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My name is Edward Bryden, and I'm a postdoc at the University of Antwerp in Belgium. I completed my PhD under the supervision of Marcus Khuri at Stony Brook University. My thesis was on the stability of the positive mass theorem in the presence of axisymmetry. Stability problems of this nature, which are also sometimes called almost rigidity results, arising from the study of geometric inequalities.

There are two ideas behind geometric inequalities First, one wants to relate local analytic information such as curvature to geometric information such as volume, or area, or the isoperimetric inequality relating the two. Typically when one does this, there is a model space for which this relationship is particularly clear. When studying curvature, these models are usually the spaces of constant sectional curvature: the sphere, the plane, and the hyperbolic plane. Consider one such example: volume growth bounds arising from lower bounds on the Ricci curvature. This result says that if a space has Ricci curvature bounded below by a constant, then it's volume growth is bounded by the volume growth present in the model space. This is the geometric inequality. The second idea is that the model space should be unique in the following sense: it should be the only space which saturates the geometric inequality. This called rigidity.

Although scalar curvature doesn't carry much information, it still has geometric consequences. There is a physical motivation coming from general relativity for this surprising fact: scalar curvature behaves a little bit like energy density. The clearest expression of this is the Positive Mass Theorem, which says that if the scalar curvature of a space is non-negative, then if we add up all of the mass present, the result will also be non-negative. This is the geometric inequality. The rigidity says that if the total mass is zero, and the scalar curvature is non-negative, then the space must be empty, and so isometric to the plane. The connection between scalar curvature and energy-density make these results very believable, but they are hard to rigorously prove.

When we ask if the Positive Mass Theorem is stable, we are asking the following natural question. If the scalar curvature is non-negative, and the mass is small, must the space be nearly empty in the sense that it is close to being flat? These questions are so interesting to me because they are so natural to ask, but have many subtleties to them. For example, the first question one might ask is also perhaps the most interesting and difficult: what does it mean for a space to be close to flat? That is, what topology are we using, if any, to measure this closeness?

In this course we will study two cases of stability: that of the Positive Mass Theorem, and that of the Torus. We will see the subtleties involved in all stages of the analysis, and hopefully it will become clear why these questions are so engaging.