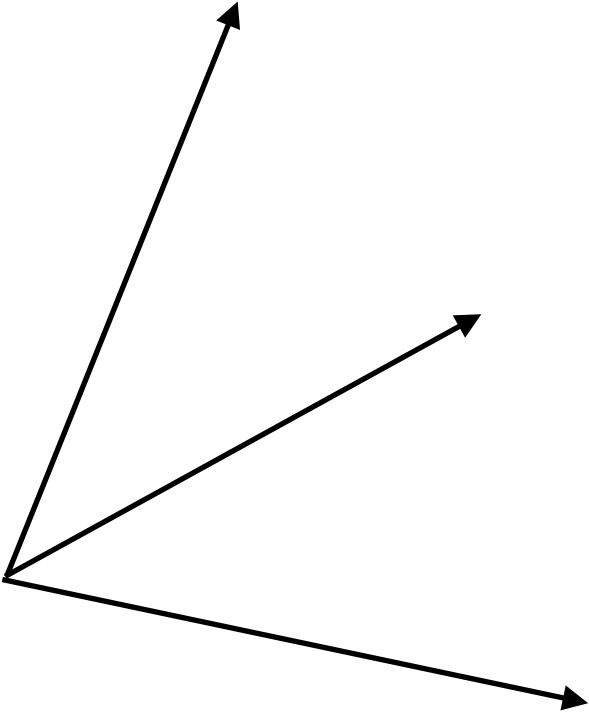
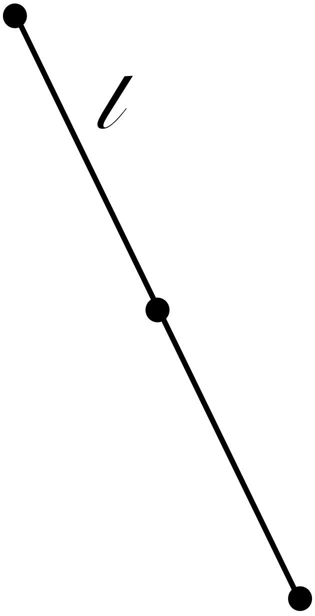
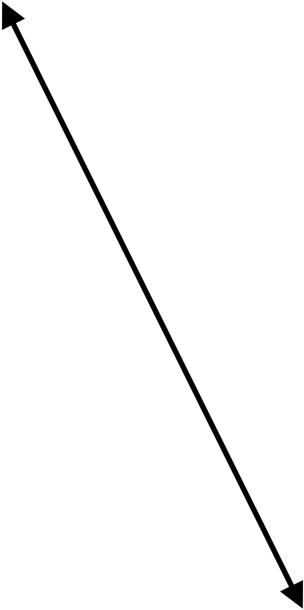
Proof. Let P be a point in the geometry. Since not all points are on the same line by Axiom 3, there must be a line ℓ which does not contain P. Then for each point on ℓ, there is a unique line containing this point and P by Axiom 4. Likewise, these three lines are all distinct (otherwise the uniqueness of Axiom 4 is violated). Finally, there is a unique line containing P which is parallel to ℓ by Axiom 5. Thus, there are at least 4 lines incident to P.

Let m be another line incident to P that was not considered before. It cannot be parallel to ℓ, otherwise the uniqueness of parallel lines would be violated. Hence, m intersects ℓ. Call their intersection Q. Then m is the unique line determined by P and Q. Since Q is on ℓ, this line was already considered above, which leads to a contradiction. The result then follows. $\square$

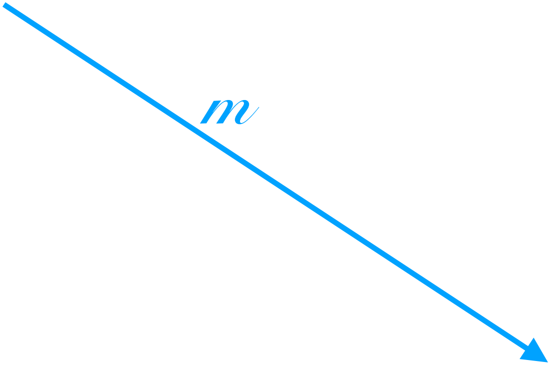




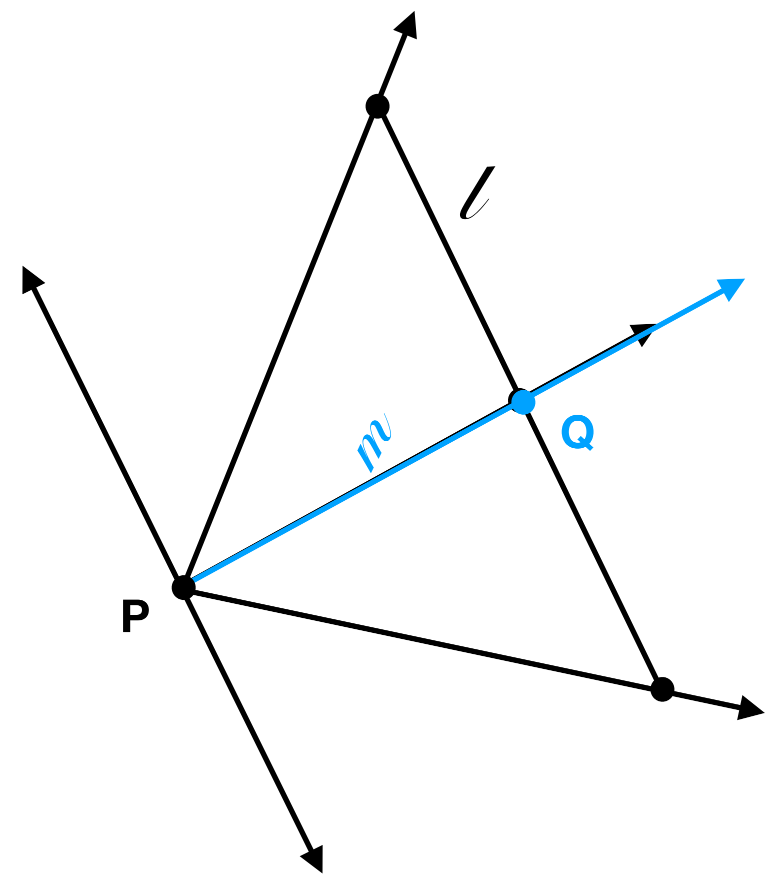
l



Q


m

P •



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