

# N. L. Carothers' Real Analysis - Summary of results

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# Chapter 1

## Compactness

### 1.1 Uniform Continuity

**Definition Uniform continuity.**  $f : (M, d) \rightarrow (N, \rho)$  is uniformly continuous if for every  $\epsilon > 0$ , there exists  $\delta > 0$  such that  $\rho(f(x), f(y)) < \epsilon$  whenever  $d(x, y) < \delta$ .

**Exercise 8.44**

Any Lipschitz map is in fact uniformly continuous. This implies that isometries are also uniformly continuous.

**Exercise 8.45**

Every map  $f : \mathbb{N} \rightarrow \mathbb{R}$  is uniformly continuous. Intuitively, this is because natural numbers never get “arbitrarily close” to each other.