ECE 278A: Digital Image Processing

Q-Sweep for Triangulation and Multi-View Projective Matching

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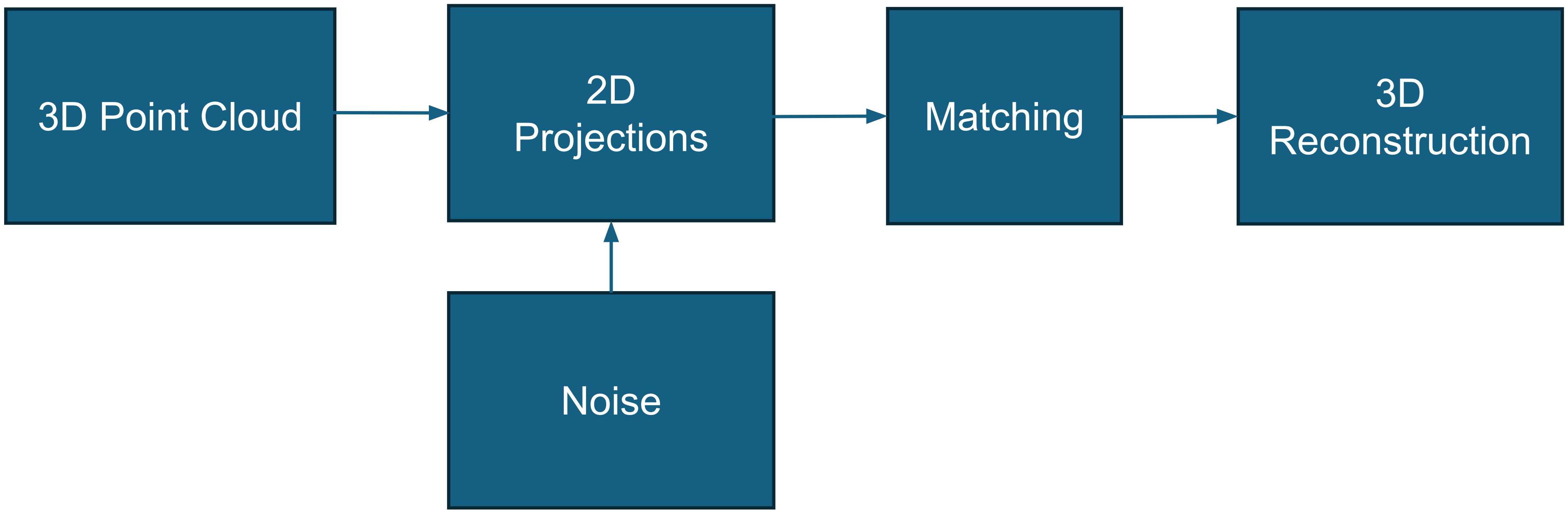
Overview

- Pipeline
- Matching
- Reconstruction
- Comparison
- Future Directions

Pipeline

Pipeline


3D Point Cloud
2D Projections
Matching
3D Reconstruction
Noise

Matching

The Matching Problem

Overview :

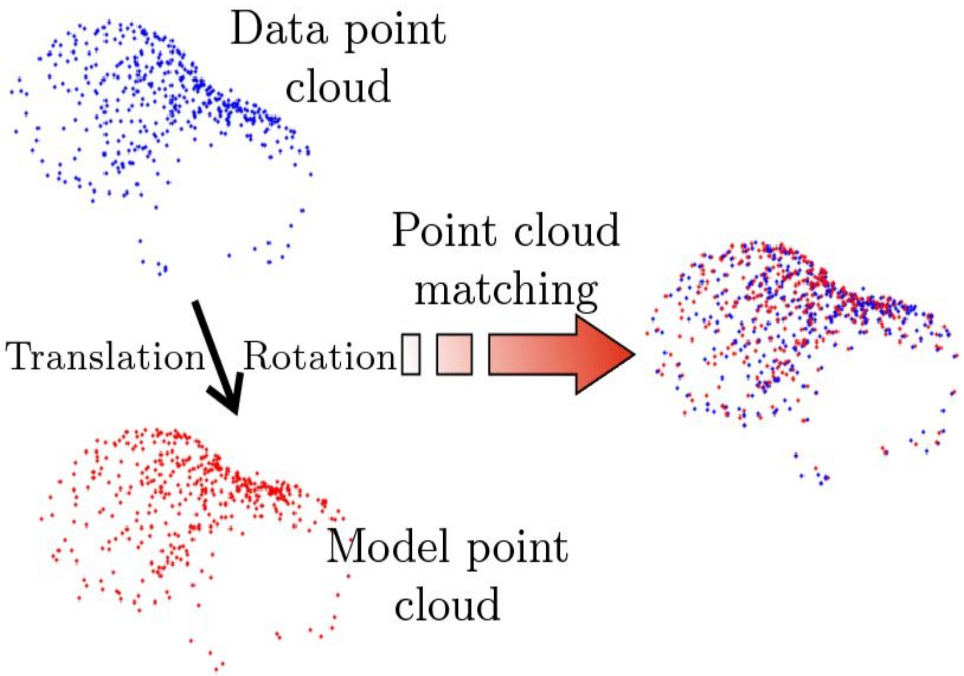
- Fundamental problem in computer vision.
- Align two or more point sets in the same coordinate system.

Objective :

- Find the optimal transformation aligning two or more point sets.

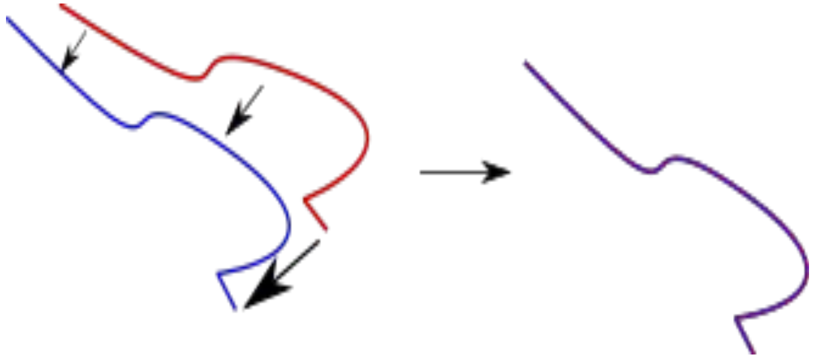
Applications :

- Robotic path planning, localization.
- 3D Reconstruction.



Iterative Closest Point

- Find the optimal rigid transformation (*R, t*), aligning one point cloud P with another Q.
- ICP is an iterative process :
- Estimate correspondences between P & Q
- Optimizes the rigid transformation between them, until convergence.
- Requires a good initial estimate.



- Mathematical formulation :

$$\min_{R,\, t} \sum_{i=1}^{M} D_i(R,\, t)^2$$

Where $D_i\,(R,\, t) = \min_{q\,\epsilon\, Q} ||Rp_i + t - q||$

Variant - Projective ICP

- Matches point sets obtained from different perspectives.
- Optimizes the projective transformation (homography) between the point sets.

$S = \{s_1, \ldots, s_{N_s}\}$, $\mathcal{M}$ is the corresponding 2D model.
Let $d_p(s, m)$ be the projective distance between point $s \in S$ and $m \in \mathcal{M}$.
Let $\mathrm{CP}(s, \mathcal{M})$ be the 'closest' point in $\mathcal{M}$ to the scene point s

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Algorithm 1 Projective ICP algorithm (PICP)

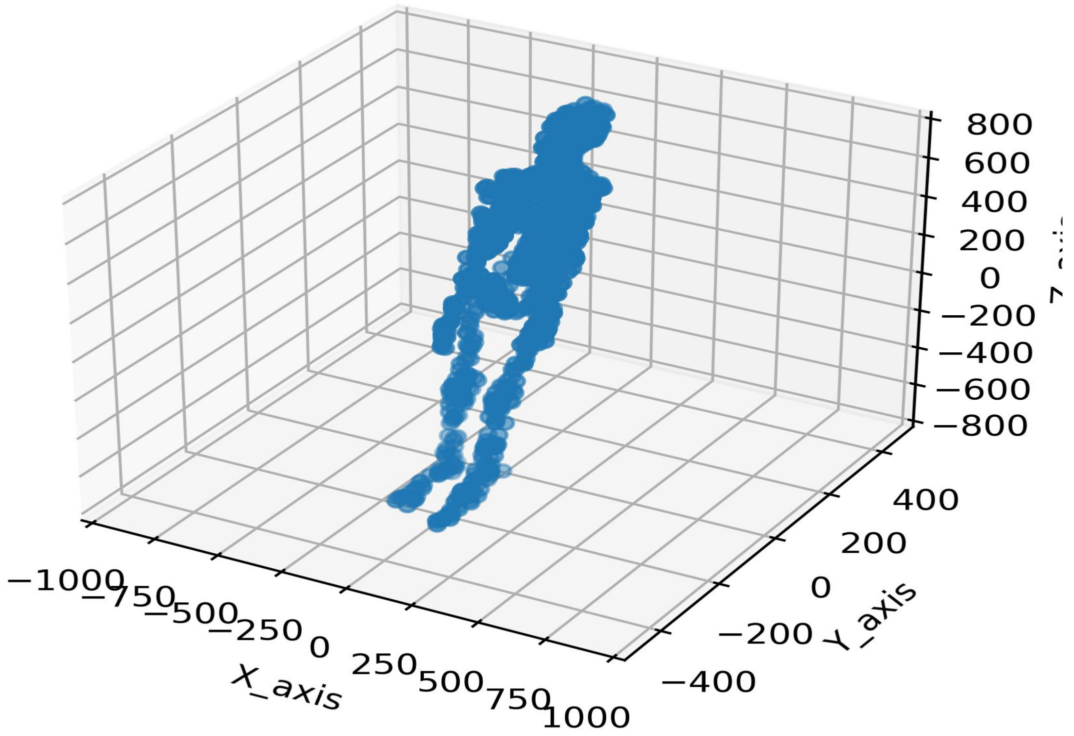
Let $\mathbf{T}^{[0]}$ be an initial estimate of the homography
Repeat for $k = 1, \ldots, k_{\max}$ or until convergence:

1. Compute the set of correspondences $\mathcal{C} = \bigcup_{i=1}^{N_s} \{(s_i, \mathrm{CP}(\mathbf{T}^{[k-1]}(s_i), \mathcal{M}))\}$.
2. Compute the new homography $\mathbf{T}^{[k]}$ between point pairs in $\mathcal{C}$

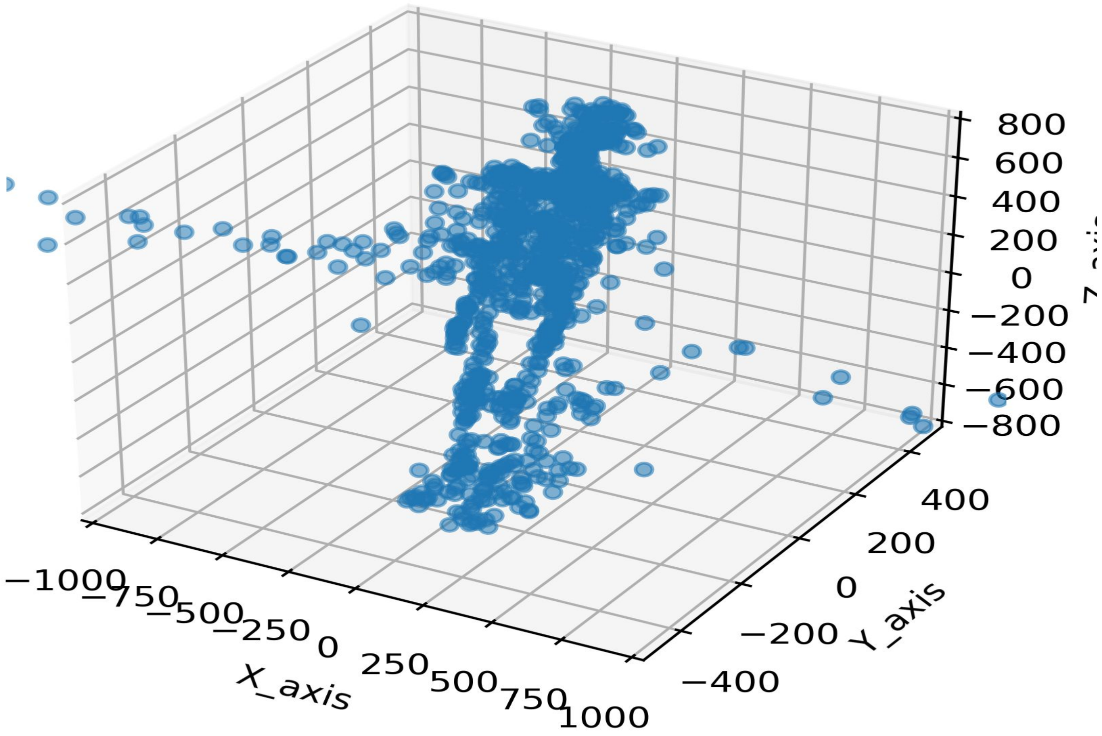
Summary

SIFT	RANSAC-SIFT	Projective-ICP
• Scale-Invariant Feature Transform • Match local features in the images • Not a good estimate due to outliers. • Limited correspondences in absence of colour gradient.	• Random Sample Consensus. • Robust homography estimate in the presence of outliers	• Use homography obtained from RANSAC-SIFT as initial estimate. • Refined homography estimate. • Correspondence between all points.

Results



Original 3D Point Cloud



Reconstructed 3D Point Cloud with PICP Matching

Reconstruction

Formulations

Given a set of N 2D image measurements,

$$\min_{\mathbf{x}\in\mathbb{R}^3} \quad \underset{i\in\{1,\ldots,N\}}{\text{maximum}} \left\| \mathbf{u}_i - \frac{\mathbf{P}_i^{1:2}\tilde{\mathbf{x}}}{\mathbf{P}_i^{3}\tilde{\mathbf{x}}} \right\|_p ,$$
$$\text{s.t.} \quad \mathbf{P}_i^3\tilde{\mathbf{x}} > 0 \ \ \forall i \in \{1,\ldots,N\},$$

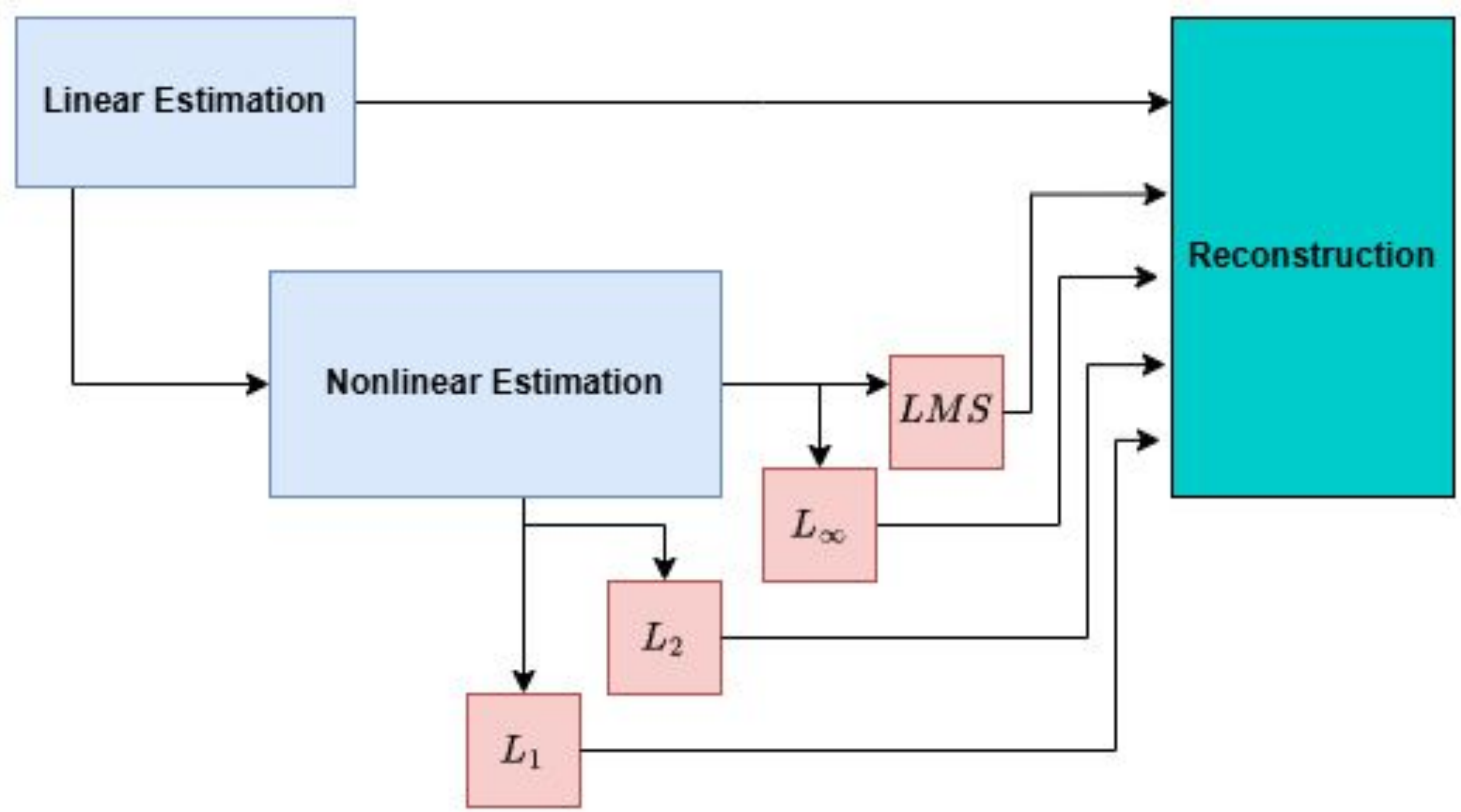
where, x is the 3D point in inhomogeneous coordinates,

u_i is the 2D point visualized by the i^{th} camera,

$P^{1:2}_i$ is the matrix containing first and second rows of the camera matrix, P

Now, we proceed to see how the model works under p = 1, 2, infinity

Reconstruction Pipeline



L-BFGS-B

(Limited-memory Broyden–Fletcher–Goldfarb–Shanno with Box constraints)

Initialization:

* Initial guess x_0.
* Objective function $f(x)$ and its gradient $\nabla f(x)$.
* Bounds $l \leq x \leq u$.

Iteration:

* Calculate $\nabla f(x_k)$ at the current position x_k.
* Approximate the inverse Hessian matrix H_k^{-1}.
* Update the position:

$$x_{k+1} = x_k - \alpha_k H_k^{-1} \nabla f(x_k)$$

  where α_k is the step size.
* Apply the box constraints: $\boxed{l \leq x \leq u.}$

Convergence criteria:

$$\|\nabla f(x_k)\| < \epsilon$$

where ϵ is a small threshold.

Least Median Squares

Given a set of N 2D image measurements,

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{R}^3} \quad & \underset{i\in\{1,\ldots,N\}}{\operatorname{median}} \left\| \mathbf{u}_i - \frac{\mathbf{P}_i^{1:2}\tilde{\mathbf{x}}}{\mathbf{P}_i^{3}\tilde{\mathbf{x}}} \right\|_p, \\ \text{s.t.} \quad & \mathbf{P}_i^3\tilde{\mathbf{x}} > 0 \;\; \forall i \in \{1,\ldots,N\} \end{aligned}$$

However, the non-differentiability of the median also complicates the usage of standard gradient-based optimization

Q-Sweep

- A technique designed for efficient, accurate depth estimation and stereo matching in computer vision tasks.
- Enhances traditional plane sweep methods by leveraging the properties of quasiconvex functions.

Advantages:

- <u>Efficiency</u>: Reduces computational complexity compared to traditional methods.
- <u>Accuracy</u>: Improves the precision of depth maps by focusing on quasiconvex properties.
- <u>Flexibility</u>: Applicable to various stereo vision tasks, including real-time applications.

Q. Zhang, T. -J. Chin and D. Suter, "Quasiconvex Plane Sweep for Triangulation with Outliers," 2017 IEEE International Conference on Computer Vision (ICCV), Venice, Italy, 2017, pp. 920-928

Q-Sweep

Algorithm 1 Q-sweep method for LMS triangulation.

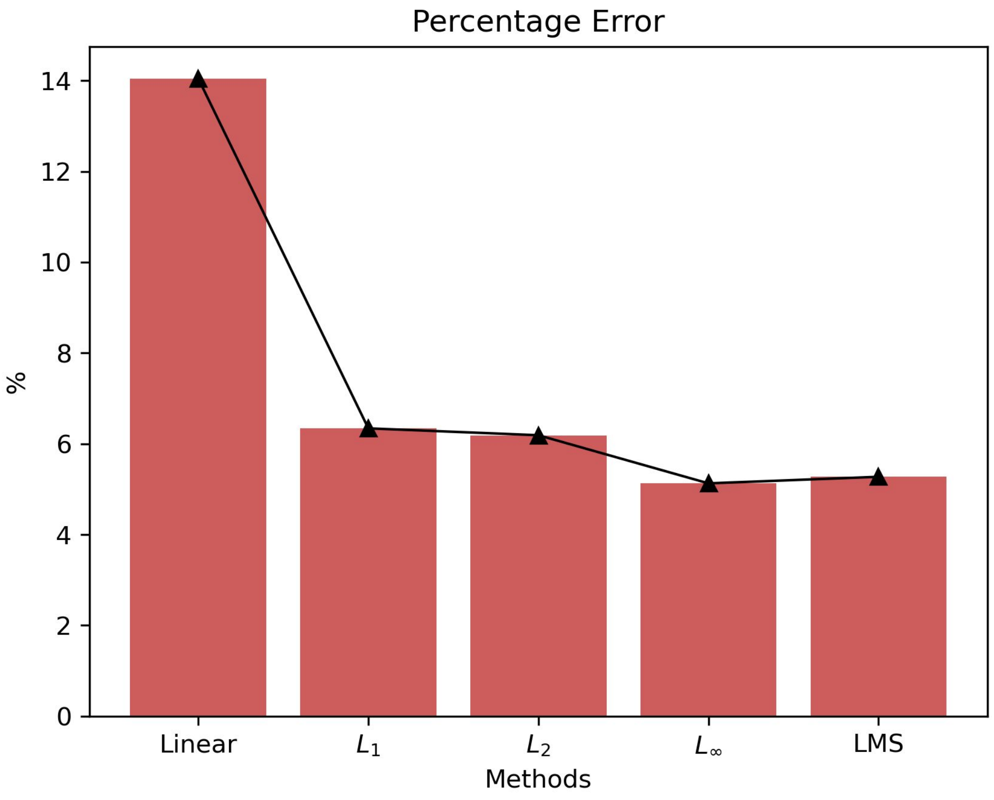
Require: Input data $\{\mathbf{A}_i, \mathbf{b}_i, \mathbf{c}_i, d_i\}_{i=1}^N$, initial soln. $\hat{\mathbf{x}}$.

$\Delta\mathbf{x} \leftarrow \texttt{DESCENTDIR}\big(\{\mathbf{A}_i, \mathbf{b}_i, \mathbf{c}_i, d_i\}_{i=1}^N, \hat{\mathbf{x}}\big)$.
while $\Delta\mathbf{x}$ is not null **do**
    $\alpha \leftarrow \texttt{STEPSIZE}\big(\{\mathbf{A}_i, \mathbf{b}_i, \mathbf{c}_i, d_i\}_{i=1}^N, \hat{\mathbf{x}}, \Delta\mathbf{x}\big)$.
    $\hat{\mathbf{x}} \leftarrow \hat{\mathbf{x}} + \alpha\Delta\mathbf{x}$.
    $\Delta\mathbf{x} \leftarrow \texttt{DESCENTDIR}\big(\{\mathbf{A}_i, \mathbf{b}_i, \mathbf{c}_i, d_i\}_{i=1}^N, \hat{\mathbf{x}}\big)$.
end while
return $\hat{\mathbf{x}}$.

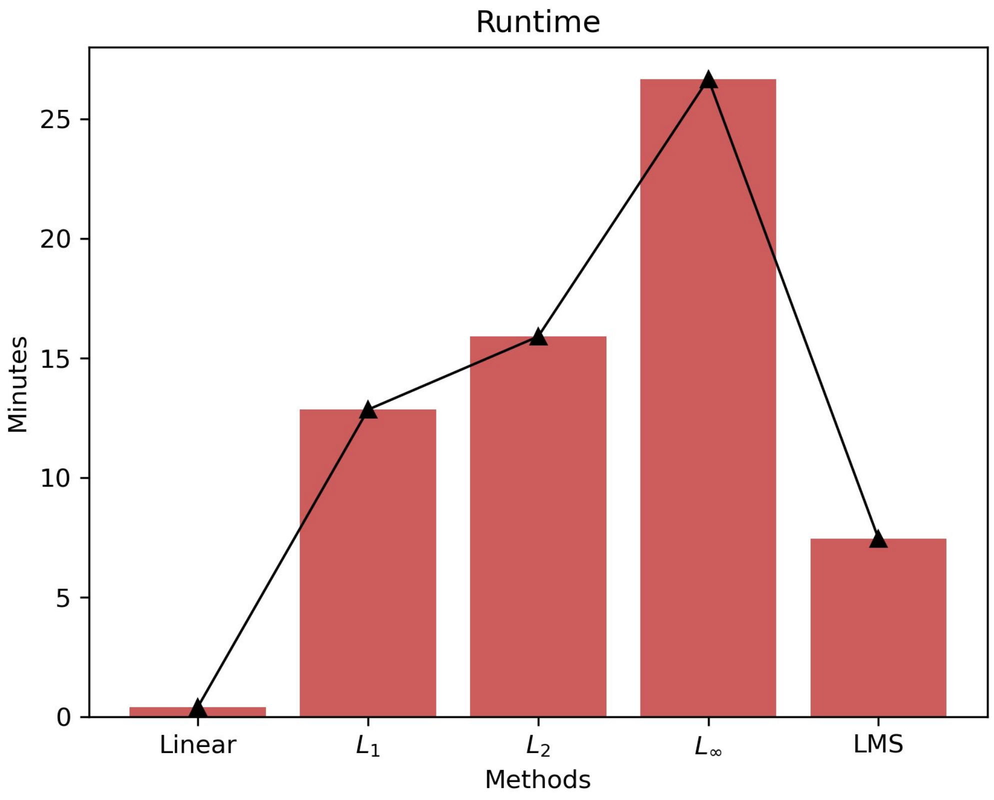
Summary

L_2 with L-BFGS-B	L_∞ with L-BFGS-B	LMS with Q-Sweep
• Least squares problem • Second-lowest runtime among all the methods • Performance worse than L_∞ in accuracy terms	• Double sorting-searching problem • Maximum runtime among all the methods • Performance best among all the methods	• Estimates p-norm formulation to fit into a plane sweeping algorithm • Least runtime among the nonlinear solutions • Performance is comparable to L_∞ method

Results



Accuracy through MAPE



Runtime (in minutes)

Discussion

Discussion

Challenges	**Future Directions**
• Slow runtime of PICP. • PICP sensitive to initial estimate which is hard to obtain in challenging environments. • Q-Sweep has slow update routine for small point clouds	• Fast and Robust PICP. • Using machine learning methods, obtain a good initial coarse matching for challenging environments. • Validating Q-Sweep for outliers with non-matched linear 3D estimations

References

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