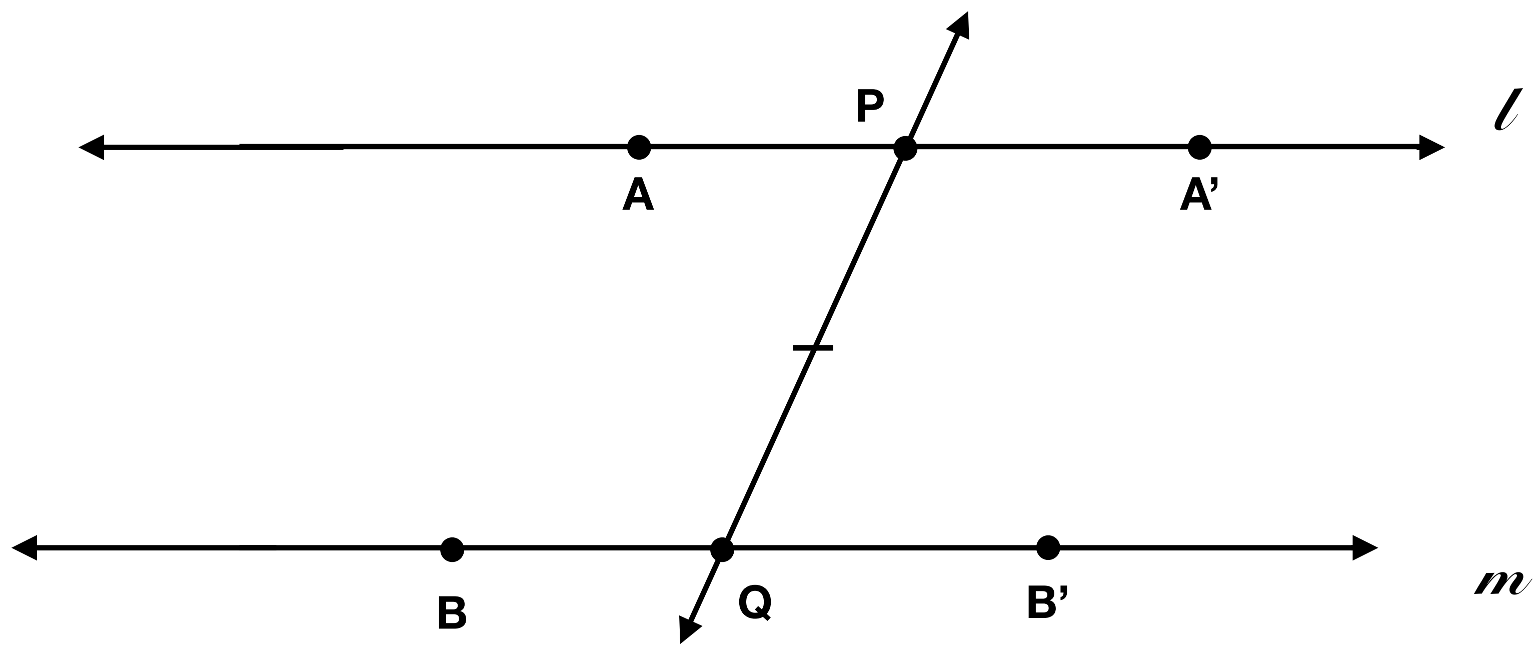
Theorem 3.2.6

Theorem 3.2.6 (Alternate Interior Angle Theorem). *If two distinct lines cut by a transversal have a pair of congruent alternate interior angles, then the two lines are parallel.*



Proof. Use the same notion as introduced in the previous definition. Thus, we will assume that $\angle APQ \cong \angle B'QP$. Suppose for the sake of contradiction that $\ell \nparallel m$. Let $R \in \ell \cap m$. Without the loss of generality, we may assume that R is on the same side of t as A and B. By segment translation, let R' be a point on $\overrightarrow{QB'}$ such that $\overline{QR'} \cong \overline{PR}$. Then $\triangle PQR \cong \triangle QPR'$ by SAS. Hence, $\angle PQB = \angle PQR \cong \angle QPR'$.

Since $\angle A'PQ$ is supplementary to $\angle APQ$ and $\angle B'QP$ is supplementary to $\angle BQP$, we conclude that $\angle A'PQ \cong \angle BQP$, since supplements of congruent angles are congruent. By transitivity of congruence, we have that $\angle A'PQ \cong \angle BQP \cong \angle QPR'$. By uniqueness of angle translation, we conclude that $\angle A'PQ = \angle R'PQ$, in particular, $\overrightarrow{PA'} = \overrightarrow{PR'}$. This shows that $R' \in \ell$. Hence, $R, R' \in \ell \cap m$. But $R \neq R'$ since they are on opposite sides of t. This contradicts line determination. Therefore, $\ell \parallel m$. $\square$

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Theorem 3.2.6 (Alternate Interior Angle Theorem). *If two distinct lines cut by a transversal have a pair of congruent alternate interior angles, then the two lines are parallel.*