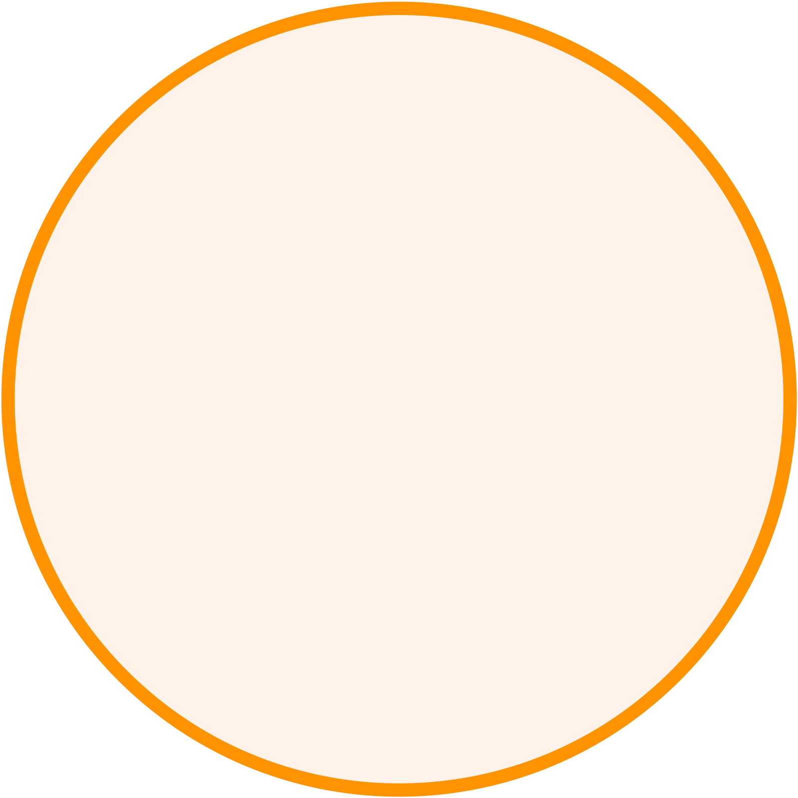

Proof. Let A, B, and C be three non-collinear points. Let ℓ and m be the two perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. We claim that ℓ and m intersect each other. If not, that is, if $\ell \parallel m$, then m intersects $\overleftrightarrow{AB}$ by Proclus' lemma. By the converse of the Alternate Interior Angle theorem, $m \perp \overleftrightarrow{AB}$ since $\ell \perp \overleftrightarrow{AB}$. But then either $\overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$ since $\overleftrightarrow{AB} \neq \overleftrightarrow{BC}$ and by the Alternate Interior Angle theorem with m as a common perpendicular. This contradicts $B \in \overleftrightarrow{AB} \cap \overleftrightarrow{BC}$. Thus, ℓ and m intersect at some point O.

By a basic SAS argument, we see that $\overline{OA} \cong \overline{OB} \cong \overline{OC}$. Thus, A, B, C are all incidence to the circle centered at O and with radius $\overline{OA}$. This circle is unique, since by a basic SSS argument shows that $\overline{PA} \cong \overline{PB} \cong \overline{PC}$ implies that $P \in \ell \cap m = \{O\}$. $\square$

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B

then

A

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• C

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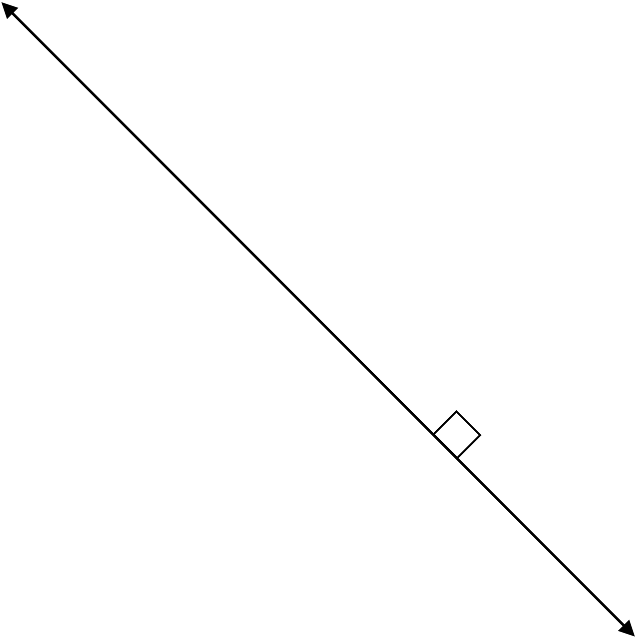
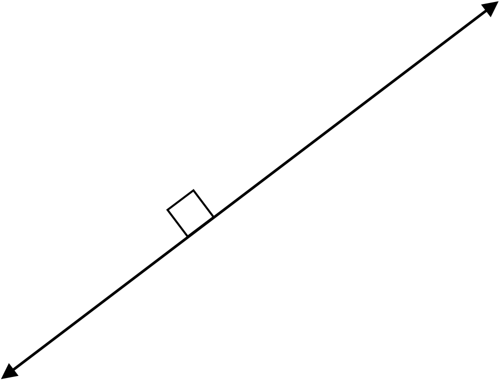
m





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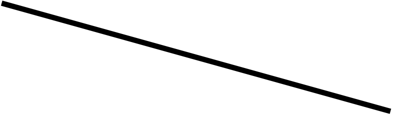

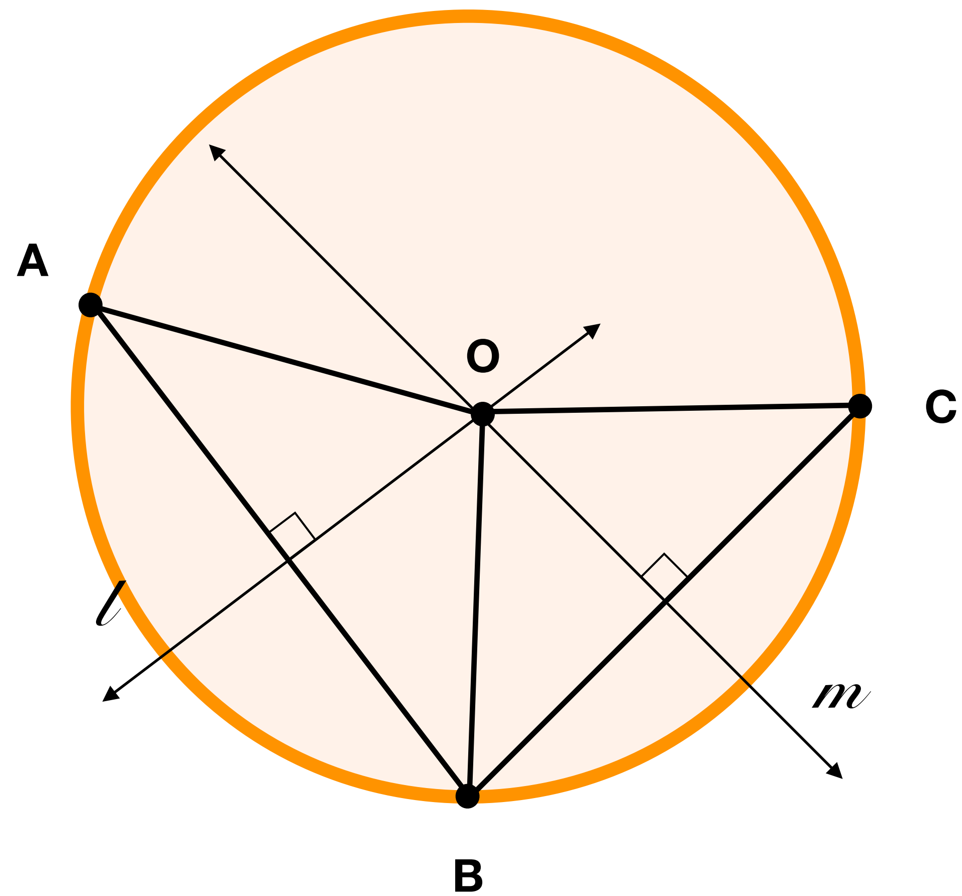

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