Geodesic Regression Characterizes 3D Shape Changes in the Female Brain During Menstruation

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Introduction

2/3 of Alzheimer's patients are women, and women are more likely to get Alzheimer's after menopause[1]. Yet, the female brain's response to sex hormone fluctuations is vastly understudied[2].

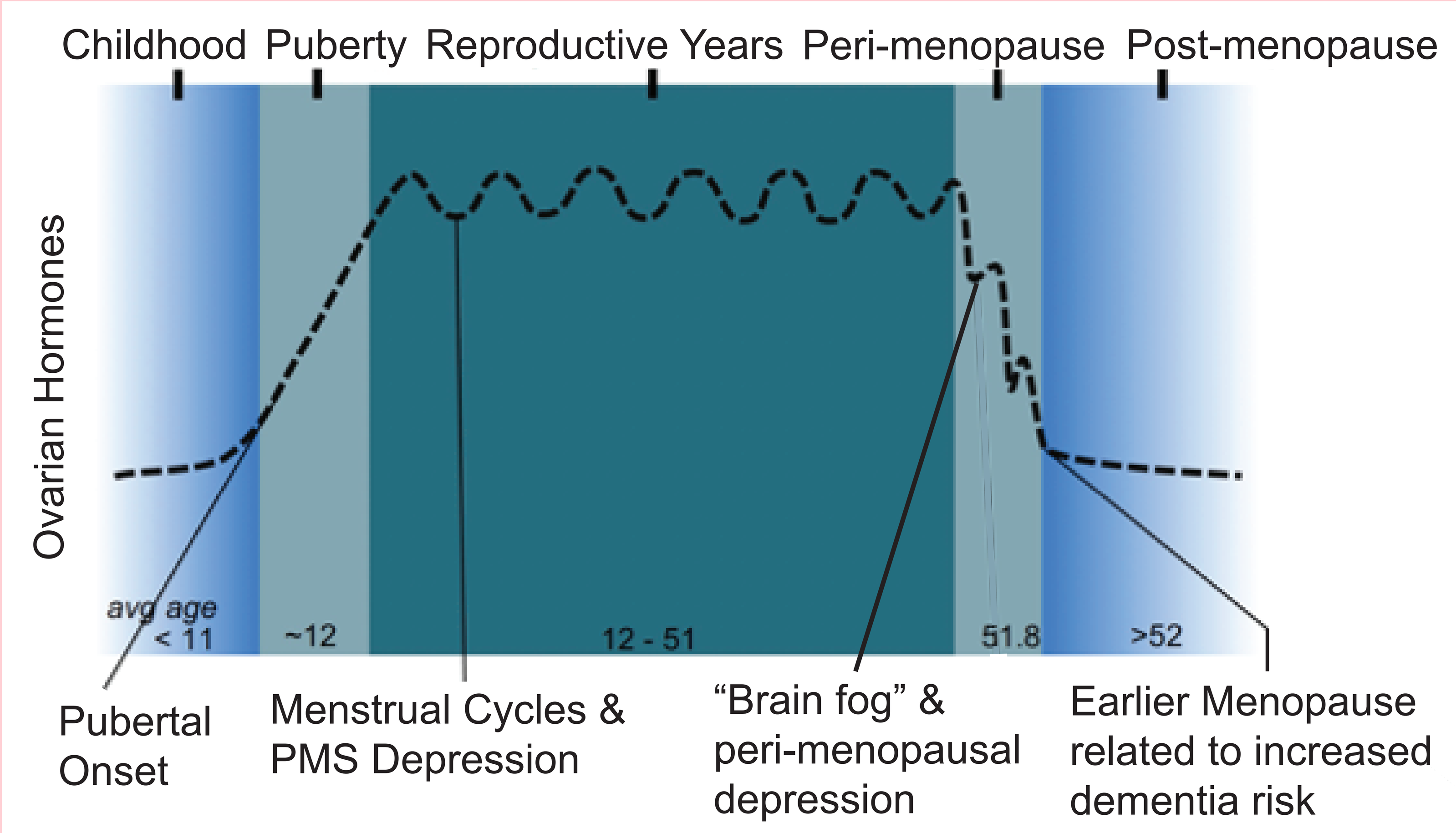


Image Credit: Laura Pritschet

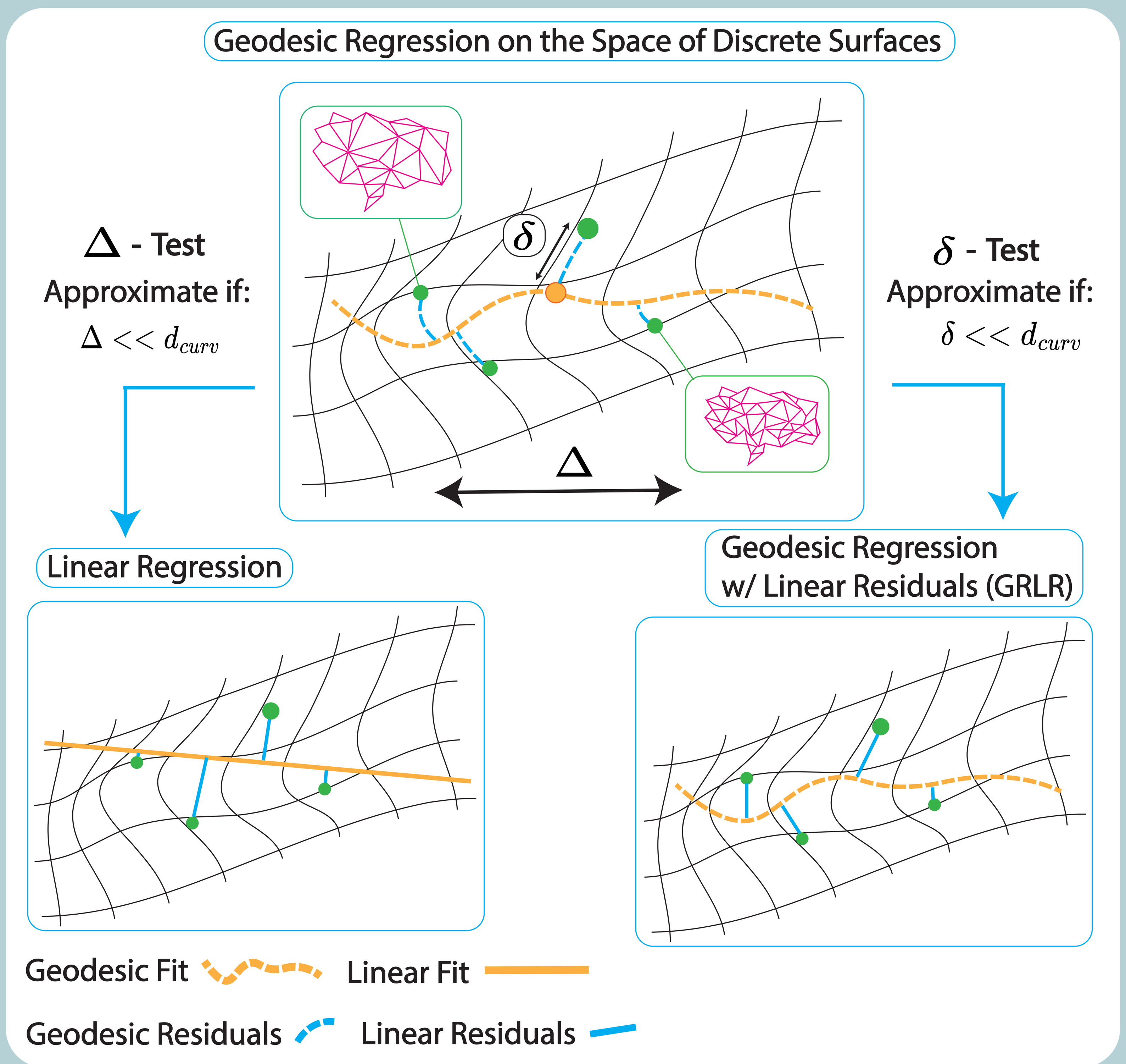
Geodesic regression (GR) on the space of discrete surfaces offers a way to characterize changes in brain shape, but this technique is too slow for practical use.

We propose methods to approximate GR and test the regimes where they can be used on the space of discrete surfaces.

3D Brain Shape ⟷ Brain Health

- **Brain surface shape can be used as a diagnostic for brain health.**
  - Healthy brains and brains with Alzheimer's are shaped differently.
  - Female brains change their shape in response to ovarian hormone fluctuations.
  - Interestingly, the hippocampal formation is the first to show signs of degeneration in Alzheimer's[6] and is also sensitive to ovarian hormone fluctuations[5].
- **Here, we build tools for 3D shape analysis to characterize brain shape changes.**
  - We investigate how brain shape changes when progesterone increases during the menstrual cycle.
- **Long term, we seek to identify "normal" and "abnormal" responses to ovarian hormones and link "abnormal" responses to specific health risks in women.**

Methods: Approximation of GR



Geodesic Regression[3]

Generative Model:

$$Y = \mathrm{Exp}(\mathrm{Exp}(p, Xv), \epsilon)$$

Loss Function:

$$(\hat{p}, \hat{v}) = \arg\min_{(p,v)} \frac{1}{2} \sum_{i=1}^{n} d(y_i, \hat{y}_i)^2,$$

$$= \arg\min_{(p,v)} \frac{1}{2} \sum_{i=1}^{n} \|\mathrm{Log}(\hat{y}_i, y_i)\|^2_{\hat{y}_i},$$

for $\hat{y}_i = \mathrm{Exp}(p, x_i v)$

Geodesic Regression w/ Linear Residuals (GRLR)

Generative Model:

$$Y = \mathrm{Exp}(p, Xv) + \epsilon$$

Loss Function:

$$(\hat{p}, \hat{v}) = \arg\min_{(p,v)} \frac{1}{2} \sum_{i=1}^{n} \|\hat{y}_i - y_i\|^2$$

for $\hat{y}_i = \mathrm{Exp}(p, x_i v)$

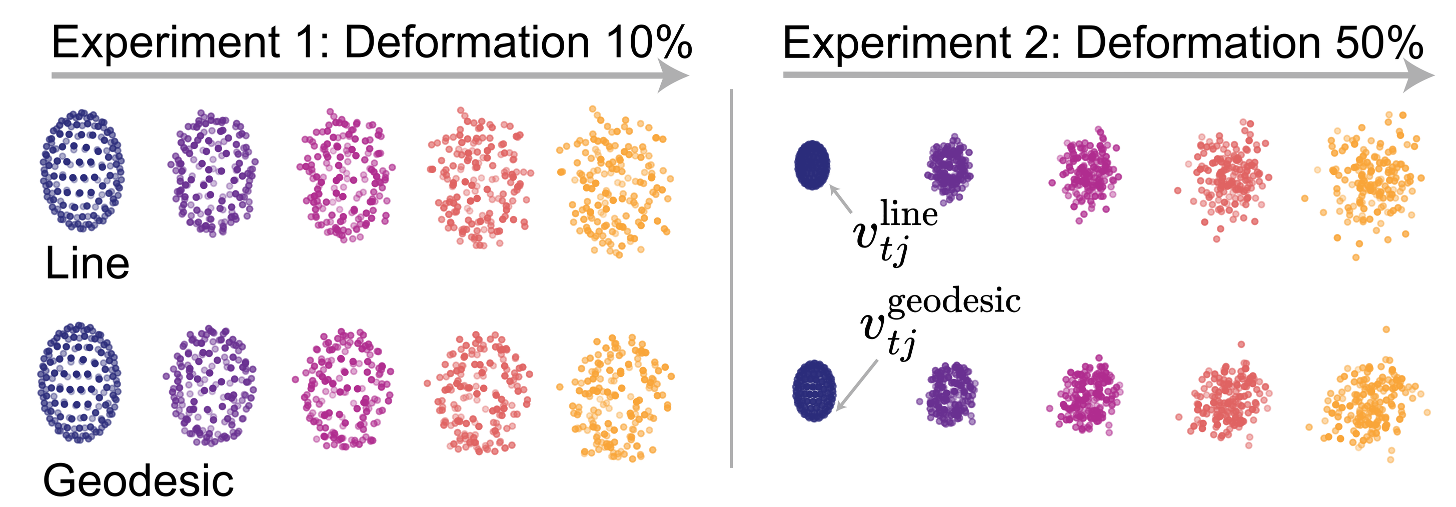
Manifold of Surface Shapes

- Equip with the second-order Sobolev metric[4]
- Distances between points are invariant to rotation & parameterization of meshes.

Definitions:

- $Y =$ data meshes y_i
- $X =$ progesterone levels
- $\epsilon =$ noise on y_i
- $\hat{p} =$ intercept of geodesic fit
- $\hat{v} =$ slope of geodesic fit
- $\hat{y}_i =$ meshes predicted by geodesic fit
- $\mathrm{Exp} =$ manifold operation "+"
- $\mathrm{Log} =$ manifold operation "-"

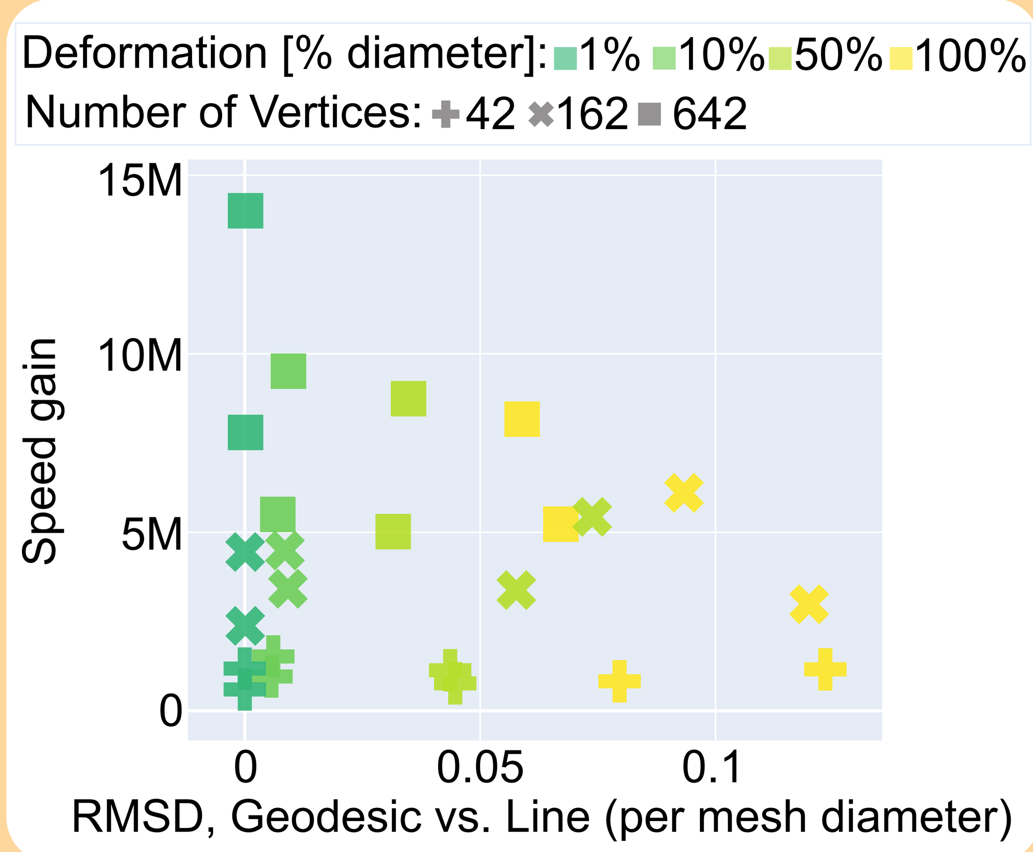
Experiments/GR Approximation Rules of Thumb



$$\mathrm{RMSD}^* = \frac{1}{D}\sqrt{\frac{1}{TN}\sum_{t=1}^{T}\sum_{j=1}^{N}\|v_{tj}^{\text{line}} - v_{tj}^{\text{geodesic}}\|^2}$$

*Root Mean Square Deviation

$v =$ vertex
$N =$ #vertices
$T =$ #meshes
$D =$ mesh diameter



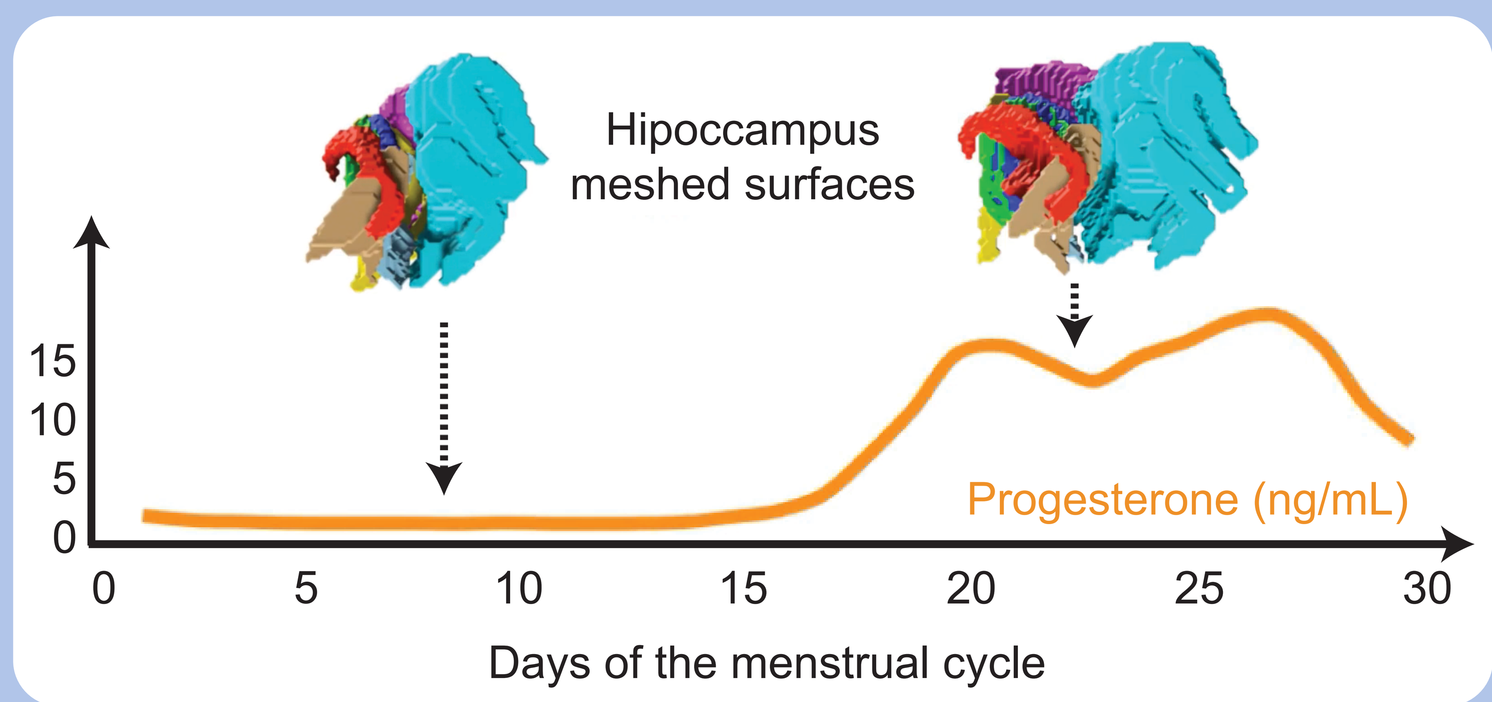
Rules of Thumb for Geodesic Regression Approximation:

Δ Test: Use Linear Regression if:
- N > 100
- and 10%-50% deformation
- and tolerance for 3% error

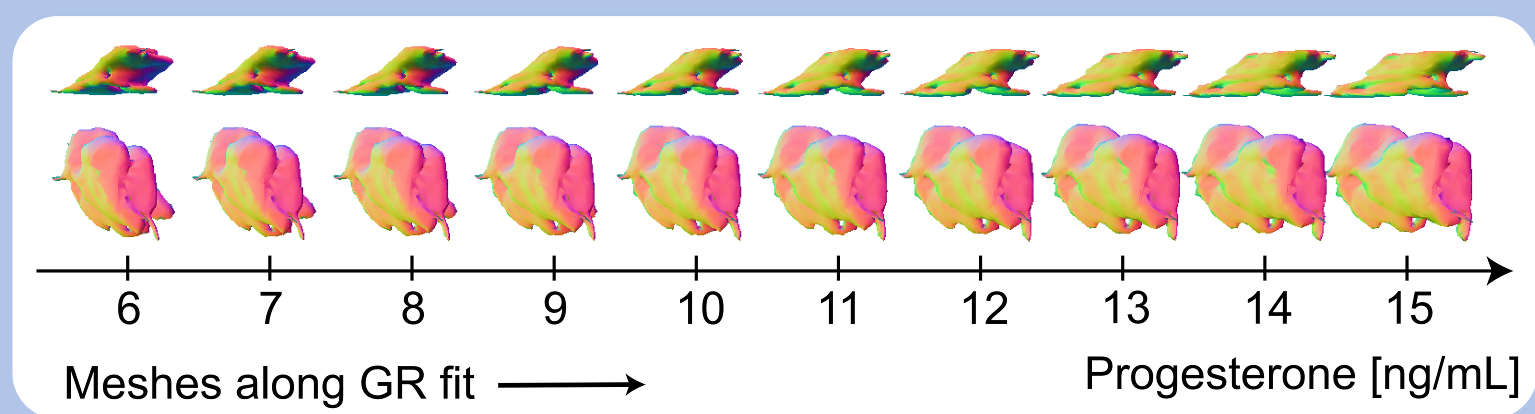
OR
- N > 40
- and 10%-50% deformation
- and tolerance for 10% error

δ Test: use Geodesic Regression with linear residuals if:
- Measurement noise on vertices is <1% of total mesh diameter
- and tolerance for 0.05% error

GR Approximation enables characterization of hippocampal formation changes over the female menstrual cycle



Dataset[5]: 3D brain magnetic resonance images (MRI): 11 images from 11 consecutive days, capturing the progesterone peak of a single female subject's natural menstrual cycle. Subject also measured hormone levels in her blood each session.



Results: Linear Regression, Adjusted $R^2 = 0.09$.
Relying on the Δ test, we show, for the first time, that an increase in progesterone is associated with a shearing deformation of the hippocampal formation and give a quantification of this deformation.

Conclusion

- We propose geodesic regression approximations for the space of discrete surfaces.
- We investigate the curvature of the space of discrete surfaces, and subsequently provide rules of thumb for when each GR approximation can be used.
- We use our GR approximations to offer **the first** quantification of shape changes in a woman's brain as a function of progesterone the menstrual cycle.

References

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