de Broglie Deterministic Dice
and emerging Relativistic Quantum Mechanics

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Abstract. Generalizing de Broglie's hypothesis, we show that the basic quantum behavior of ordinary field theory can be retrieved in a semi-classical and geometrical way from the assumption of intrinsic periodicity of elementary systems. The geometrodynamical description of interactions that arises in from this approach provides an intuitive interpretation of Maldacena's conjecture and it turns out to be of the same type of the one prescribed by general relativity.

1. Introduction

Similarly to a particle in a box or to a vibrating string, relativistic fields can be quantized by imposing their characteristic de Broglie periodicities as constraints [1, 2, 3, 4, 5, 6, 7]. In this way elementary systems can be thought of as "de Broglie internal clocks", that is, as relativistic fields with intrinsic dynamical de Broglie time periodicities T_t. Even for a light particle such as the electron, this intrinsic time periodicity (also know as *zitterbewegung*) is extremely fast ($T_t \lesssim 10^{-21}s$), many orders of magnitude away from the present experimental time resolution. As in *dice* rolling too fast with respect to a given time resolution, these de Broglie internal clocks can only be described statistically. It can be shown that the effective statistical description emerging from such periodic dynamics matches (without fine tunings) ordinary relativistic Quantum Mechanics (QM) [1]. The idea is similar to the "stroboscopic quantization" [8] or to the 't Hooft determinism [9]. At the same time the underling classical-relativistic physics seems to solve fundamental conceptual difficulties of the canonical quantum theory. In particular they do not involve any local-hidden-variable so that we can actually speak about deterministic quantization. Our assumption of dynamical periodic fields can be regarded as a combination of Newton's law of inertia and the de Broglie-Planck hypothesis of periodic matter waves: *elementary isolated systems must be supposed to have persistent periodicities as long as they do not interact.* To the energy of an elementary system $\bar{E} = \hbar\bar{\omega}$ we associate an intrinsic time periodicity is $T_t = 2\pi/\bar{\omega}$. Furthermore T_t must be considered to be dynamical, since it is related to the energy through the de Broglie like relation $T_t = h/\bar{E}$. This dynamical behavior of the periodicities guarantees full consistence with relativistic causality and time ordering. Interactions can be introduced in the theory as variations of the de Broglie periodicities of the fields. They can be equivalently formulated in terms of geometric deformations of the compact space-time dimensions of fields with Periodic Boundary Conditions (PBCs), in a way that mimics the same geometrodynamics of General Relativity (GR). The good behaviors of the theory arise from the fact that it adds a cyclic property to the usual notion of relativistic

time. After all time can only be defined by counting the number of cycles of isolated phenomena *supposed* to be periodic[1] such as the Cs atomic clock ($T_{Cs} \sim 10^{-10}s$). This approach turns out to be of particular interest in addressing the problem of the arrow of time.

2. Cyclic fields

The relativistic generalization of Newton's law of inertia states that every isolated elementary system has persistent four-momentum $\bar{p}_\mu = \{\bar{E}/c, \bar{\mathbf{p}}\}$. On the other hand, the de Broglie-Planck formulation of QM prescribes that the four-momentum must be associated with the four-angular-frequency $\bar{\omega}_\mu = \bar{p}_\mu c/\hbar$ of a corresponding field. Here we will assume that every elementary system is described in terms of intrinsically periodic fields whose periodicities are the usual de Broglie-Planck periodicities $T^\mu = \{T_t, \vec{\lambda}_x/c\}$. As Newton's law of inertia doesn't imply that every point particle moves on a straight line, our assumption of intrinsic periodicities does not mean that the physical world should appear to be periodic. In fact, the four-periodicity T^μ is fixed dynamically by the four-momentum through the de Broglie-Planck relation

$$c\bar{p}_\mu T^\mu = h \,. \tag{1}$$

Thus the variation of four-momentum that occurs during interactions corresponds to a retarded variation of the intrinsic periodicities of the fields. This will guarantee time ordering and relativistic causality.

Considering for the sake of simplicity only the case of periodicity $T_t(\mathbf{p})$ along the time coordinate, the cyclic field turns out to be decomposed as a tower of frequency eigenmodes

$$\Phi(\mathbf{x}, t) = \sum_n A_n \phi_n(\mathbf{x}) u_n(t) \,, \qquad \text{where} \qquad u_n(t) = e^{-i\omega_n(\bar{\mathbf{p}})t} \,. \tag{2}$$

Its quantized angular frequency spectrum is

$$\omega_n(\mathbf{p}) = n\bar{\omega}(\bar{\mathbf{p}}) = n\frac{2\pi}{T_t(\bar{\mathbf{p}})} \,. \tag{3}$$

The eigenmodes correspond to the harmonic modes of a string vibrating in a compact dimension with compactification length $T_t(\mathbf{p})$ and PBCs, *i.e.* in a cyclic dimension. Such a system can be imagined as a so called "de Broglie internal clock", that is, a periodic phenomenon whose time periodicity is fixed dynamically by the inverse of its energy $T_t(\mathbf{p}) = h/\bar{E}(\mathbf{p})$. Thus, according to the de Broglie-Planck relation eq.(1), we have a quantized energy spectrum

$$E_n(\mathbf{p}) = n\bar{E}(\mathbf{p}) = n\frac{h}{T_t(\mathbf{p})} \,. \tag{4}$$

The dependence on the spatial momentum $\bar{\mathbf{p}}$ means that the time periodicity depends on the reference frame. Indeed $T_t(\mathbf{p})$ is a time interval, it transforms dynamically according to Lorentz covariance.

In a relativistic framework it is easy to figure out that time periodicity induces periodicities on the spatial dimensions as well. They must necessarily be considered to have a consistent Lorentz covariant theory. Since in this case the whole physical information of the system is contained in a single four-period T^μ, our intrinsically four-periodic free field can be described by the following action in compact four dimensions with PBCs

$$\mathcal{S}_{\lambda_s} = \oint_0^{T^\mu} d^4 x \mathcal{L}_{\lambda_s}(\partial_\mu \Phi, \Phi) \,. \tag{5}$$

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[1] *"By a clock we understand anything characterized by a phenomenon passing periodically through identical phases so that we must assume, by the principle of sufficient reason, that all that happens in a given period is identical with all that happens in an arbitrary period."*   A. Einstein [10]

It is important to note that PBCs (represent by the circle in the volume integral symbol) minimize the action at the boundaries, in particular of the compact time dimension. Therefore they have the same formal validity as the usual (Synchronous) BCs assumed in ordinary field theory (fixed variations at the time boundaries, *i.e.* $\delta\Phi|_\Sigma = 0$). This is an essential feature because it guarantees that all the symmetries of the relativistic theory are preserved (as in usual field theory), even though we are assuming PBCs. In particular it guarantees that the theory is Lorentz covarince (the theory is based upon relativistic waves with retarded propagators [1]). For instance we can assume a generic Lorentz transformation

$$dx^\mu \to dx'^\mu = \Lambda^\mu_\nu \, dx^\nu \,, \qquad \bar{p}_\mu \to \bar{p}'_\mu = \Lambda^\nu_\mu \, \bar{p}_\nu \,. \tag{6}$$

By definition, the four-periodicity T^μ is such that the phase of the field is invariant under four-periodic translations $\exp\left[-ix^\mu \bar{p}_\mu/\hbar\right] = \exp\left[-i(x^\mu + cT^\mu)\bar{p}_\mu/\hbar\right]$. In this way we see that the four-periodicity is actually a contravariant four-vector. It transforms under Lorentz transformations in the following way

$$T^\mu \to T'^\mu = \Lambda^\mu_\nu \, T^\nu \tag{7}$$

so that the phase of the field is a scalar quantity under Lorentz transformations - de Broglie phase harmony. This can also be inferred by noticing that after the transformation of variables eq.(6), the free action eq.(5) turns out to be

$$\mathcal{S}_{\lambda_s} = \oint_0^{T'^\mu} d^4x' \mathcal{L}_{\lambda_s}(\partial'_\mu \Phi, \Phi) \,. \tag{8}$$

Therefore, in the new reference system, the new four-periodicity T'^μ of the field is actually given by eq.(7), that is, eq.(8) describes a system with four-momentum $\bar{p}'_\mu$ given by eq.(6).

The proper-time periodicity of massive particles, also known as the "de Broglie periodic phenomenon", is represented by $T_\tau = T_t(0)$. Since T_μ is a tangent vector, we can introduced the mnemonic notation $c\bar{p}_\mu = h/T^\mu$, so that it is easy to figure out that it transforms in the relativistic way according to the following constraint

$$\frac{1}{T_\tau^2} = \frac{1}{T_t^2(\mathbf{p})} - \frac{c^2}{\vec{\lambda}_x^2(\mathbf{p})} = \frac{1}{T_\mu}\frac{1}{T^\mu} \,. \tag{9}$$

It can be derived for instance by dividing the relativistic dispersion relation $\bar{M}^2c^4 = \bar{E}^2(\bar{\mathbf{p}}) - \bar{\mathbf{p}}^2c^2 = \bar{p}^\mu \bar{p}_\mu c^2$ by the Planck constant, *i.e.* by using eq.(1).

If we give energy to a field through interaction or we move away from the rest frame we have a relativistic deformation of its intrinsic time periodicity, as shown in fig.(1.b) and eq.(7). Therefore every level of the quantized energy spectrum transforms according to a relativistic dispersion relation. The resulting quantized energy spectrum of the field is

$$E_n(\bar{\mathbf{p}}) = nE(\bar{\mathbf{p}}) = n\sqrt{\bar{M}^2c^4 - \bar{\mathbf{p}}^2c^2} \,. \tag{10}$$

Thus, we find that such periodic fields have the same energy spectrum as the ordinary second quantized fields after normal ordering, as shown in fig.(1.a). In this way we see that this quantization prescription represents a generalization to relativistic fields of the semi-classical quantization of a particle in a box. This approach has also interesting analogies with the Matsubara [11], Kaluza-Klein (KK) [12, 13] and *zitterbewegung* models.

It must be noticed from eq.(9) that the proper-time periodicity T_τ is related to the mass of the field $T_\tau = T_t(0) = h/\bar{M}c^2$. Moreover, since the proper-time τ is proportional to the world-line parameter $s = c\tau$, we find that in this theory the world-line parameter is a cyclic variable with compactification length

$$\lambda_s = cT_\tau = \frac{h}{\bar{M}c} \,. \tag{11}$$

It actually corresponds to the Compton wavelength of matter fields.

The proper-time periodicity (Compton time) fixes the upper bound of the time periodicity, $T_\tau \geq T_t(\bar{\mathbf{p}})$, since the mass is the lower bond of the energy, $\bar{M}c^2 \leq \bar{E}(\bar{\mathbf{p}})$. The heavier the mass the faster the proper-time periodicity. For instance, even a light particle such as the electron has (in a generic reference frame) intrinsic time periodicity faster than $10^{-21}s$, *i.e.* faster than its proper-time periodicity. It should be noted that this is many orders of magnitude away from the Cs atomic clock whose periodicity is by definition of the order of $10^{-10}s$ and it is extremely fast even if compared with the present experimental resolution in time ($\sim 10^{-17}s$).
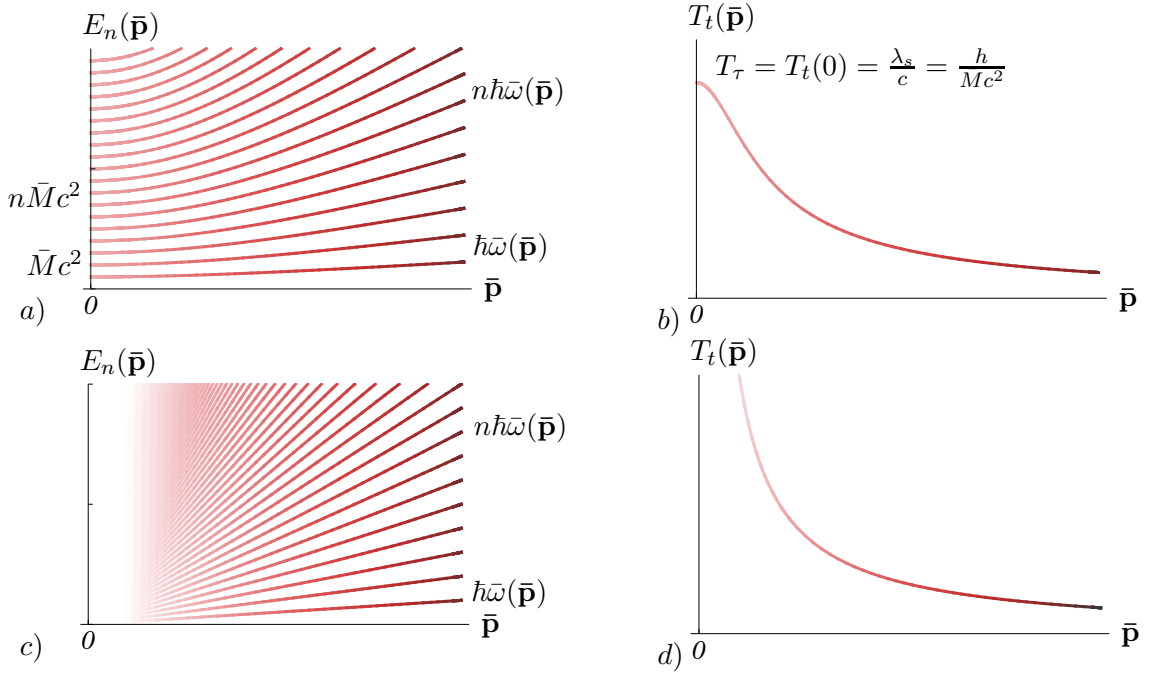

Figure 1. Dispersion relation $a)$ - $c)$ - of the quantized energy spectrum $E_n(\mathbf{p})$ and $b)$ - $d)$ - the corresponding time periodicity $T_t(\mathbf{p})$ of a massive - massless - cyclic field.

It is interesting to point out that since the world-line parameter s is compact, we have a deep analogy with ordinary string theory where one of the two world-sheet parameters is supposed to be compact or periodic. Thus our field theory can be classified as a particular kind of string theory - the periodic field eq.(2) is a vibrating string. We will also see that our field theory turns out to be "dual" to an extra-dimensional field theory whose compactification length is λ_s.

3. Correspondence to Quantum Field Theory

Now we shortly show that this description provides a remarkable matching with the canonical formulation of QM as well as with the Feynman Path Integral (FPI) formulation [1, 2, 3, 4, 5, 6, 7]. The evolution along the compact time dimension is described by the so called bulk equation of motions $(\partial_t^2 + \omega_n^2)\phi_n(x,t) = 0$ - for the sake of simplicity in this section we assume a single spatial dimension x. Thus the time evolution of the energy eigenmodes can be written as first order differential equations $i\hbar\partial_t\phi_n(x,t) = E_n\phi_n(x,t)$. The periodic field eq.(2) is a sum of on-shell standing waves. Actually this is the typical case where a Hilbert space can be defined. In fact, the energy eigenmodes form a complete set with respect to the inner product

$$\langle\phi|\chi\rangle \equiv \int_0^{\lambda_x} \frac{dx}{\lambda_x}\phi^*(x)\chi(x)\,. \tag{12}$$

Therefore we can define the Hilbert eigenstates as $\langle x|\phi_n\rangle \equiv \phi_n(x)/\sqrt{\lambda_x}$. On this base we can formally build a Hamiltonian operator $\mathcal{H}|\phi_n\rangle \equiv \hbar\omega_n|\phi_n\rangle$ and a momentum operator $\mathcal{P}|\phi_n\rangle \equiv -\hbar k_n|\phi_n\rangle$, where $k_n = n\bar{k} = nh/\lambda_x$. Thus the time evolution of a generic state $|\phi(0)\rangle \equiv \sum_n a_n|\phi_n\rangle$ is actually described by the familiar Schrödinger equation

$$i\hbar\partial_t|\phi(t)\rangle = \mathcal{H}|\phi(t)\rangle. \tag{13}$$

Moreover the time evolution is given by the usual time evolution operator $\mathcal{U}(t';t) = \exp[-\frac{i}{\hbar}\mathcal{H}(t-t')]$ which turns out to be a Markovian operator: $\mathcal{U}(t'';t') = \prod_{m=0}^{N-1}\mathcal{U}(t'+t_{m+1};t'+t_m-\epsilon)$ where $N\epsilon = t''-t'$.

From the fact that the spatial coordinate is in this theory a cyclic variable, by using the definition of the expectation value of an observable $\hbar\partial_x F(x)$ between two generic initial and final states $|\phi_i\rangle$ and $|\phi_f\rangle$ of this Hilbert space, and after integrating by parts eq.(12), we find

$$\langle\phi_f|\hbar\partial_x F(x)|\phi_i\rangle = i\langle\phi_f|\mathcal{P}F(x) - F(x)\mathcal{P}|\phi_i\rangle . \tag{14}$$

Assuming now that the observable is such that $F(x) = x$ [14] we obtain the usual commutation relation of ordinary QM: $[x,\mathcal{P}] = i\hbar$. With this result we have checked the correspondence to canonical QM.

Furthermore, it is possible to prove the correspondence to the FPI formulation. In fact, it is sufficient to plug the completeness relation of the energy eigenmodes in between the elementary time evolutions of the unitary (Markovian) operator. With these elements at hand and proceeding in a completely standard way we find the usual FPI in phase space,

$$\mathcal{Z} = \lim_{N\to\infty}\int_0^{\lambda_x}\left(\prod_{m=1}^{N-1}dx_m\right)\prod_{m=0}^{N-1}\langle\phi|e^{-\frac{i}{\hbar}(\mathcal{H}\Delta\epsilon_m - \mathcal{P}\Delta x_m)}|\phi\rangle . \tag{15}$$

This important result has been obtained without any further assumption than PBCs — see [2] for the generalization to interactions. It can be understood intuitively in a graphical way. For the sake of simplicity we again consider only time periodicity. We have to imagine a cylinder where there is an infinite set of possible classical - straight - paths with different winding numbers that link every given initial and final points. If we imagine opening this cylinder, the paths with different winding number are lines, but the initial and final points are no more the same — they differ by periods — fig.(2.a). Since these evolutions are unitary and invariant under periodic translations, we can ideally cut these paths in elementary parts, translate the resulting elementary paths by periods and combine them, fig.(2.b). In the resulting combination of paths the degeneracy under periodic translations has been removed; all the resulting paths obtained from this combination of elementary periodic paths have the same initial and final points. By iterating this procedure, fig.(2.c), it is easy to see that it reproduces — see [4] for a mathematical proof — the variation around a give classical paths of the usual Feynman formulation, see [1, 4]. In this way we can figure out that with cyclic dimensions there are many possible evolutions of a field from an initial configuration to a final configuration, that can self-interfere, similarly to the non-classical path of the FPI. However there is a fundamental conceptual difference with respect to the usual Feynman formulation, in fact now all these possible paths are classical paths, *i.e.* they are equivalent to classical periodic paths. This means that in this path integral formulation it is not necessary to relax the classical variational principle in order to have self-interference.

Since we have essentially standing waves, it is easy to see that there is an underlying Heisenberg uncertainty relation [1, 4]. Moreover, since periodicity means that the only possible energy eigenmodes are those with an integer number of cycles, we obtain the Bohr-Sommerfeld quantization condition (for instance the periodicity condition $E_n T_t = nh$ can be more in general written as $\oint E_n dt = nh$). This allows us to solve many non-relativistic quantum problems [1, 4].
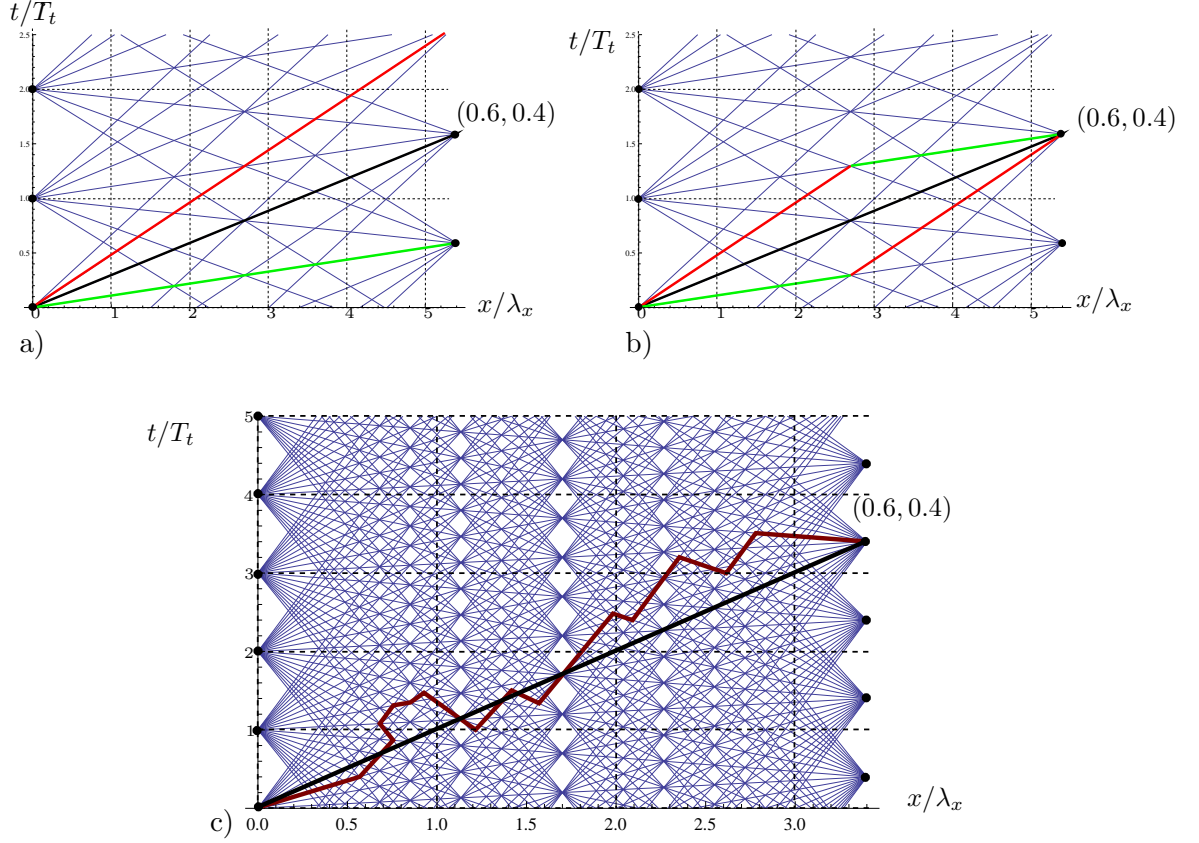
Figure 2. Periodic paths a) can be combined b) in order to reproduce Feynman paths c).

The non-quantum limit of a massive field, that is the non-relativistic single particle description, is obtained by setting the mass to infinity so that, as shown in [1, 4], in an effective classical limit, only the first level of the energy spectrum must be considered. This leads to a consistent interpretation of the wave-particle duality and of the double slit experiment. The quantities describing the first energy level are addressed by the bar sign. For instance, the Lagrangian of the fundamental mode $\bar{\Phi}(x)$ is $\bar{\mathcal{L}}_{\lambda_s}(\partial_\mu \bar{\Phi}(x), \bar{\Phi}(x))$.

On the other hand a massless field has infinite Compton wavelength and thus an infinite proper-time periodicity, fig.(1.d). Its quantum limit is at high frequency where the PBCs are important. In this limit we have discretized energy spectrum, in agreement with the ordinary description of the black-body radiation (no UV catastrophe), fig.(1.c). The opposite limit is when the time periodicity tends to infinity and we get back a continuous energy spectrum.

A remarkable property of this theory is that QM emerges from PBCs without involving any local-hidden-variable. This suggests that the theory can in principle violates Bell's inequality and we can actually speak about determinism. The idea is similar to the stroboscopic quantization or to the 't Hooft determinism. In fact, the intrinsic time periodicities are typically extremely fast. Thus, we inevitably have a too low revolution in time, so that at every observation the system appears in an aleatoric phase of its evolution, just like a clock observed under a stroboscopic light. Thus, the de Broglie intrinsic clocks of elementary particles can be imagined as a "de Broglie deterministic dice", that is dice rolling with time periodicities T_t. Since such dice roll too fast with respect to our time resolution, we can only predict the outcomes statistically. The results presented in this section show us that the statistical description associated to intrinsically fast periodic phenomena actually matches ordinary QM.

4. Matching with relativistic geometrodynamics

To introduce interactions we must bear in mind that the four-periodicity T^μ is fixed by the inverse of the four-momentum $\bar{p}_\mu$ according to the de Broglie-Planck relation eq.(1).

As already said, an isolated elementary system (*i.e.* free field) has persistent four-momentum. On the other hand, an elementary system under a generic interaction scheme can be described locally in terms of corresponding variations of its four-momentum with respect to the free case

$$\bar{p}_\mu \to \bar{p}'_\mu(x) = e^a_\mu(x)\bar{p}_a \,. \tag{16}$$

In other words we describe interactions in terms of the so call tetrad (or virebein) $e^a_\mu(x)$. Thus the interaction scheme eq.(16) turns out to be encoded in the corresponding local variation of the space-time periodicities

$$T^\mu \to T'^\mu(x) = e^\mu_a(x)T^a \,. \tag{17}$$

This is the corresponding deformation of the compactification lengths of a periodic field as well (in the case of sufficiently smooths or exponential deformations we approximate the compactification length with the local periodicity). Roughly speaking, interactions can be thought of as stretching of the compact dimensions of the theory. Equivalently, the interaction eq.(16) turns out to be encoded in the corresponding curved space-time background

$$\eta_{\mu\nu} \to g_{\mu\nu}(x) = e^a_\mu(x)e^b_\nu(x)\eta_{ab} \,. \tag{18}$$

This result can be double checked by considering the transformation of space-time variables

$$dx_\mu \to dx'_\mu(x) = e^a_\mu(x)dx_a \,. \tag{19}$$

This transformation of variables (diffeomorphism) with determinant of the Jacobian $\sqrt{-g(x)}$, the free action eq.(5) turns out to be

$$\mathcal{S}_{\lambda_s} \sim \oint^{e^\mu_a(x)T^a} d^4x\sqrt{-g(x)}\mathcal{L}_{\lambda_s}(e^a_\mu(x)\partial_a\Phi(x), \Phi(x)) \,. \tag{20}$$

Therefore, the periodic field which minimizes this action has four-periodicity T'^μ, eq.(17), or equivalently has four-momentum $\bar{p}_\mu$, eq.(16). We conclude that a field under the interaction scheme eq.(16) is described locally by the solutions of the bulk equations of motion with PBCs on the deformed compact background eq.(18) and compactification lengths eq.(17).

This geometrodynamical approach to interactions is interesting because it actually mimics the usual geometrodynamical approach of GR. In fact, if we suppose a weak Newton potential $V(\mathbf{x}) = -GM_\odot/|\mathbf{x}|c^2 \ll 1$, we find that the energy in a gravitational well varies (with respect to the free case) as $\bar{E} \to \bar{E}' \sim \left(1 + GM_\odot/|\mathbf{x}|c^2\right)\bar{E}$. According to eq.(17) or eq(1), this means that the de Broglie clocks in a gravitational well are slower with respect to the free clocks $T_t \to T'_t \sim \left(1 - GM_\odot/|\mathbf{x}|c^2\right)T_t$. Thus we have a gravitational redshift $\bar{\omega} \to \bar{\omega}' \sim \left(1 + GM_\odot/|\mathbf{x}|c^2\right)\bar{\omega}$. With this schematization of interactions we have retrieved two important predictions of GR.

Besides this, [15], we must also consider the analogous variation of spatial momentum $\bar{\mathbf{p}} \to \bar{\mathbf{p}}' \sim \left(1 + GM_\odot/|\mathbf{x}|c^2\right)\bar{\mathbf{p}}$, and the corresponding variation of spatial periodicities $\vec{\lambda} \to \vec{\lambda}' \sim \left(1 - GM_\odot/|\mathbf{x}|c^2\right)\vec{\lambda}$. According to the relation eq.(18), this means that the weak Newtonian interaction turns out to be encoded in the usual Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM_\odot}{|\mathbf{x}|c^2}\right)dt^2 - \left(1 + \frac{2GM_\odot}{|\mathbf{x}|c^2}\right)d|\mathbf{x}|^2 - |\mathbf{x}|^2 d\Omega^2 \,, \tag{21}$$

We have found that the geometrodynamical approach to interactions actually can be used to describe linearized gravity and that the geometrodynamics of the compact space-time dimensions correspond to the usual relativistic ones.

As well known, see for instance [15], it is possible to retrieve ordinary GR from a linear formulation by including self-interactions. More naively, as we will show in detail in a forthcoming paper, we can add "by hand" a kinetic term to the Lagrangian (with appropriate coupling), in order to describe the dynamics of the metric $g_{\mu\nu}$ which is the new *d.o.f.* of the theory eq.(22). Thus, neglecting quantum corrections, we can replace the classical limit $\sqrt{-g}\bar{\mathcal{L}}_{\lambda_s}$ of the Lagrangian in eq.(22) with the Hilbert-Einstein Lagrangian

$$\bar{\mathcal{L}}_{HE} = \sqrt{-g}\left[-\frac{g^{\mu\nu}\mathcal{R}_{\mu\nu}}{16\pi G_N} + \bar{\mathcal{L}}_{\lambda_s}(e^a_\mu \partial_a \bar{\Phi}(x), \bar{\Phi}(x))\right] . \tag{22}$$

It can be shown that this procedure is parallel to what we usually do in electromagnetism when we add the term $-F_{\mu\nu}F^{\mu\nu}/4e^2$ to describe the kinematics of the gauge field. Intuitively, because of its geometrical meaning, the Ricci tensor is the correct mathematical object to describe the variations of the space-time compactification lengths in different space-time points. Bearing in mind eq.(1), we note that actually such a kinetic term must encode the content of four-momentum in different space-time points. By varying the metric (neglecting issues related to the variation of boundary terms of the Hilbert-Einstein action and related BCs [16]) eq.(22) yields the usual Einstein equation $\mathcal{R}^{\mu\nu} = -8\pi G_N \mathcal{T}^{\mu\nu}$. With these simple and heuristic arguments we have shown that field theory in compact space-time is in agreement not only with special relativity but also with GR. In forthcoming papers we will show that, by writing eq.(16) as a minimal substitution, such a geometrodynamical approach to interactions can be also used to describe ordinary gauge interactions. Gauge fields will turn out to "tune" the variation of periodicities, allowing a semi-classical interpretation of superconductivity [2].

5. "Virtual" Extra Dimension (VXD)

Our geometrodynamical description of interactions provides an intuitive interpretation of AdS/CFT whose essential meaning is summarized by the Witten's words: "In this description, quantum phenomena [...] are coded in classical geometry" [17]. To understand this point we should note that our field theory in compact space-time is "dual" to an extra dimensional field theory, see [1, 4, 3]. For instance we may note that the energy spectrum eq.(10) in the rest frame $(\bar{\mathbf{p}} \equiv 0)$ reproduces the usual KK mass tower $M_n = E_n(0)/c^2 = n\bar{M} = nh/\lambda_s c$, see fig.(1.a). Practically, the recipe to obtain our periodic fields from a corresponding extra dimensional field on a flat five dimensional metric $(dS^2 = dx_\mu dx^\mu - ds^2)$ and with zero five dimensional mass $(dS^2 = 0)$, is to identify the compact extra dimension s with our compact world-line parameter $s = c\tau$ and integrating it out, see [1, 4, 3] for more detail. In this way we get back the usual four-dimensional Minkowskian metric $ds^2 = dx_\mu dx^\mu$. In particular we find that the compactification length of the eXtra Dimension (XD) corresponds to the compactification length λ_s of the compact world-line parameter s, *i.e.* to the Compton wavelength eq.(11). When we identify the XD with the world-line parameter we say that it is a "Virtual" eXtra Dimension (VXD) and the KK mode are the "virtual" modes of the field[2]. Thus, in our theory, the KK modes are not different elementary fields as in the usual KK scenario but collective modes of the same 4D periodic field, similarly to the holographic prescription. They play the role of energy eigenmodes. Finally, since periodic fields reproduce QM and, on the other hand, they turn out to be dual to extra dimensional fields, we infer that the classical geometrodynamics on a VXD could be used to describe quantum behaviors, similarly to AdS/CFT.

Shortly, in the approximation of massless fields, to describe the interaction scheme eq.(16) through the VXD formalism we have to generalize the deformed metric eq.(18) to the following

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[2] Note that in the original Kaluza's formulation [12] the XD was introduced not like an "real" XD but as a "mathematical trick", whereas in the original Klein's proposal [13] the assumption of cyclic (*i.e.* compact with PBCs) XD was used as a semi-classical quantization condition.

deformed metric in VXD

$$g_{MN}(s) \sim \begin{pmatrix} g_{\mu\nu}(x(s)) & 0 \\ 0 & 1 \end{pmatrix} , \tag{23}$$

where the capital letters label the 5D Lorentz indices. Thus we expect to find out that the classical geometrodynamics of the field in such a deformed metric with VXD reproduces the quantum behaviors of the corresponding interaction scheme eq.(16). Similarly to the Maldacena conjecture, this correspondence can be summarized by the mnemonic relation

$$\mathcal{Z} = \int_0^{\lambda_x} \mathcal{D}x \; e^{\frac{i}{\hbar}\mathcal{S}(s,s')[\dot{x},x]} \leftrightarrow e^{\frac{i}{\hbar}\mathcal{S}^{VXD}(s,s')} . \tag{24}$$

For instance [4], we can consider a collider experiment where the Quark-Gluon-Plasma (QGP) can be approximated as a volume of massless fields with high four-momentum[3]. As predicted by a simple Bjorken hydrodynamical model [18], or by thinking of the analogy of QCD with a thermodynamical system [19], the four-momentum of the fields decays exponentially and conformally as the QGP radiates hadronically and electromagnetically[4]: $\bar{p}_\mu \to \bar{p}'_\mu(s) \simeq e^{-ks}\bar{p}_\mu$. - in natural units. The QGP freeze-out is, in first approximation, encoded in the tetrad $e^a_\mu(s) \simeq \delta^a_\mu e^{-ks}$. This means that the space-time periodicities of the fields have a conformal and exponential dilatation $T^\mu \to T'^\mu(s) \simeq e^{ks}T^\mu$. Indeed the time periodicity $T'_t(s)$ is the usual conformal parameter of AdS/CFT and the QGP freeze-out is effectively be described by the virtual AdS metric

$$dS^2 \simeq e^{-2ks}dx_\mu dx^\mu - ds^2 \equiv 0 . \tag{25}$$

According to our description, it is easy to prove [20, 21] that the propagation of classical fields an warped virtual metric with infinite VXD describes the quantum behavior of a massless theory such as CFT. A finite VXD gives mass to the virtual KK modes breaking the conformal invariance. Their collective classical propagation can described by using holography [20, 21], [5]

$$\Pi^{Holo}(q^2) \sim -\frac{q^2}{2kg_5^2} \log \frac{q^2}{\Lambda^2} , \tag{26}$$

which assuming $1/kg_{5^2} = N_c/12\pi^2$, can be matched to the quantum two point function of QCD. The mass spectrum of the virtual KK modes can be interpreted as an hadronic spectrum. Thus we obtain semi-classically the quantum running of the strong coupling constant

$$\frac{1}{e^2_{eff}(q)} \sim \frac{1}{e^2} - \frac{N_c}{12\pi^2} \log \frac{q}{\Lambda} . \tag{27}$$

These are fundamental characteristics of the AdS/QCD models.

6. Conclusions

To understand the conceptual meaning of this theory we have to note that time can only be defined by counting the number of cycles of phenomena *supposed* to be periodic. Only by assuming periodicity we can ensure that the duration of the unit of time is always the same; in the past, in the present and in the future, [10]. In particular, the usual - non compact - time axis ($t \in \mathbb{R}$) is defined with reference to the Cs-133 clock whose characteristic periodicity

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[3] This example is of particular interest because of the recent data from LHC

[4] The simplest Bjorken kinetic model [18] describing the exponential gradient of temperature of the QGP freeze-out can be regarded as the analogous of Newton's law of cooling in QCD thermodynamics.

[5] To evaluate the low energies effective propagator we assume Neumann BCs at the UV scales Λ and boundary field $A_\mu(q)$ at the IR scale μ, in the hypothesis of Euclidean momentum q such that $\Lambda \gg |q| \gg \mu$. [21, 4].

$\sim 10^{-10}s$. Hence, for a consistent formulation of natural laws we believe that there must be an assumption of periodicity in physics. We explicitly introduce this assumption by imposing on every free field its de Broglie intrinsic periodicity as a constraint. As shown in detail in [1], see also [4, 5, 6], and as we have summarized in this paper, under this assumption ordinary QM emerges as an effective description of the underlying deterministic field theory by a process of "information loss". It is important to note that in our case there are no local-hidden-variables, time being a physical variable that can not be eliminated from the theory, and the PBCs being an element of non locality. This suggests that the theory is not constrained by Bell's or similar theorems; it can in principle violate Bell's inequality and reproduce the predictions of QM. This allows us to speculate about a scenario where, by observing the de Broglie dice of electrons with time resolution greater that the ZHz, it could be possible to resolve the underlying deterministic cyclic electrodynamics and in principle predict the outcomes of the quantum dice.

These cyclic fields can therefore be identified with the so call "de Broglie internal clocks" or "de Broglie periodic phenomena". Similarly to a calendar or to a stopwatch where every moment in time is determined by the combination of the phases of periodic cycles (typically: years, months, days, hours, minutes and seconds), every value of our external temporal axes (defined in terms of the "ticks" of the Cs-133 atomic clock) is characterized by a unique combination of the phases of the de Broglie internal clocks, *i.e.* by the "ticks" of all the elementary particles constituting the system under investigation. In this scenario the long time scales are provided by massless fields with low frequencies (low energies implies long time periodicities). The combination of two or more clocks - *i.e.* a non elementary system - with irrational ratio of periodicities, gives ergodic - or even more chaotic - evolutions. Furthermore the clocks can vary their periodicity through interactions (exchange of energy) as their periodicities depend on the reference systems according to the relativistic laws. Once the de Broglie internal clocks are fixed to be clockwise or anticlockwise, the flow of time is determined by the combinations of their "ticks" and the variations of their periodicities through interactions. This means that the external time axis can be in principle dropped and the flow of time can be effectively described in terms of the "ticks" of these de Broglie internal clocks. In this scenario the flow of time doesn't depend on the assumption of clockwise or anticlockwise rotations of the de Broglie internal clocks. The resulting formulation of the flow of time is particularly interesting for the problem of the time arrow in physics.

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