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Let B and B' be midpoints of AC, and let $\preceq$ be an ordering of $\overleftrightarrow{AC}$ such that $A \preceq C$. Without the loss of generality, we may assume that $B' \preceq B$. In particular, $A - B' - B - C$. Then

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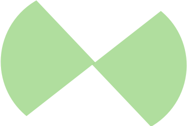
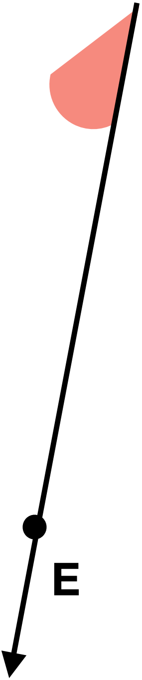
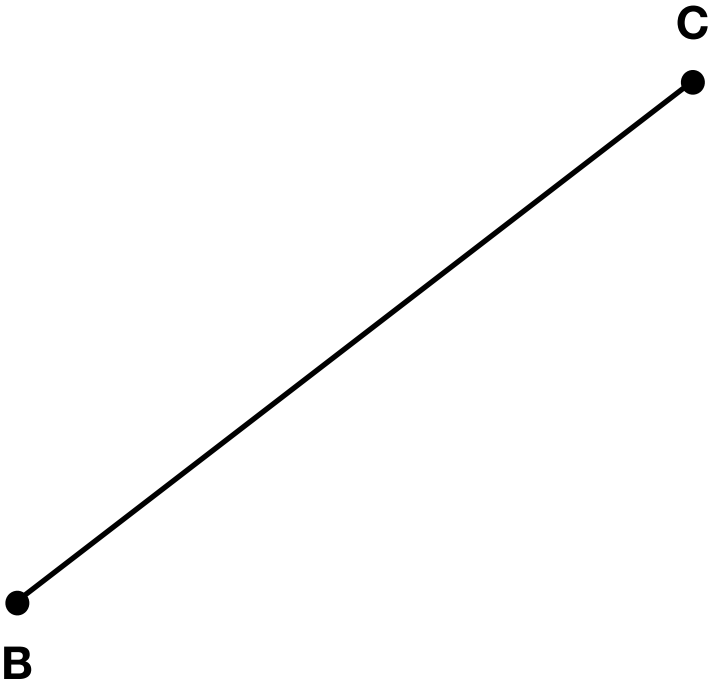
Hence, $\overline{AB'} \cong \overline{AB}$. But by the uniqueness of Segment Translation, we conclude that $B = B'$, that is, midpoints are unique. $\square$


D




E


C
B

A •

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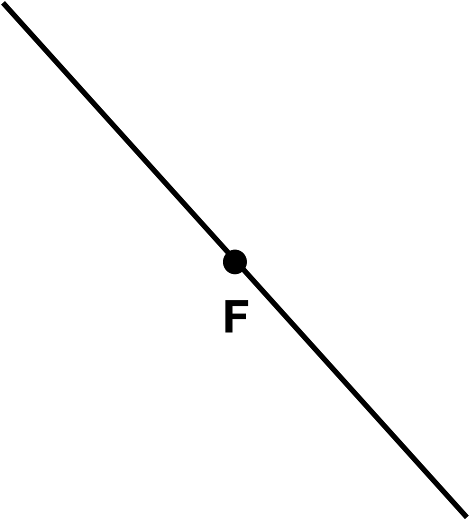
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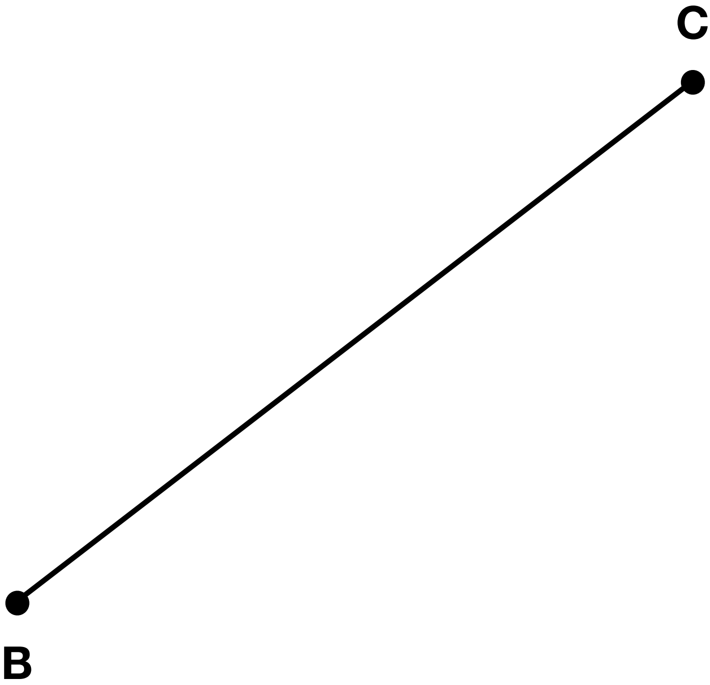
C
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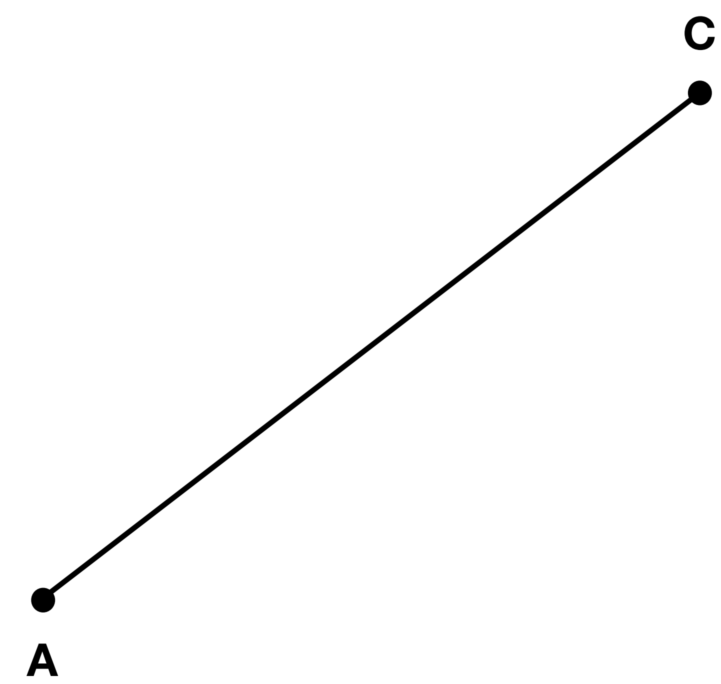

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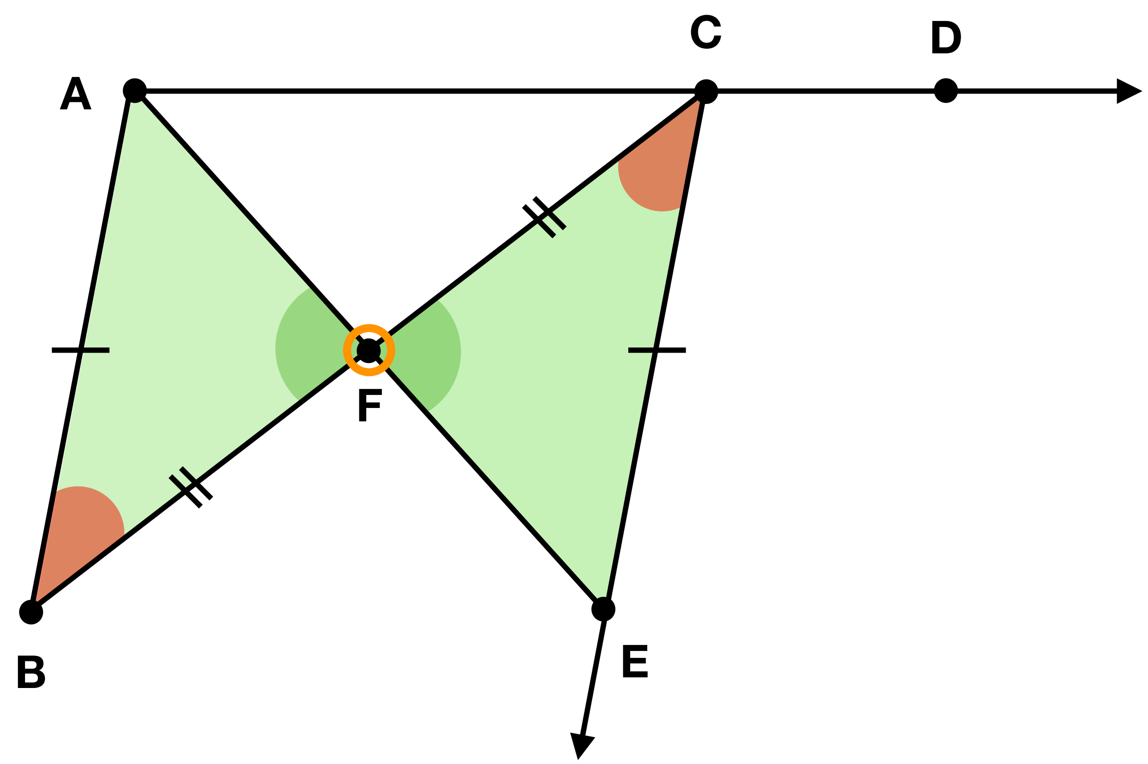

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