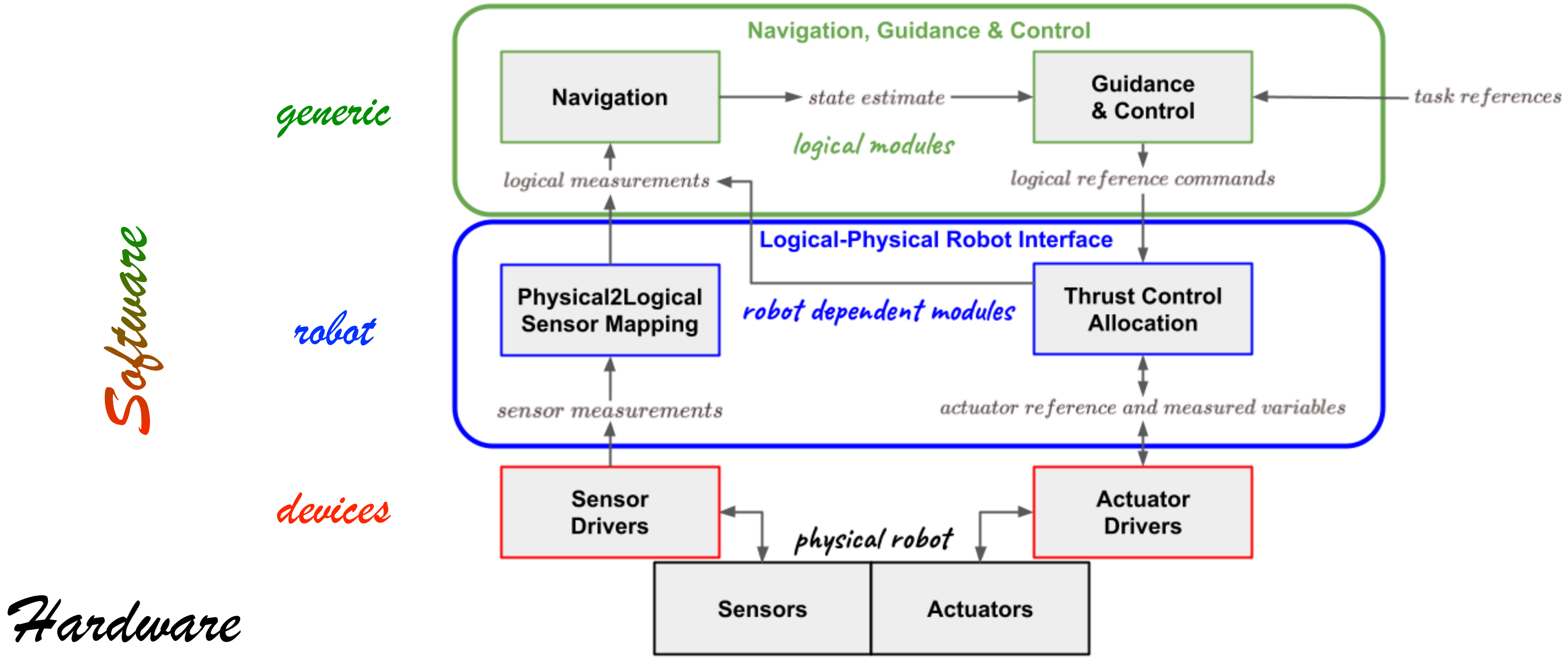
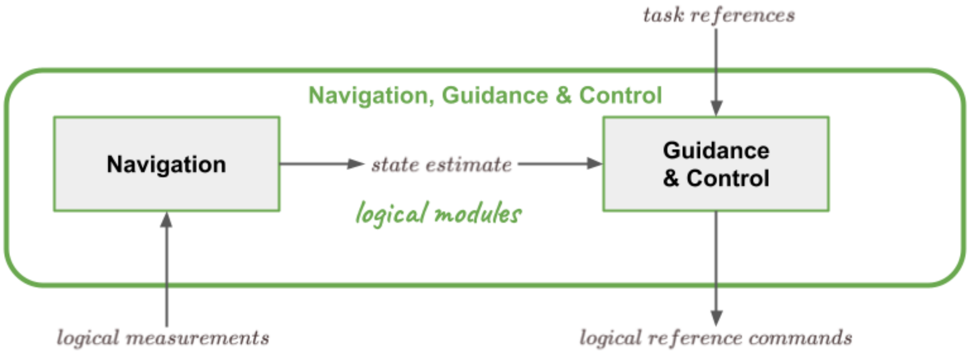
Navigation for UMVs

Massimo Caccia
Consiglio Nazionale delle Ricerche
Istituto di Ingegneria del Mare

AMV NGC architecture



AMV Navigation, Guidance and Control



- **Navigation** estimates the motion of the vehicle
- **Guidance** handles the vehicle's kinematics executing the desired motion task functions
- **Control** handles the vehicle's dynamics implementing linear and angular velocity controllers

massimo.caccia@cnr.it

Measurement error

- difference between a measured value of a quantity and its true value
  - **random error**: is caused by inherently unpredictable fluctuations in the readings of a measurement apparatus or in the experimenter's interpretation of the instrumental reading. Random errors show up as different results for ostensibly the same repeated measurement. They can be estimated by comparing multiple measurements, and reduced by averaging multiple measurements (Wikipedia)
  - **systematic error**: is predictable and typically constant or proportional to the true value. If the cause of the systematic error can be identified, then it usually can be eliminated. Systematic errors are caused by imperfect calibration of measurement instruments or imperfect methods of observation, or interference of the environment with the measurement process, and always affect the results of an experiment in a predictable direction. Incorrect zeroing of an instrument leading to a zero error is an example of systematic error in instrumentation. (Wikipedia)

massimo.caccia@cnr.it

Which are the effects of measurement noise (random error)?

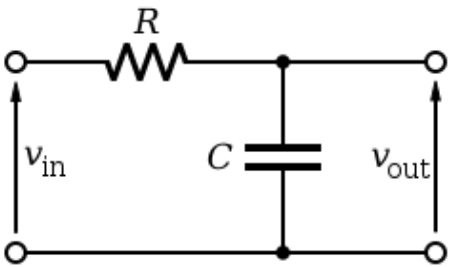
- if you provide sensor measurement as direct feed-back to your control system, measurement noise gets part of the error thus generating a corresponding noise in the proportional part of the control action, thus increasing the stress of the actuators
  - Note: if the noise is, as in many cases, at zero-mean it does not affect the integral component of the control action
- **noise is amplified by the derivative operation**
  - thus introducing strong vibrations in the derivative part of the control action (e.g. depth-heave control)

we need to smooth the signal!

massimo.caccia@cnr.it

Low Pass filtering

- A **low-pass filter** is a **filter** that **passes** signals with a **frequency lower** than a selected cutoff **frequency** and attenuates signals with frequencies higher than the cutoff **frequency** (Wikipedia)



$$v_{\text{in}}(t) - v_{\text{out}}(t) = R\,i(t)$$

$$Q_c(t) = C\,v_{\text{out}}(t)$$

$$i(t) = \frac{\mathrm{d}\,Q_c}{\mathrm{d}\,t}$$

$$v_{\text{in}}(t) - v_{\text{out}}(t) = RC\frac{\mathrm{d}\,v_{\text{out}}}{\mathrm{d}\,t}.$$

$$x_i - y_i = RC\,\frac{y_i - y_{i-1}}{\Delta_T}$$

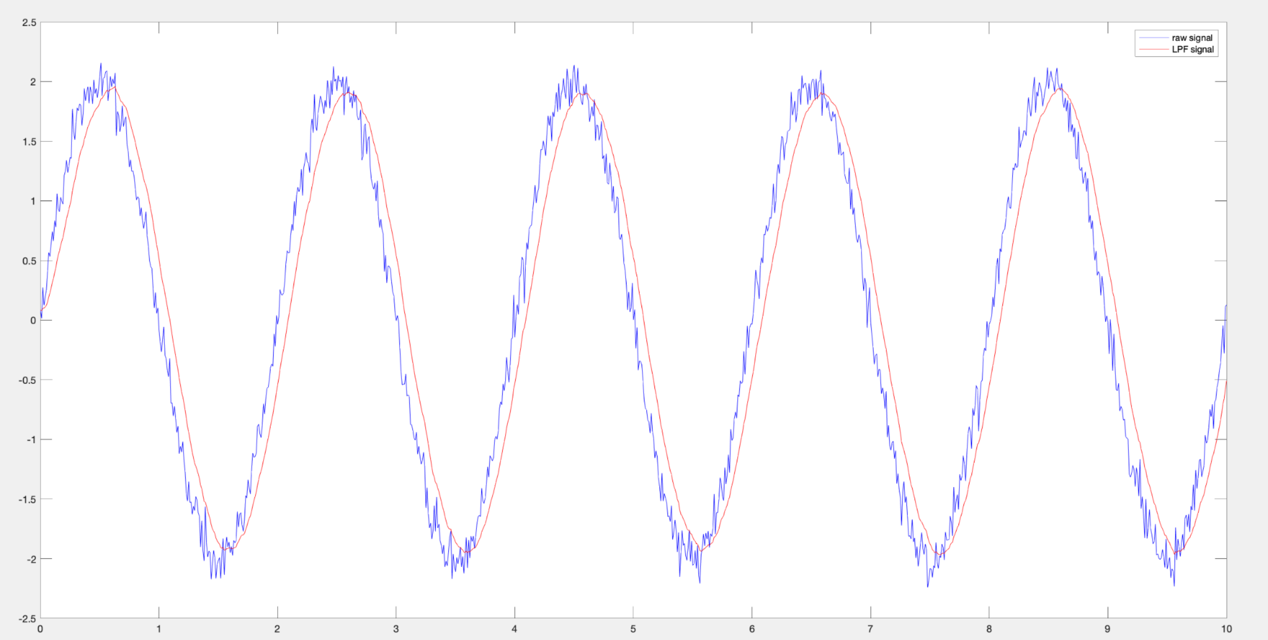
$$y_i = \overbrace{x_i\left(\frac{\Delta_T}{RC + \Delta_T}\right)}^{\text{Input contribution}} + \overbrace{y_{i-1}\left(\frac{RC}{RC + \Delta_T}\right)}^{\text{Inertia from previous output}}.$$

memory/prediction

ISTITUTO DI INGEGNERIA DEL MARE
INSTITUTE OF MARINE ENGINEERING
massimo.caccia@cnr.it

Low Pass filtering (RC LPF)

$$y_i = \alpha x_i + (1-\alpha) y_{i-1} \qquad \text{where} \qquad \alpha := \frac{\Delta_T}{RC + \Delta_T}$$



causal LP filtering of the measurements introduces delay

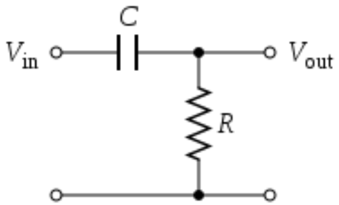
massimo.caccia@cnr.it

Which are the effects of measurement bias?

- bias in speed/acceleration measurements that are integrated has to be filtered by High Pass filtering

High Pass filtering

- A **high-pass filter** (**HPF**) is a filter that passes signals with a frequency higher than a certain cutoff frequency and attenuates signals with frequencies lower than the cutoff frequency



$$\begin{cases} V_{out}(t) = I(t)\,R \\ Q_c(t) = C\,\left(V_{in}(t) - V_{out}(t)\right) \\ I(t) = \frac{\mathrm{d}\,Q_c}{\mathrm{d}\,t} \end{cases}$$

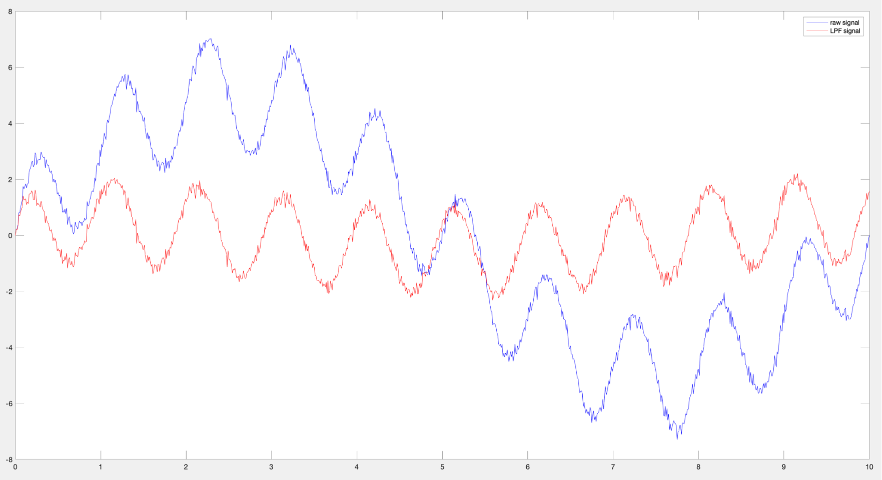
$$V_{out}(t) = \overbrace{C\left(\frac{\mathrm{d}\,V_{in}}{\mathrm{d}\,t} - \frac{\mathrm{d}\,V_{out}}{\mathrm{d}\,t}\right)}^{I(t)} R = RC\left(\frac{\mathrm{d}\,V_{in}}{\mathrm{d}\,t} - \frac{\mathrm{d}\,V_{out}}{\mathrm{d}\,t}\right)$$

massimo.caccia@cnr.it

High Pass filtering (RC HPF)

$$y_i = \overbrace{\frac{RC}{RC + \Delta_T} y_{i-1}}^{\text{Decaying contribution from prior inputs}} + \overbrace{\frac{RC}{RC + \Delta_T} (x_i - x_{i-1})}^{\text{Contribution from change in input}}$$

$$y_i = \alpha y_{i-1} + \alpha (x_i - x_{i-1}) \qquad \text{where} \qquad \alpha \triangleq \frac{RC}{RC + \Delta_T}$$



ISTITUTO DI INGEGNERIA DEL MARE
INSTITUTE OF MARINE ENGINEERING
massimo.caccia@cnr.it

Filtering (measurement noise reduction) & Control

- causal LP filtering of the measurements introduces **delay**
- increasing the control gain, that is commonly done to reduce error and obtain a fast response, in the presence of **delay** makes the closed-loop system unstable
- **noise reduction vs. estimate delay**

until now we have considered the measurements as a signal in time with no information on how it evolves

massimo.caccia@cnr.it

Systems and state variables

- A **system** S accepts **inputs** u, that can not control, and transform them into **outputs** y
- **Memoryless** systems: the output depends only by the current input
- **Dynamic systems**: the output depends by the present and past values of the inputs
- **State** variables x embed the memory of the system, i.e. describe enough to determine its future behaviour in absence of any external input

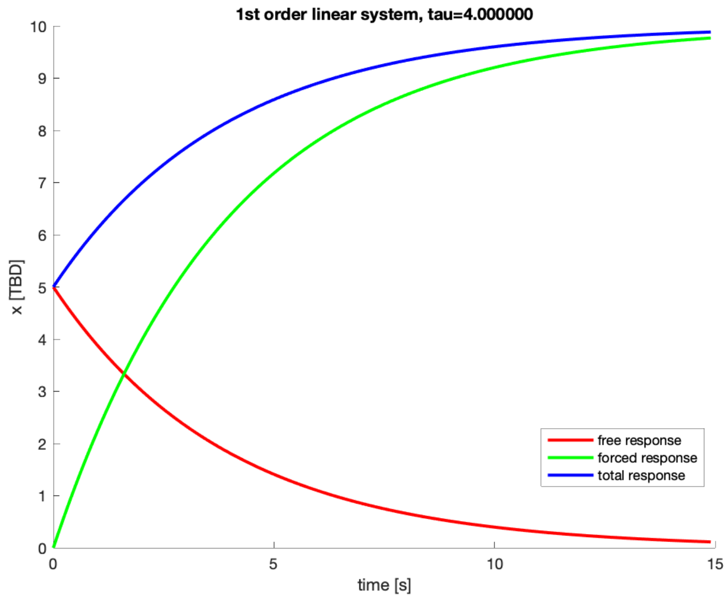
$$\dot{x} = f(x) + g(u)$$
$$y = h(x)$$

- f : state-transition model
- g : control-input model
- h : observation model

massimo.caccia@cnr.it

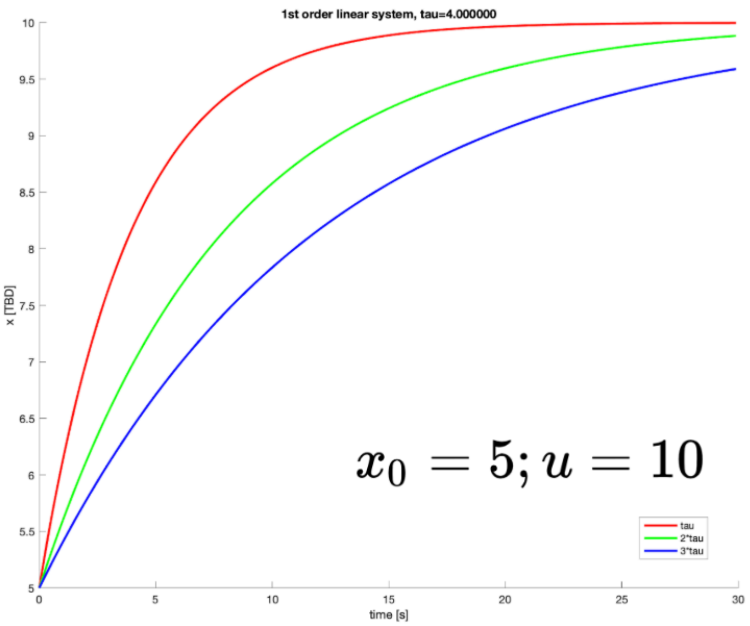
First Order Linear Systems

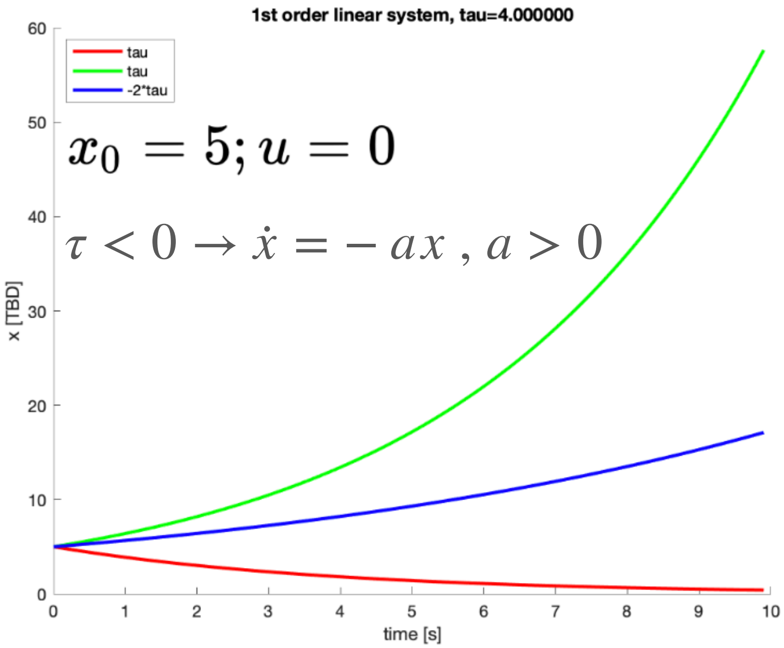
- $\tau\dot{x} = -x + u$, $x(t_0) = x_0$
  - τ : time constant
- free response: $u(t) = 0$
  - $x_0(t) = Ae^{-\frac{t}{\tau}}$, $x_0(0) = A$
- forced response, e.g. $u(t) = 1(k)$
  - $x_u(t) = k\left(1 - e^{-\frac{t}{\tau}}\right)$, $x_u(0) = 0$
- superposition principle
  - $x(t) = x_0(t) + x_u(t) = Ae^{-\frac{t}{\tau}} + k\left(1 - e^{-\frac{t}{\tau}}\right)$



First Order Linear Systems: what happens changing τ

- $\tau \dot{x} = -x + u$, $x(t_0) = x_0$, τ : time constant





the system state explodes

ISTITUTO DI INGEGNERIA DEL MARE
INSTITUTE OF MARINE ENGINEERING
massimo.caccia@cnr.it

Discrete Time First Order Linear System

$$x_k = F_k x_{k-1} + B_k u_k + w_k$$
$$y_k = H_k x_k + \nu_k$$

- F_k : state-transition model
- B_k : control-input model
- H_k : observation model
- w_k : process noise assumed such that $Q_k : w_k \sim \mathcal{N}(0, Q_k)$
- ν_k : observation noise assumed such that $R_k : w_k \sim \mathcal{N}(0, R_k)$

massimo.caccia@cnr.it

Discrete Time First Order Linear System: filtering

$$\hat{x}_{k|k} = \hat{x}\left(k, I(k)\right)$$

- a posteriori state estimate at time k given observations up to and including at time k

$$P_{k|k} = P\left(k, I(k)\right)$$

- a posteriori estimate covariance matrix (a measure of the estimated accuracy of the state estimate)

$$\hat{x}\left(k, I(k)\right) = \underbrace{\hat{x}\left(k, I(k-1)\right)}_{\text{predicted state}} + \underbrace{G_k}_{\text{gain}} \underbrace{\left[y(k) - \underbrace{\hat{y}\left(k, \hat{x}\left(k, I(k-1)\right)\right)}_{\text{predicted observation}}\right]}_{\text{innovation}}$$

massimo.caccia@cnr.it

Linear Kalman Filter

- Linear system and measurement equations (observations)
  - $x_k = F_k x_{k-1} + B_k u_k + w_k$
  - $y_k = H_k x_k + \nu_k$
- Prediction
  - predicted state estimate : $\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B_k u_k$
  - predicted estimate covariance : $P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$
- Innovation
  - innovation : $\tilde{y}_k = y_k - H_k \hat{x}_{k|k-1}$
  - innovation covariance : $S_k = H_k P_{k|k-1} H_k^T + R_k$

Linear Kalman Filter

- Update
  - optimal Kalman gain : $K_k = P_{k-1|k} H_k^T S_k^{-1}$
  - updated state estimate : $\hat{x}_{k|k} = \hat{x}_{k-1|k} + K_k \tilde{y}_k$
  - updated estimate covariance : $P_{k|k} = \left(I - K_k H_k\right) P_{k-1|k}$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left[y(k) - H_k \hat{x}_{k|k-1} \right]$$

predicted state gain predicted observation

innovation

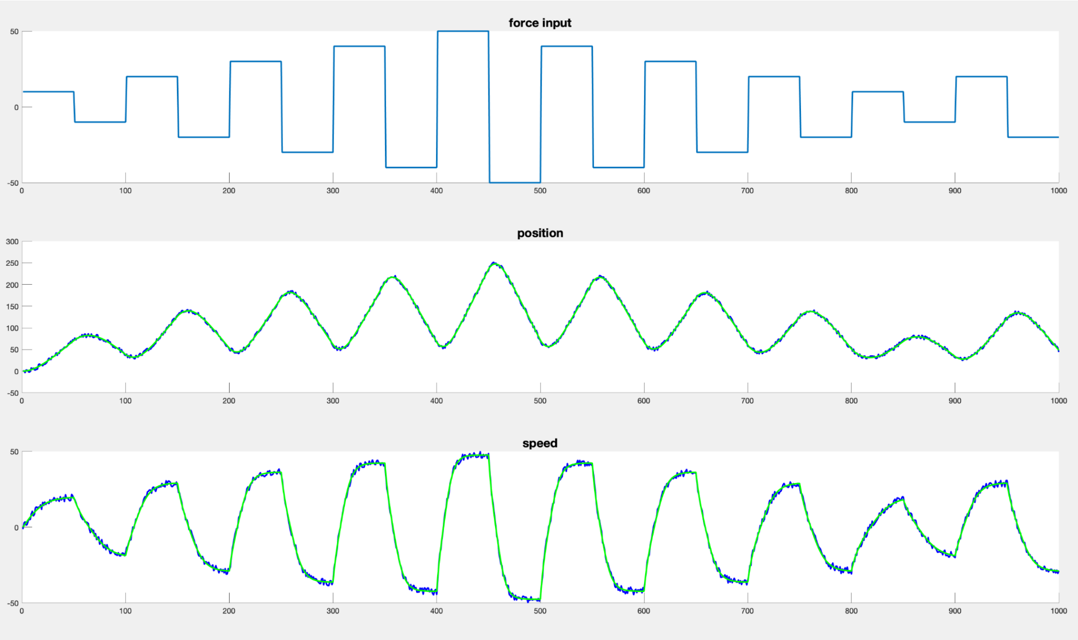
massimo.caccia@cnr.it

Example: quadratic drag system (e.g. heave motion)

$$x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$$

$$x_2(k) = x_2(k-1) + \left[-k_{\dot{x}} x_2(k-1) - k_{\dot{x}|\dot{x}|} x_2(k-1)\,|\,x_2(k-1)\,| + u_k\right] \Delta t + \sigma(k)$$

$$y(k) = x_1(k) + \nu(k)$$



massimo.caccia@cnr.it

Example: Linear Kalman filter

$$x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$$
$$x_2(k) = x_2(k-1) + \sigma(k)$$
$$y(k) = x_1(k) + \nu(k)$$

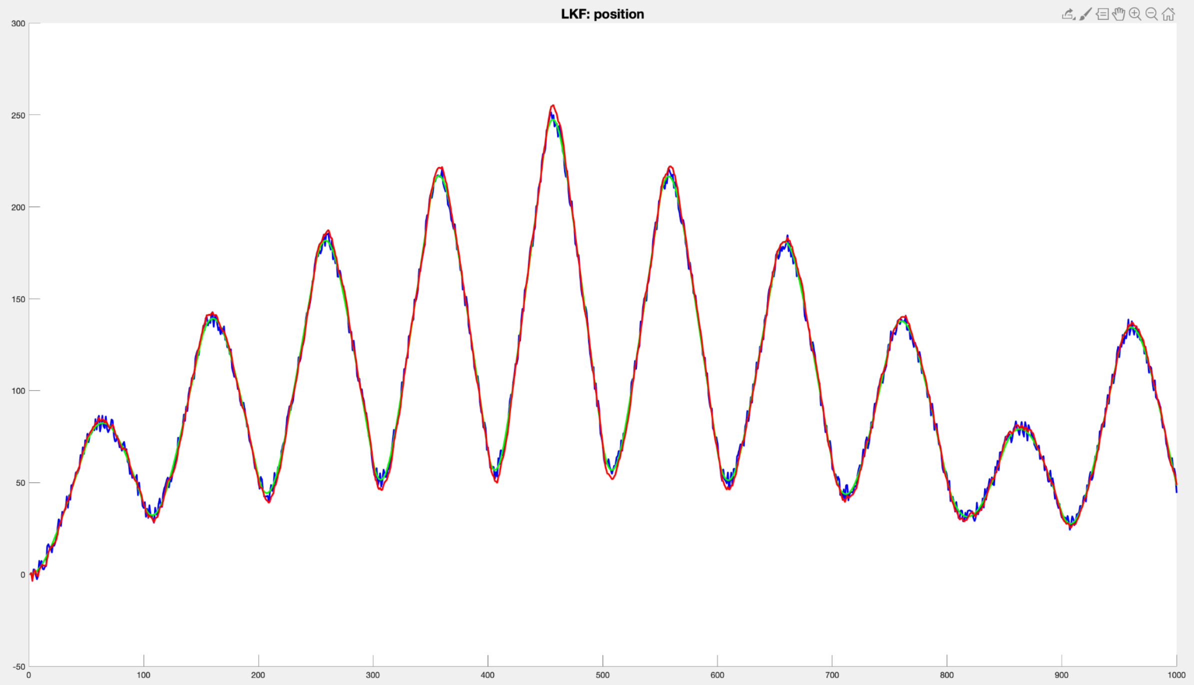
- position is measured
- pure kinematic model, no knowledge of the dynamics
    - control input is neglected
    - system noise models the possibility of velocity changes

$$\hat{x}_2 = \hat{\dot{x}} \rightarrow \hat{x}_2 \neq \dot{x}_1$$

massimo.caccia@cnr.it

Example: Linear Kalman filter - estimated position

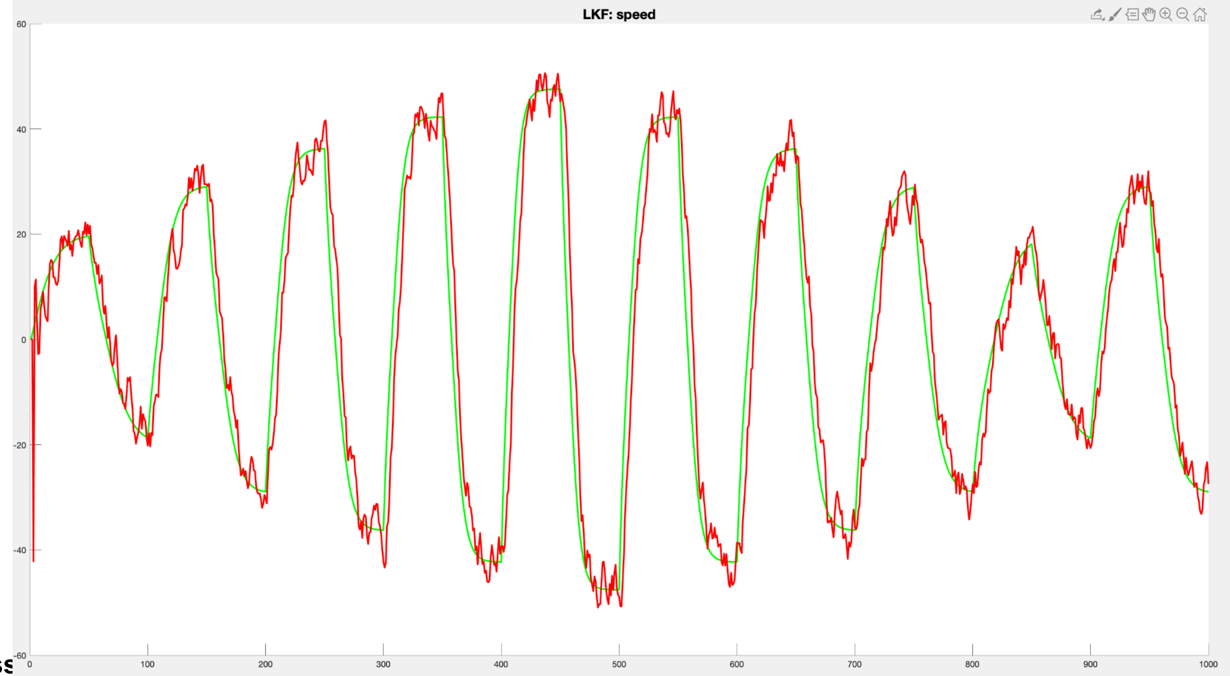
$$x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$$
$$x_2(k) = x_2(k-1) + +\sigma(k)$$
$$y(k) = x_1(k) + \nu(k)$$



Example: Linear Kalman filter - estimated velocity

$$x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$$
$$x_2(k) = x_2(k-1) + +\sigma(k)$$
$$y(k) = x_1(k) + \nu(k)$$

delay and noise in the velocity estimate



Extended Kalman Filter

- Nonlinear system and measurement equations (observations)
  - $x_k = f\left(x_{k-1}\, , \, u_k\right) + w_k$
  - $y_k = h\left(x_k\right) + \nu_k$
- State transition matrix

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}\, , \, u_k}$$

- Observation matrix

$$H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k|k-1}}$$

massimo.caccia@cnr.it

Extended Kalman Filter

- Nonlinear system and measurement equations (observations)
  - $x_k = f\left(x_{k-1}\ , u_k\right) + w_k$
  - $y_k = h\left(x_k\right) + \nu_k$
- Prediction
  - predicted state estimate : $\hat{x}_{k|k-1} = f\left(\hat{x}_{k-1|k-1}\ , u_k\right)$
  - predicted estimate covariance : $P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$
- Innovation
  - innovation : $\tilde{y}_k = y_k - h\left(\hat{x}_{k|k-1}\right)$
  - innovation covariance : $S_k = H_k P_{k|k-1} H_k^T + R_k$

massimo.caccia@cnr.it

Linear Kalman Filter

- Update
  - near-optimal Kalman gain : $K_k = P_{k-1|k} H_k^T S_k^{-1}$
  - updated state estimate : $\hat{x}_{k|k} = \hat{x}_{k-1|k} + K_k \tilde{y}_k$
  - updated estimate covariance : $P_{k|k} = \left(I - K_k H_k\right) P_{k-1|k}$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left[y(k) - h(\hat{x}_{k|k-1}) \right]$$

predicted state gain

predicted observation

innovation

massimo.caccia@cnr.it

Example: Extended Kalman filter

$$x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$$

$$x_2(k) = x_2(k-1) + \left[-k_{\dot{x}} x_2(k-1) - k_{\dot{x}|\dot{x}|} x_2(k-1)\,|\,x_2(k-1)\,| + u_k\right] \Delta t + \sigma(k)$$

$$y(k) = x_1(k) + \nu(k)$$

- position is measured
- dynamics model
    - control input is considered
    - nonlinear transition matrix
    - system noise models actuation and dynamics uncertainty

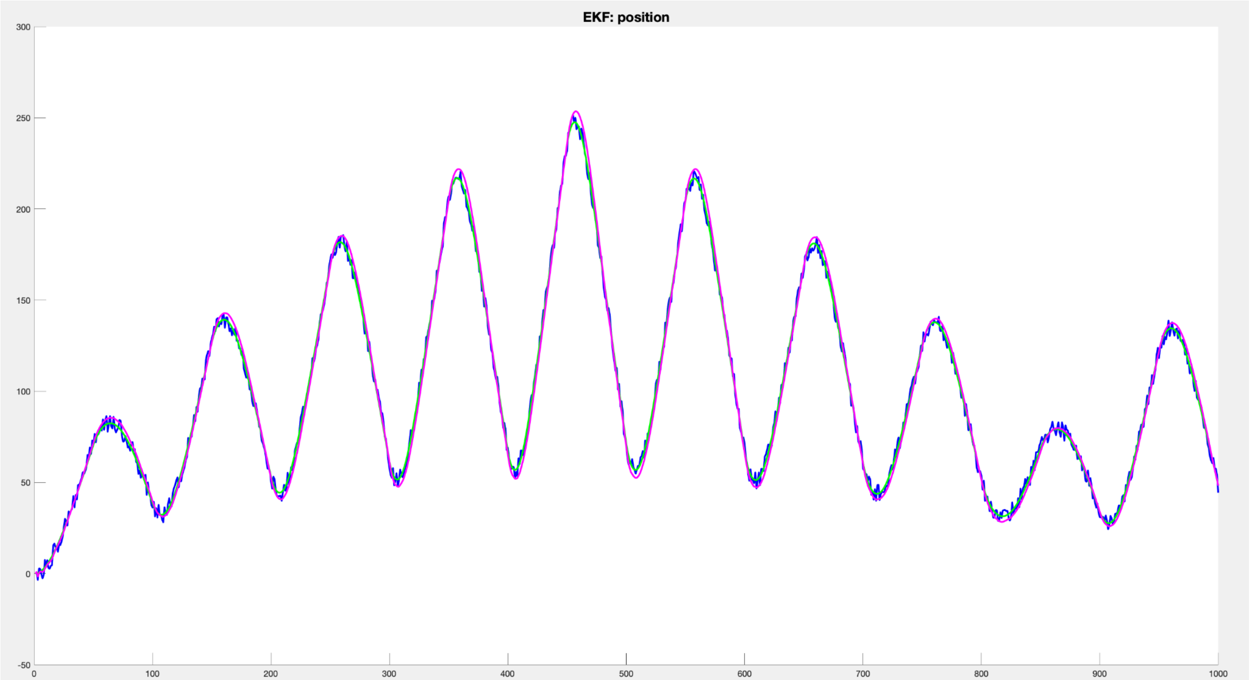
massimo.caccia@cnr.it

Example: Extended Kalman filter - estimated position

$$x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$$

$$x_2(k) = x_2(k-1) + \left[-k_{\dot{x}} x_2(k-1) - k_{\dot{x}|\dot{x}|} x_2(k-1)\,|x_2(k-1)| + u_k \right] \Delta t + \sigma(k)$$

$$y(k) = x_1(k) + \nu(k)$$



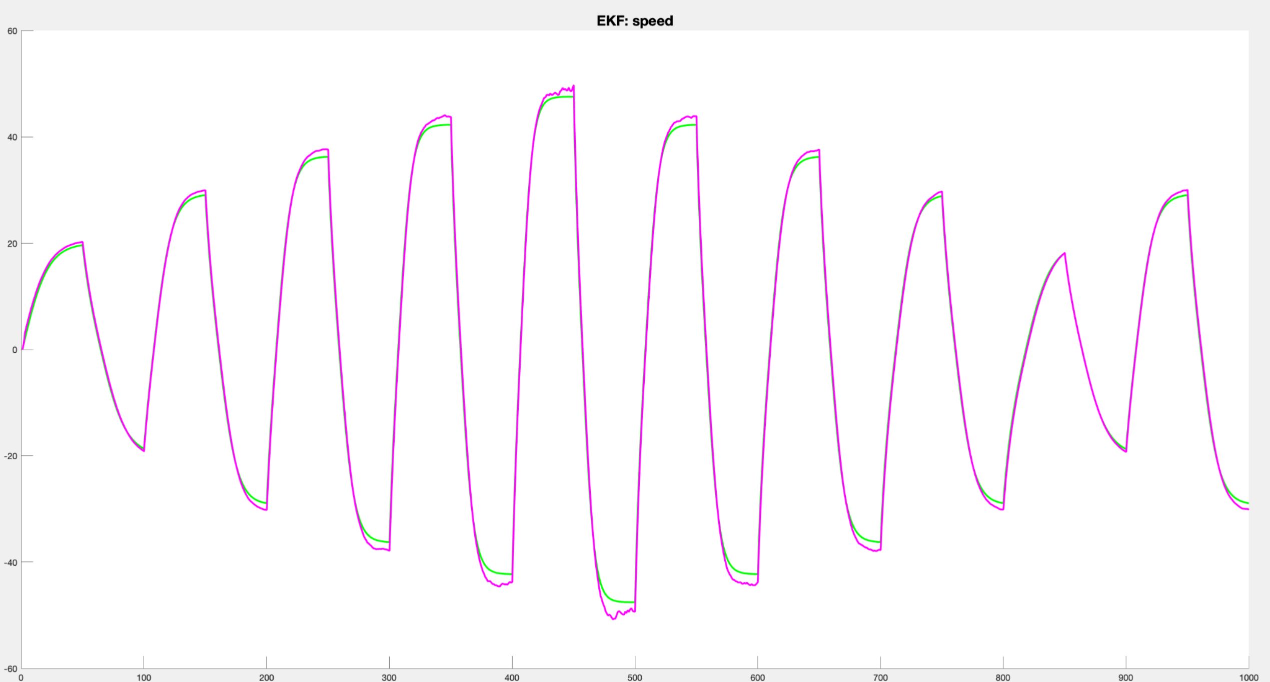
Example: Extended Kalman filter - estimated velocity

$$x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$$

$$x_2(k) = x_2(k-1) + \left[-k_{\dot{x}} x_2(k-1) - k_{\dot{x}|\dot{x}|} x_2(k-1)\,|\,x_2(k-1)\,| + u_k\right] \Delta t + \sigma(k)$$

$$y(k) = x_1(k) + \nu(k)$$

small offset,
but no delay and low noise in the velocity estimate



Multirate Extended Kalman Filter

- Nonlinear system and measurement equations (observations)
  - $x_k = f\left(x_{k-1}\ , u_k\right) + w_k$
  - $y_k = h_k\left(x_k\right) + \nu_k$
- Time-variant observation matrix (measurement channels)
  - at each time k only a subset of the possible measurements is available

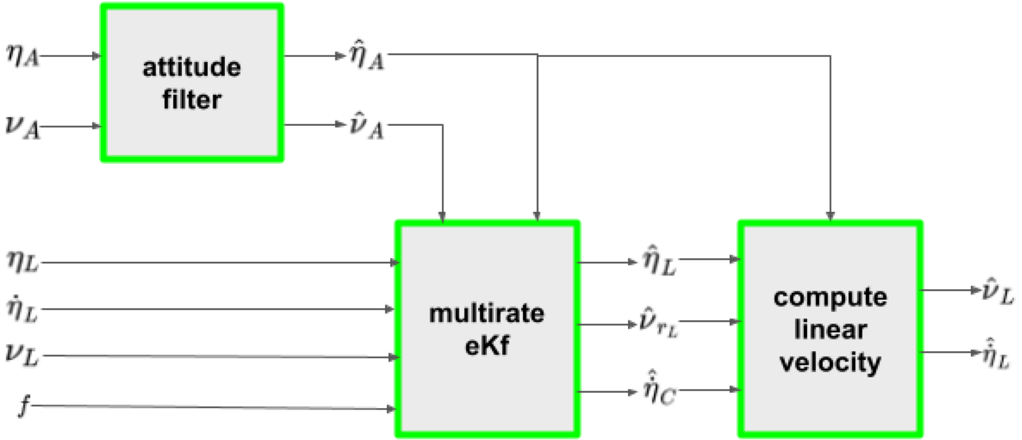
$$h_k(x_k) = \left[h_1^T(x_k)\ ,\ \ldots\ ,\ h_1^T(x_k)\right]^T\ ,\ size(y_k) \leq \sum_{i=1}^{m} size\left(h_i(x_k)\right)$$

  - at each step innovation has the size of available measurements
  - when no measurements are available a pure prediction step is executed

massimo.caccia@cnr.it

Example: ASV navigation system

- $\eta_L = \left[x\ ,\ y\right]^T$ $\qquad\qquad \eta_A = \psi$
- $\nu_L = [u\ ,\ v]^T$ $\qquad\qquad \nu_A = r$
- $\dot{\eta}_L = \left[U\cos\chi\ ,\ U\sin\chi\right]^T \leftarrow \left[U\ ,\ \chi\right]^T$ $\qquad\qquad f = [X\ ,\ Y]^T$



ISTITUTO DI INGEGNERIA DEL MARE
INSTITUTE OF MARINE ENGINEERING
massimo.caccia@cnr.it

ASV yaw motion: Linear Kalman Filter

$$\psi(k) = x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$$
$$r(k) = x_2(k) = x_2(k-1) + \sigma(k)$$
$$y(k) = \left[y_1(k)\ ,\ y_2(k)\right]^T = \left[x_1(k)\ ,\ x_2(k)\right]^T) + \nu(k)$$

- heading is measured (low noise)
- yaw rate is measured (low noise)
- pure kinematic model, no knowledge of the dynamics
    - control input is neglected
    - system noise models the possibility of velocity changes

massimo.caccia@cnr.it

ASV horizontal motion: multirate Extended Kalman Filter

- augmented state :

- $\psi(k) = x_1(k) = x_1(k-1) + x_2(k-1)\Delta t$

$r(k) = x_2(k) = x_2(k-1) + \sigma(k)$

$y(k) = \left[y_1(k)\ ,\ y_2(k)\right]^T = \left[x_1(k)\ ,\ x_2(k)\right]^T) + \nu(k)$

- heading is measured (low noise)
- yaw rate is measured (low noise)
- pure kinematic model, no knowledge of the dynamics
    - control input is neglected
    - system noise models the possibility of velocity changes

massimo.caccia@cnr.it

ASV horizontal motion: multirate Extended Kalman Filter

- augmented state with linear velocities with respect to the water in the body-fixed reference frame and sea current

$$[x\ ,\ y\ ,\ u_r\ ,\ v_r\ ,\ \dot{x}_C\ ,\ \dot{y}_C]^T$$

$$\begin{aligned}
\dot{x} &= u_r \cos\hat{\psi} - v_r \sin\hat{\psi} + \dot{x}_C \\
\dot{y} &= u_r \sin\hat{\psi} + v_r \cos\hat{\psi} + \dot{y}_C \\
\dot{u}_r &= \tilde{X}_u u_r + \tilde{X}_{u|u|}|u_r|u_r + \frac{m_v}{m_u} v_r \hat{r} + \tilde{m}_{r^2}\hat{r}^2 + \frac{1}{m_u} X \\
\dot{v}_r &= \tilde{X}_v v_r + \tilde{X}_{v|v|}|v_r|v_r - \frac{m_u}{m_v} u_r \hat{r} + \frac{1}{m_v} Y \\
\ddot{x}_C &= 0 \\
\ddot{y}_C &= 0
\end{aligned} \tag{39}$$

where $m_u = (m - X_{\dot{u}})$, $m_v = (m - Y_{\dot{v}})$, $\tilde{X}_u = \frac{X_u}{m_u}$, $\tilde{X}_{u|u|} = \frac{X_{u|u|}}{m_u}$, $\tilde{m}_{r^2} = \frac{(mx_g - Y_{\dot{r}})}{m_u}$, $\tilde{X}_v = \frac{X_v}{m_v}$, and $\tilde{X}_{v|v|} = \frac{X_{v|v|}}{m_v}$.

massimo.caccia@cnr.it

ASV horizontal motion: multirate Extended Kalman Filter

- Linearised state transition matrix

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & \cos\hat{\psi}\Delta t & -\sin\hat{\psi}\Delta t & \Delta t & 0 \\ 0 & 1 & \sin\hat{\psi}\Delta t & \cos\hat{\psi}\Delta t & 0 & \Delta t \\ 0 & 0 & 1 + \left(\tilde{X}_u + 2\tilde{X}_{u|u|}\,|u_r|\right)\Delta t & \dfrac{m_v}{m_u}\hat{r}\Delta t & 0 & 0 \\ 0 & 0 & -\dfrac{m_u}{m_v}\hat{r}\Delta t & 1 + \left(\tilde{X}_v + 2\tilde{X}_{v|v|}\,|v_r|\right)\Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

massimo.caccia@cnr.it

ASV horizontal motion: multirate Extended Kalman Filter

- Control input matrix

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_u}\Delta t & 0 \\ 0 & \frac{1}{m_v}\Delta t \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

massimo.caccia@cnr.it

ASV horizontal motion: multirate Extended Kalman Filter

- GNSS: GGA sentence (position)

$$x_{GNSS} = x + \Delta x \cos\hat{\psi} - \Delta y \sin\hat{\psi}$$
$$y_{GNSS} = y + \Delta x \sin\hat{\psi} + \Delta y \cos\hat{\psi}$$

- Linearised observation matrix

$$\mathbf{H}_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

massimo.caccia@cnr.it

ASV horizontal motion: multirate Extended Kalman Filter

- GNSS: VTG sentence (speed)

$$\dot{x}_{GNSS} = u_r \cos\hat{\psi} - v_r \sin\hat{\psi} + \dot{x}_C - \hat{r}\left(\Delta x \sin\hat{\psi} + \Delta y \cos\hat{\psi}\right)$$
$$\dot{y}_{GNSS} = u_r \sin\hat{\psi} + v_r \cos\hat{\psi} + \dot{y}_C + \hat{r}\left(\Delta x \cos\hat{\psi} - \Delta y \sin\hat{\psi}\right)$$

- Linearised observation matrix

$$\mathbf{H}_{U\chi} = \begin{bmatrix} 0 & 0 & \frac{\partial U}{\partial u_r} & \frac{\partial U}{\partial v_r} & \frac{\partial U}{\partial \dot{x}_C} & \frac{\partial U}{\partial \dot{y}_C} \\ 0 & 0 & \frac{\partial \chi}{\partial u_r} & \frac{\partial \chi}{\partial v_r} & \frac{\partial \chi}{\partial \dot{x}_C} & \frac{\partial \chi}{\partial \dot{y}_C} \end{bmatrix}$$

massimo.caccia@cnr.it

ASV horizontal motion: multirate Extended Kalman Filter

- DVL

$$u_{DVL} = u_r + \dot{x}_C \cos\psi + \dot{y}_C \sin\psi - r\Delta y$$
$$v_{DVL} = v_r - \dot{x}_C \sin\psi + \dot{y}_C \cos\psi + r\Delta x$$

- Linearised observation matrix

$$\mathbf{H}_{uv} = \begin{bmatrix} 0 & 0 & 1 & 0 & \cos\hat{\psi} & \sin\hat{\psi} \\ 0 & 0 & 0 & 1 & -\sin\hat{\psi} & \cos\hat{\psi} \end{bmatrix}$$

massimo.caccia@cnr.it

ASV horizontal motion: multirate Extended Kalman Filter

- Observation matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{xy} \\ \mathbf{H}_{uv} \\ \mathbf{H}_{U\chi} \end{bmatrix}$$

massimo.caccia@cnr.it