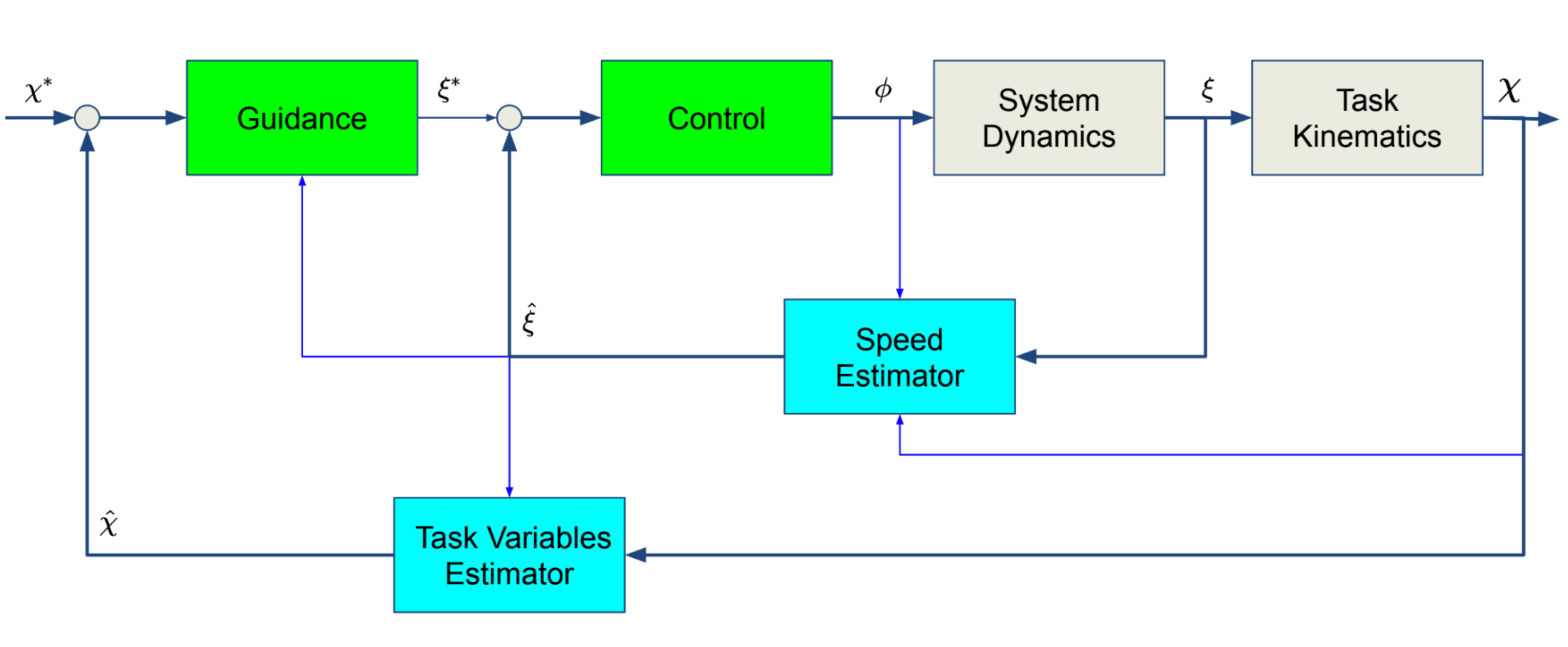

Guidance and Control of UMVs

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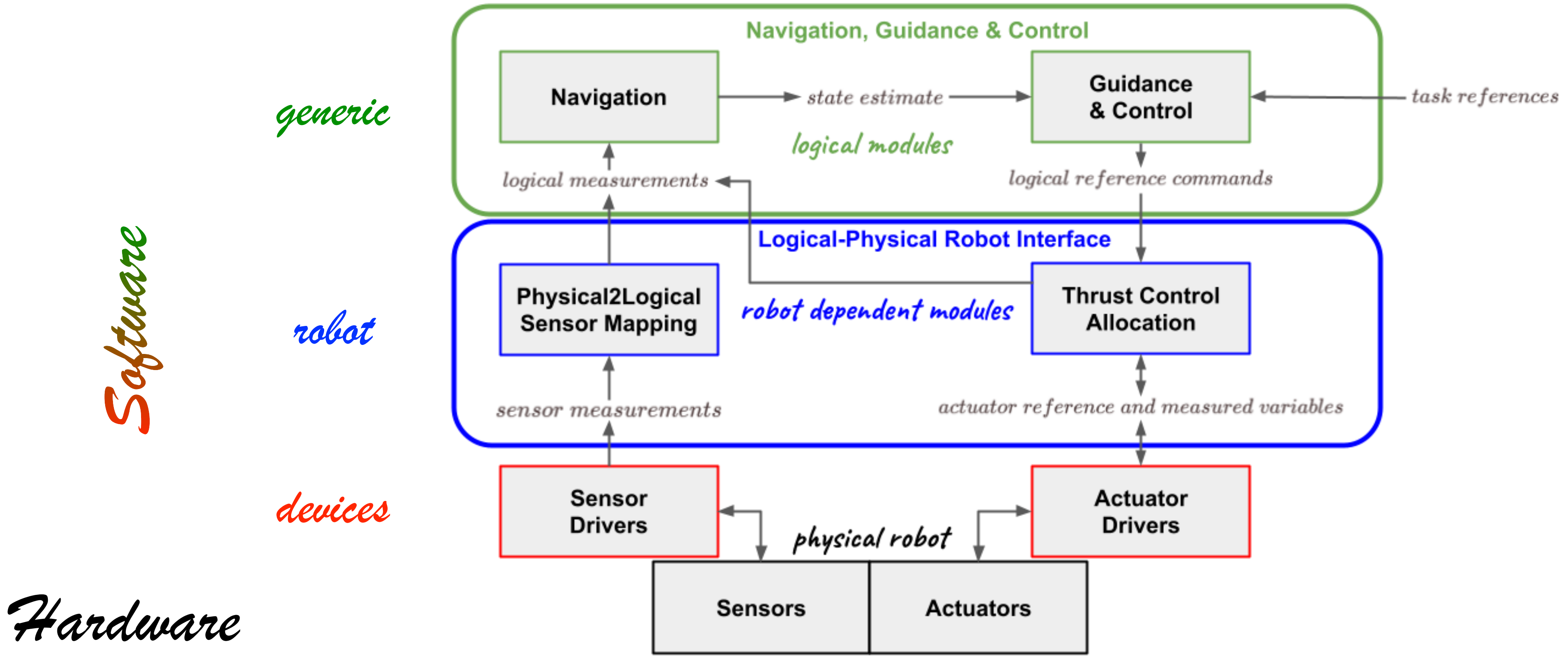
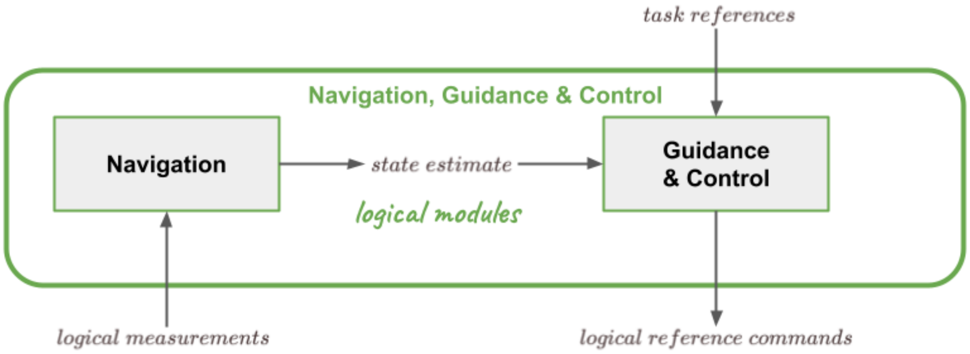
AMV NGC architecture



AMV Navigation, Guidance and Control



- **Navigation** estimates the motion of the vehicle
- **Guidance** handles the vehicle's kinematics executing the desired motion task functions
- **Control** handles the vehicle's dynamics implementing linear and angular velocity controllers

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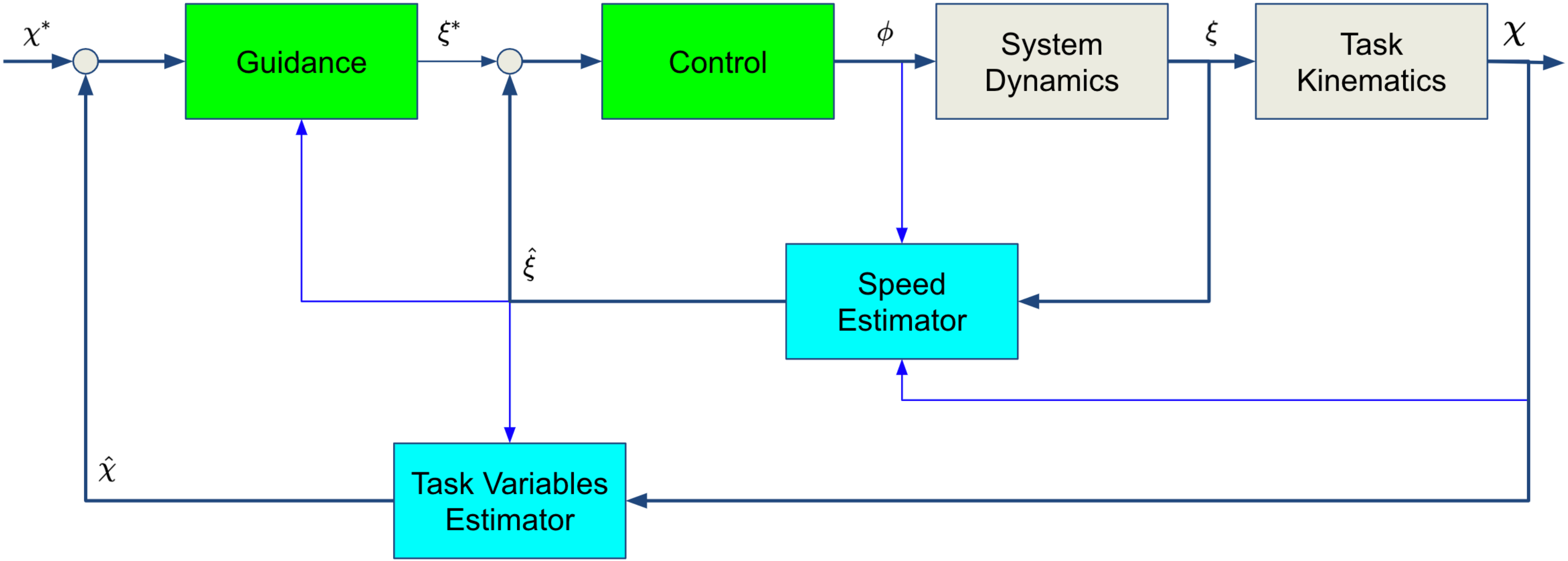
Dual loop guidance and control architecture



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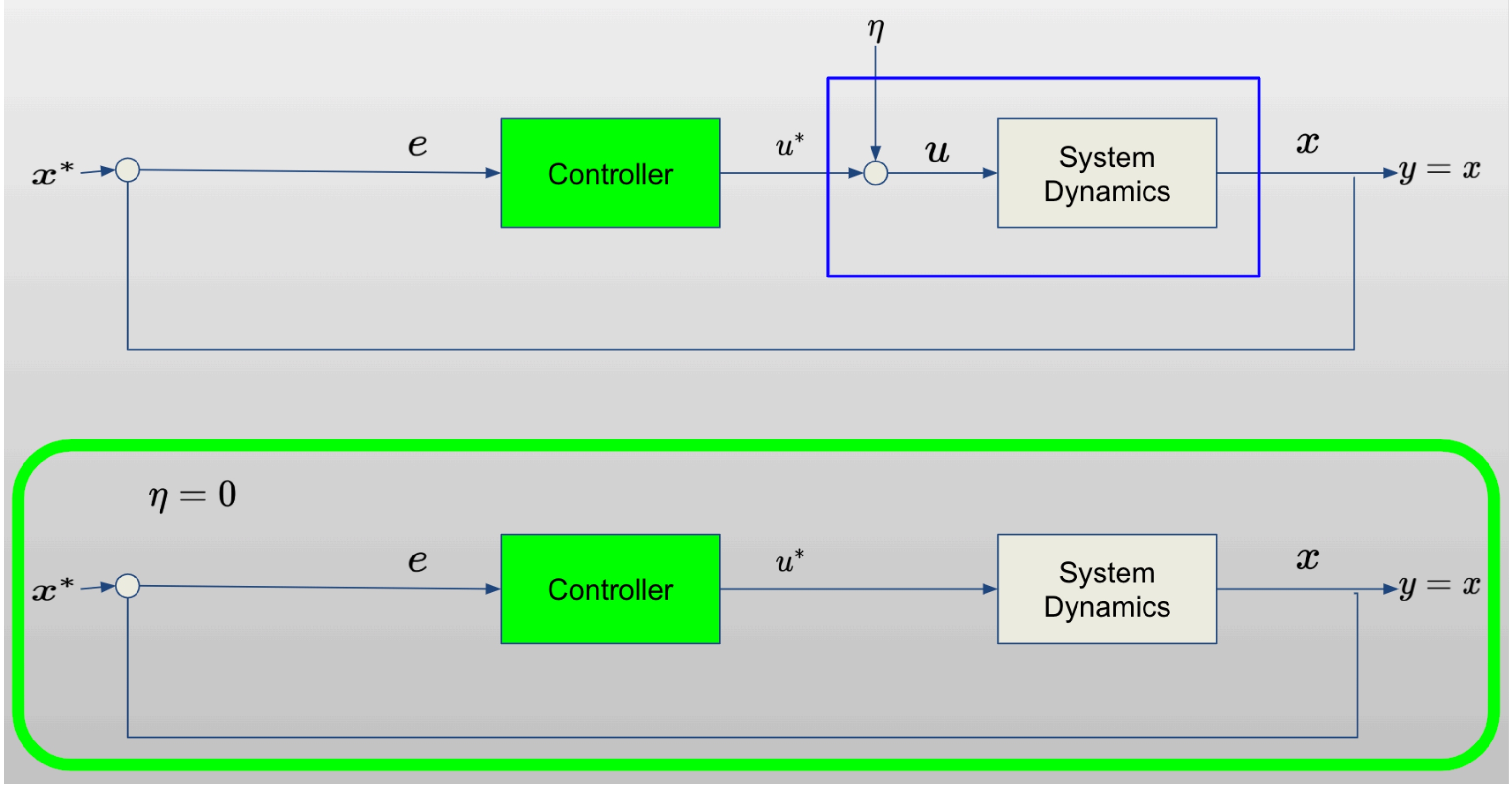
Closed-loop control system



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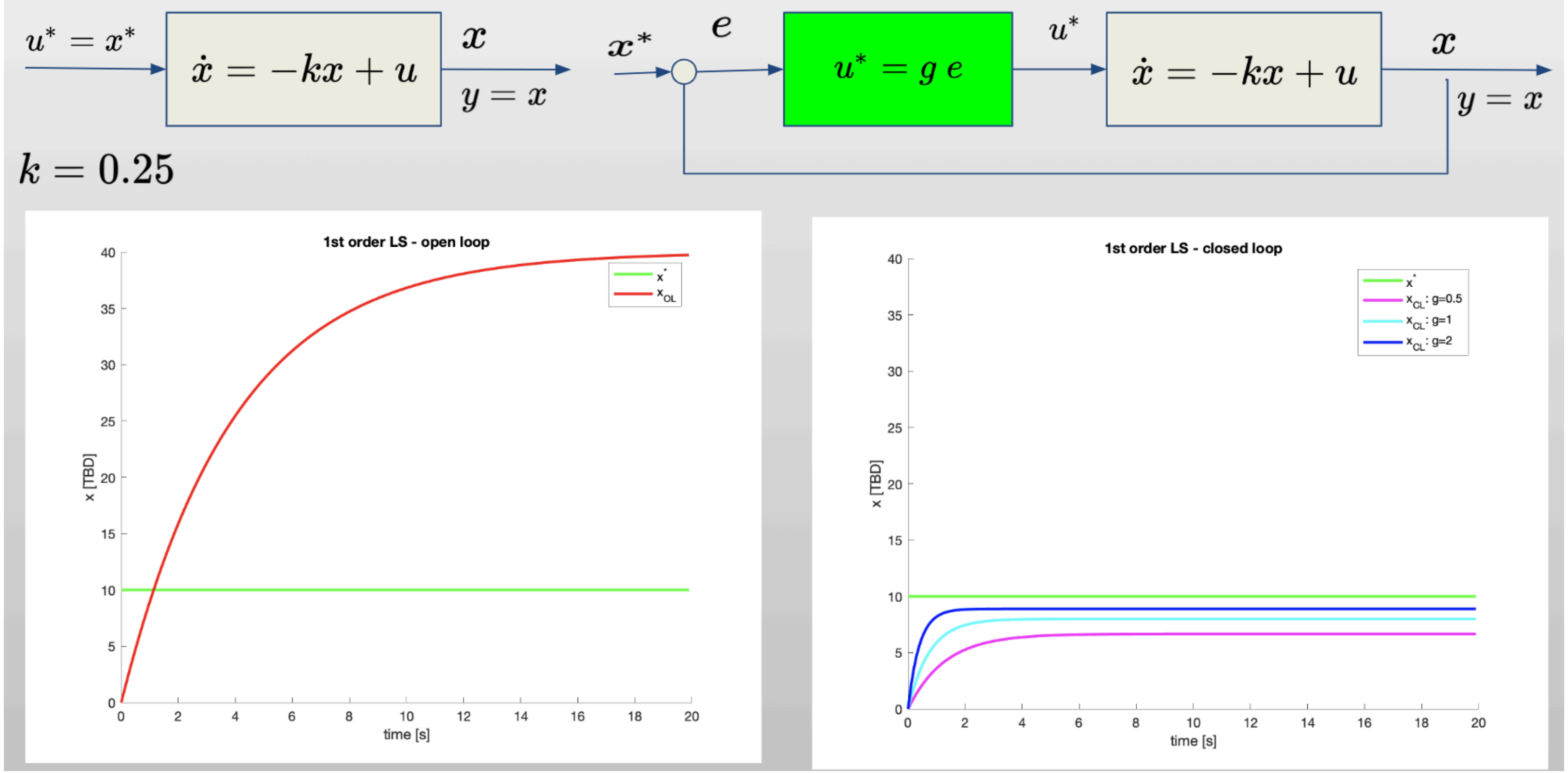
Open-loop vs. closed-loop system



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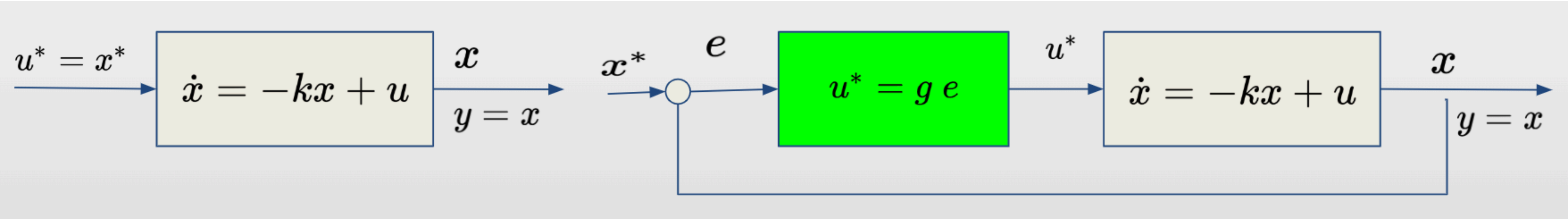
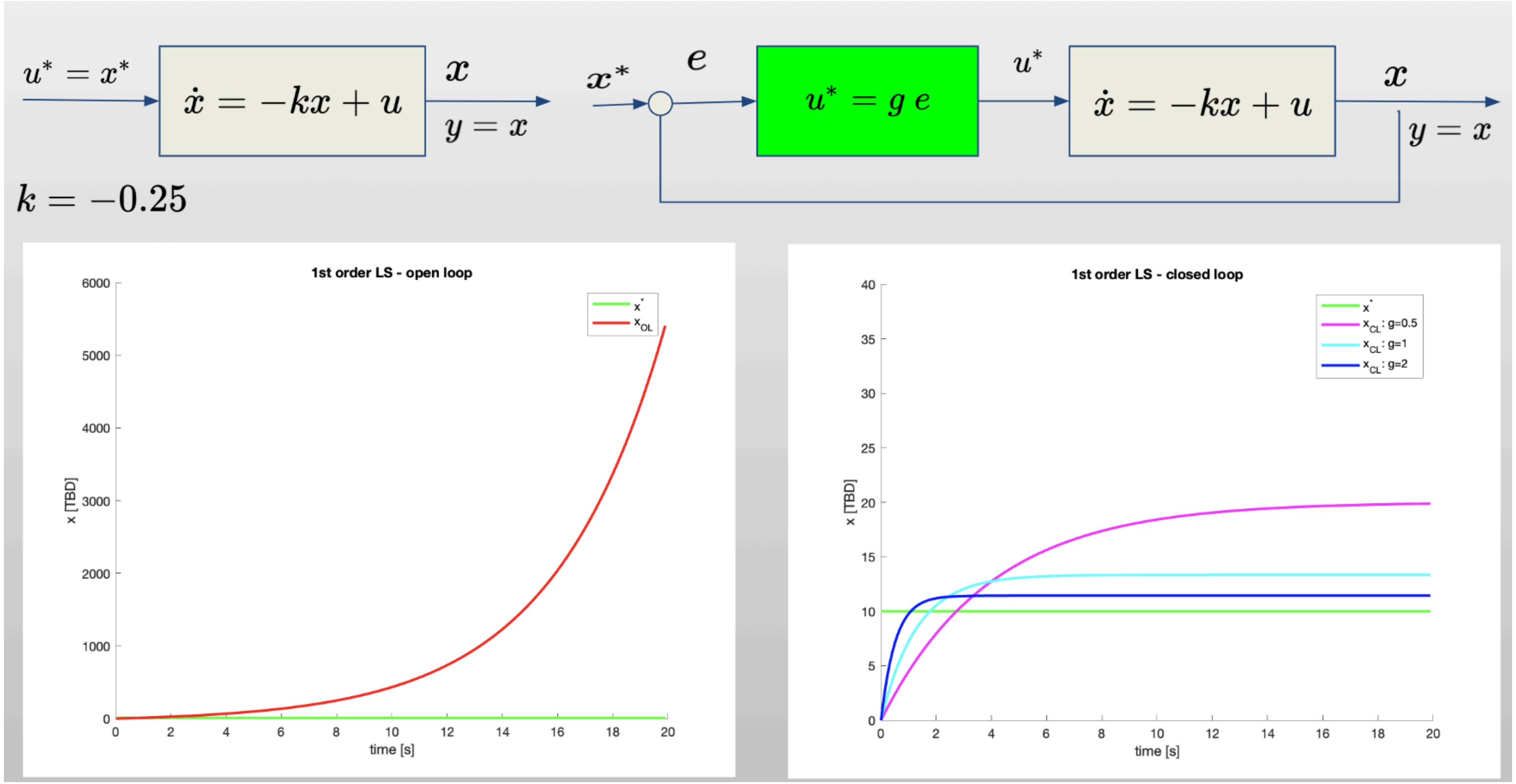
Open-loop vs. closed-loop system



- increasing the feedback gain reduces
  - the time constant of the closed-loop system
  - the steady-state error of the closed-loop system

Open-loop vs. closed-loop system



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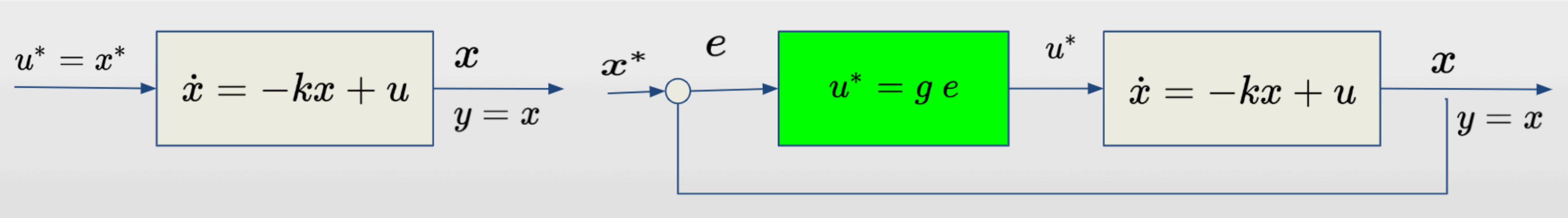
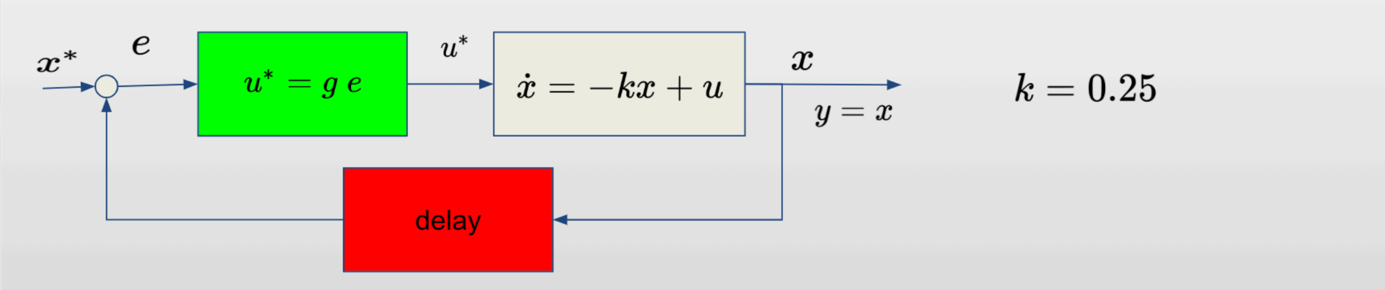
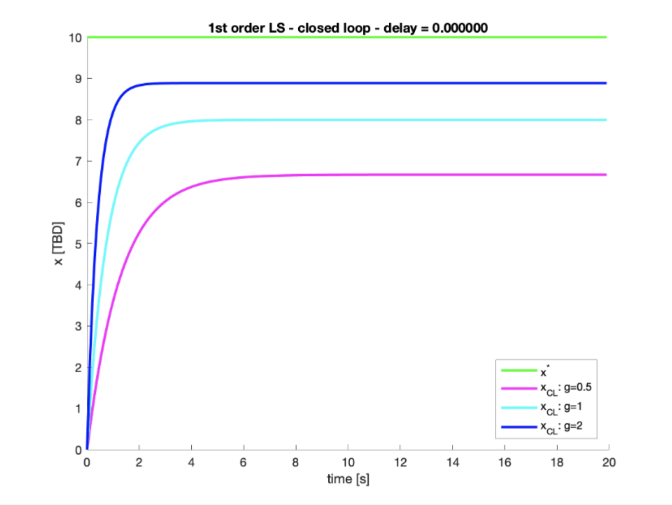
Open-loop vs. closed-loop system

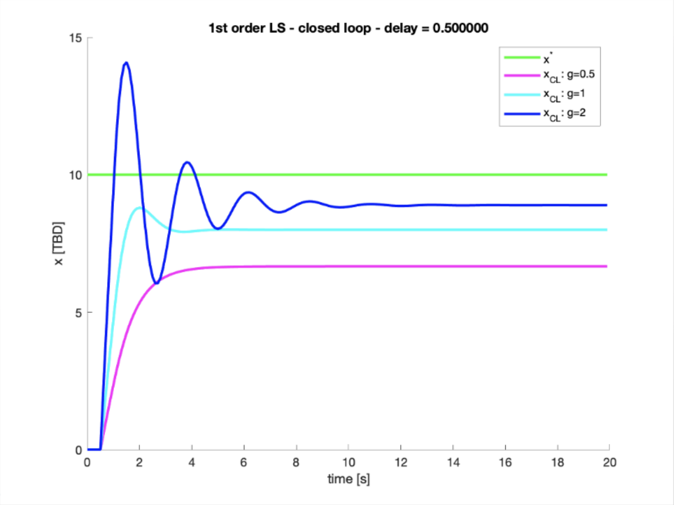


- **the feedback stabilises the system**

Closed-loop system with delay







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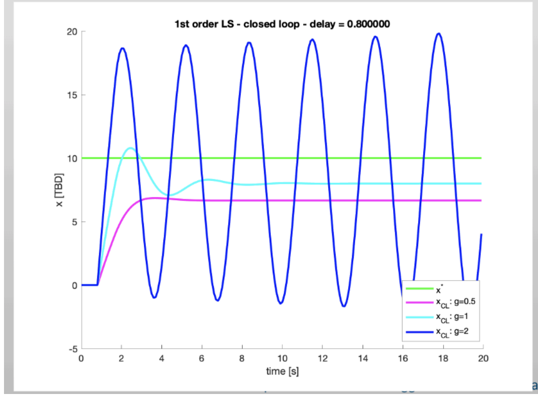
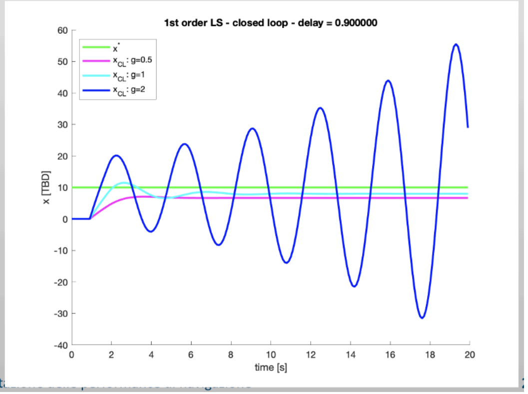
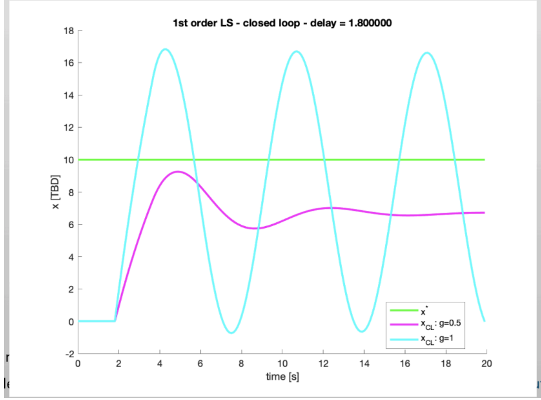
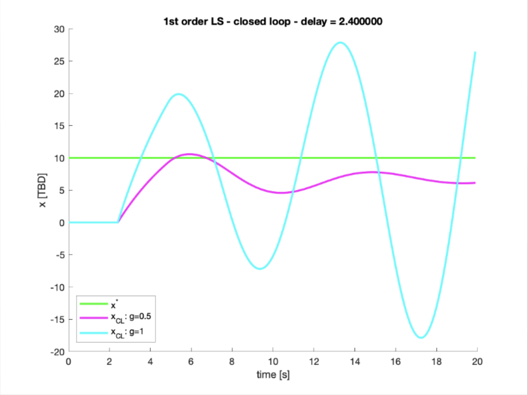
Closed-loop system with delay









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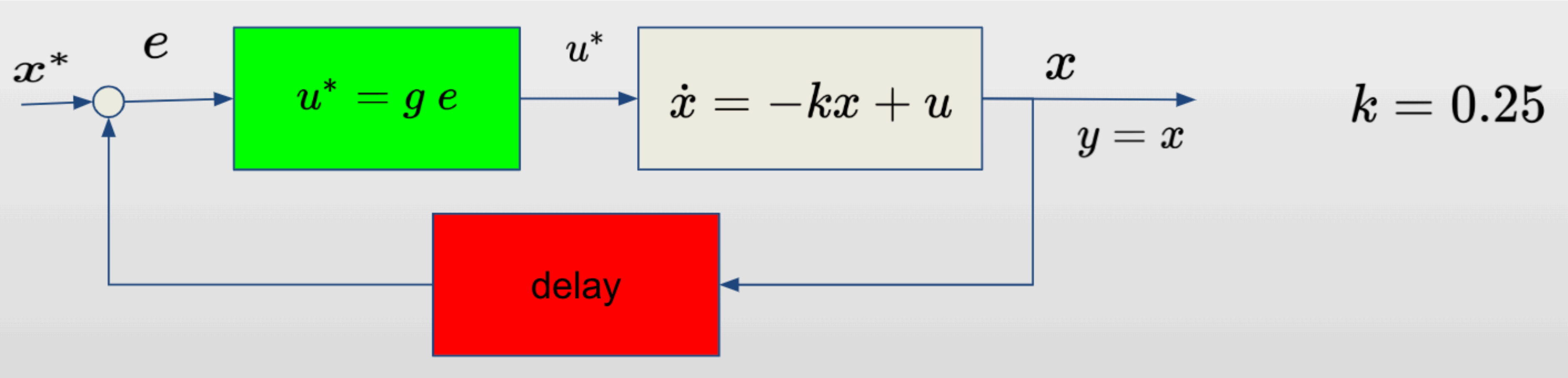
Closed-loop system with delay



- **delay makes the closed-loop system unstable**
- **increasing the gain in the presence of delay makes the closed-loop system unstable**

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System equilibrium and stability

$$\dot{x} = f(x) \;,\; x(t_0) = x_0$$

- **Lyapunov stability** of an equilibrium means that solutions starting "close enough" to the equilibrium remain "close enough" forever
- **Asymptotic stability** means that solutions that start close enough not only remain close enough but also eventually converge to the equilibrium

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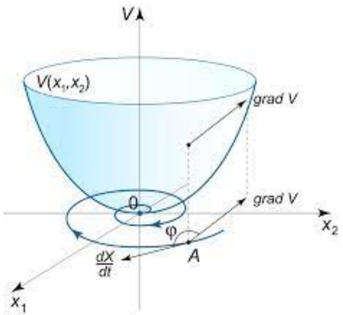
Stability analysis

- **Problem statement**: analyse the stability of a time invariant system with an equilibrium point at x=0

- **Stability: Lyapunov's Direct Method**
  - **Candidate Lyapunov function**:
    $$V(x) \ : \ V(x) > 0 \ , 0 < \| \, x \, \| \ \ for\ some\ r$$
  - $\dot{V}(x)$ derivative along the trajectories of the system $\dot{x} = f\left(x(t)\right)$
  - $V(x)$ is a **Lyapunov function** of $\dot{x} = f\left(x(t)\right)$
    if $\dot{V}(x) \ : \ \dot{V}(x) \leq 0 \ , 0 < \| \, x \, \| \ \ for\ some\ r$



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Lyapunov stability analysis

- **Lyapunov Theorem for Local Stability**
  - If there exists a Lyapunov function of system $\dot{x} = f\left(x(t)\right)$ then $x = 0$ is a **stable equilibrium point in the sense of Lyapunov**.
  - If in addition $\dot{V}(x) \ : \ \dot{V}(x) < 0\ ,\ 0 <\| x \| \ \ for\ some\ r$ then $x = 0$ is an **asymptotically stable equilibrium point**

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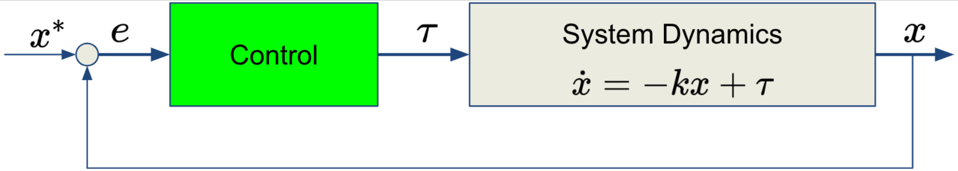
Handling dynamics: velocity control

Hp. Linear system

$$\dot{x} = -kx + \tau$$



Candidate Lyapunov function $V(x) = \frac{1}{2}(x^* - x)^2$

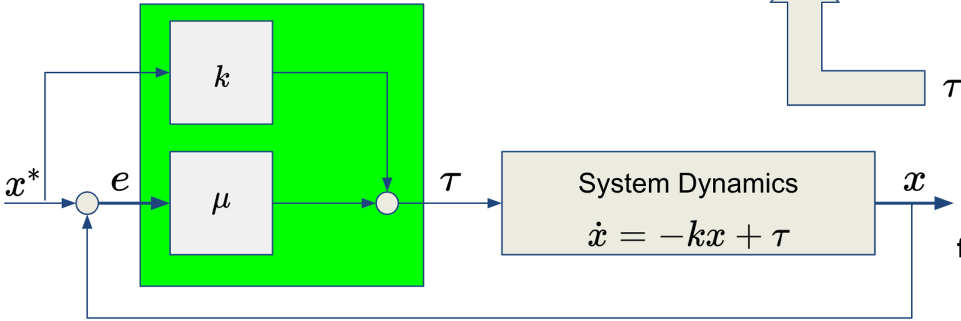
$$\dot{V}(x) = -(x^* - x)\dot{x} = -(x^* - x)(-kx + \tau)$$

$$\dot{V}(x) = -(k + \mu)(x^* - x)^2$$

$$\tau = \boxed{kx^*} + \boxed{\mu(x^* - x)}$$

feed-forward

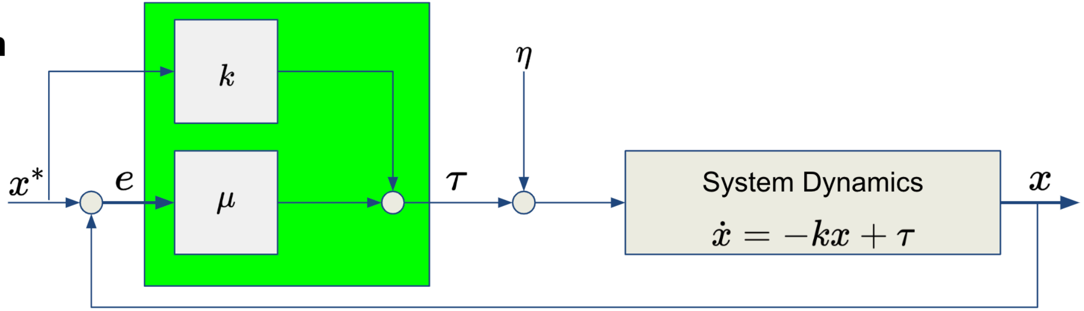
feed-back proportional



Velocity control: disturbance compensation

Hp. Linear system

$$\dot{x} = -kx + \tau + \eta$$



Proportional feed-back control $\tau = kx^* + \mu(x^* - x)$

$$\dot{x} = -kx + kx^* + \mu(x^* - x) + \eta = (k + \mu)(x^* - x) + \eta$$

Equilibrium $\dot{x} = 0$ $x = x^* + \boxed{\frac{\eta}{k+\mu}}$ **error is not zero**

error goes to zero when the proportional gain goes to infinity

$$\boxed{\lim_{\mu \to \infty} \frac{\eta}{k+\mu} = 0}$$

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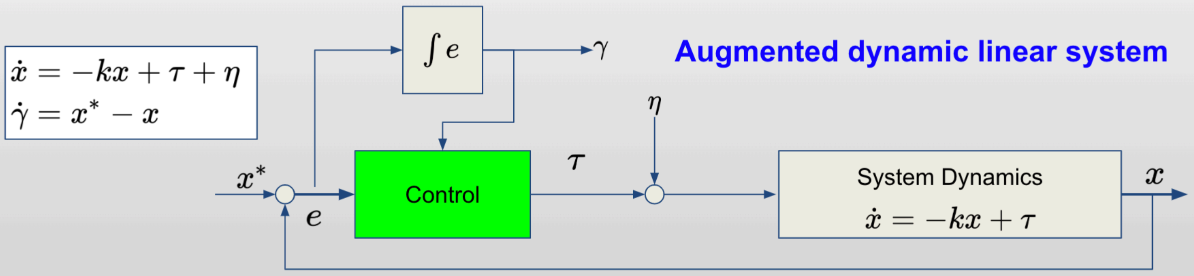
Velocity control: disturbance compensation



Candidate Lyapunov function $V(x,\gamma) = \frac{1}{2}(x^* - x)^2 + \frac{1}{2}\lambda(\gamma + \frac{\eta}{\lambda})^2$

$$\dot{V}(x,\gamma) = (kx - \tau + \lambda\gamma)(x^* - x) \Rightarrow \dot{V}(x,\gamma) = -(k+\mu)(x^* - x)^2$$

$$\tau = \boxed{kx^*} + \boxed{\mu(x^* - x)} + \boxed{\lambda\gamma}$$

feed-forward

feed-back proportional

integral

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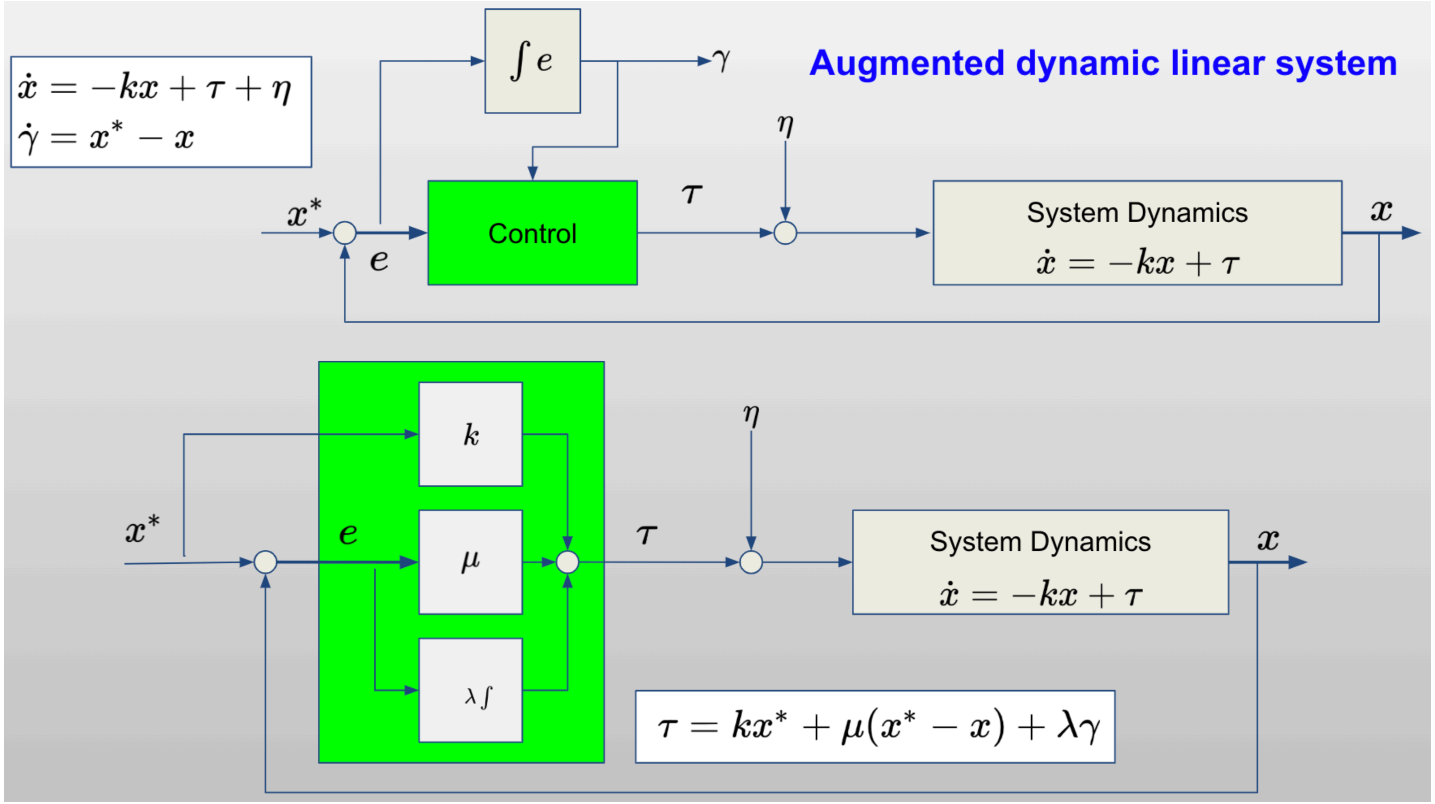
Velocity control: disturbance compensation



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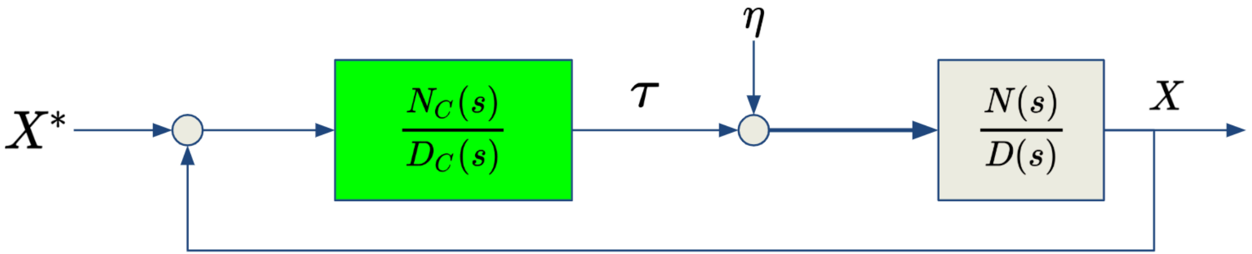
Disturbance compensation using Laplace transformation



$$X(s) = \frac{N(s)}{D(s)}\eta(s) + \frac{N(s)}{D(s)}\frac{N_C(s)}{D_C(s)}\left(X^*(s) - X(s)\right)$$

$$X(s) = \boxed{\frac{D_C(s)N(s)}{D(s)D_C(s)+N(s)N_C(s)}}\eta(s) + \frac{N_C(s)N(s)}{D(s)D_C(s)+N(s)N_C(s)}X^*(s)$$

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Disturbance compensation using Laplace transformation

Final Value Theorem $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$

Step function input and noise $X(s) = \frac{X^*}{s}\ ,\ \eta(s) = \frac{\eta^*}{s}$

$$\lim_{s\to 0} sX(s) = \lim_{s\to 0} \frac{D_C(s)N(s)}{D(s)D_C(s)+N(s)N_C(s)} s\frac{\eta^*}{s} + \lim_{s\to 0} \frac{N_C(s)N(s)}{D(s)D_C(s)+N(s)N_C(s)} s\frac{X^*}{s}$$

$$D_C(s) = s\bar{D}_C(s)$$

$$\lim_{s\to 0} sX(s) = \lim_{s\to 0} \cancelto{0}{\frac{s\bar{D}_C(s)N(s)}{D(s)s\bar{D}_C+N(s)N_C(s)}} s\frac{\eta^*}{s} + \lim_{s\to 0} \frac{N_C(s)N(s)}{D(s)s\bar{D}_C+N(s)N_C(s)} s\frac{X^*}{s} = X^*$$

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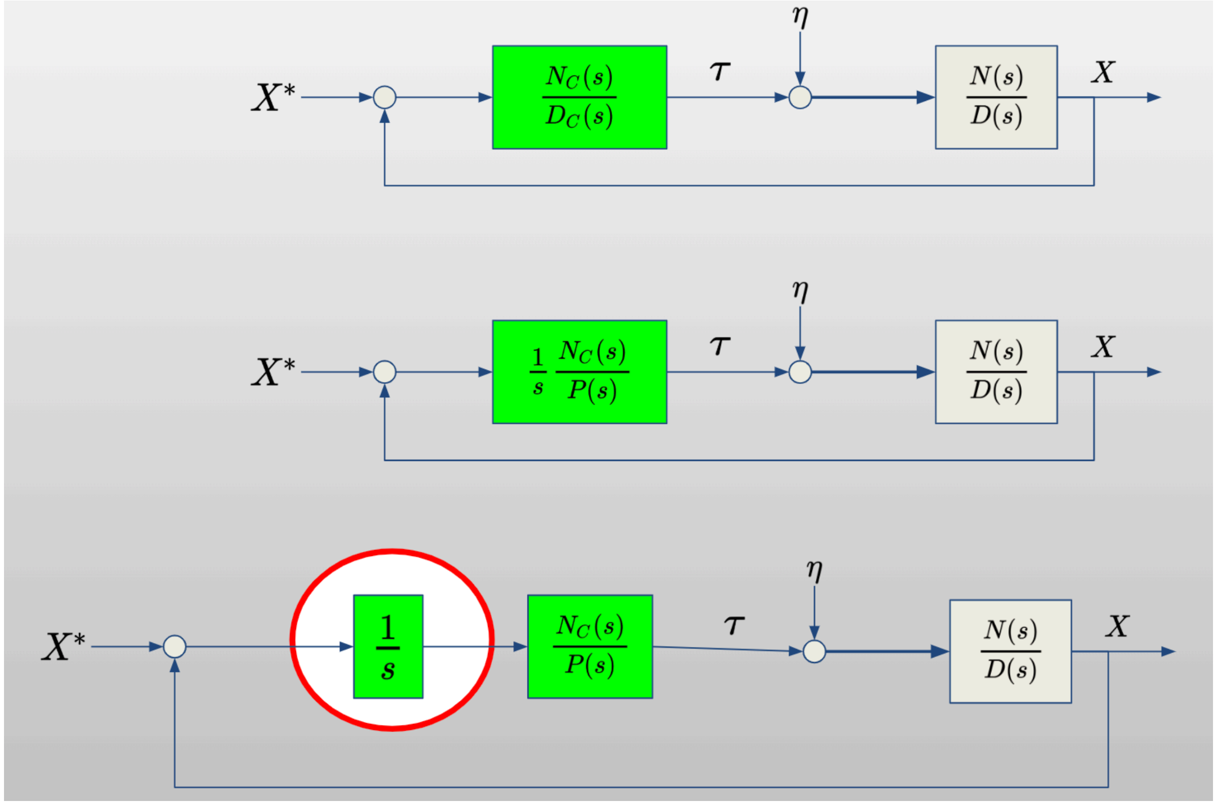
Disturbance compensation using Laplace transformation



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AMV dynamics: practical 1 d.o.f. uncoupled model

$$m_\xi \dot{\xi} = -k_\xi \xi - k_{\xi\|\xi\|} \xi\|\xi\| + \phi_\xi + \eta_\xi$$

$$m_\xi \dot{\xi} \approx -k_\xi \xi + \phi_\xi + \eta_\xi \ , \ at\ low\ speed$$

Linearised system dynamics

$$\dot{\xi}(\xi, \phi_\xi) \approx \dot{\xi}(\bar{\xi}, \bar{\phi}_\xi) + \frac{\partial \dot{\xi}}{\partial \xi}(\bar{\xi}, \bar{\phi}_\xi)(\xi - \bar{\xi}) + \frac{\partial \dot{\xi}}{\partial \phi_\xi}(\bar{\xi}, \bar{\phi}_\xi)(\phi_\xi - \bar{\phi}_\xi)$$

AMV Control system design: velocity control

- Given a constant operating point ξ^* the controller provides:
  - ϕ^* : feed-forward constant control value yielding zero error
  - ϕ_δ : feed-back control action, which assigns a desired characteristic equation to the closed-loop linearised system
- Controller : $\phi_\xi = \phi^* + \phi_\delta$
- For every value of the reference speed ξ^* in the operating range, a feed-forward control value $\phi^* = \left(k_\xi + k_{\xi\|\xi\|} \parallel \xi^* \parallel\right) \xi^*$ is applied, and the nonlinear state equation is linearised about the desired operating point $\left(\xi^*\ ,\ \phi^*\right)$

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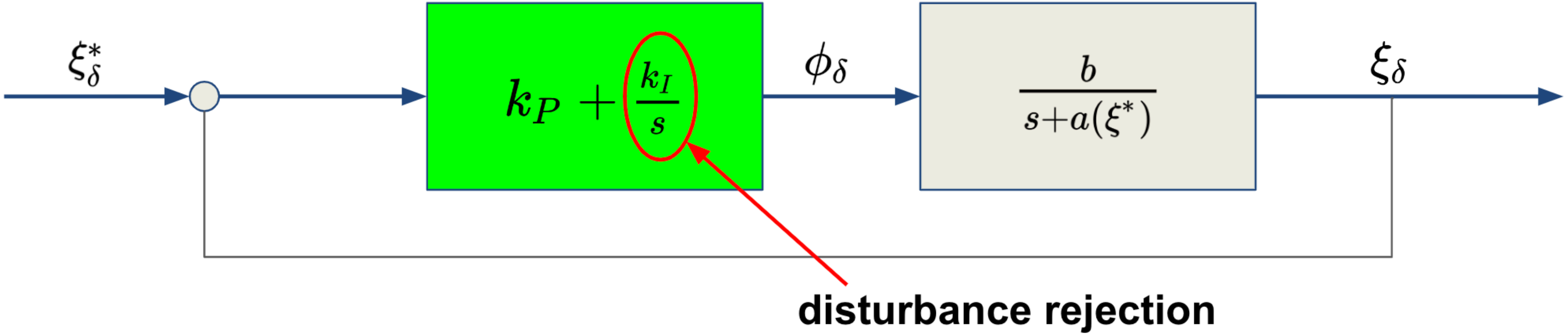
AMV Control system design: velocity control



closed-loop transfer function

$$\frac{bk_P s + bk_I}{s^2 + [a(\xi^*) + bk_P]s + bk_I}$$

closed-loop characteristic equation

$$s^2 + 2\sigma s + \sigma^2 + \omega_n^2 = 0$$

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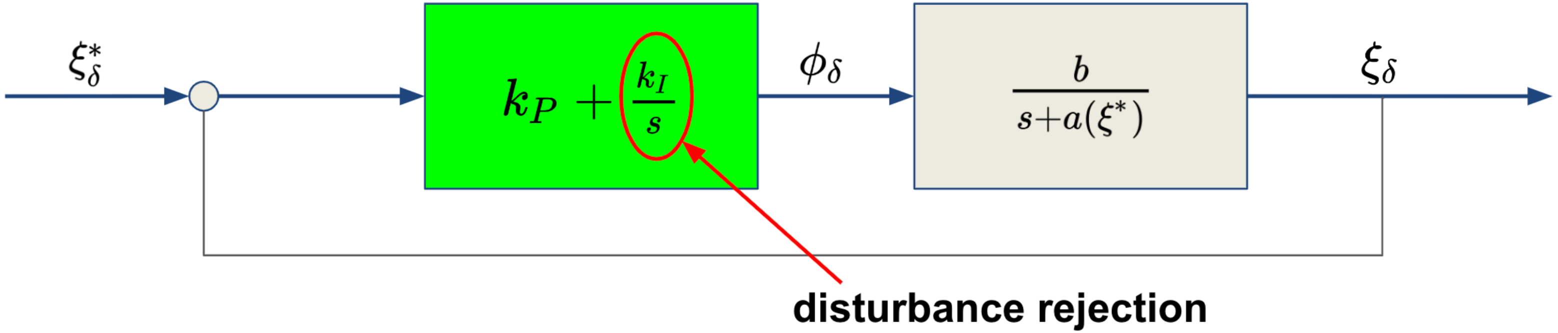
AMV Control system design: velocity control



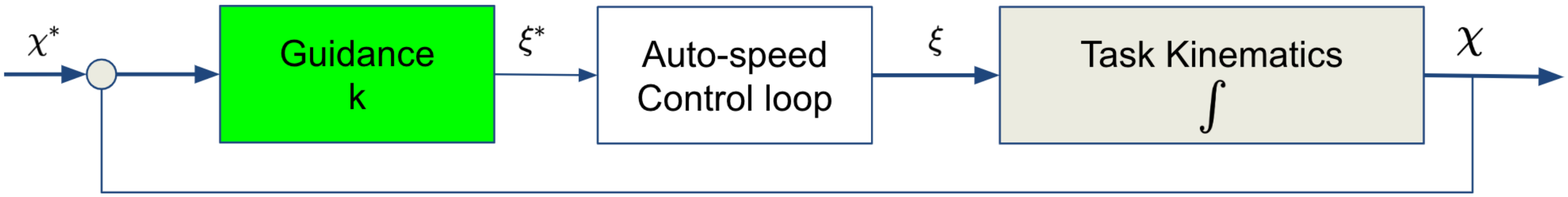
$$\frac{bk_P s + bk_I}{\boxed{s^2 + [a(\xi^*) + bk_P]s + bk_I}}$$

$$\boxed{s^2 + 2\sigma s + \sigma^2 + \omega_n^2 = 0}$$

Gain-Scheduling controller

$$\boxed{\begin{aligned} k_P &= m_\xi \left[2\sigma - a(\xi^*)\right] = 2m_\xi - k_\xi - 2k_{\xi|\xi|} |\xi^*| \\ k_I &= m_\xi \left(\sigma^2 + \omega_n^2\right) \end{aligned}}$$

AMV Guidance system design: position control



guidance transfer function $$\frac{k}{s+k}$$

ideal guidance time constant $$\tau = \frac{1}{k}$$

- in practice, the auto-speed control loop has a finite dynamics
- the proportional gain of the position guidance task function is chosen such that its time constant is from 2 to 10 times slower than the inner loop dynamics

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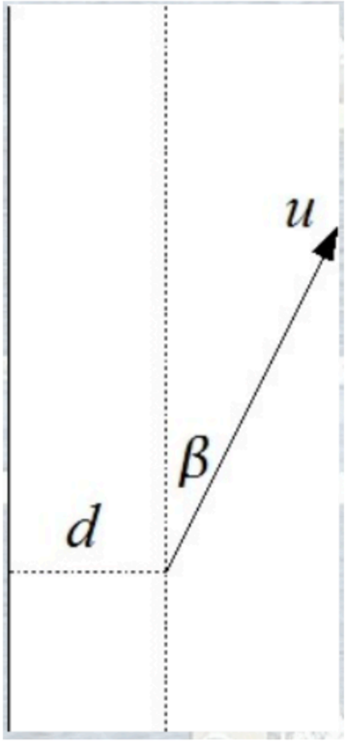
AMV Guidance system design: straight line-following



$$\dot{d} = u \sin\beta + \eta \approx u\beta + \eta \,, \;\; u > 0$$
$$\dot{\beta} = r$$
$$\ddot{d} \approx \dot{u}\beta + u\dot{\beta} + \dot{\eta} = ur$$

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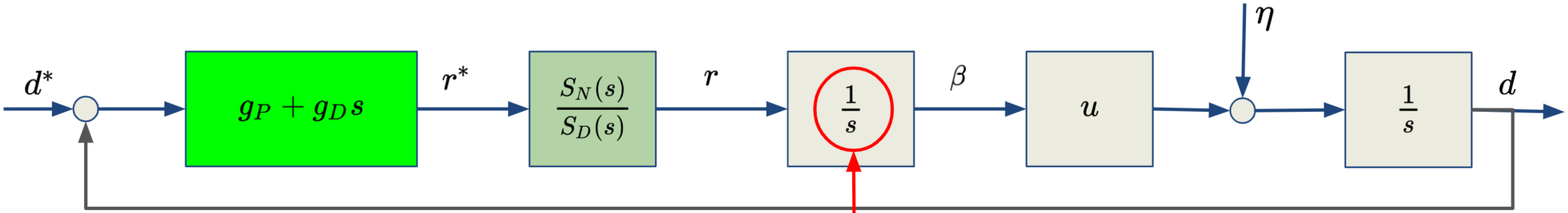
AMV Guidance system design: straight line-following



Closed-loop control $S_N(s) = p_1 s + p_0$

$$S_D(s) = q_2 s^2 + q_1 s + q_0$$

The integrator embedded in the system kinematics naturally compensate external disturbance

$$D = \frac{u S_N(s)(g_P + g_D s)}{s^2 S_D(s) + u S_N(s)(g_P + g_D s)} D^* + \frac{s S_D(s)}{s^2 S_D(s) + u S_N(s)(g_P + g_D s)} \eta$$

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Guidance tasks

- Point stabilisation
  - to stabilise the vehicle zeroing the position and orientation (in the absence of currents) error with respect to a given target point
- Trajectory tracking
  - the vehicle is required to track a time-parameterised reference
- Path following
  - the vehicle is required to converge to and follow a path, without any temporal specification
- Path tracking
  - the vehicle is required to track a target that moves along a predefined path
  - it is possible to separate the spatial and temporal constraints, giving priority to the former one, i.e. the vehicle tries to move along the path and then to zero the range from the target

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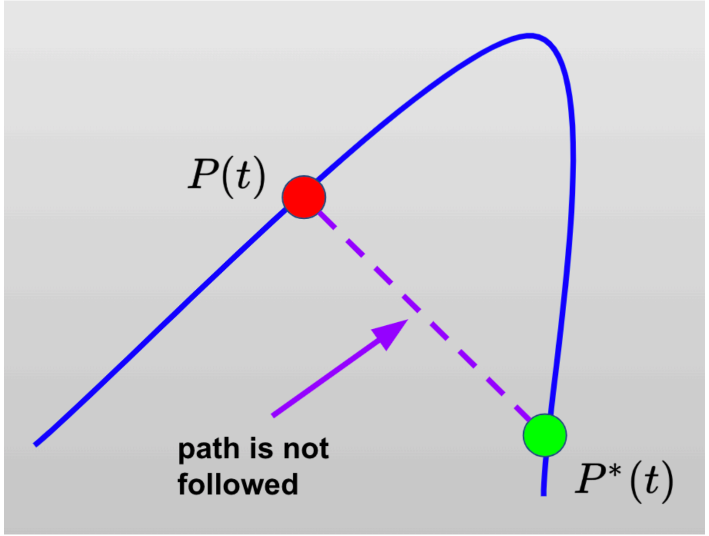
Trajectory tracking

- the vehicle is required to track a time-parameterised reference
  - priority is given to be in the time-scheduled point at time t with respect to exactly follow the desired path



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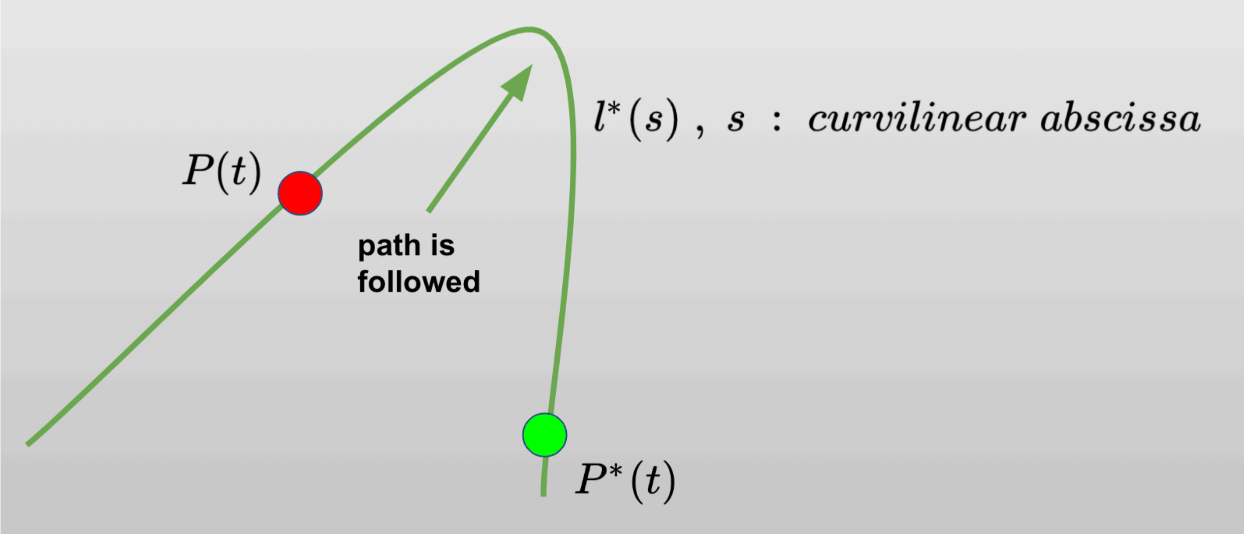
Path following

- the vehicle is required to converge to and follow a path, without any temporal specification



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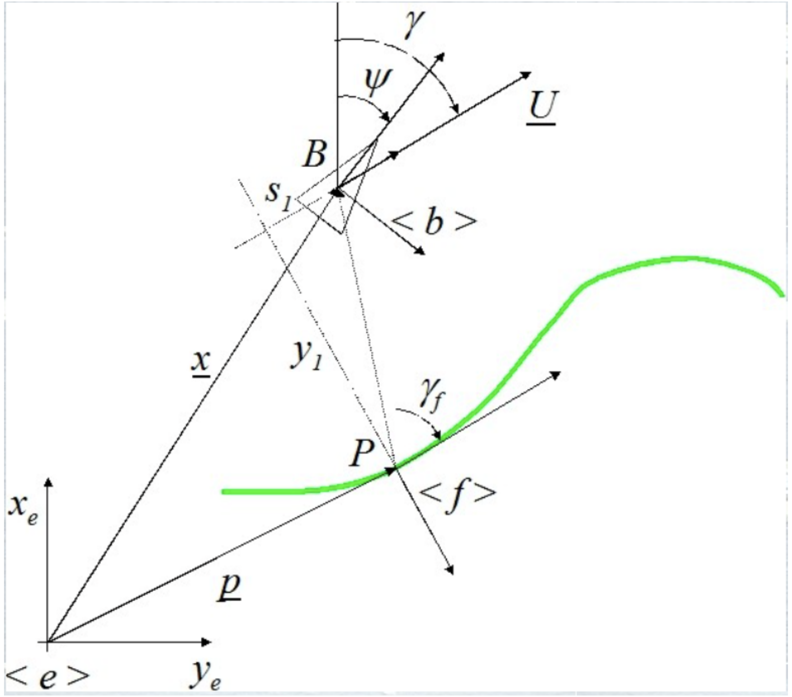
Path-following: nomenclature and virtual target



$$\dot{y}_1 = -c_c \dot{s}\, s_1 + U \sin \beta$$
$$\dot{\beta} = \gamma - c_c \dot{s}$$
$$\dot{\gamma} = r\eta(t)$$

- Serret-Frenet frame <f> in P
  virtual target moving along the path

s	curvilinear abscissa of an arbitrary point on the path P
$\dot{s}$	speed of P in <f>
γ_f	tangent to the path in P
$c_c = \frac{\dot{\gamma}_f}{\dot{s}}$	curvature of the path in P
$[s_1 \;,\; y_1]$	vehicle coordinates in the Serret-Frenet frame <f>
$\beta = \gamma - \gamma_f$	approach angle to the path in P

$$\eta(t) = \frac{u_r^2}{U^2} + \frac{u_r}{U^2}(\dot{x}_C \cos\psi + \dot{y}_C \sin\psi)$$

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Path-following: controlling the vehicle yaw rate

- a desired approach angle is defined as a function of the range from the tangent line to the path in the virtual target position

$$\varphi(y_1) : \|\varphi(y_1)\| < \frac{\pi}{2} \;,\; y_1\varphi(y_1) \leq 0 \;,\; \varphi(0) = 0 \qquad \boxed{\varphi(y_1) := -\psi_a \tanh(k_\varphi y_1)}$$

- the vehicle heads to the desired path and remains over it once reached

Candidate Lyapunov function $\quad V(\beta) = \frac{1}{2}(\beta - \varphi)^2$

$$\dot{V}(\beta) = (\dot{\beta} - \dot{\varphi})(\beta - \varphi) = [r\eta(t) - c_c\dot{s}](\beta - \varphi) \Rightarrow \dot{V}(\beta) = -k_1(\beta - \varphi)^2$$

$$\boxed{r^* = \frac{1}{\eta(t)}[\dot{\varphi} - k_1(\beta - \varphi) + c_c\dot{s}]}$$

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Path-following: controlling the virtual target velocity

- consider the motion of the feedback control system restricted to the set E such that

$$\dot{V}(\beta) = 0, \ i.e. \ \beta = \varphi$$

Candidate Lyapunov function $V_E(s_1, y_1) = \frac{1}{2}(s_1^2 + y_1^2)$

$$\dot{V}_E(s_1, y_1) = [U \cos\beta - \dot{s}]s_1 + U y_1 \sin\beta \Longrightarrow \dot{V}_E(s_1, y_1) = -k_2 s_1^2 + U y_1 \sin\beta$$

speed

angle of attack

$$\boxed{\dot{s}^* = U \cos\beta + k_2 s_1}$$

abscissa of the projected vehicle with respect to the virtual frame

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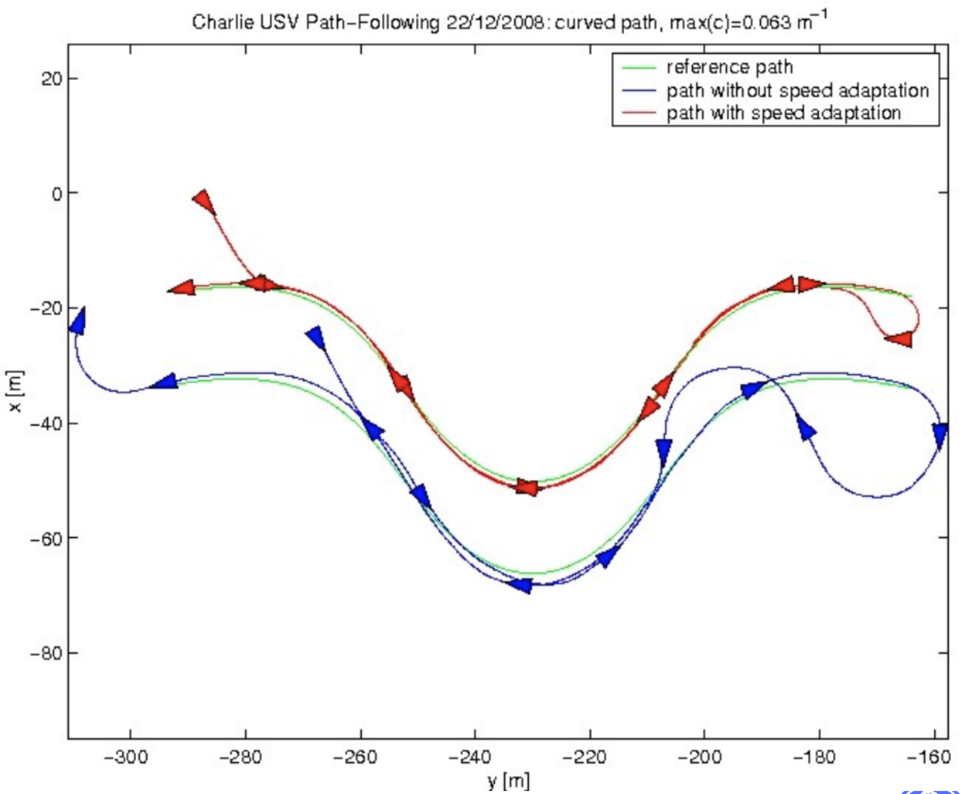
Path-following: experimental results



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Target following (path tracking)

- **Problem** : given a target T moving along a path $\lambda = \lambda(s)$ with linear velocity $u_T = \xi > 0$, with curvilinear abscissa s_T in direction ψ_T, determine appropriate control laws for the linear velocity u and angular velocity r of the vehicle V such that it follows the path λ with zero distance from the target t.

  For simplicity, the vehicle V, whose orientation is denoted by ψ, is modelled as a unicycle, and it is assumed that the target T moves along a straight path.

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Target following (path tracking)

- **Proposed solution** : consider a virtual target P moving along the line λ with abscissa s_P and a Serret-Frenet frame $< f >$ associated with it.
  The velocity of P with respect to $< f >$ is $[\dot{s},0]^T$, while the coordinates of the vehicle V are $[s_V, y_V]^T$. The approach angle of the vehicle V to the reference line λ is given by $\beta = \psi - \psi_T$.
  The kinematics of the vehicle V and the target T in the coordinates (s, y) are given as follows:
  $\dot{s}_V = -\dot{s} + u\cos\beta$
  $\dot{y}_V = u\sin\beta$
  $\dot{\beta} = r$
  $\dot{s}_T = \xi - \dot{s}$
  The objective is to have $s_V = 0$, $y_V = 0$, $s_T = 0$.

Target following (path tracking)

- **Step 1** : vehicle V follows a virtual target P moving along the line λ

  $$r^* = \dot{\phi} - k_1 \left(\beta - \phi \right)$$

  $$\dot{s}^* = u \cos \beta + k_2 s_V$$

- **Step 2** : to compute the velocity u of vehicle V to reach target T

  Given the Lyapunov function $V = \dfrac{1}{2} \left(\beta - \phi \right)$ and the corresponding set E_1 defined by $\dot{V} = 0$ and defined the set E_2 as $\dot{V} = 0 \wedge \dot{V}_{E_1} = 0$, let us study the motion of the closed-loop control system, constrained to E_2, considering the Lyapunov function $V_{E_2} = \dfrac{1}{2} s_T^2$.

  Differentiating with respect to time we obtain $V_{E_2} = \left(\xi - \dot{s} \right) s_T = \left(\xi - u \cos \beta - k_2 s_V \right) s_T$ so the control law $u^* = \dfrac{1}{\cos \beta} \left(\xi + k_3 s_T - k_2 s_V \right)$ guarantees $\dot{V}_{E_2} < 0$, assuming $\beta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right)$

Cooperative target following (e.g. along a straight line)

- **Problem** : given a target T moving on a path $\lambda = \lambda(s)$ with linear velocity $u_T = \xi > 0$, where s is the curvilinear abscissa, and N vehicles V_i, modelled as unicycles, determine appropriate control laws for the linear velocity u_i and rotational velocity r_i of each vehicle so that it follows a reference path λ with a distance error with respect to the target T equal to 0.

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Cooperative target following (e.g. along a straight line)

- **Step 1** : each vehicle V_i follows the corresponding virtual target P_i

  Consider the target T moving along the reference line λ with curvilinear abscissa $s_T = s_0$ such that $\dot{s}_T = \dot{s}_0 = u_T = u_0 = \xi > 0$ and a Serret-Frenet frame $< f_0 >$ associated with it. For each vehicle V_i , $i = 1 \ldots N$ consider a virtual target P_i moving along the path λ with curvilinear abscissa $s_i = 1 \ldots N$ and a Serret-Frenet frame $< f_0 >$ associated with it. Let $\left[s_{V_i} , y_{V_i} \right]$ be the coordinates of the vehicle V_i in the corresponding triplet $< f_i >$ associated with the virtual target P_i.

  Each vehicle V_i follows its virtual target P_i moving along the path λ according to the kinematic control law

$$r_i^* = \dot{\phi}\left(y_{V_i}\right) - k_1 \left[\beta_i - \phi\left(y_{V_i}\right)\right]$$

$$\dot{s}_i = u_i \cos \beta_i + k_2 s_{V_i}$$

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Cooperative target following (e.g. along a straight line)

- **Step 2** : virtual targets P_i cooperative motion

  given an ordered list of virtual targets P_i , $i = 1 \ldots N$ associated with the corresponding vehicles V_i , $i = 1 \ldots N$ and a target $T = P_0$ moving at speed $u_T = u_0$ determine for each vehicle an appropriate control law for the speed reference u_i^* so that each virtual target P_i follows the previous virtual target P_{i-1}.

  The virtual target P_{i-1}, moving at the speed $\dot{s}_{i-1}$ with coordinate x equal to s_{T_i} in the Serret-Frenet frame associated to P_i is seen at each step as the target for the vehicle V_i, whose convergence to the target itself is guaranteed by the control law $u_i^* = \dfrac{1}{\cos \beta} \left(\dot{s}_{i-1} + k_3 s_{T_i} - k_2 s_{V_i} \right)$

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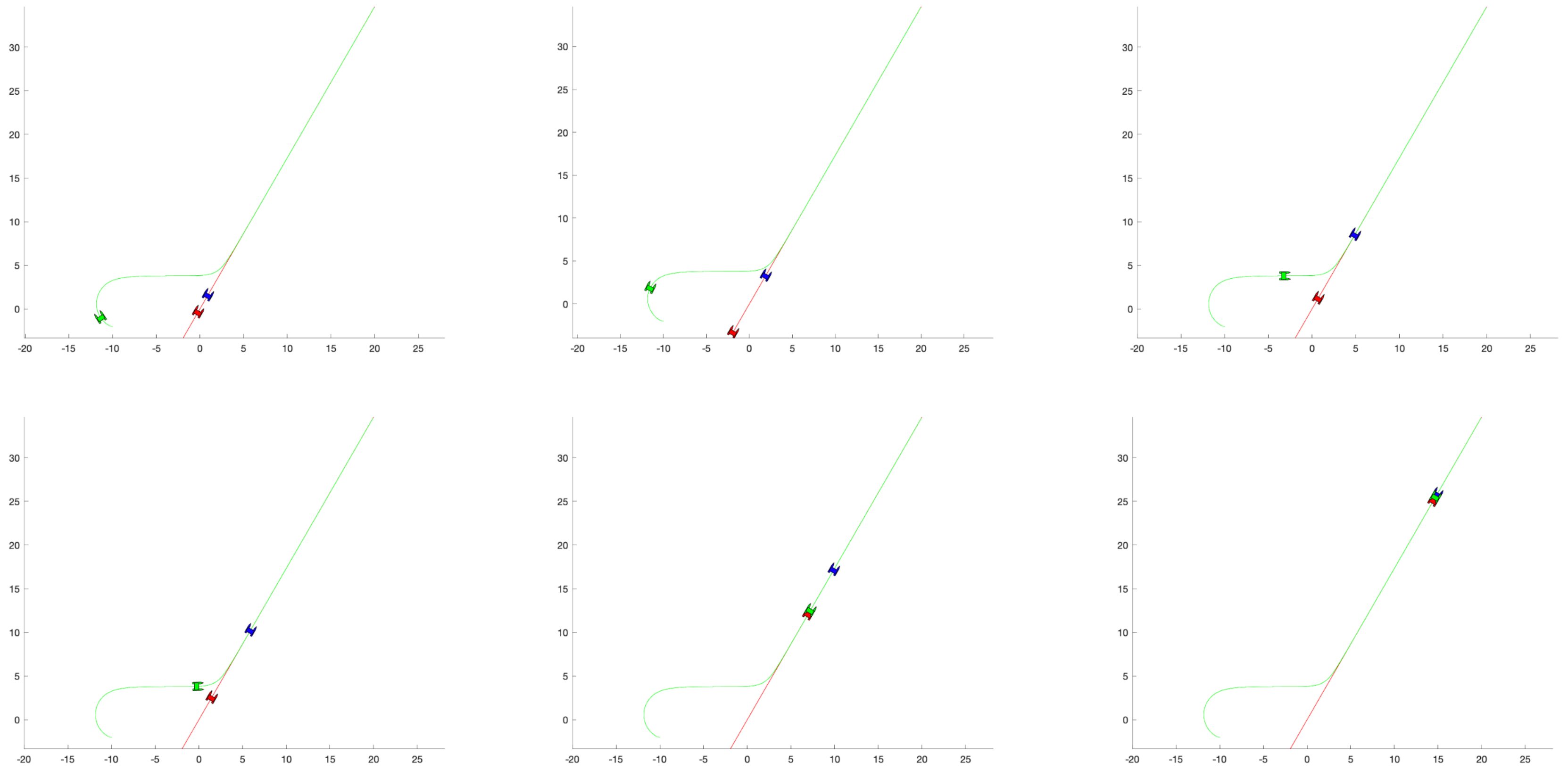
Cooperative target following (e.g. along a straight line)

$$u_i^* = \frac{1}{\cos\beta}\left(\dot{s}_{i-1} + k_3 s_{T_i} - k_2 s_{V_i}\right)$$

$$r_i^* = \dot{\phi}\left(y_{V_i}\right) - k_1\left[\beta_i - \phi\left(y_{V_i}\right)\right]$$

$$\dot{s}_i = u_i \cos\beta_i + k_2 s_{V_i}$$

Cooperative target following: an example



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