Lab 04

Least squares

Motivation

- optimize transformations - But how?
- Use more common points!
- Leads to overdetermined system of linear equations!

Overdetermined system

- more equations than unknowns
- -> usually no solution

- Goal: Find parameters that fit best to *all* equations.

Least squares method

- finds "best" parameters
- "best" = variation between observed and calculated values is minimal

Minimize sum:

$$S(O,s)=\sum_{i=1}^{n}\left(P_{i,X}-f_{s,1}\left(\begin{pmatrix}P_{i,x}\\P_{i,y}\end{pmatrix}\right)\right)^2+\left(P_{i,Y}-f_{s,2}\left(\begin{pmatrix}P_{i,x}\\P_{i,y}\end{pmatrix}\right)\right)^2$$

$$P_i \in O \text{ for all } 1 \leq i \leq n \text{ with } P_i=\left(\begin{pmatrix}x\\y\end{pmatrix},\begin{pmatrix}X\\Y\end{pmatrix}\right)$$

Task 1 - Mathematical Background

- Find a simple and visual example for the difficultys of an overdetermined systems

- Minimum: Describe the mathematical approach of solving such a system in a clear and simple way
- mathematical derivation of formula

- Don't simply state formulae but explain what their purpose is.

Task 2 - Workflow and calculation

- Describe how you would solve the task step by step (workflow)
- Keep it simple: Basic steps and formulae
- Perform the calculations
- State your results clearly! If you upload an R Script state your results in the answer field or give **extensive** comments in the script

Formula

O_2 is Vector of known target coordinates composed of Xt1, Yt1, Xt2, Yt2, Xt3, Yt3, ... , Xtn, Ytn

$$s = (A^T \cdot A)^{-1} \cdot A^T \cdot O_2 \text{ where } O_2 = \begin{pmatrix} P_{1,X} \\ P_{1,Y} \\ P_{2,X} \\ P_{2,Y} \\ \vdots \\ P_{n,X} \\ P_{n,Y} \end{pmatrix}$$

s is the vector of fitted parameters

Task 3 - Use your results!

- Insert your calculated parameters into the transformation formula.

- Test them by using two **new** control points

- Compare your results with those from lab 3 and comment on the differences

Task 4 - Transformations

- Describe the difference between similarity, affine and polynomial transformation

R and huge numbers

- R can't handle "huge" numbers -> overflow

- shrink the coordinates measured from the shape file

- substract certain value (for example):
    - 3,400,000 for the x-values
    - 5,700,000 for the y-values

R - Matrix calculations

A and B are matrices:

- transpose matrix A: t(M)

- multiplication of A and B: A%*%B

- calculate inverse matrix (e.g. A^(-1)): solve (A)

Questions?

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