Bohm's Quantum Potential

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The **Bohm** or **Bohmian interpretation** of quantum mechanics, which Bohm called the **causal**, or later, the **ontological interpretation**, is an interpretation postulated by David Bohm in 1952 as an alternative to the standard Copenhagen interpretation. The Bohm interpretation grew out of the search for an alternative model based on the assumption of hidden variables. Its basic formalism corresponds in the main to Louis de Broglie's pilot-wave theory of 1927. Consequently it is sometimes called the **de Broglie-Bohm theory**.

The Bohm interpretation is causal but not local and is non-relativistic. It has not been disproven, but there are other schemes (such as the Copenhagen interpretation) that give the same theoretical predictions, so are equally confirmed by the experimental results.

The Bohm interpretation is based on these principles:

- **Every particle travels in a definite path**

- **We do not know what that path is**

- **The state of N particles is affected by a 3N dimensional field, which guides the motion of the particles**

  De Broglie called this the **pilot wave**; Bohm called it the **ψ-field**. This field has a piloting influence on the motion of the particles. The *quantum potential* is derived from the **ψ-field**.

- **This 3N dimensional field satisfies the Schrödinger equation**

  Mathematically, the field corresponds to the wavefunction of conventional quantum mechanics, and evolves according to the Schrödinger equation. The positions of the particles do not affect the wave function.

- **Each particle's momentum p is** $\nabla S(x, t)$

- **The particles form a statistical ensemble, with probability density**
  $$\rho(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2$$

Reformulating the Schrödinger equation

Some of Bohm's insights are based on a reformulation of the Schrödinger equation; instead of using the wavefunction $\psi(\mathbf{x}, t)$, he defines the wavefunction as

$$\psi(\mathbf{x}, t) = R(\mathbf{x}, t)e^{iS(\mathbf{x}, t)/\hbar}$$

and solves it for the (real) magnitude function $R(\mathbf{x},t)$ and (real) phase function $S(\mathbf{x},t)$.

The [Schrödinger equation](#) for one particle of mass **m** is

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) + V(\mathbf{x})\psi(\mathbf{x},t),$$

where the [wavefunction](#) $\psi(\mathbf{x},t)$ is a complex function of the spatial coordinate $\mathbf{x}$ and time *t*. It can then be split into two coupled equations by expressing it in terms of *R* and *S*:

$$\frac{\partial R(\mathbf{x},t)}{\partial t} = \frac{-1}{2m}[R(\mathbf{x},t)\nabla^2 S(\mathbf{x},t) + 2\nabla R(\mathbf{x},t)\cdot\nabla S(\mathbf{x},t)]$$
$$\frac{\partial S(\mathbf{x},t)}{\partial t} = -\left[V + \frac{1}{2m}(\nabla S(\mathbf{x},t))^2 - \frac{\hbar^2}{2m}\frac{\nabla^2 R(\mathbf{x},t)}{R(\mathbf{x},t)}\right].$$

The probability density $\rho(\mathbf{x},t)$ is a real function defined as the magnitude of the wave function:

$$\rho(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2 = R^2(\mathbf{x},t)$$

Thus

$$\psi(\mathbf{x},t) = \sqrt{\rho(\mathbf{x},t)}e^{iS(\mathbf{x},t)/\hbar}.$$

Therefore we can substitute $\rho(\mathbf{x},t)$ for $R^2(\mathbf{x},t)$ and get:

$$-\frac{\partial\rho(\mathbf{x},t)}{\partial t} = \nabla\cdot\left(\rho(\mathbf{x},t)\frac{\nabla S(\mathbf{x},t)}{m}\right) \qquad (1)$$
$$-\frac{\partial S(\mathbf{x},t)}{\partial t} = V(\mathbf{x}) + Q(\mathbf{x},t) + \frac{1}{2m}(\nabla S(\mathbf{x},t))^2 \qquad (2)$$

where

$$Q(\mathbf{x},t) = -\frac{\hbar^2}{2m}\frac{\nabla^2 R(\mathbf{x},t)}{R(\mathbf{x},t)} = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(\mathbf{x},t)}}{\sqrt{\rho(\mathbf{x},t)}}$$

Bohm called the function $Q(\mathbf{x},t)$ the *quantum potential*.

We can use the same argument on the many-particle [Schrödinger equation](#):

$$i\hbar\frac{\partial\psi(\mathbf{x_1},\mathbf{x_2},...,t)}{\partial t} = \sum_i \frac{-\hbar^2}{2m_i}\nabla_i^2\psi(\mathbf{x_1},\mathbf{x_2},..,t) + V(\mathbf{x_1},\mathbf{x_2},..)\psi(\mathbf{x_1},\mathbf{x_2},...,t),$$

where the *i*-th particle has mass m_i and position coordinate $\mathbf{x_i}$ at time *t*. The [wavefunction](#) $\psi(\mathbf{x_1},\mathbf{x_2},...,t)$ is a complex function of all the $\mathbf{x_i}$ and time *t*. ∇_i is the grad operator with respect to $\mathbf{x_i}$, i.e. of the *i*-th particle's position coordinate. As before the probability density $\rho(\mathbf{x_1},\mathbf{x_2},...,t)$ is a real function defined by

$$\rho(\mathbf{x_1},\mathbf{x_2},..,t) = |\psi(\mathbf{x_1},\mathbf{x_2},..,t)|^2.$$

As before, we can define a real function $S(\mathbf{x_1},\mathbf{x_2},...,t)$ to be the [complex phase](#), so that we can define a similar relationship to the 1-particle example:

$$\psi(\mathbf{x_1},\mathbf{x_2},..,t) = \sqrt{\rho(\mathbf{x_1},\mathbf{x_2},..,t)}e^{iS(\mathbf{x_1},\mathbf{x_2},..,t)/\hbar}.$$

We can use the same argument to express the Schrödinger equation in terms of $\rho(\mathbf{x_1},\mathbf{x_2},..,t)$ and $S(\mathbf{x_1},\mathbf{x_2},..,t)$:

$$-\frac{\partial\rho(\mathbf{x_1},\mathbf{x_2},..,t)}{\partial t} = \sum_i \nabla_i\cdot\left(\rho(\mathbf{x_1},\mathbf{x_2},..,t)\frac{\nabla_i S(\mathbf{x_1},\mathbf{x_2},..,t)}{m_i}\right) \qquad (3)$$

$$-\frac{\partial S(\mathbf{x_1},\mathbf{x_2},..,t)}{\partial t} = V(\mathbf{x_1},\mathbf{x_2},..)+Q(\mathbf{x_1},\mathbf{x_2},..,t)$$

$$+\sum_i\frac{1}{2m_i}(\nabla_i S(\mathbf{x_1},\mathbf{x_2},..,t))^2 \qquad (4)$$

where

$$Q(\mathbf{x_1},\mathbf{x_2},..,t) = -\sum_i\frac{\hbar^2}{2m_i}\frac{\nabla_i^2 R(\mathbf{x_1},\mathbf{x_2},..,t)}{R(\mathbf{x_1},\mathbf{x_2},..,t)} = -\sum_i\frac{\hbar^2}{2m_i}\left(\frac{\nabla_i^2\rho(\mathbf{x_1},\mathbf{x_2},..,t)}{2\rho(\mathbf{x_1},\mathbf{x_2},..,t)} - \left(\frac{\nabla_i\rho(\mathbf{x_1},\mathbf{x_2},..,t)}{2\rho(\mathbf{x_1},\mathbf{x_2},..,t)}\right)^2\right).$$

One-particle formalism

In his 1952 paper, Bohm starts from the reformulated Schrödinger equation. He points out that in equation (2):

$$-\frac{\partial S(\mathbf{x},t)}{\partial t} = V(\mathbf{x}) + Q(\mathbf{x},t) + \frac{1}{2m}(\nabla S(\mathbf{x},t))^2,$$

if one represents the world of classical physics by setting $\hbar$ to zero (which results in Q becoming zero) then $S(\mathbf{x},t)$ is the solution to the [Hamilton-Jacobi equation](#). He

quotes a theorem that says that if an ensemble of particles (which follow the equations of motion) have trajectories that are normal to a surface of constant *S*, then they are normal to all surfaces of constant *S*, and that $\nabla S(\mathbf{x}, t)/m$ is the velocity of any particle passing point **X** at time *t*.

Therefore, we can express equation (1) as:

$$-\frac{\partial \rho(\mathbf{x}, t)}{\partial t} = \nabla \cdot (\rho(\mathbf{x}, t)\mathbf{v}) .$$

This equation shows that it is consistent to express $\rho(\mathbf{x}, t)$ as the probability density because $\rho(\mathbf{x}, t)\mathbf{v}$ is then the mean current of particles, and the equation expresses the conservation of probability.

Of course, $\hbar$ is non-zero. Bohm suggests that we still treat the particle velocity as $\nabla S(\mathbf{x}, t)/m$. The movement of a particle is described by equation (2):

$$-\frac{\partial S(\mathbf{x}, t)}{\partial t} = V(\mathbf{x}) + Q(\mathbf{x}, t) + \frac{1}{2m}(\nabla S(\mathbf{x}, t))^2$$

where

$$Q(\mathbf{x}, t) = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 \rho(\mathbf{x}, t)}{2\rho(\mathbf{x}, t)} - \left(\frac{\nabla \rho(\mathbf{x}, t)}{2\rho(\mathbf{x}, t)}\right)^2\right).$$

V is the classical potential, which influences the particle's movement in the ways described by the classical laws of motion. *Q* also has the form of a potential; it is known as the *quantum potential*. It influences particles in ways that are specific to quantum theory. Thus the particle is moving under the influence of a quantum potential *Q* as well as the classical potential *V*.

Many-particle formalism

The momentum of Bohm's *i*-th particle's "hidden variable" is defined by

$$\mathbf{p_i} = m_i\mathbf{v_i} = \nabla_i S \qquad (3)$$

and the particles' total energy as $E = -\partial S/\partial t$; equation (1) is the [continuity equation](#) for probability with

$$\mathbf{j_i} = \rho\mathbf{v_i} = \rho\frac{\mathbf{p_i}}{m_i} = \rho\frac{\nabla_i S}{m_i} ,$$

and equation (4) is a statement that total energy is the sum of the potential energy, quantum potential and the kinetic energies.