

N. L. Carothers' Real Analysis - Summary of results

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Contents

1 Compactness	5
1.1 Uniform Continuity	5

Chapter 1

Compactness

1.1 Uniform Continuity

Definition Uniform continuity. $f : (M, d) \rightarrow (N, \rho)$ is uniformly continuous if for every $\epsilon > 0$, there exists $\delta > 0$ such that $\rho(f(x), f(y)) < \epsilon$ whenever $d(x, y) < \delta$.

Exercise 8.44

Any Lipschitz map is in fact uniformly continuous. This implies that isometries are also uniformly continuous.

Exercise 8.45

Every map $f : \mathbb{N} \rightarrow \mathbb{R}$ is uniformly continuous. Intuitively, this is because natural numbers never get “arbitrarily close” to each other.