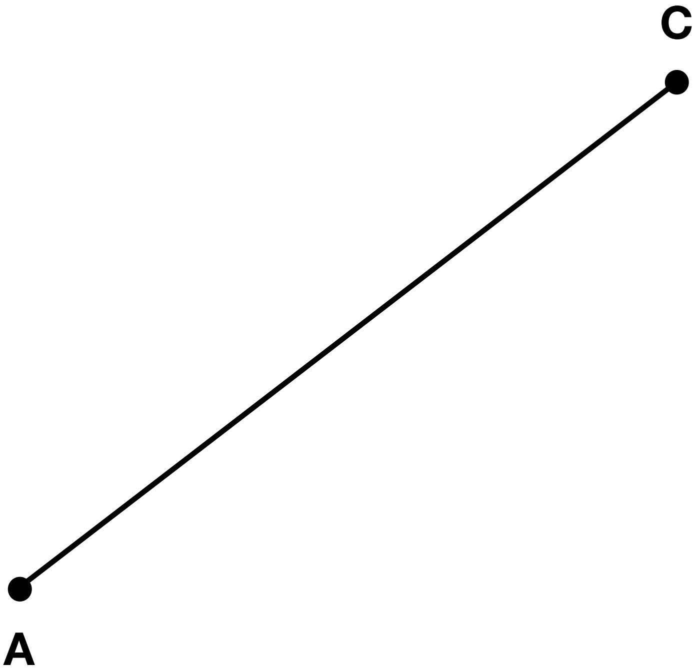
Finally, the angles $\angle AFB \cong \angle EFC$ since they are vertical angles (remember that $A - F - E$ and $B - F - C$). Therefore, $\triangle AFB \cong \triangle EFC$ by AAS. In particular, $\overline{BF} \cong \overline{FC}$. This provides the existence of midpoints.

Let B and B' be midpoints of AC, and let $\preceq$ be an ordering of $\overleftrightarrow{AC}$ such that $A \preceq C$. Without the loss of generality, we may assume that $B' \preceq B$. In particular, $A - B' - B - C$. Then

$$\overline{AB'} \leq \overline{AB} \cong \overline{BC} \leq \overline{B'C} \cong \overline{AB'}.$$

Hence, $\overline{AB'} \cong \overline{AB}$. But by the uniqueness of Segment Translation, we conclude that $B = B'$, that is, midpoints are unique. $\square$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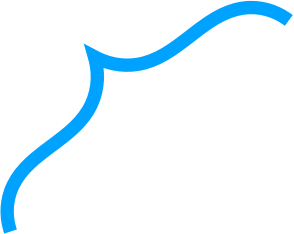

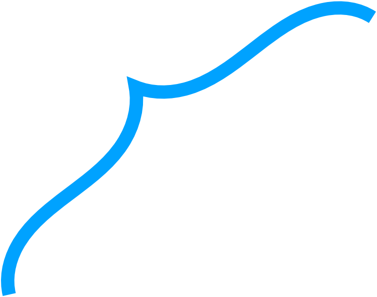

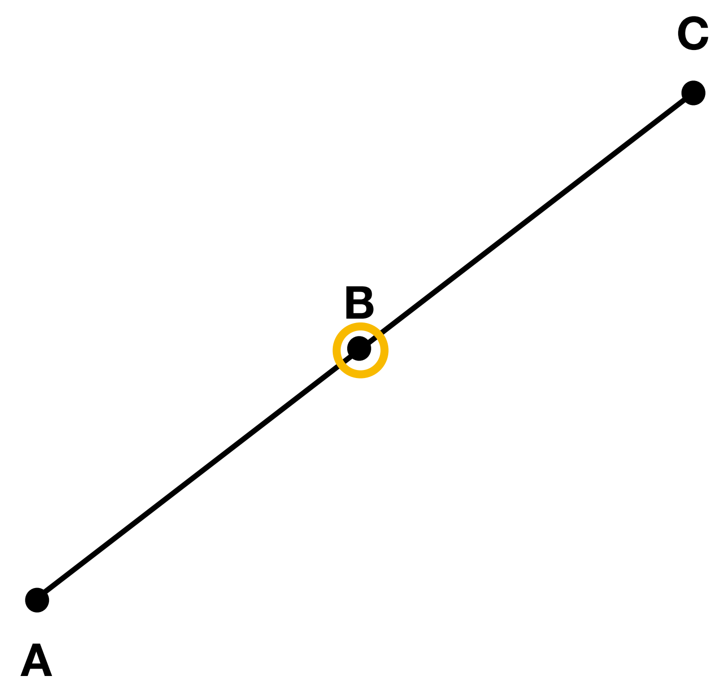

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