

Theorem 6.8.5

Theorem 6.8.5 (The Obtuse Angle Hypothesis). *In elliptic geometry, the summit angles of a Saccheri quadrilateral are obtuse.*

Proof. Let $\square ABCD$ be a Saccheri quadrilateral with altitude \overline{EF} with $F \in \overleftrightarrow{AB}$. By the elliptic parallel postulate, the lines \overleftrightarrow{AB} and \overleftrightarrow{DC} must intersect at some point, say P . Relative to the ideal line \overleftrightarrow{AD} , we may say that $F - B - P$ and $E - C - P$.

Since $\triangle EFP$ is a double-right triangle, Theorem 6.8.4 shows that \overline{EP} and \overline{FP} are polar lengths. Also, $\triangle BPC$ is a right, with $m\angle C = 90^\circ$. By Theorem 6.8.4, $\angle BCP$ is acute since $\overline{BP} < \overline{FP}$ (relative to \overleftrightarrow{AD}). Then its supplement, $\angle BCE$ is obtuse. \square

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