

Theorem 1.3.14

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Proof. Let P be a point in the geometry. Since not all points are on the same line by Axiom 3, there must be a line ℓ which does not contain P . Then for each point on ℓ , there is a unique line containing this point and P by Axiom 4. Likewise, these three lines are all distinct (otherwise the uniqueness of Axiom 4 is violated). Finally, there is a unique line containing P which is parallel to ℓ by Axiom 5. Thus, there are at least 4 lines incident to P .

Let m be another line incident to P that was not considered before. It cannot be parallel to ℓ , otherwise the uniqueness of parallel lines would be violated. Hence, m intersects ℓ . Call their intersection Q . Then m is the unique line determined by P and Q . Since Q is on ℓ , this line was already considered above, which leads to a contradiction. The result then follows. \square

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