# Theorem 4.2.2

**Theorem 4.2.2.** *In Euclidean geometry, three non-collinear points determine a unique circle.*

*Proof.* Let $A$, $B$, and $C$ be three non-collinear points. Let $\ell$ and $m$ be the two perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. We claim that $\ell$ and $m$ intersect each other. If not, that is, if $\ell \parallel m$, then $m$ intersects $\overleftrightarrow{AB}$ by Proclus' lemma. By the converse of the Alternate Interior Angle theorem, $m \perp \overleftrightarrow{AB}$ since $\ell \perp \overleftrightarrow{AB}$. But then $\overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$ since $\overleftrightarrow{AB} \neq \overleftrightarrow{BC}$ and by the Alternate Interior Angle theorem with $m$ as a common perpendicular. This contradicts $B \in \overleftrightarrow{AB} \cap \overleftrightarrow{BC}$. Thus, $\ell$ and $m$ intersect at some point $O$.

By a basic SAS argument, we see that $\overline{OA} \cong \overline{OB} \cong \overline{OC}$. Thus, $A$, $B$, $C$ are all incidence to the circle centered at $O$ and with radius $\overline{OA}$. This circle is unique, since a basic SSS argument shows that $\overline{PA} \cong \overline{PB} \cong \overline{PC}$ implies that $P \in \ell \cap m = \{O\}$. □

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