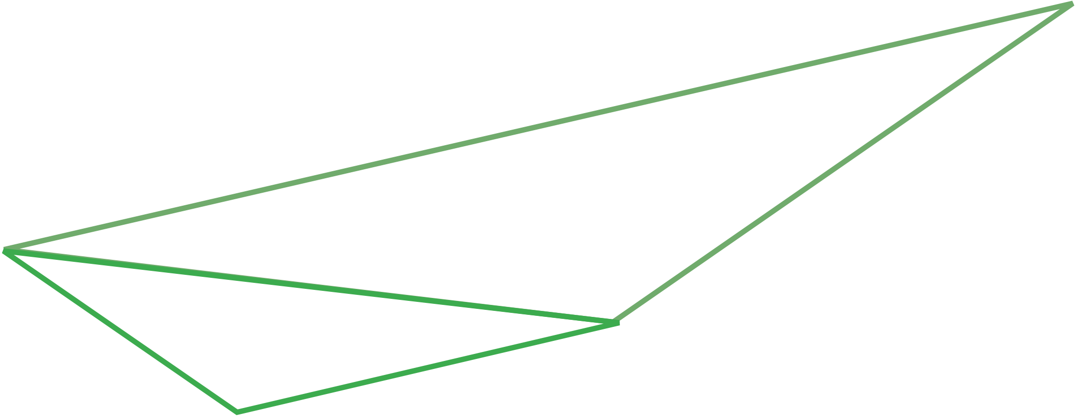
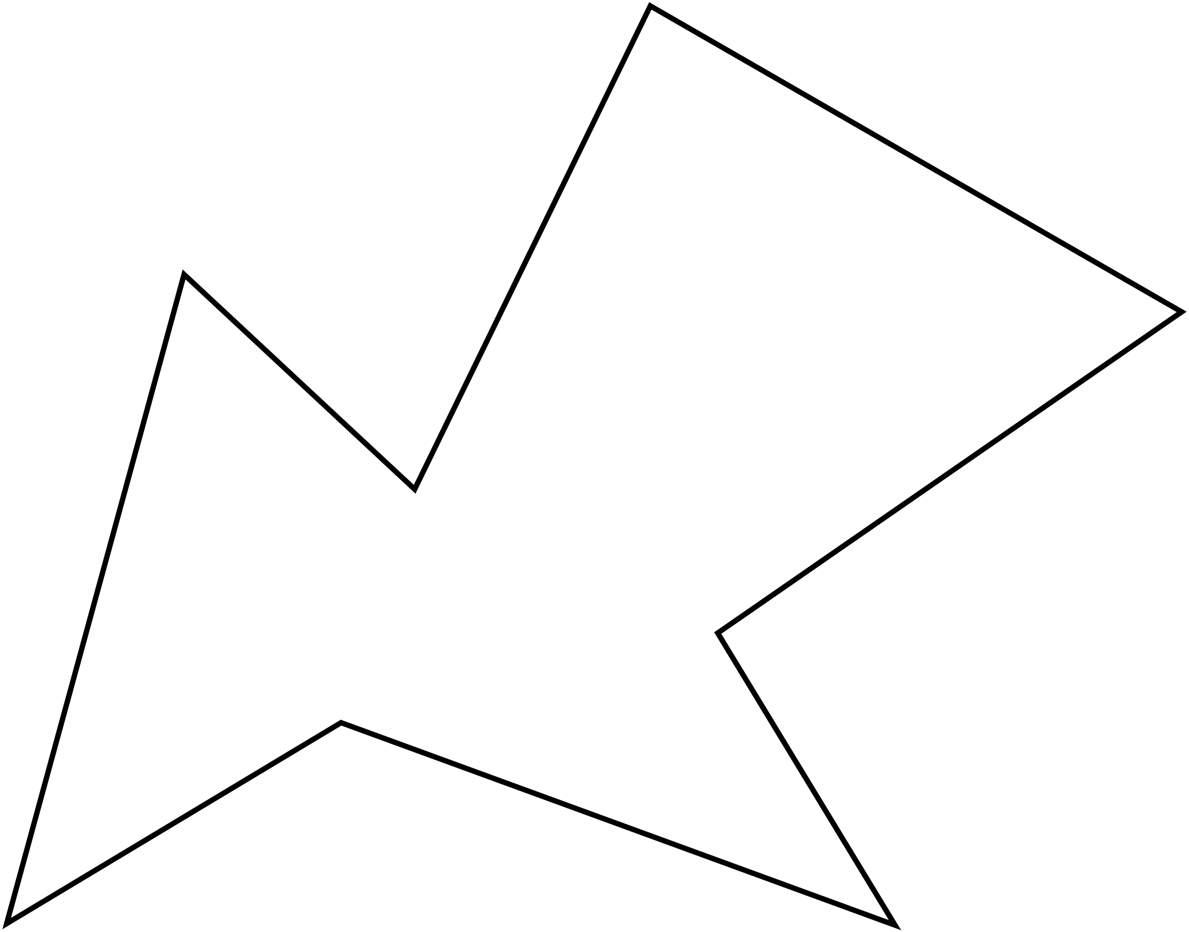
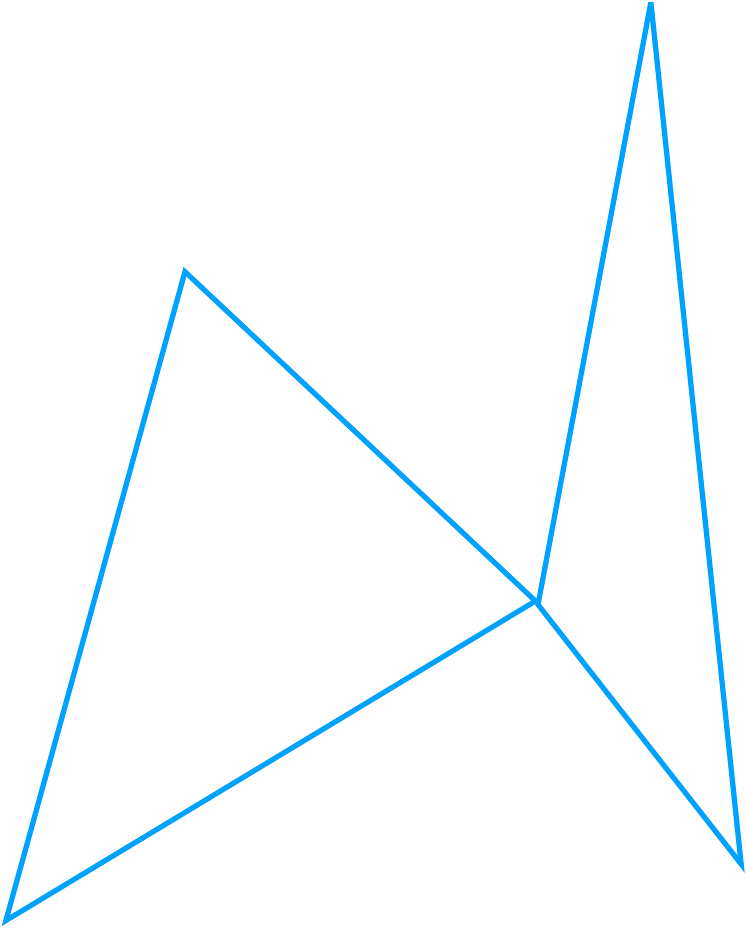
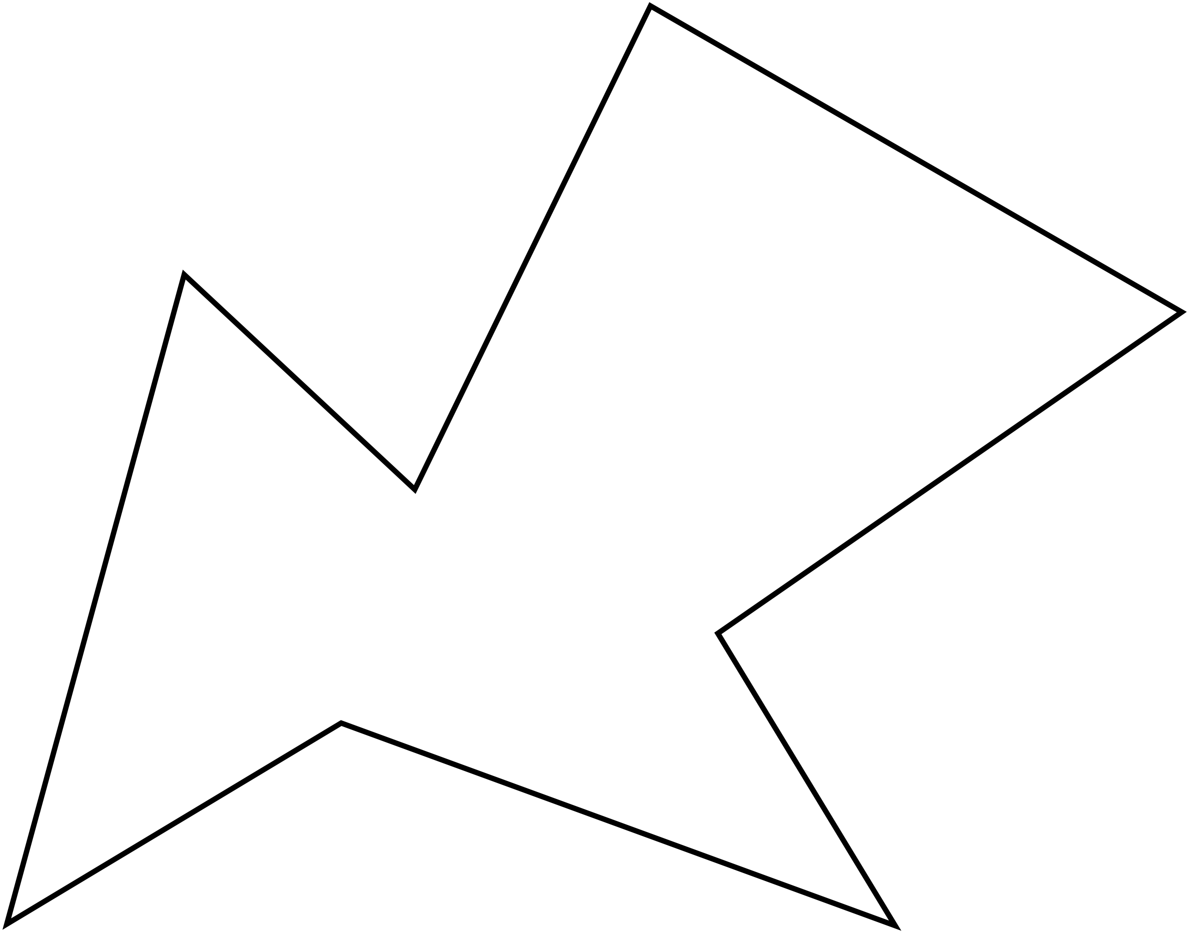
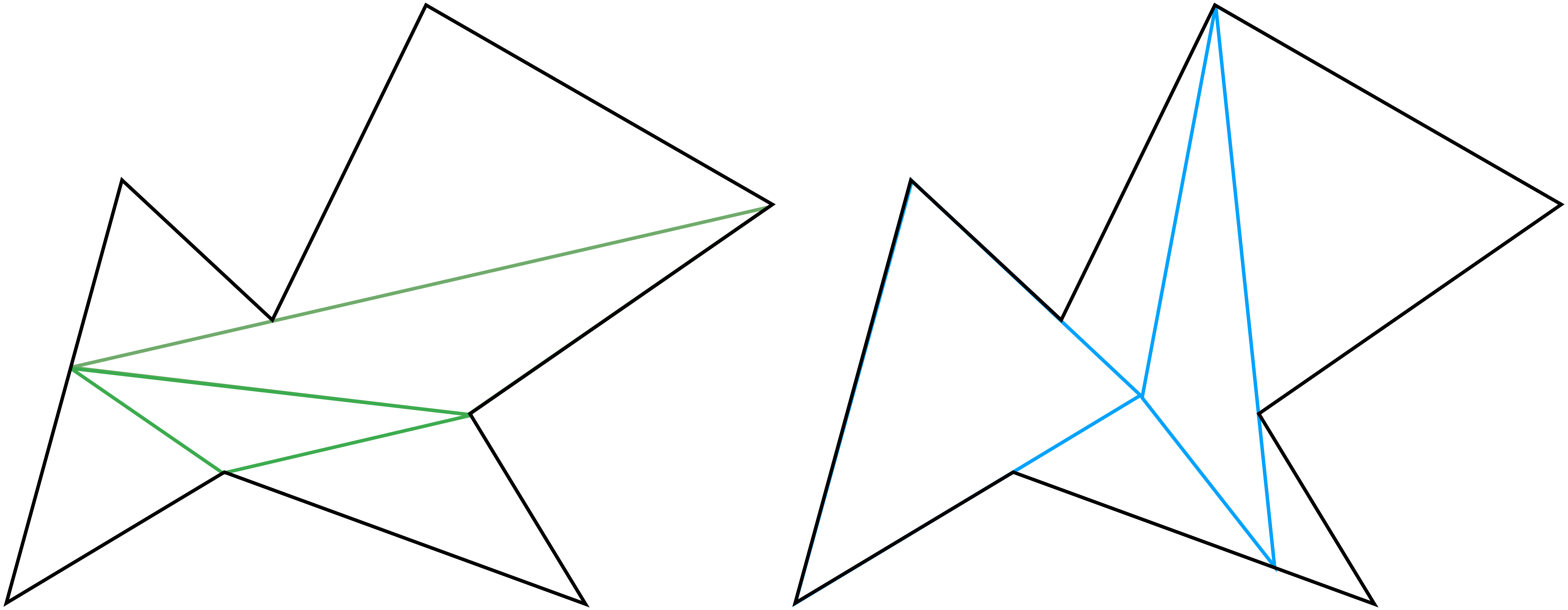


Proof. The proof relies on finding a common refinement of each partition, that is, a **refinement** $\mathcal{R}$ of a partition $\mathcal{P}$ of triangles is a partition of the original polygon such that each triangle in $\mathcal{P}$ is a union of adjacent, non-overlapping triangles of $\mathcal{R}$. In other words, the refinement $\mathcal{R}$ is a union of partitions of triangles for each triangle in $\mathcal{P}$. It is a combinatorially argument that any two partitions of triangles of a common polygon have a common refinement. The details rely primarily upon corresponding all the possible intersections and symmetric differences of two overlapping triangles and why each intersection or symmetric difference can be partitioned into triangles. A double induction argument then follows. The complete details are left to the reader. When a common refinement is obtained, the additivity of defects of triangles implies that the two orignal partitions of the polygon will have the same defect as the common refinement. Therefore, the defect of a polygon is well-defined. □









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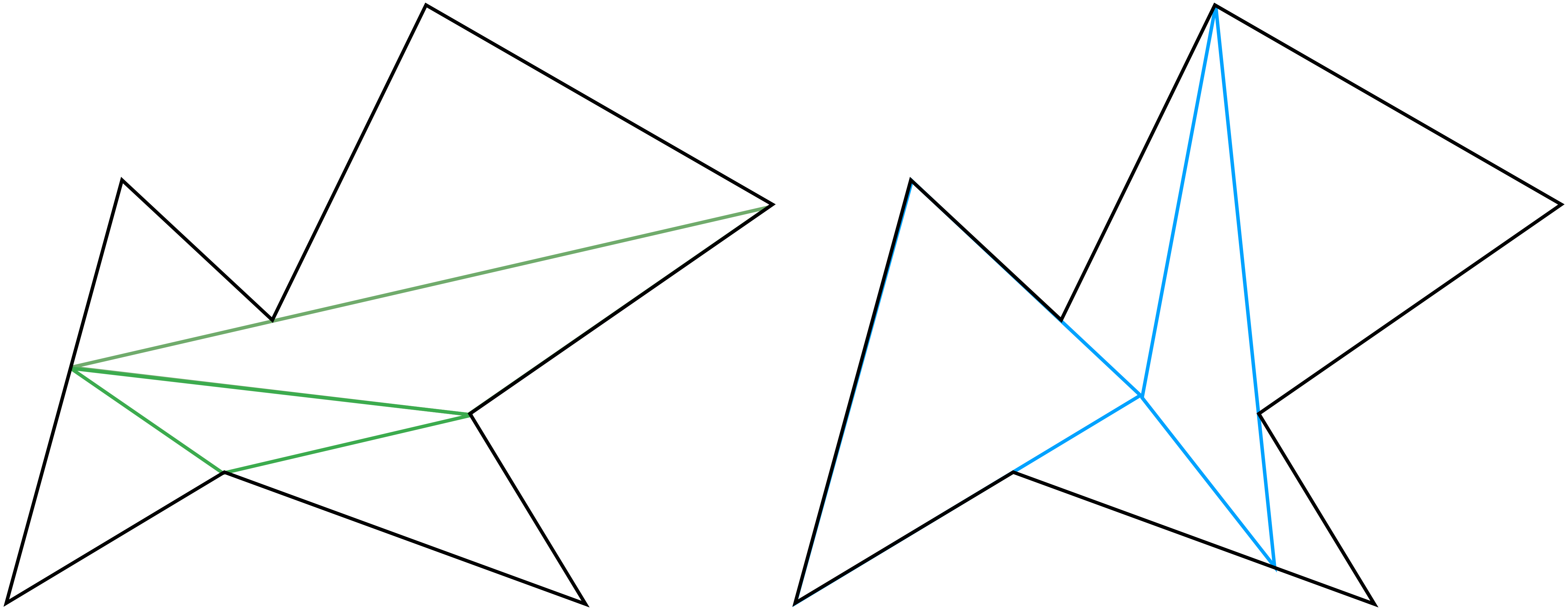

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