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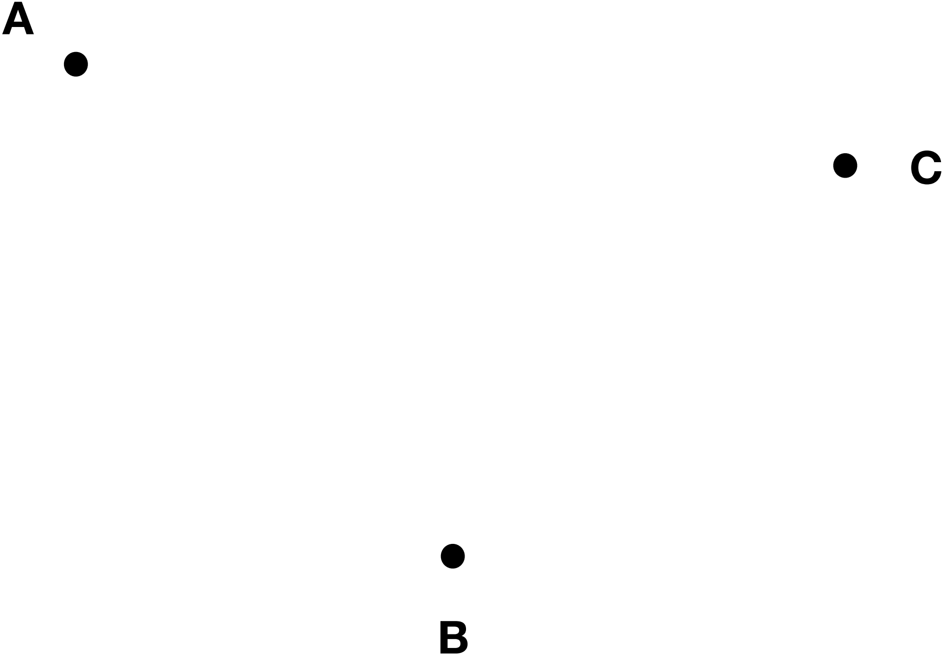
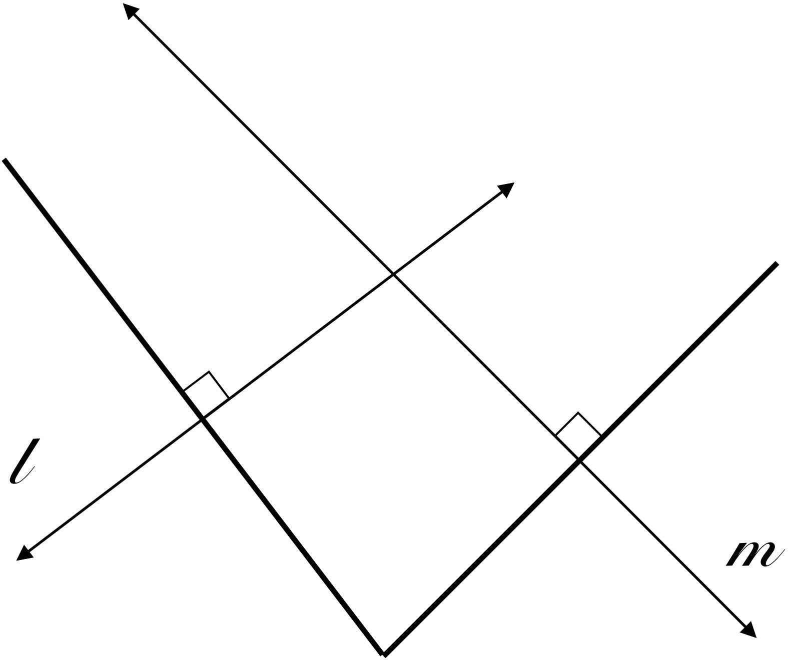
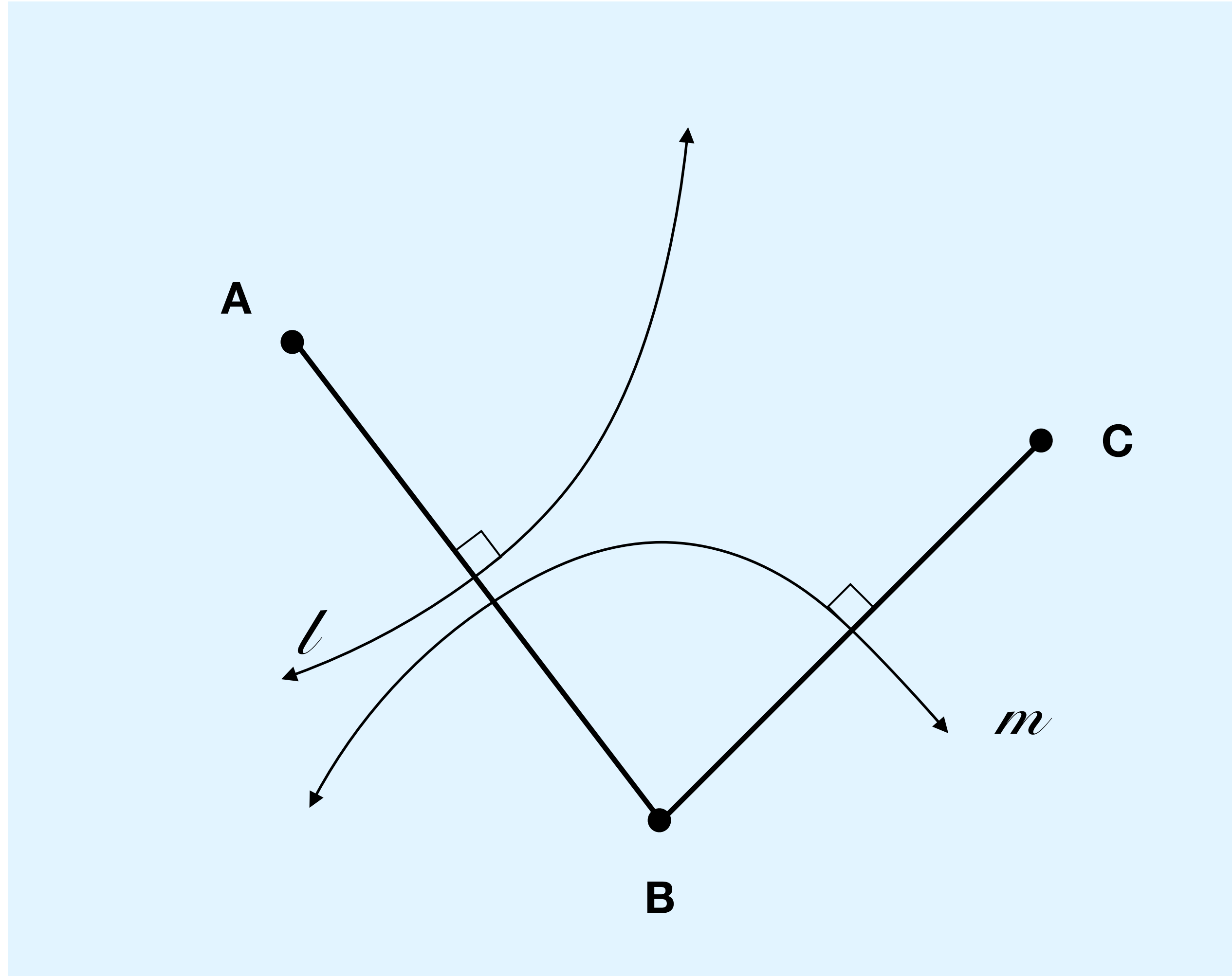

A
C
B


l
m

then



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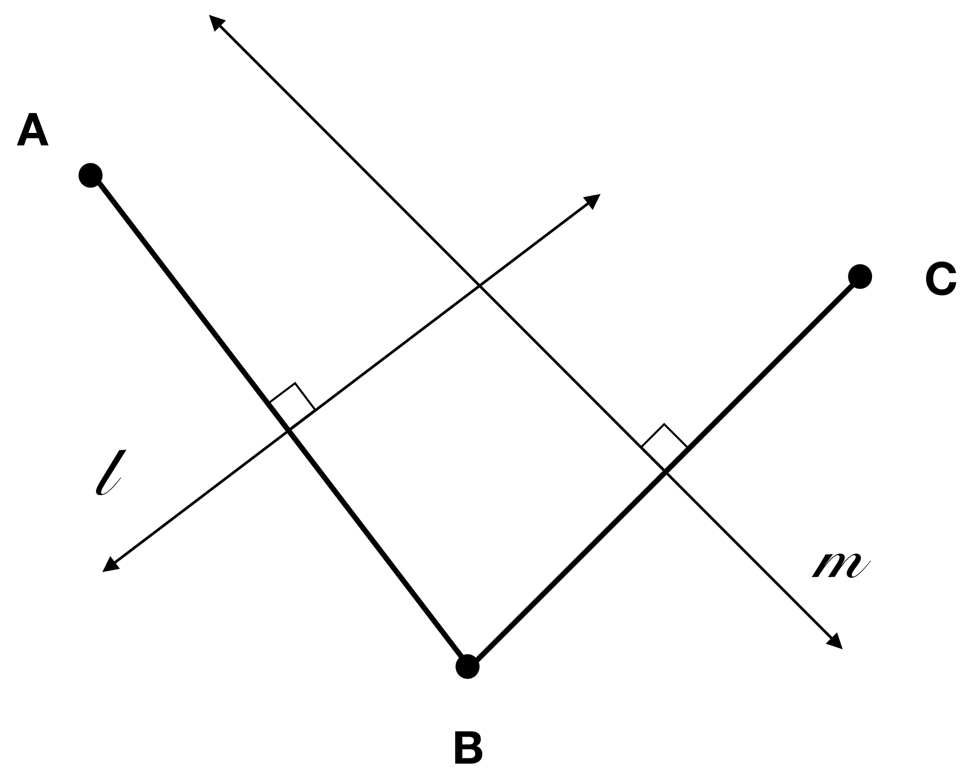

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