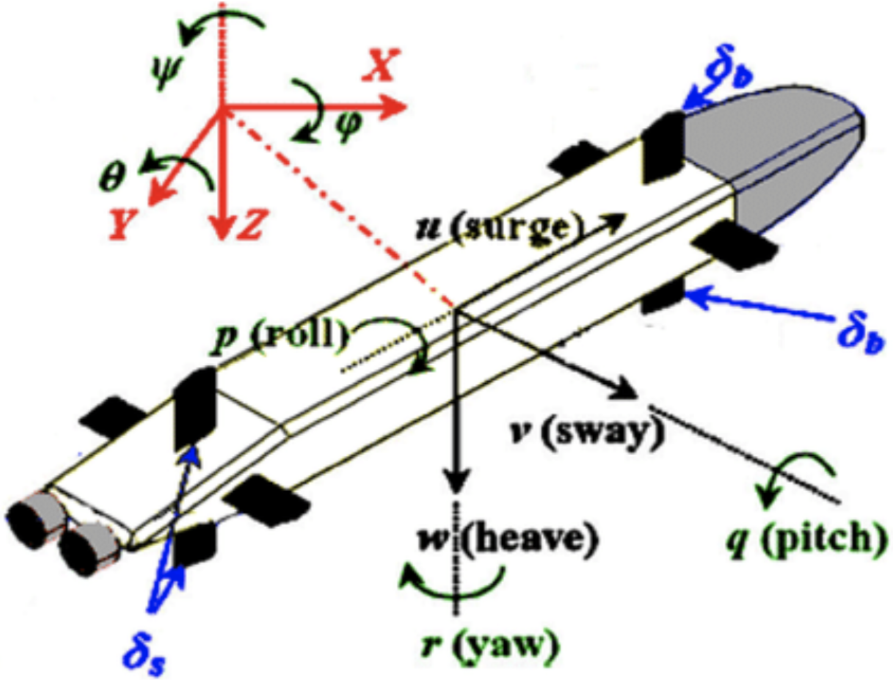
- **Underwater/Surface Vehicle can be modelled as a rigid body**
  - the physics of a rigid body floating in a fluid in known

$$\nu_1 = [u\ v\ w]^T \qquad \nu_2 = [p\ q\ r]^T$$

$$\eta_1 = [x\ y\ z]^T \qquad \eta_2 = [\phi\ \theta\ \psi]^T$$

$$F = [X\ Y\ Z]^T \qquad T = [K\ M\ N]^T$$

$$r_G = [x_G\ y_G\ z_G]^T$$



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Dynamics of a rigid body moving in a fluid

$$\dot{\boldsymbol{\eta}}_1 = \mathbf{J}_1\left(\boldsymbol{\eta}_2\right)\boldsymbol{\nu}_1$$

$$\mathbf{J}_1\left(\boldsymbol{\eta}_2\right) = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\theta s\psi s\phi & -c\psi s\phi + s\theta s\psi c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

$$\mathbf{M}_{RB}\dot{\nu} + \mathbf{C}_{RB}\left(\nu\right)\nu + \mathbf{M}_A\dot{\nu}_r + \mathbf{N}\left(\nu_r\right)\nu_r = \tau$$

$$\mathbf{N}\left(\nu_r\right) = \mathbf{C}_A\left(\nu_r\right) + \mathbf{D} + \mathbf{D}_n\left(\nu_r\right)$$

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Added mass

- In fluid mechanics, **added mass** or **virtual mass** is the inertia added to a system because an accelerating or decelerating body must move (or deflect) some volume of surrounding fluid as it moves through it. Added mass is a common issue because the object and surrounding fluid cannot occupy the same physical space simultaneously. For simplicity this can be modeled as some volume of fluid moving with the object, though in reality "all" the fluid will be accelerated, to various degrees. (source: Wikipedia)
- added mass in not the mass of the water occupied by the system

ASV dynamics model

- Consider a simplified 3 dot model: surge, sway, yaw

$$\mathbf{M}_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_g \\ 0 & mx_g & I_z \end{bmatrix}, \mathbf{M}_A = -\begin{bmatrix} X_{\dot{u}} & 0 & 0 \\ 0 & Y_{\dot{v}} & Y_{\dot{r}} \\ 0 & Y_{\dot{r}} & N_{\dot{r}} \end{bmatrix} \rightarrow \mathbf{M} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix}$$

$$\mathbf{M}\dot{\nu}_r + \mathbf{C}_{RB}(\nu_r)\nu_r + \mathbf{N}(\nu_r)\nu_r = \tau$$

$$\mathbf{M}\dot{\nu}_r + \mathbf{C}_{RB}(\nu_r)\nu_r + \mathbf{C}_A(\nu_r)\nu_r + \mathbf{D}\nu_r + \mathbf{D}_n(\nu_r)\nu_r = \tau$$

- Dynamics is written as a function of the velocity with respect to the water

ASV dynamics: Coriolis acceleration

$$\mathbf{C}_{RB} = \begin{bmatrix} 0 & -mr & -mrx_g \\ mr & 0 & 0 \\ mrx_g & 0 & 0 \end{bmatrix}, \mathbf{C}_A = \begin{bmatrix} 0 & 0 & Y_{\dot{v}}v_r + Y_{\dot{r}}r \\ 0 & 0 & -X_{\dot{u}}u_r \\ -Y_{\dot{v}}v_r - Y_{\dot{r}}r & X_{\dot{u}}u_r & 0 \end{bmatrix} \rightarrow \mathbf{C} = \begin{bmatrix} 0 & -mr & -mrx_g + Y_{\dot{v}}v_r + Y_{\dot{r}}r \\ mr & 0 & -X_{\dot{u}}u_r \\ mrx_g - Y_{\dot{v}}v_r - Y_{\dot{r}}r & X_{\dot{u}}u_r & 0 \end{bmatrix}$$

ASV dynamics: linear and quadratic drag

$$\mathbf{D} = -\begin{bmatrix} X_u & 0 & 0 \\ 0 & Y_v & Y_r \\ 0 & N_v & N_r \end{bmatrix}, \mathbf{D}_n(\nu_r) = -\begin{bmatrix} X_{u|u|}\left|u_r\right| & 0 & 0 \\ 0 & Y_{v|v|}\left|v_r\right| + Y_{v|r|}\left|r\right| & Y_{r|v|}\left|v_r\right| + Y_{r|r|}\left|r\right| \\ 0 & N_{v|v|}\left|v_r\right| + N_{v|r|}\left|r\right| & N_{r|v|}\left|v_r\right| + N_{r|r|}\left|r\right| \end{bmatrix}$$

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ASV dynamics: speed equation

$$\left(m - X_{\dot{u}}\right)\dot{u}_r = X_u u_r + X_{u|u|}\,|u_r|\,u_r + \left(m - Y_{\dot{v}}\right) v_r r + \left(m x_g - Y_{\dot{r}}\right) r^2 + X$$

Simplified model, neglecting v_r and r^2

$$\left(m - X_{\dot{u}}\right)\dot{u}_r = X_u u_r + X_{u|u|}\,|u_r|\,u_r + X$$

- Parameters
  - mass, including added mass
  - linear drag coefficient
  - quadratic drag coefficient

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ASV dynamics: steering equations

$$\left(m - Y_{\dot{v}}\right)\dot{v}_r + \left(mx_g - Y_{\dot{r}}\right)\dot{r} = Y_v v_r + Y_r r + Y_{v|v|}|v_r|v_r + Y_{v|r|}|r|v_r + Y_{r|v|}|v_r|r + Y_{r|r|}|r|r - \left(m - X_{\dot{u}}\right)u_r r + Y$$

$$\left(mx_g - Y_{\dot{r}}\right)\dot{v}_r + \left(I_z - N_{\dot{r}}\right)\dot{r} = N_v v_r + N_r r + N_{v|v|}|v_r|v_r + N_{v|r|}|r|v_r + N_{r|v|}|v_r|r + N_{r|r|}|r|r - \left(mx_g - Y_{\dot{r}}\right)u_r r - \left(X_{\dot{u}} - Y_{\dot{v}}\right)u_r v_r + N$$

Simplified model, neglecting cross-coupling drag terms

$$\left(m - Y_{\dot{v}}\right)\dot{v}_r + \left(mx_g - Y_{\dot{r}}\right)\dot{r} = Y_v v_r + Y_{v|v|}|v_r|v_r - \left(m - X_{\dot{u}}\right)u_r r + Y$$

$$\left(mx_g - Y_{\dot{r}}\right)\dot{v}_r + \left(I_z - N_{\dot{r}}\right)\dot{r} = N_r r + N_{r|r|}|r|r - \left(mx_g - Y_{\dot{r}}\right)u_r r - \left(X_{\dot{u}} - Y_{\dot{v}}\right)u_r v_r + N$$

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ASV dynamics: steering equations - simplified models

Uncoupled model

$$\left(m - Y_{\dot{v}}\right)\dot{v}_r = Y_v v_r + Y_{v|v|}\left|v_r\right|v_r - \left(m - X_{\dot{u}}\right)u_r r + Y$$

$$\left(I_z - N_{\dot{r}}\right)\dot{r} = N_r r + N_{r|r|}\left|r\right|r - \left(m x_g - Y_{\dot{r}}\right)u_r r - \left(X_{\dot{u}} - Y_{\dot{v}}\right)u_r v_r + N$$

Steering equation neglecting sway motion

$$\left(I_z - N_{\dot{r}}\right)\dot{r} = N_r r + N_{r|r|}\left|r\right|r - \left(m x_g - Y_{\dot{r}}\right)u_r r + N$$

surge rate affects yaw dynamics

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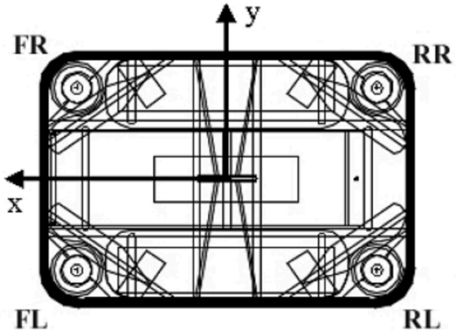
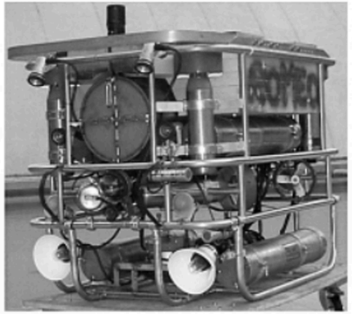
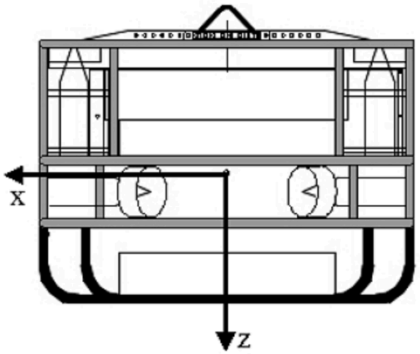
ROV dynamics: practical 1 dof model - Romeo ROV







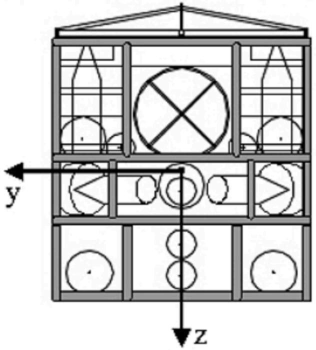


TABLE I
THRUST MAPPING MODES

thrust mapping mode	criterion	thruster forces				motion direction with $\eta_\xi = 1$
vertical motion		τ_{VFL}	τ_{VFR}	τ_{VRL}	τ_{VRR}	
vertical all pitch, roll, heave	$\min J = \frac{1}{2}\tau_V^T\tau_V$	$\frac{-L-M+Z}{4}$	$\frac{L-M+Z}{4}$	$\frac{-L+M+Z}{4}$	$\frac{L+M+Z}{4}$	-
vertical all	$\min J = \frac{1}{2}\tau_V^T\tau_V$ $L = 0$ $M = 0$	$\frac{Z}{4}$	$\frac{Z}{4}$	$\frac{Z}{4}$	$\frac{Z}{4}$	downward heave
horizontal motion		τ_{HFL}	τ_{HFL}	τ_{HRL}	τ_{HRR}	
horizontal all	$\min J = \frac{1}{2}\tau_H^T\tau_H$	$\frac{X+Y+N}{4}$	$\frac{X-Y-N}{4}$	$\frac{-X+Y-N}{4}$	$\frac{-X-Y+N}{4}$	-
surge front	$\tau_{HRL} + \tau_{HRR} = 0$ $Y = 0$	$\frac{X}{2} + \frac{N}{4}$	$\frac{X}{2} - \frac{N}{4}$	$-\frac{N}{4}$	$\frac{N}{4}$	forward surge
surge rear	$\tau_{HFL} + \tau_{HFR} = 0$ $Y = 0$	$\frac{N}{4}$	$-\frac{N}{4}$	$-\frac{X}{2} - \frac{N}{4}$	$-\frac{X}{2} + \frac{N}{4}$	backward surge
sway left	$\tau_{FR} + \tau_{RR} = 0$ $X = 0$	$\frac{Y}{2} + \frac{N}{4}$	$-\frac{N}{4}$	$\frac{Y}{2} - \frac{N}{4}$	$\frac{N}{4}$	right sway
sway right	$\tau_{FL} + \tau_{RL} = 0$ $X = 0$	$\frac{N}{4}$	$-\frac{Y}{2} - \frac{N}{4}$	$-\frac{N}{4}$	$-\frac{Y}{2} + \frac{N}{4}$	left sway
yaw front-left rear-right	$\tau_{HFR} + \tau_{HRL} = 0$ $X = 0$ $Y = 0$	$\frac{N}{2}$	0	0	$\frac{N}{2}$	clockwise yaw
yaw front-right rear-left	$\tau_{HFL} + \tau_{HRR} = 0$ $X = 0$ $Y = 0$	0	$-\frac{N}{2}$	$-\frac{N}{2}$	0	counter-clockwise yaw

ROV dynamics: practical 1 dof model

$$m_\xi \dot{\xi} = -k_\xi \xi - k_{\xi|\xi|} \xi |\xi| + \eta_\xi \phi_\xi^n + \nu_\xi$$

- ϕ_ξ^n : nominal actuators action
- the inertia and drag coefficients are assumed to be independent from how the total thrust is distributed on the vehicle's thrusters (thrust mapping μ)
- $\eta_\xi = \eta_\xi(\mu)$: the thruster installation coefficient is function of the thrust mapping
  - "the installation coefficients of a thruster take into account the differences in force that the thruster provides when operating in the proximity of the ROV, as opposed to when it is tested in open water." (Goheen and Jefferys)

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ROV dynamics: practical 1 dof model

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Onboard sensor-based identification

- **Identification of drag and thruster installation coefficients**
  - **steady-state manoeuvres**

    $$0 = -k_\xi \xi - k_{\xi|\xi|}\xi\,|\,\xi\,| + \eta_\xi \phi_\xi^n + \nu_\xi$$

  - unknown parameters: k_ξ , $k_{\xi|\xi|}$, η_ξ , ν_ξ
  - Since the model is homogeneous in the parameters the thrust mapping $\eta_\xi(\mu)$ must be known must be known for at least one mapping μ.

    Generally, it is not difficult to heuristically find a specific thrust mapping for which can be reasonably thought to be 1, i.e., a mapping in which thrusters operate in open water as during the thrust tunnel identification experiments.

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Identification of drag and thruster installation coefficients

- control input :

$$y = \left[\phi^*_{\xi_1} \quad \cdots \quad \phi^*_{\xi_{n^*}} \quad 0 \quad \cdots \quad \cdots \quad \cdots \quad 0\right]^T, \quad y \in \mathbb{R}^{n^*+\sum_{\mu=1}^{M} n(m_\mu),1}$$

- unknown parameters :

$$\theta = \left[k_\xi \quad k_{\xi|\xi|} \quad \bar{\nu} \quad \eta_\xi(\mu)\,\big|_{\mu=1} \quad \cdots \quad \eta_\xi(\mu)\,\big|_{\mu=1}\right]^T, \quad \theta \in \mathbb{R}^{3+M,1}$$

- measurements : $H \in \mathbb{R}^{n^*+\sum_{\mu=1}^{M} n(\mu),3+M}$

Measurements: H matrix

$$H = \begin{bmatrix} \xi_1^* & \xi_1^* \left| \xi_1^* \right| & -1 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \xi_{n^*}^* & \xi_{n^*}^* \left| \xi_{n^*}^* \right| & -1 & 0 & \cdots & \cdots & 0 \\ \xi_1 \left.\right|_{\mu=1} & \xi_1 \left| \xi_1 \right|_{\mu=1} & -1 & -\phi_{\xi_1}^n(\mu) \left.\right|_{\mu=1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \xi_{n(\mu)} \left.\right|_{\mu=1} & \xi_{n(\mu)} \left| \xi_{n(\mu)} \right| \left.\right|_{\mu=1} & -1 & -\phi_{\xi_{n(\mu)}}^n(\mu) \left.\right|_{\mu=1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \xi_1 \left.\right|_{\mu=M} & \xi_1 \left| \xi_1 \right|_{\mu=M} & -1 & 0 & \cdots & 0 & -\phi_{\xi_1}^n(\mu) \left.\right|_{\mu=M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \xi_{n(\mu)} \left.\right|_{\mu=M} & \xi_{n(\mu)} \left| \xi_{n(\mu)} \right| \left.\right|_{\mu=M} & -1 & 0 & \cdots & 0 & -\phi_{\xi_{n(\mu)}}^n(\mu) \left.\right|_{\mu=M} \end{bmatrix}$$

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Least Square Estimate

- system : $y = H\theta$
- Least Square estimate : $\hat{\theta} = (H^T H)^{-1} H^T y$
- standard deviation of the estimate : $\hat{\sigma}_\theta = \sqrt{diag\left[(H^T H)^{-1}\sigma_\epsilon^2\right]}$
- measurement noise variance : $\hat{\sigma}_\epsilon^2 = \dfrac{(y - H\hat{\theta})^T (y - H\hat{\theta})}{dim(y) - dim(\theta)}$
- percentile parameter error : $100(\hat{\sigma}_\theta / |\hat{\theta}|$

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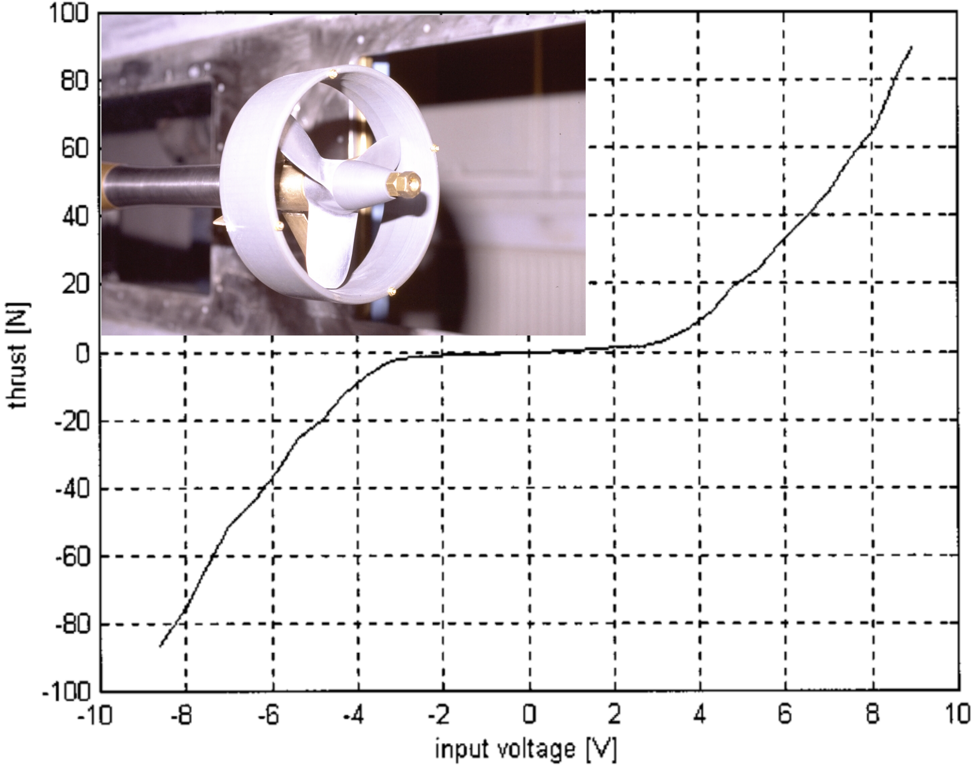
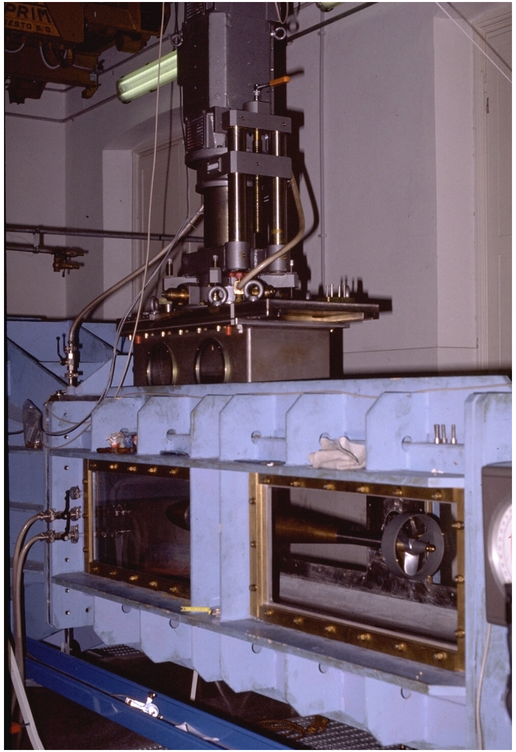
ROV heave dynamics identification: preliminary tests



- thrust tunnel actuator identification



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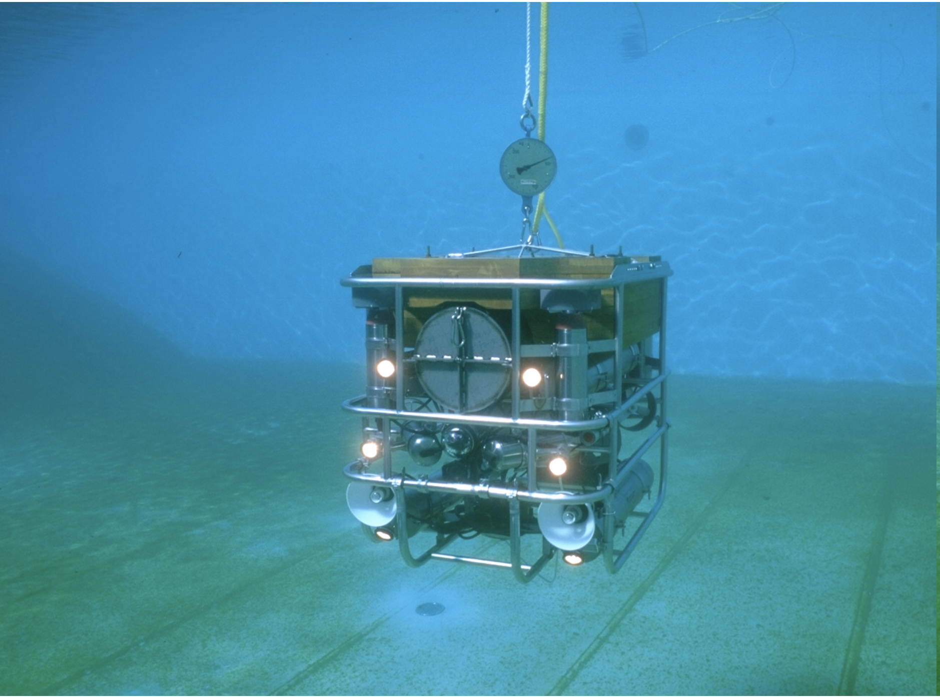

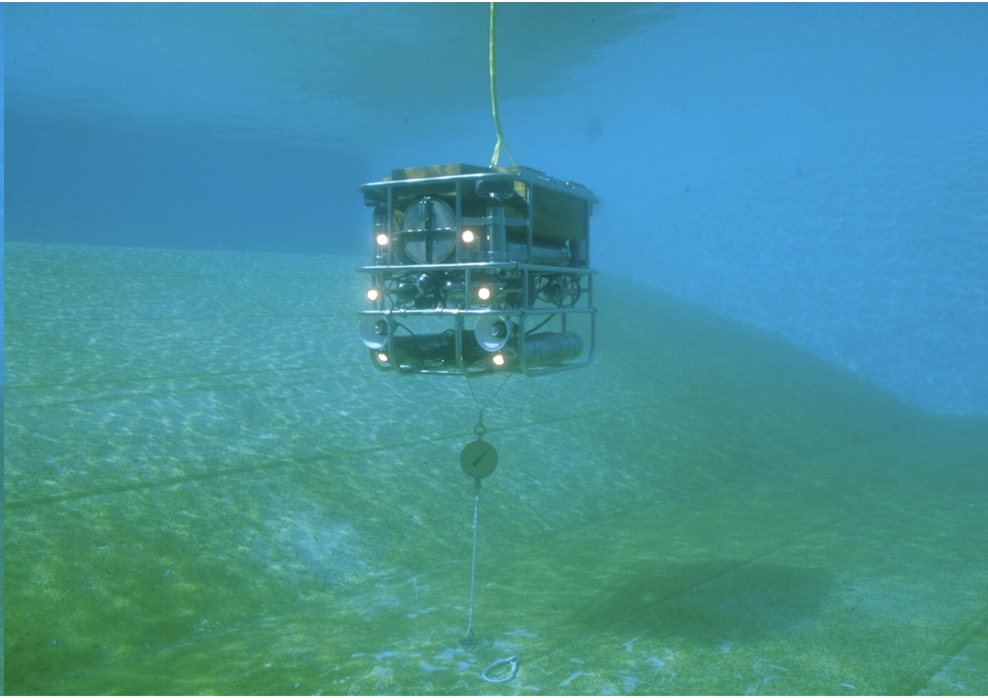

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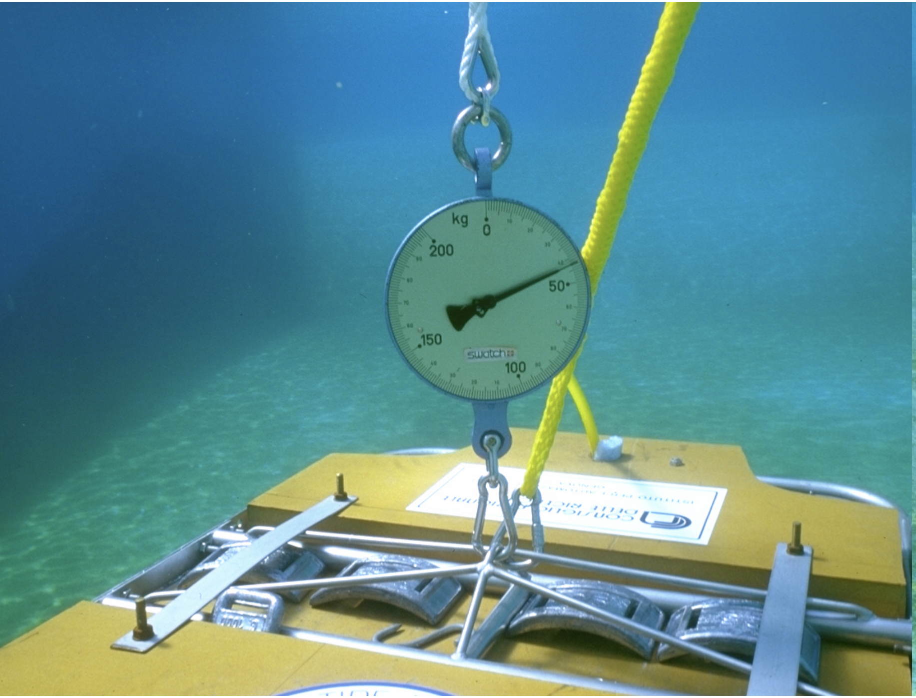
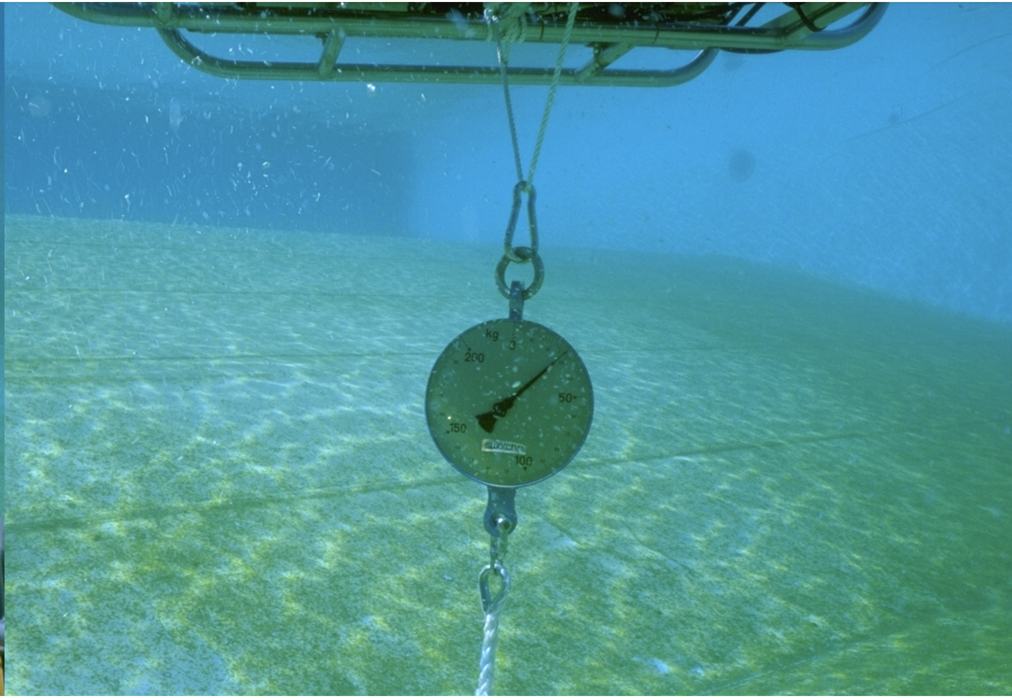
ROV heave dynamics identification: preliminary tests

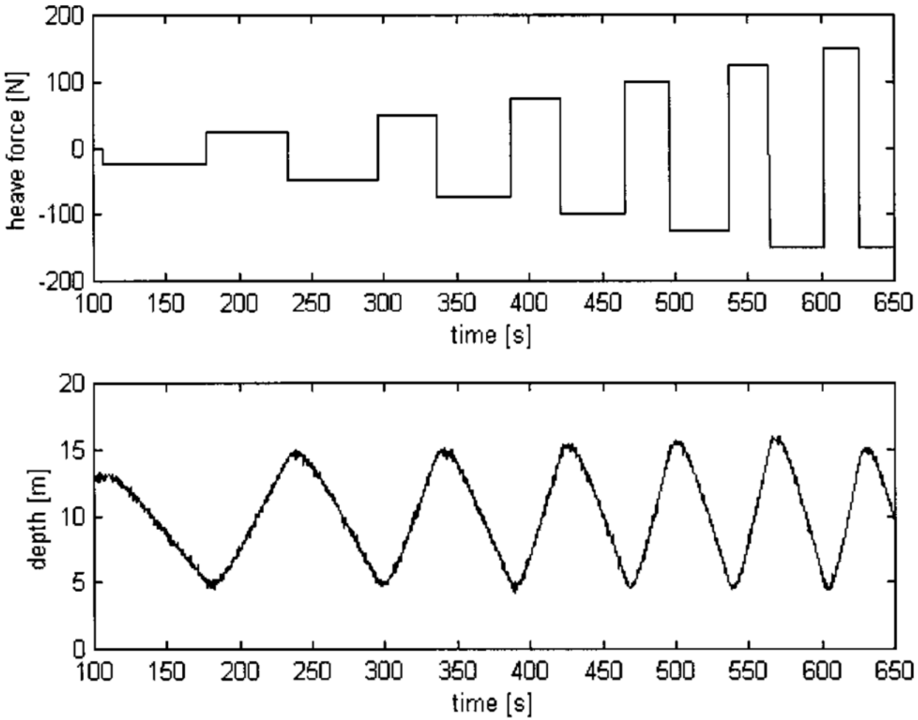
efficiency reduction of about 40% when pulling up in static conditions





Heave drag identification: steady-state experiments

Five different experiments, numbered 0–4, have been performed with inputs of the kind shown in figure, each with a different vehicle weight. During experiments 0 and 1, the vehicle was positive, during experiment 2 it was roughly neutral, and in the last two experiments it was negative. Weight was changed by adding on ROMEO’s top, during each experiment, one diver’s lead weight (about 0.7 kg in water) which reasonably does not affect the hydrodynamic derivatives but only the overall weight.



Heave drag identification: steady-state model

$$k_w w + k_{w|w|} w\,|\,w\,| - \eta_w \phi_w^n - W - i\Delta W$$

$$i = 0\ ,\ \ \ldots\ \ ,4 \qquad \begin{cases} \eta_w = 1\ ,\forall \phi_w^n \le 0 \\ \eta_w < 1\ ,\forall \phi_w^n < 0 \end{cases}$$

- w : heave velocity
- W : weight-buoyancy force in experiment 0
- ΔW : weight added to Romeo in each experiment

unknown parameter vector : $\theta = \left[k_w\ ,\ k_{w|w|}\ ,\ W\ ,\ \Delta W\ ,\ \eta_w\right]^T$

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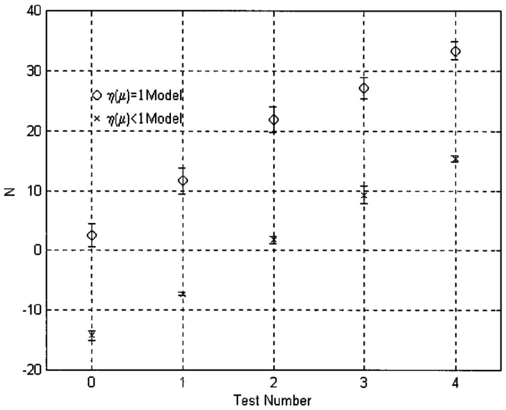
Heave drag identification: results

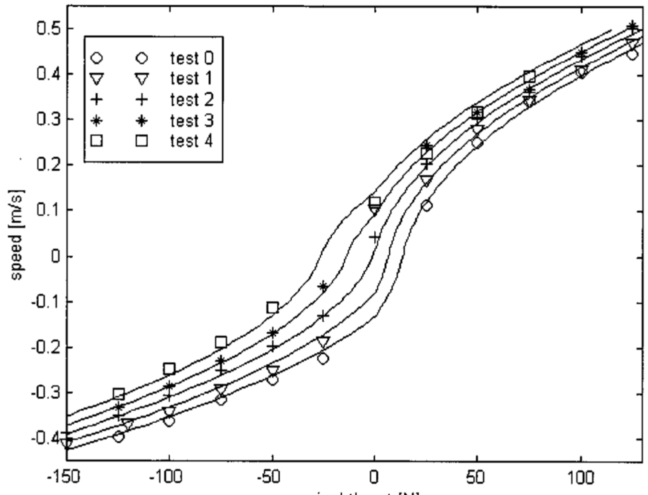
TABLE II

ROMEO HEAVE MODEL: ESTIMATED DRAG, THRUSTER INSTALLATION COEFFICIENTS, AND WEIGHT-BOUYANCY FORCES

	k_w [Ns/m]	$k_{w\|w\|}$ [Ns^2/m^2]	W [N]	ΔW [N]	η_w
$\hat{\theta}$	44.7	430.3	-13.5	7.1	0.56
$\hat{\sigma}_\theta$	8.8	20.8	1.3	0.4	0.023
$100\hat{\sigma}_\theta / \lvert\hat{\theta}\rvert$	19.7 %	4.8 %	9.6 %	5.6 %	4.1 %

0.6 in pool tests





Yaw drag identification: steady-state experiments

- yaw front-left rear-right mapping mode
  - to enable the thrusters to work in open water when positive torque was applied and with remarkable interactions with the hull in case of negative torque ($\eta_r^{hull} < 1$).
- horizontal all mapping mode
  - to enable the evaluation of $\eta_r^{hull-propeller}$, a second horizontal thruster installation coefficient that takes into account both the propeller hull and the propeller–propeller interactions occurring among horizontal thrusters on the same side of the vehicle.
- $\theta = \left[k_r \, , \, k_{r|r|} \, , \, \eta_r^{hull} \, , \, \eta_r^{hull-propeller}\right]^T$

Yaw drag identification: results

TABLE III
ROMEO YAW MODEL: ESTIMATED DRAG AND THRUSTER INSTALLATION COEFFICIENTS

		k_r [Nms/rad]	$k_{r\|r\|}$ [Nms²/rad²]	η_r^{hull}	$\eta_r^{hull-propeller}$
Linear and quadratic yaw model	$\hat{\theta}$	20.5	49.5	0.60	0.60
	$\hat{\sigma}_\theta$	2.6	7.2	0.03	0.025
	$100\hat{\sigma}_\theta/\|\hat{\theta}\|$	12.7 %	14.5 %	5 %	4.2 %
Linear and quadratic yaw model at low speed	$\hat{\theta}$	23.8	30.8	0.67	0.61
	$\hat{\sigma}_\theta$	3.5	20.7	0.048	0.036
	$100\hat{\sigma}_\theta/\|\hat{\theta}\|$	14.7 %	67.2 %	7.2 %	5.9 %
Linear yaw model at low speed	$\hat{\theta}$	28.6	-	0.68	0.62
	$\hat{\sigma}_\theta$	1.3	-	0.049	0.036
	$100\hat{\sigma}_\theta/\|\hat{\theta}\|$	4.5 %	-	7.2 %	5.8 %

$r < 10\ deg/s$