2019 IE 535 PROGRAMMING PROJECT

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Name: Geonsik Yu **Purdue ID: 0030931371**
Models chosen: Model 12 (Alloy Blending) and Model 27 (Hydrological Model)

1. LP formulations

Model 12: Alloy (Blending)

A company desires to blend a new alloy of 40 percent tin, 35 percent zinc, and 25 percent lead from several available allows having the following properties:

	Alloy				
Property	1	2	3	4	5
% tin	60	25	45	20	50
% zinc	10	15	45	50	40
% lead	30	60	10	30	10
Cost ($/lb)	19	17	23	21	25

The objective is to determine the proportions of these alloys that should be blended to produce the new alloy at a minimum cost. Formulate the linear programming model for this problem.

- **Decision variables**
  $x_i :=$ The proportion of alloy type i in the new alloy.

- **LP formulation**

$$\begin{aligned}
\text{Minimize} \quad & 19x_1 + 17x_2 + 23x_3 + 21x_4 + 25x_5 & \\
\text{subject to} \quad & 60x_1 + 25x_2 + 45x_3 + 20x_4 + 50x_5 = 40 & \text{(Tin Percentage)} \\
& 10x_1 + 15x_2 + 45x_3 + 50x_4 + 40x_5 = 35 & \text{(Zinc Percentage)} \\
& 30x_1 + 60x_2 + 10x_3 + 30x_4 + 10x_5 = 25 & \text{(Lead Percentage)} \\
& x_i \geq 0 \quad \text{for } i \in \{1, \ldots, 5\} & \text{(Nonnegativity)}
\end{aligned}$$

Model 27: Hydrological Model

From hydrological considerations, **the expected runoff depends on the precipitation during that period and the previous two periods** and b_0, b_1, b_2 are the coefficients that are required to be estimated. These coefficients have to satisfy the following constraints from hydrological considerations: $b_0 + b_1 + b_2 = 1$, where $b_0 > b_1 > b_2 > 0$. **(Suppose that we have a very small positive number ϵ to avoid strict inequality.)**

Obtain the best estimates for b_0, b_1, b_2, (these are decision variables)
(Q1) where objective is to minimize the sum of absolute deviations: $\sum_i |R_i - b_0p_i - b_1p_{i-1} - b_2p_{i-2}|$
(Q2) where objective is to minimize the maximum absolute deviation: $\max_i |R_i - b_0p_i - b_1p_{i-1} - b_2p_{i-2}|$.

- **(Q1: minimize the sum of absolute deviations) LP formulation**
  Auxiliary Variables Introduced: $A_1 \dots A_{12}$ (A_i for absolute value function of i-th period)

$$
\begin{array}{lll}
\text{Minimize} & \sum_{i=3}^{12} A_i & \\
\text{subject to} & A_3 \geq 1.0 - 5.7 \times b_0 - 4.4 \times b_1 - 3.8 \times b_1 & (Period3) \\
& A_3 \geq -(1.0 - 5.7 \times b_0 - 4.4 \times b_1 - 3.8 \times b_1) & (Period3) \\
& A_4 \geq 2.1 - 5.2 \times b_0 - 5.7 \times b_1 - 4.4 \times b_1 & (Period4) \\
& A_4 \geq -(2.1 - 5.2 \times b_0 - 5.7 \times b_1 - 4.4 \times b_1) & (Period4) \\
& A_5 \geq 3.7 - 7.7 \times b_0 - 5.2 \times b_1 - 5.7 \times b_1 & (Period5) \\
& A_5 \geq -(3.7 - 7.7 \times b_0 - 5.2 \times b_1 - 5.7 \times b_1) & (Period5) \\
& A_6 \geq 4.2 - 6.0 \times b_0 - 7.7 \times b_1 - 5.2 \times b_1 & (Period6) \\
& A_6 \geq -(4.2 - 6.0 \times b_0 - 7.7 \times b_1 - 5.2 \times b_1) & (Period6) \\
& A_7 \geq 4.3 - 5.4 \times b_0 - 6.0 \times b_1 - 7.7 \times b_1 & (Period7) \\
& A_7 \geq -(4.3 - 5.4 \times b_0 - 6.0 \times b_1 - 7.7 \times b_1) & (Period7) \\
& A_8 \geq 4.4 - 5.7 \times b_0 - 5.4 \times b_1 - 6.0 \times b_1 & (Period8) \\
& A_8 \geq -(4.4 - 5.7 \times b_0 - 5.4 \times b_1 - 6.0 \times b_1) & (Period8) \\
& A_9 \geq 4.3 - 5.5 \times b_0 - 5.7 \times b_1 - 5.4 \times b_1 & (Period9) \\
& A_9 \geq -(4.3 - 5.5 \times b_0 - 5.7 \times b_1 - 5.4 \times b_1) & (Period9) \\
& A_{10} \geq 4.2 - 2.5 \times b_0 - 5.5 \times b_1 - 5.7 \times b_1 & (Period10) \\
& A_{10} \geq -(4.2 - 2.5 \times b_0 - 5.5 \times b_1 - 5.7 \times b_1) & (Period10) \\
& A_{11} \geq 3.6 - 0.8 \times b_0 - 2.5 \times b_1 - 5.5 \times b_1 & (Period11) \\
& A_{11} \geq -(3.6 - 0.8 \times b_0 - 2.5 \times b_1 - 5.5 \times b_1) & (Period11) \\
& A_{12} \geq 2.7 - 0.4 \times b_0 - 0.8 \times b_1 - 2.5 \times b_1 & (Period12) \\
& A_{12} \geq -(2.7 - 0.4 \times b_0 - 0.8 \times b_1 - 2.5 \times b_1) & (Period12) \\
& b_i \geq \epsilon; \quad b_0 - b_1 \geq \epsilon; \quad b_1 - b_2 \geq \epsilon; & \\
& A_i\text{'s free or non-negative (either way is fine)} & [8pt]
\end{array}
$$

- **(Q1: Modified Version)**

$$
\begin{array}{lll}
\text{Minimize} & \sum_{i=3}^{12} A_i & \\
\text{subject to} & A_3 + 5.7 \times b_0 + 4.4 \times b_1 + 3.8 \times b_1 \geq 1.0 & (Period3) \\
& A_3 - 5.7 \times b_0 - 4.4 \times b_1 - 3.8 \times b_1 \geq -1.0 & (Period3) \\
& A_4 + 5.2 \times b_0 + 5.7 \times b_1 + 4.4 \times b_1 \geq 2.1 & (Period4) \\
& A_4 - 5.2 \times b_0 - 5.7 \times b_1 - 4.4 \times b_1 \geq -2.1 & (Period4) \\
& A_5 + 7.7 \times b_0 + 5.2 \times b_1 + 5.7 \times b_1 \geq 3.7 & (Period5) \\
& A_5 - 7.7 \times b_0 - 5.2 \times b_1 - 5.7 \times b_1 \geq -3.7 & (Period5) \\
& A_6 + 6.0 \times b_0 + 7.7 \times b_1 + 5.2 \times b_1 \geq +4.2 & (Period6) \\
& A_6 - 6.0 \times b_0 - 7.7 \times b_1 - 5.2 \times b_1 \geq -4.2 & (Period6) \\
& A_7 + 5.4 \times b_0 + 6.0 \times b_1 + 7.7 \times b_1 \geq 4.3 & (Period7) \\
& A_7 - 5.4 \times b_0 - 6.0 \times b_1 - 7.7 \times b_1 \geq -4.3 & (Period7) \\
& A_8 + 5.7 \times b_0 + 5.4 \times b_1 + 6.0 \times b_1 \geq 4.4 & (Period8) \\
& A_8 - 5.7 \times b_0 - 5.4 \times b_1 - 6.0 \times b_1 \geq -4.4 & (Period8) \\
& A_9 + 5.5 \times b_0 + 5.7 \times b_1 + 5.4 \times b_1 \geq 4.3 & (Period9) \\
& A_9 - 5.5 \times b_0 - 5.7 \times b_1 - 5.4 \times b_1 \geq -4.3 & (Period9) \\
& A_{10} + 2.5 \times b_0 + 5.5 \times b_1 + 5.7 \times b_1 \geq 4.2 & (Period10) \\
& A_{10} - 2.5 \times b_0 - 5.5 \times b_1 - 5.7 \times b_1 \geq -4.2 & (Period10) \\
& A_{11} + 0.8 \times b_0 + 2.5 \times b_1 + 5.5 \times b_1 \geq 3.6 & (Period11) \\
& A_{11} - 0.8 \times b_0 - 2.5 \times b_1 - 5.5 \times b_1 \geq -3.6 & (Period11) \\
& A_{12} + 0.4 \times b_0 + 0.8 \times b_1 + 2.5 \times b_1 \geq 2.7 & (Period12) \\
& A_{12} - 0.4 \times b_0 - 0.8 \times b_1 - 2.5 \times b_1 \geq -2.7 & (Period12) \\
& b_i \geq \epsilon; \quad b_0 - b_1 \geq \epsilon; \quad b_1 - b_2 \geq \epsilon; & \\
& A_i\text{'s free or non-negative (either way is fine)} & [8pt]
\end{array}
$$

- **(Q2: minimize the max of absolute deviations) LP formulation**
  Auxiliary Variables Introduced: A only.

$$
\begin{array}{lll}
\text{Minimize} & A & \\
\text{subject to} & A \geq 1.0 - 5.7 \times b_0 - 4.4 \times b_1 - 3.8 \times b_1 & (Period3) \\
& A \geq -(1.0 - 5.7 \times b_0 - 4.4 \times b_1 - 3.8 \times b_1) & (Period3) \\
& A \geq 2.1 - 5.2 \times b_0 - 5.7 \times b_1 - 4.4 \times b_1 & (Period4) \\
& A \geq -(2.1 - 5.2 \times b_0 - 5.7 \times b_1 - 4.4 \times b_1) & (Period4) \\
& A \geq 3.7 - 7.7 \times b_0 - 5.2 \times b_1 - 5.7 \times b_1 & (Period5) \\
& A \geq -(3.7 - 7.7 \times b_0 - 5.2 \times b_1 - 5.7 \times b_1) & (Period5) \\
& A \geq 4.2 - 6.0 \times b_0 - 7.7 \times b_1 - 5.2 \times b_1 & (Period6) \\
& A \geq -(4.2 - 6.0 \times b_0 - 7.7 \times b_1 - 5.2 \times b_1) & (Period6) \\
& A \geq 4.3 - 5.4 \times b_0 - 6.0 \times b_1 - 7.7 \times b_1 & (Period7) \\
& A \geq -(4.3 - 5.4 \times b_0 - 6.0 \times b_1 - 7.7 \times b_1) & (Period7) \\
& A \geq 4.4 - 5.7 \times b_0 - 5.4 \times b_1 - 6.0 \times b_1 & (Period8) \\
& A \geq -(4.4 - 5.7 \times b_0 - 5.4 \times b_1 - 6.0 \times b_1) & (Period8) \\
& A \geq 4.3 - 5.5 \times b_0 - 5.7 \times b_1 - 5.4 \times b_1 & (Period9) \\
& A \geq -(4.3 - 5.5 \times b_0 - 5.7 \times b_1 - 5.4 \times b_1) & (Period9) \\
& A \geq 4.2 - 2.5 \times b_0 - 5.5 \times b_1 - 5.7 \times b_1 & (Period10) \\
& A \geq -(4.2 - 2.5 \times b_0 - 5.5 \times b_1 - 5.7 \times b_1) & (Period10) \\
& A \geq 3.6 - 0.8 \times b_0 - 2.5 \times b_1 - 5.5 \times b_1 & (Period11) \\
& A \geq -(3.6 - 0.8 \times b_0 - 2.5 \times b_1 - 5.5 \times b_1) & (Period11) \\
& A \geq 2.7 - 0.4 \times b_0 - 0.8 \times b_1 - 2.5 \times b_1 & (Period12) \\
& A \geq -(2.7 - 0.4 \times b_0 - 0.8 \times b_1 - 2.5 \times b_1) & (Period12) \\
& b_i \geq \epsilon; \quad b_0 - b_1 \geq \epsilon; \quad b_1 - b_2 \geq \epsilon; & \\
& A \text{ free or non-negative (either way is fine)} & [8pt]
\end{array}
$$

- **(Q2: Modified Version)**

$$
\begin{array}{lll}
\text{Minimize} & A & \\
\text{subject to} & A + 5.7 \times b_0 + 4.4 \times b_1 + 3.8 \times b_1 \geq 1.0 & (Period3) \\
& A - 5.7 \times b_0 - 4.4 \times b_1 - 3.8 \times b_1 \geq -1.0 & (Period3) \\
& A + 5.2 \times b_0 + 5.7 \times b_1 + 4.4 \times b_1 \geq 2.1 & (Period4) \\
& A - 5.2 \times b_0 - 5.7 \times b_1 - 4.4 \times b_1 \geq -2.1 & (Period4) \\
& A + 7.7 \times b_0 + 5.2 \times b_1 + 5.7 \times b_1 \geq 3.7 & (Period5) \\
& A - 7.7 \times b_0 - 5.2 \times b_1 - 5.7 \times b_1 \geq -3.7 & (Period5) \\
& A + 6.0 \times b_0 + 7.7 \times b_1 + 5.2 \times b_1 \geq +4.2 & (Period6) \\
& A - 6.0 \times b_0 - 7.7 \times b_1 - 5.2 \times b_1 \geq -4.2 & (Period6) \\
& A + 5.4 \times b_0 + 6.0 \times b_1 + 7.7 \times b_1 \geq 4.3 & (Period7) \\
& A - 5.4 \times b_0 - 6.0 \times b_1 - 7.7 \times b_1 \geq -4.3 & (Period7) \\
& A + 5.7 \times b_0 + 5.4 \times b_1 + 6.0 \times b_1 \geq 4.4 & (Period8) \\
& A - 5.7 \times b_0 - 5.4 \times b_1 - 6.0 \times b_1 \geq -4.4 & (Period8) \\
& A + 5.5 \times b_0 + 5.7 \times b_1 + 5.4 \times b_1 \geq 4.3 & (Period9) \\
& A - 5.5 \times b_0 - 5.7 \times b_1 - 5.4 \times b_1 \geq -4.3 & (Period9) \\
& A + 2.5 \times b_0 + 5.5 \times b_1 + 5.7 \times b_1 \geq 4.2 & (Period10) \\
& A - 2.5 \times b_0 - 5.5 \times b_1 - 5.7 \times b_1 \geq -4.2 & (Period10) \\
& A + 0.8 \times b_0 + 2.5 \times b_1 + 5.5 \times b_1 \geq 3.6 & (Period11) \\
& A - 0.8 \times b_0 - 2.5 \times b_1 - 5.5 \times b_1 \geq -3.6 & (Period11) \\
& A + 0.4 \times b_0 + 0.8 \times b_1 + 2.5 \times b_1 \geq 2.7 & (Period12) \\
& A - 0.4 \times b_0 - 0.8 \times b_1 - 2.5 \times b_1 \geq -2.7 & (Period12) \\
& b_i \geq \epsilon; \quad b_0 - b_1 \geq \epsilon; \quad b_1 - b_2 \geq \epsilon; & \\
& A \text{ free or non-negative (either way is fine)} & [8pt]
\end{array}
$$

- Underlying assumptions in Model 27

  1. No model fitting for period 1 and period 2: Because there is no precipitation records for the previous year, we cannot test the two period which requires the previous year's precipitations.
  2. Strict inequalities: to implement strict inequality, I introduce a small enough positive constant ϵ to the problem.

2. Source Codes (Simplex)

File list:

- **MySimplex.py**: includes simplex class and its member functions.
- **model12_mySimplex.py**: includes the problem construction and solver calls from MySimplex.py for model 12.
- **model27_mySimplex_Q1.py**: includes the problem construction and solver calls from MySimplex.py for model 27 Q1.
- **model27_mySimplex_Q2.py**: includes the problem construction and solver calls from MySimplex.py for model 27 Q2.
- **model12_cplex.py**: includes the problem construction and solver calls from IBM ILOG Cplex library for model 12.
- **model27_cplex_Q1.py**: includes the problem construction and solver calls from IBM ILOG Cplex library for model 27 Q2.
- **model27_cplex_Q2.py**: includes the problem construction and solver calls from IBM ILOG Cplex library for model 27 Q2.

: Source codes follow from the next page.

```python
############################################################################
# File Name: mySimplex.py                                                  #
# Author: Geonsik Yu, Purdue University, IE Dept                           #
#                                                                          #
# - Implement simplex method with the following requirements satisfied:    #
#                                                                          #
# Programming Requirements:                                                #
# (R1) Design a routine to do the conversion to the standard form LP.      #
# (R2) Design a routine to detect if the LP at hand is feasible or not.    #
#      (15 pt)                                                             #
# (R3) Design a routine to detect if the coefficient matrix associated with#
#    the standard form of the LP has full row rank; and if not, how to     #
#    remove the redundant constraints. (15 pt)                             #
# (R4) Design a routine to check if a basic feasible solution (BFS) with   #
#     the identity matrix as the corresponding basis is readily available. #
#     If yes, you can just go ahead to use the BFS to start your simplex   #
#     method;                                                              #
#     otherwise, use the methods we learned in class to start your simplex #
#     with an identity matrix as the initial basis. (15 pt)                #
# (R5) Your code needs to implement one rule to prevent the simplex method #
#     from cycling. (15 pt)                                                #
# (R6) Termination: your code needs to be able to handle both cases at     #
#     termination (6-1) a finite optimal solution or (6-2) unbounded(15 pt)#
############################################################################

from builtins import object
import numpy as np
from numpy import dot, zeros
from numpy.linalg import matrix_rank, norm
import sys

### Numpy printing parameters setting.
np.set_printoptions(precision = 4, linewidth = 200, threshold=sys.maxsize)
### Global threshold value for checking zero value with floating point errors.
THRESHOLD = 1e-8

class SimplexProblem(object):
    def __init__(self):
        ############################################################################
        # Constructor: This function constructs an LP problem object               #
        ############################################################################

        ## Attribute that will be initialized by setObjectiveDirection() function.
        self._objDirection = None     # Decides whether the given problem is maximization or minimization.
        self._objectiveValue = 0

        ## Attributes that will be initialized by setVariables() function.
        self._objCoeffs = None        # Coefficient values in the objective function for variables.
        self._varNames = None         # Names for variables.
        self._varLowerbounds = None   # Lowerbounds for variables.
        self._varCount = None         # Number of non-slack, non-artificial objCoeffs.
        self._reducedCosts = None     # Reduced costs

        # Attributes that will be initialized by addConstraint function.
        self._conNames = []         # List of names for all the constraints.
        self._AMatrix = []          # Coefficient Matrix "A"
        self._ineqDirs = []         #
        self._RHSs = []
        self._basisSet = []

        # Attributes for Two step method.
        self._twoStep = False
        self._2nd_objectiveValue = 0
        self._reducedSecondary = []   # Secondary reduced costs

    def debug(self):
        ############################################################################
        # This debug function prints out internal attributes.                      #
        ############################################################################
        print("# of original variables: ", self._varCount)
        print(self._varNames)
        print(self._reducedCosts)
        if self._twoStep == True:
            print(self._reducedSecondary)
        tmp = np.array(self._AMatrix)
        for idx in range(len(self._AMatrix)):
            print(tmp[idx], "||", self._RHSs[idx])
        print("Basis: ", self._basisSet)
        print("--------------------------------------")

    def setObjectiveDirection(self, Max = True):
        self._objDirection = Max
        ############################################################################
        # This function initializes the direction of optimization: If the problem  #
        # is on maximization, Max = True (Default), otherwise, Max = False.        #
        ############################################################################

    def setVariables(self, Names, ObjCoeffs, Lowerbounds=None):
        ############################################################################
        # This function initializes the decision variable set, their names, their  #
        # coefficients in the objective function, and their upper & lower bounds.  #
        ############################################################################
        self._objCoeffs = ObjCoeffs
        self._varNames = Names
        self._varLowerbounds = Lowerbounds

```

```python
    def addConstraint(self, Name, rowVec, ineq_dir, RHS):
        ############################################################################
        # This function adds one constraint at a time. Each constraint requires to #
        # have coefficient vector with the same order with the variable vector, one#
        # inequality direction, and one right-hand side constant value.            #
        ############################################################################
        self._conNames.append(Name)
        self._AMatrix.append(rowVec)
        self._ineqDirs.append(ineq_dir)
        self._RHSs.append(RHS)

    def printCurrentStatus(self, Tableau):
        print("------------------------------------------------------------------")
        print("Current Basic Variables: ", ' '.join(self._basisSet) )
        print("Current Objective Value: ", repr(Tableau[0,-1]) )
        for idx in range(Tableau.shape[1]-1):
            value = 0.0
            if self._varNames[idx] in self._basisSet:
                value = Tableau[1+self._basisSet.index(self._varNames[idx]), -1]
            print(self._varNames[idx], " := ", repr(value))

    def setup(self):
        ############################################################################
        # Related to the requirement (R1).                                         #
        # (1) Transform the objective direction into "Minimization" and            #
        #     negate the signs of objective coefficients.                          #
        # (2) Transform the input constraint:                                      #
        #     2-1) [LHS <= RHS form]: pass.                                        #
        #     2-2) [LHS >= RHS form]: change to [-LHS <= -RHS].                    #
        #     2-3) [LHS = RHS form]: split into [LHS <= RHS form]                  #
        #                                   and [-LHS <= -RHS form].               #
        # (3) Check the resulting RHS's from (2).                                  #
        #     IF all the RHS's are positive:                                       #
        #          Go add (+1 coeff) slack variables for each constraints.         #
        #          Check the problem type as "COMMON".                             #
        #          Check the problem feasibility as "FEASIBLE"                     #
        #     Else:                                                                #
        #          Go add (+1 coeff) slack variables for constraints with positive #
        #             RHS's (name S_i),                                            #
        #             add (-1 coeff) slack variables for constraints with negative #
        #             RHS's (name S_i), and                                        #
        #             add (+1 coeff) artificial variables for constraints with     #
        #             negative RHS's (name A_i).                                   #
        #          Check the problem type as "TWOSTEP".                            #
        ############################################################################
        self._varCount = len(self._varNames)
        if self._objDirection == True:
            self._reducedCosts = self._objCoeffs
        else:
            self._reducedCosts = [-1*ele for ele in self._objCoeffs]

        for idx in range(len(self._conNames)):
            if self._ineqDirs[idx] == 'G':
                self._AMatrix[idx] = [-1*ele for ele in self._AMatrix[idx]]
                self._RHSs[idx] = -1*self._RHSs[idx]
            elif self._ineqDirs[idx] == 'E':
                self._AMatrix.append( [-1*ele for ele in self._AMatrix[idx]] )
                self._RHSs.append( -1*self._RHSs[idx] )

        s_ind, a_ind = 1, 1
        for idx in range(len(self._AMatrix)):
            self._varNames.append("S"+repr(s_ind))
            self._reducedCosts.append(0)
            for jdx in range(len(self._AMatrix)):
                if jdx == idx:
                    self._AMatrix[jdx].append(+1)
                else:
                    self._AMatrix[jdx].append(0)
            s_ind += 1

        for idx in range(len(self._RHSs)):
            if self._RHSs[idx] < 0:
                self._varNames.append("A"+repr(a_ind))
                self._reducedCosts.append(0)
                self._AMatrix[idx] = [-1*ele for ele in self._AMatrix[idx]]
                self._RHSs[idx] = -1*self._RHSs[idx]
                for jdx in range(len(self._AMatrix)):
                    if jdx == idx:
                        self._AMatrix[jdx].append(+1)
                    else:
                        self._AMatrix[jdx].append(0)
                self._basisSet.append("A"+repr(a_ind))
                a_ind += 1
            else:
                self._basisSet.append(self._varNames[self._varCount+idx])

        if a_ind < 1.0001: ## "COMMON" case.
            returnStr = "COMMON"
        else: ## "TWOSTEP" case.
            self._twoStep = True
            for name in self._varNames:
                if name[0] == "A":
                    self._reducedSecondary.append(-1)
                else:
                    self._reducedSecondary.append(0)
            returnStr = "TWOSTEP"
        return returnStr
```

```python

    def buildTableau(self):
        ##########################################################################
        # This function builds tableau for both "COMMON" and "TWOSTEP" cases       #
        # and reduce the constraints (non-zeroth rows of the tableau) using the    #
        # reduced row echelon form.                                                #
        ##########################################################################
        Tableau = []
        if self._twoStep == True:
            Tableau.append( self._reducedSecondary )
        else:
            Tableau.append( self._reducedCosts )
        Tableau[-1].append( 0 )

        for idx in range(len(self._AMatrix)):
            Tableau.append( self._AMatrix[idx] )
            Tableau[-1].append( self._RHSs[idx] )

        Tableau = np.array(Tableau)
        #Tableau[1:,], tempIndices = self.reduceProblemMatrix(Tableau[1:,])
        tempConMat, tempIndices = self.reduceProblemMatrix(Tableau[1:,])
        tempConMat = np.vstack([Tableau[0,:], tempConMat])
        Tableau = tempConMat

        ## Reorder basis based on the indices from the reducedProblemMatrix
        tempList = [self._basisSet[i] for i in tempIndices]
        self._basisSet = tempList

        print("Initialize the given LP problem:")
        if self._twoStep:
            Tableau = self.initZeroth(Tableau)
            print("TWO STEP METHOD CALLED:")
        else:
            print("VANILLA SIMPLEX METHOD CALLED:")

        self.printCurrentStatus(Tableau)
        return Tableau

    def initZeroth(self, Tableau):
        ##########################################################################
        # This function initializes the 0th row of the tableau for basic columns.  #
        # It assumes that for the input tableau's basic columns, they are all one- #
        # hot vectors except for the zero-th element. This function simply finds   #
        # basic columns with -1 for the zero-th element, and add pivot rows to the #
        # zero-th row.                                                             #
        ##########################################################################
        colIndices = []
        for idx in range(Tableau.shape[1]):
            if (Tableau[0, idx] != 0 and self._varNames[idx] in self._basisSet):
                colIndices.append(idx)
        for idx in colIndices:
            rowIdx = np.argmax(Tableau[:, idx])
            ratio = -1*Tableau[0, idx]/Tableau[rowIdx, idx]
            Tableau[0, :] += ratio*Tableau[rowIdx, :]

        print("Initialize the zero-th row:")
        self.printCurrentStatus(Tableau)
        return Tableau

    def solve(self, Tableau):
        # Related to the requirement (R2).
        # Related to the requirement (R4).
        if self._twoStep == True:
            ## IF the problem type is "TWOSTEP":
            #     (a) Put a secondary objective function with (+1 coeff) for all artificial variables.
            #     (b) Run tableau simplex steps until it meets one of the [Tableau Simplex's Termination Conditions]
            #     (c) IF the optimal objective value from (b) is zero with all zero artificial values, then process to the 2nd step
            #         Otherwise, end procedure and say the original LP is not feasible.
            terminationFlag = 0
            print("Run the 1st step of Two Step Method")
            while(terminationFlag == 0):
                Tableau, terminationFlag = self.run_oneIteration(Tableau)
                self.printCurrentStatus(Tableau)
            if Tableau[0,-1] < THRESHOLD:
                ## When 1st phase of Two step method ends with feasibility guaranteed.
                ## (1) Setup 2nd phase tableau.
                removalIndices = [self._varNames.index(ele) for ele in self._varNames if ele[0]=="A"]
                Tableau = np.delete(Tableau, removalIndices, 1)
                for idx in range(Tableau.shape[1]-1):
                    Tableau[0,idx] = self._reducedCosts[idx]
                Tableau[0,-1] = 0
                ## (2) Restore 0 reduced costs for the basic columns.
                print("Run the Vanilla Simplex Method or the 2st step of Two Step Method")
                Tableau = self.initZeroth(Tableau)
                terminationFlag = 0
                while(terminationFlag == 0):
                    Tableau, terminationFlag = self.run_oneIteration(Tableau)
                    self.printCurrentStatus(Tableau)
            else:
                ## When 1st phase of Two step method ends otherwise.
                # [Tableau Simplex's Termination Conditions]
                # (T0) When 1st phase of Two step method ends otherwise.
                #    - "No feasible solution"
                print("- Terminate Procedure: No feasible solution for this problem.")
        else:
            ## IF the problem type is "COMMON":
            ## Run tableau simplex steps until it meets one of the [Tableau Simplex's Termination Conditions]
```

```
            while(terminationFlag == 0):
                Tableau, terminationFlag = self.run_oneIteration(Tableau)
                self.printCurrentStatus(Tableau)
            self.printCurrentStatus(Tableau)
        return Tableau

    def run_oneIteration(self, Tableau):
        ############################################################################
        # (1) Find a positive zeroth row element in the tableau with smallest index#
        #     value. Set the corresponding variable as "Entering Variable" in      #
        #     this iteration.                                                      #
        #     If there is no such element in the zeroth row, then terminate the    #
        #     iteration.                                                           #
        #     If there is no such element that (2) can find an element, then       #
        #     terminate the iteration.                                             #
        # (2) Find a non-negative element from the selected column with the minimun#
        #     ratio (rhs/ele). Set the corresponding variable as "Leaving Variable"#
        #     in this iteration.                                                   #
        #     If there exists a tie, then select the one with the smaller index.   #
        #     If there is no such element in the column, go to (1) and find another#
        #     column.                                                              #
        # (3) Multiply the pivot row by ratios (-Ele/pivotEle) and add it to the   #
        #     corresponding rows, respectively.                                    #
        #                                                                          #
        # Related to the requirement (R6)                                          #
        ############################################################################
        terminationType = 0 # 0 - No termination
                            # 1 - Termination with optimal corner.
                            # 2 - Termination with unboundedness.
        pivotCol = None
        pivotRow = None
        failCol = []

        Once = False
        pivotCol = self.findCol(Tableau, failCol)

        if pivotCol == None:
            if Once == False:
                # [Tableau Simplex's Termination Conditions]
                # (T1) If all the reduced costs are negative.
                #    - "Optimal corner point found"
                #    - Return the optimal solution and the optimal objective value.
                print("- Terminate Procedure: Optimal Value = ", Tableau[0,-1])
                terminationType = 1
            else:
                # [Tableau Simplex's Termination Conditions]
                # (T2) If for all positive reduced cost columns, there are only non-positive elememts.
                #    - "Unboundedness found"
                #    - Return "-inf" for minimization problems and "+inf" for maximization problems.
                print("- Terminate Procedure: Unboundedness")
                terminationType = 2
        else:
            Once = True
            pivotRow = self.findRow(Tableau[:, pivotCol], Tableau[:, -1])
            if pivotRow == None:
                failCol.append(pivotCol)

            self._basisSet[pivotRow-1] = self._varNames[pivotCol]

            Tableau[pivotRow, :] = Tableau[pivotRow, :] / Tableau[pivotRow, pivotCol]

            for idx in range(Tableau.shape[0]):
                if idx != pivotRow:
                    ratio = -1*Tableau[idx, pivotCol]/Tableau[pivotRow, pivotCol];
                    Tableau[idx, :] += ratio * Tableau[pivotRow, :]
        return (Tableau, terminationType)

    def findCol(self, Tableau, failCol):
        ############################################################################
        # This function returns the pivot column for simplex iteration.            #
        # Check the non-optimality condition from the last column because we always#
        # want to remove the artificial column first for the TWOSTEP level.        #
        # To achieve consistency in terms of the Blend's rule, we choose the       #
        # HIGHEST-numbered nonbasic column with a negative (reduced) cost.         #
        # Related to the requirement (R5): Blends' rule.                           #
        ############################################################################
        pivotCol = None
        #for idx in range(Tableau.shape[1]-1):
        for idx in reversed(range(Tableau.shape[1]-1)):
            if (Tableau[0, idx] > THRESHOLD and idx not in failCol):
                pivotCol = idx
                break;
        return pivotCol

    def findRow(self, PivotCol, RHSs):
        ############################################################################
        # This function returns the pivot row when pivot column is given in simplex#
        # We choose the row with the lowest ratio between the (transformed) right #
        # hand side and the coefficient in the pivot tableau where the coefficient #
        # is greater than zero.                                                    #
        # To achive consistency in terms of the blend's rule, when the minimum     #
        # ratio is shared by several rows, we choose the row with HIGHEST index.   #
        # Related to the requirement (R5): Blends' rule.                           #
        ############################################################################
        pivotRow = None
        minRatio = float("Inf")
        #for idx in reversed(range(1, PivotCol.size)):
```

```python
        for idx in range(1, PivotCol.size):
            ## Selects larger index when it falls in tie situation.
            if PivotCol[idx] > 0:
                if RHSs[idx]/PivotCol[idx] <= minRatio:
                    minRatio = RHSs[idx]/PivotCol[idx]
                    pivotRow = idx
        return pivotRow


    def reduceProblemMatrix(self, ProblemMatrix):
        ############################################################################
        # This function returns the problem matrix [A;b] after removing row vector #
        # redundancy.                                                              #
        # Related to the requirement (R3).                                         #
        ############################################################################
        rref_results = self.rref(ProblemMatrix)

        rowFlags = [False]*rref_results[0].shape[0]
        # Check each rref matrix's row whether there is non-zero element and make a T/F flag array for it.
        for i in range(rref_results[0].shape[0]):
            for j in range(rref_results[0].shape[0]):
                if abs(rref_results[0][i][j]) < THRESHOLD:
                    rowFlags[i] = True
                    break;

        remainderIndices = []
        for idx in range(len(rowFlags)):
            if rowFlags[idx] == False:
                break;
            remainderIndices.append(rref_results[1][idx])
        return (ProblemMatrix[remainderIndices,:], remainderIndices)

    def rref(self, ProblemMatrix):
        ############################################################################
        # This function returns (1) the reduced row echelon form (RREF) of the     #
        # problem matrix and (2) the list of original row indices from the input   #
        # matrix of each row of RREF. * ProblemMatrix = [A; b]                     #                         #
        # Reference Source:                                                        #
        # https://stackoverflow.com/questions/7664246/python-built-in-function-to- #
        #  do-matrix-reduction/7665269#7665269                                     #
        ############################################################################
        Matrix = ProblemMatrix.copy()
        rows, cols = Matrix.shape
        r = 0
        pivots_pos = []
        row_exchanges = np.arange(rows)
        for c in range(cols):
            ## Find the pivot row:
            pivot = np.argmax (np.abs (Matrix[r:rows,c])) + r
            m = np.abs(Matrix[pivot, c])
            if m <= THRESHOLD:
                ## Skip column c, making sure the approximately zero terms are actually zero.
                Matrix[r:rows, c] = np.zeros(rows-r)
            else:
                ## keep track of bound variables
                pivots_pos.append((r,c))

                if pivot != r: ## Swap current row and pivot row
                    Matrix[[pivot, r], c:cols] = Matrix[[r, pivot], c:cols]
                    row_exchanges[[pivot,r]] = row_exchanges[[r,pivot]]

                ## Normalize pivot row
                Matrix[r, c:cols] = Matrix[r, c:cols] / Matrix[r, c];
                ## Eliminate the current column
                v = Matrix[r, c:cols]

                if r > 0: ## Above (before row r):
                    ridx_above = np.arange(r)
                    Matrix[ridx_above, c:cols] = Matrix[ridx_above, c:cols] - np.outer(v, Matrix[ridx_above, c]).T
                if r < rows-1: ## Below (after row r):
                    ridx_below = np.arange(r+1,rows)
                    Matrix[ridx_below, c:cols] = Matrix[ridx_below, c:cols] - np.outer(v, Matrix[ridx_below, c]).T
                r += 1
            if r == rows: ## Check if done
                break;
        return (Matrix, row_exchanges)
```

```python
###########################################################################
# File Name: model12_mySimplex.py                                         #
# Author: Geonsik Yu, Purdue University, IE Dept                          #
# LP problem (Model 12: Alloy Blending) from:                             #
# https://sites.math.washington.edu/~burke/crs/407/models/m12.html        #
###########################################################################
import MySimplex

## STEP 1. Set up what we need. -----------------------------------------------------------
## Declare variable names
variables = ["x1", "x2", "x3", "x4", "x5"]
## Delare a list of coefficients  of each variable in the objective function (same order)
obj_coeffs = [19.0, 17.0, 23.0, 21.0, 25.0]
## Delare a list of lowerbounds of each variable
lowerbounds = [0.0, 0.0, 0.0, 0.0, 0.0]

## Declare contraint names
constraint_names = ["Tin(%)", "Zinc(%)", "Lead(%)"]
## Delare a list of RHS constants of each constraints
righthand = [40.0, 35.0, 25.0]
## Delare a list of inequality directions of each constraints
senses = ['E', 'E', 'E']
## Set coefficients of each variables in each constraints:
lin_expr = [[60.0, 25.0, 45.0, 20.0, 50.0],
            [10.0, 15.0, 45.0, 50.0, 40.0],
            [30.0, 60.0, 10.0, 30.0, 10.0]]

## STEP 2. Generate LP problem object -----------------------------------------------------
## Generate an LP problem framework
problem = MySimplex.SimplexProblem()
## Set objective as minimization
problem.setObjectiveDirection( Max=False )
## Set variables and objective function
problem.setVariables( Names=variables, ObjCoeffs=obj_coeffs, Lowerbounds=lowerbounds )
## Set constraints
for idx in range(len(lin_expr)):
    problem.addConstraint( Name = constraint_names[idx]
                                            , rowVec = lin_expr[idx]
                                            , ineq_dir = senses[idx]
                                            , RHS = righthand[idx] )

## STEP 3. Solve the problem --------------------------------------------------------------
problem.setup()
Tableau = problem.buildTableau()
Tableau = problem.solve(Tableau)
```

```python
###########################################################################
# File Name: model27_mySimplex_Q1.py                                      #
# Author: Geonsik Yu, Purdue University, IE Dept                          #
# LP problem (Model 27: Hydrological Model) from:                         #
# https://sites.math.washington.edu/~burke/crs/407/models/m27.html        #
###########################################################################
import MySimplex

def oneHot(length, hotIdx):
    hotVec = [0.0]*length
    hotVec[hotIdx] = 1.0
    return hotVec

## STEP 1. Set up what we need. -----------------------------------------------------------
## Declare small constant epsilon for strict inequality removal:
EPSILON = 0.000000000001
## Declare variable names:
variables = ["b0", "b1", "b2", "x3", "x4", "x5", "x6", "x7", "x8", "x9", "x10", "x11", "x12"]
## Delare a list of coefficients  of each variable in the objective function (same order):
obj_coeffs = 3*[0.0] + 10*[1.0]
## Delare a list of lowerbounds of each variable:
#lowerbounds = 3*[EPSILON] + 10*[-float("inf")]
lowerbounds = 3*[EPSILON] + 10*[0]
## Declare contraint names:
constraint_names = ["Period 3-(1)", "Period 3-(2)",
                                    "Period 4-(1)", "Period 4-(2)",
                                    "Period 5-(1)", "Period 5-(2)",
                                    "Period 6-(1)", "Period 6-(2)",
                                    "Period 7-(1)", "Period 7-(2)",
                                    "Period 8-(1)", "Period 8-(2)",
                                    "Period 9-(1)", "Period 9-(2)",
                                    "Period 10-(1)", "Period 10-(2)",
                                    "Period 11-(1)", "Period 11-(2)",
                                    "Period 12-(1)", "Period 12-(2)",
                                    "b2", "b1 - b2", "b0 - b1",
                                    "b0+b1+b2"]
## Declare a list of RHS constants of each constraints:
righthand = [1.0, -1.0, 2.1, -2.1, 3.7, -3.7, 4.2, -4.2, 4.3, -4.3,
                     4.4, -4.4, 4.3, -4.3, 4.2, -4.2, 3.6, -3.6, 2.7, -2.7,
                     EPSILON, EPSILON, EPSILON, 1.0]

## Declare a list of inequality directions of each constraints:
senses = 23*['G'] + ['E']

## Declare and complete a coefficient matrix for the constraints:
Mat = []
Precip = [3.8, 4.4, 5.7, 5.2, 7.7, 6.0, 5.4, 5.7, 5.5, 2.5, 0.8, 0.4]
for i in range(2, 12):
    tmp1 = [Precip[i], Precip[i-1], Precip[i-2]] + oneHot(length=10, hotIdx=(i-2))
    tmp2 = [-Precip[i], -Precip[i-1], -Precip[i-2]] + oneHot(length=10, hotIdx=(i-2))
    Mat.append(tmp1)
    Mat.append(tmp2)
Mat.append([0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])
Mat.append([0.0, 1.0,-1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])
Mat.append([1.0,-1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])
Mat.append([1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])

## Set coefficients of each variables in each constraints:
lin_expr = []
for row in Mat:
    print(row)
    lin_expr.append( row )

## STEP 2. Generate LP problem object -----------------------------------------------------
## Generate an LP problem framework
problem = MySimplex.SimplexProblem()
## Set objective as minimization
problem.setObjectiveDirection( Max=False )
```

```
## Set variables and objective function
problem.setVariables( Names=variables, ObjCoeffs=obj_coeffs, Lowerbounds=lowerbounds )
## Set constraints
for idx in range(len(lin_expr)):
    problem.addConstraint( Name = constraint_names[idx]
                                          , rowVec = lin_expr[idx]
                                          , ineq_dir = senses[idx]
                                          , RHS = righthand[idx] )

## STEP 3. Solve the problem --------------------------------------------------------------
problem.setup()
Tableau = problem.buildTableau()
Tableau = problem.solve(Tableau)
```

```
###########################################################################
# File Name: model27_mySimplex_Q2.py                                      #
# Author: Geonsik Yu, Purdue University, IE Dept                          #
# LP problem (Model 27: Hydrological Model) from:                         #
# https://sites.math.washington.edu/~burke/crs/407/models/m27.html        #
###########################################################################
import MySimplex

## STEP 1. Set up what we need. ----------------------------------------------------------------
## Declare small constant epsilon for strict inequality removal:
EPSILON = 0.000000000001
## Declare variable names:
variables = ["b0", "b1", "b2", "x3"]
## Delare a list of coefficients  of each variable in the objective function (same order):
obj_coeffs = 3*[0.0] + [1.0]
## Delare a list of lowerbounds of each variable:
#lowerbounds = 3*[EPSILON] + 10*[-float("inf")]
lowerbounds = 3*[EPSILON] + [0.0]
## Declare contraint names:
constraint_names = ["Period 3-(1)", "Period 3-(2)",
                                    "Period 4-(1)", "Period 4-(2)",
                                    "Period 5-(1)", "Period 5-(2)",
                                    "Period 6-(1)", "Period 6-(2)",
                                    "Period 7-(1)", "Period 7-(2)",
                                    "Period 8-(1)", "Period 8-(2)",
                                    "Period 9-(1)", "Period 9-(2)",
                                    "Period 10-(1)", "Period 10-(2)",
                                    "Period 11-(1)", "Period 11-(2)",
                                    "Period 12-(1)", "Period 12-(2)",
                                    "b2", "b1 - b2", "b0 - b1",
                                    "b0+b1+b2"]
## Declare a list of RHS constants of each constraints:
righthand = [1.0, -1.0, 2.1, -2.1, 3.7, -3.7, 4.2, -4.2, 4.3, -4.3,
                     4.4, -4.4, 4.3, -4.3, 4.2, -4.2, 3.6, -3.6, 2.7, -2.7,
                     EPSILON, EPSILON, EPSILON, 1.0]

## Declare a list of inequality directions of each constraints:
senses = 23*['G'] + ['E']

## Declare and complete a coefficient matrix for the constraints:
Mat = []
Precip = [3.8, 4.4, 5.7, 5.2, 7.7, 6.0, 5.4, 5.7, 5.5, 2.5, 0.8, 0.4]
for i in range(2, 12):
    tmp1 = [Precip[i], Precip[i-1], Precip[i-2], 1]
    tmp2 = [-Precip[i], -Precip[i-1], -Precip[i-2], 1]
    Mat.append(tmp1)
    Mat.append(tmp2)
Mat.append([0.0, 0.0, 1.0, 0.0])
Mat.append([0.0, 1.0,-1.0, 0.0])
Mat.append([1.0,-1.0, 0.0, 0.0])
Mat.append([1.0, 1.0, 1.0, 0.0])

## Set coefficients of each variables in each constraints:
lin_expr = []
for row in Mat:
    print(row)
    lin_expr.append( row )

## STEP 2. Generate LP problem object ----------------------------------------------------------
## Generate an LP problem framework
problem = MySimplex.SimplexProblem()
## Set objective as minimization
problem.setObjectiveDirection( Max=False )
## Set variables and objective function
problem.setVariables( Names=variables, ObjCoeffs=obj_coeffs, Lowerbounds=lowerbounds )
## Set constraints
for idx in range(len(lin_expr)):
```

```
    problem.addConstraint( Name = constraint_names[idx]
                                                   , rowVec = lin_expr[idx]
                                                   , ineq_dir = senses[idx]
                                                   , RHS = righthand[idx] )

## STEP 3. Solve the problem -------------------------------------------------------------
problem.setup()
Tableau = problem.buildTableau()
Tableau = problem.solve(Tableau)
```

```python
#############################################################################
# File Name: model12_cplex.py                                               #
# Author: Geonsik Yu, Purdue University, IE Dept                            #
# LP problem (Model 12: Alloy Blending) from:                               #
# https://sites.math.washington.edu/~burke/crs/407/models/m12.html          #
#############################################################################
import cplex

## STEP 1. Set up what we need. ---------------------------------------------------------------
## Declare variable names
variables = ["x1", "x2", "x3", "x4", "x5"]
## Delare a list of coefficients  of each variable in the objective function (same order)
obj_coeffs = [19.0, 17.0, 23.0, 21.0, 25.0]
## Delare a list of upperbounds of each variable
upperbounds = [cplex.infinity, cplex.infinity, cplex.infinity, cplex.infinity, cplex.infinity]
## Delare a list of lowerbounds of each variable
lowerbounds = [0.0, 0.0, 0.0, 0.0, 0.0]
## Declare contraint names
constraint_names = ["Tin(%)", "Zinc(%)", "Lead(%)"]
## Delare a list of RHS constants of each constraints
righthand = [40.0, 35.0, 25.0]
## Delare a list of inequality directions of each constraints
senses = ['E', 'E', 'E']
## Set coefficients of each variables in each constraints:
lin_expr = [cplex.SparsePair(ind=variables, val=[60.0, 25.0, 45.0, 20.0, 50.0]),
            cplex.SparsePair(ind=variables, val=[10.0, 15.0, 45.0, 50.0, 40.0]),
            cplex.SparsePair(ind=variables, val=[30.0, 60.0, 10.0, 30.0, 10.0])]

## STEP 2. Generate LP problem object -----------------------------------------------------
## Generate an LP problem
problem = cplex.Cplex()
## Set objective as minimization
problem.objective.set_sense( problem.objective.sense.minimize )
## Set variables and objective function
problem.variables.add( obj=obj_coeffs, ub=upperbounds, lb=lowerbounds, names=variables )
## Set constraints
problem.linear_constraints.add(lin_expr = lin_expr, senses = senses, rhs = righthand, names = constraint_names)
## Solve the problem
problem.solve()

## STEP 3. Print out results --------------------------------------------------------------
numrows = problem.linear_constraints.get_num()
numcols = problem.variables.get_num()

print("Solution status = "+ repr(problem.solution.get_status())+ ": " +repr(problem.solution.status[problem.solution.get_status()]))
print("Solution value  = "+ repr(problem.solution.get_objective_value()))

x = problem.solution.get_values()
shadow_price = problem.solution.get_dual_values()
for i in range(numcols):
    print("Variable " + variables[i] + ": Value = " + repr(x[i]))
for i in range(numrows):
    print("Constraint " + constraint_names[i] + ": Shadow Price = " + repr(shadow_price[i]))
```

```python
##############################################################################
# File Name: model27_cplex_Q1.py                                             #
# Author: Geonsik Yu, Purdue University, IE Dept                             #
# LP problem (Model 27: Hydrological Model) from:                            #
# https://sites.math.washington.edu/~burke/crs/407/models/m27.html          #
##############################################################################
import cplex

def oneHot(length, hotIdx):
    hotVec = [0.0]*length
    hotVec[hotIdx] = 1.0
    return hotVec

## STEP 1. Set up what we need. ------------------------------------------------------------
## Declare small constant epsilon for strict inequality removal:
EPSILON = 0.000000000001
## Declare variable names:
variables = ["b0", "b1", "b2", "A3", "A4", "A5", "A6", "A7", "A8", "A9", "A10", "A11", "A12"]
## Delare a list of coefficients  of each variable in the objective function (same order):
obj_coeffs = 3*[0.0] + 10*[1.0]
## Delare a list of upperbounds of each variable:
upperbounds = 13*[cplex.infinity]
## Delare a list of lowerbounds of each variable:
#lowerbounds = 3*[EPSILON] + 10*[-cplex.infinity]
lowerbounds = 3*[EPSILON] + 10*[0]

## Declare contraint names:
constraint_names = ["Period 3-(1)", "Period 3-(2)",
                                    "Period 4-(1)", "Period 4-(2)",
                                    "Period 5-(1)", "Period 5-(2)",
                                    "Period 6-(1)", "Period 6-(2)",
                                    "Period 7-(1)", "Period 7-(2)",
                                    "Period 8-(1)", "Period 8-(2)",
                                    "Period 9-(1)", "Period 9-(2)",
                                    "Period 10-(1)", "Period 10-(2)",
                                    "Period 11-(1)", "Period 11-(2)",
                                    "Period 12-(1)", "Period 12-(2)",
                                    "b2", "b1 - b2", "b0 - b1",
                                    "b0+b1+b2"]
## Declare a list of RHS constants of each constraints:
righthand = [1.0, -1.0, 2.1, -2.1, 3.7, -3.7, 4.2, -4.2, 4.3, -4.3,
                     4.4, -4.4, 4.3, -4.3, 4.2, -4.2, 3.6, -3.6, 2.7, -2.7,
                     EPSILON, EPSILON, EPSILON, 1.0]

## Declare a list of inequality directions of each constraints:
senses = 23*['G'] + ['E']

## Declare and complete a coefficient matrix for the constraints:
Mat = []
Precip = [3.8, 4.4, 5.7, 5.2, 7.7, 6.0, 5.4, 5.7, 5.5, 2.5, 0.8, 0.4]
for i in range(2, 12):
    tmp1 = [Precip[i], Precip[i-1], Precip[i-2]] + oneHot(length=10, hotIdx=(i-2))
    tmp2 = [-Precip[i], -Precip[i-1], -Precip[i-2]] + oneHot(length=10, hotIdx=(i-2))
    Mat.append(tmp1)
    Mat.append(tmp2)
Mat.append([0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])
Mat.append([0.0, 1.0,-1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])
Mat.append([1.0,-1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])
Mat.append([1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])

## Set coefficients of each variables in each constraints:
lin_expr = []
for row in Mat:
    print(row)
    lin_expr.append( cplex.SparsePair(ind=variables, val=row) )

## STEP 2. Generate LP problem object ------------------------------------------------------
## Generate an LP problem
problem = cplex.Cplex()
## Set objective as minimization
problem.objective.set_sense( problem.objective.sense.minimize )
## Set variables and objective function
problem.variables.add( obj=obj_coeffs, ub=upperbounds, lb=lowerbounds, names=variables )
## Set constraints
problem.linear_constraints.add(lin_expr = lin_expr, senses = senses, rhs = righthand, names = constraint_names)
## Solve the problem
problem.solve()

## STEP 3. Print out results ---------------------------------------------------------------
numrows = problem.linear_constraints.get_num()
numcols = problem.variables.get_num()

print("Solution status = "+ repr(problem.solution.get_status())+ ": " +repr(problem.solution.status[problem.solution.get_status()]))
print("Solution value  = "+ repr(problem.solution.get_objective_value()))

x = problem.solution.get_values()
shadow_price = problem.solution.get_dual_values()
for i in range(numcols):
    print("Variable " + variables[i] + ": Value = " + repr(x[i]))
for i in range(numrows):
    print("Constraint " + constraint_names[i] + ": Shadow Price = " + repr(shadow_price[i]))
```

```python
##########################################################################
# File Name: model27_cplex_Q2.py                                         #
# Author: Geonsik Yu, Purdue University, IE Dept                         #
# LP problem (Model 27: Hydrological Model) from:                        #
# https://sites.math.washington.edu/~burke/crs/407/models/m27.html       #
##########################################################################
import cplex

## STEP 1. Set up what we need. -------------------------------------------------------------
## Declare small constant epsilon for strict inequality removal:
EPSILON = 0.000000000001
## Declare variable names:
variables = ["b0", "b1", "b2", "A"]
## Delare a list of coefficients  of each variable in the objective function (same order):
obj_coeffs = 3*[0.0] + [1.0]
## Delare a list of upperbounds of each variable:
upperbounds = 4*[cplex.infinity]
## Delare a list of lowerbounds of each variable:
#lowerbounds = 3*[EPSILON] + [-cplex.infinity]
lowerbounds = 3*[EPSILON] + [0.0]
## Declare contraint names:
constraint_names = ["Period 3-(1)", "Period 3-(2)",
                                    "Period 4-(1)", "Period 4-(2)",
                                    "Period 5-(1)", "Period 5-(2)",
                                    "Period 6-(1)", "Period 6-(2)",
                                    "Period 7-(1)", "Period 7-(2)",
                                    "Period 8-(1)", "Period 8-(2)",
                                    "Period 9-(1)", "Period 9-(2)",
                                    "Period 10-(1)", "Period 10-(2)",
                                    "Period 11-(1)", "Period 11-(2)",
                                    "Period 12-(1)", "Period 12-(2)",
                                    "b2", "b1 - b2", "b0 - b1",
                                    "b0+b1+b2"]
## Declare a list of RHS constants of each constraints:
righthand = [1.0, -1.0, 2.1, -2.1, 3.7, -3.7, 4.2, -4.2, 4.3, -4.3,
                     4.4, -4.4, 4.3, -4.3, 4.2, -4.2, 3.6, -3.6, 2.7, -2.7,
                     EPSILON, EPSILON, EPSILON, 1.0]

## Declare a list of inequality directions of each constraints:
senses = 23*['G'] + ['E']

## Declare and complete a coefficient matrix for the constraints:
Mat = []
Precip = [3.8, 4.4, 5.7, 5.2, 7.7, 6.0, 5.4, 5.7, 5.5, 2.5, 0.8, 0.4]
for i in range(2, 12):
    tmp1 = [Precip[i], Precip[i-1], Precip[i-2], 1]
    tmp2 = [-Precip[i], -Precip[i-1], -Precip[i-2], 1]
    Mat.append(tmp1)
    Mat.append(tmp2)
Mat.append([0.0, 0.0, 1.0, 0.0])
Mat.append([0.0, 1.0,-1.0, 0.0])
Mat.append([1.0,-1.0, 0.0, 0.0])
Mat.append([1.0, 1.0, 1.0, 0.0])

## Set coefficients of each variables in each constraints:
lin_expr = []
for row in Mat:
    lin_expr.append( cplex.SparsePair(ind=variables, val=row) )

## STEP 2. Generate LP problem object -------------------------------------------------------
## Generate an LP problem
problem = cplex.Cplex()
## Set objective as minimization
problem.objective.set_sense( problem.objective.sense.minimize )
## Set variables and objective function
problem.variables.add( obj=obj_coeffs, ub=upperbounds, lb=lowerbounds, names=variables )
## Set constraints
problem.linear_constraints.add(lin_expr = lin_expr, senses = senses, rhs = righthand, names = constraint_names)
## Solve the problem
problem.solve()

## STEP 3. Print out results ----------------------------------------------------------------
numrows = problem.linear_constraints.get_num()
numcols = problem.variables.get_num()

print("Solution status = "+ repr(problem.solution.get_status())+ ": " +repr(problem.solution.status[problem.solution.get_status()]))
print("Solution value  = "+ repr(problem.solution.get_objective_value()))

x = problem.solution.get_values()
shadow_price = problem.solution.get_dual_values()
for i in range(numcols):
    print("Variable " + variables[i] + ": Value = " + repr(x[i]))
for i in range(numrows):
    print("Constraint " + constraint_names[i] + ": Shadow Price = " + repr(shadow_price[i]))
```

4. MySimplex and Cplex Solver Outputs

Run model12_mySimplex.py

```
Geonsikui-MacBook-Pro-2:IE535_Simplex geonsik$ python model12_mySimplex.py
Initialize the given LP problem:
Initialize the zero-th row:
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S2 A1 A2 A3
Current Objective Value:  100.0
x1  :=  0.0x2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0S1  :=  40.0S2  :=  35.0S3  :=  25.0S4  :=  0.0S5  :=
0.0S6  :=  0.0A1  :=  40.0A2  :=  35.0A3  :=  25.0
TWO STEP METHOD CALLED:
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S2 A1 A2 A3
Current Objective Value:  100.0
x1  :=  0.0x2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0S1  :=  40.0S2  :=  35.0S3  :=  25.0S4  :=  0.0S5  :=
0.0S6  :=  0.0A1  :=  40.0A2  :=  35.0A3  :=  25.0
Run the 1st step of Two Step Method
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S2 x5 A2 A3
Current Objective Value:  20.0
x1  :=  0.0x2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.8S1  :=  0.0S2  :=  3.0S3  :=  17.0S4  :=  0.0S5  :=  0.
0S6  :=  0.0A1  :=  0.0A2  :=  3.0A3  :=  17.0
------------------------------------------------------------------
Current Basic Variables:  S4 S3 S2 x5 A2 A3
Current Objective Value:  20.0
x1  :=  0.0x2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.8S1  :=  0.0S2  :=  3.0S3  :=  17.0S4  :=  0.0S5  :=  0.
0S6  :=  0.0A1  :=  0.0A2  :=  3.0A3  :=  17.0
------------------------------------------------------------------
Current Basic Variables:  S4 S3 S2 x5 x4 A3
Current Objective Value:  14.705882352941176
x1  :=  0.0x2  :=  0.0x3  :=  0.0x4  :=  0.08823529411764706x5  :=  0.7647058823529412S1  :=  0.0S2  :=  0.0S3
  :=  14.705882352941176S4  :=  0.0S5  :=  0.0S6  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  14.705882352941176
------------------------------------------------------------------
Current Basic Variables:  S4 S3 S5 x5 x4 A3
Current Objective Value:  14.705882352941176
x1  :=  0.0x2  :=  0.0x3  :=  0.0x4  :=  0.08823529411764706x5  :=  0.7647058823529412S1  :=  0.0S2  :=  0.0S3
  :=  14.705882352941176S4  :=  0.0S5  :=  0.0S6  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  14.705882352941176
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S5 x5 x4 A3
Current Objective Value:  14.705882352941176
x1  :=  0.0x2  :=  0.0x3  :=  0.0x4  :=  0.08823529411764706x5  :=  0.7647058823529412S1  :=  0.0S2  :=  0.0S3
  :=  14.705882352941176S4  :=  0.0S5  :=  0.0S6  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  14.705882352941176
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S5 x5 x4 x2
Current Objective Value:  -1.7763568394002505e-15
x1  :=  0.0x2  :=  0.25x3  :=  0.0x4  :=  0.125x5  :=  0.625S1  :=  0.0S2  :=  0.0S3  :=  0.0S4  :=  0.0S5  :=
  0.0S6  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0
- Terminate Procedure: Optimal Value =  -1.7763568394002505e-15
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S5 x5 x4 x2
Current Objective Value:  -1.7763568394002505e-15
x1  :=  0.0x2  :=  0.25x3  :=  0.0x4  :=  0.125x5  :=  0.625S1  :=  0.0S2  :=  0.0S3  :=  0.0S4  :=  0.0S5  :=
  0.0S6  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0
```

```
Run the Vanilla Simplex Method or the 2st step of Two Step Method
Initialize the zero-th row:
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S5 x5 x4 x2
Current Objective Value:  22.5
x1  :=  0.0x2  :=  0.25x3  :=  0.0x4  :=  0.125x5  :=  0.625S1  :=  0.0S2  :=  0.0S3  :=  0.0S4  :=  0.0S5  :=
  0.0S6  :=  0.0
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S2 x5 x4 x2
Current Objective Value:  22.5
x1  :=  0.0x2  :=  0.25x3  :=  0.0x4  :=  0.125x5  :=  0.625S1  :=  0.0S2  :=  0.0S3  :=  0.0S4  :=  0.0S5  :=
  0.0S6  :=  0.0
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S2 x5 x3 x2
Current Objective Value:  21.599999999999998
x1  :=  0.0x2  :=  0.3x3  :=  0.5x4  :=  0.0x5  :=  0.19999999999999996S1  :=  0.0S2  :=  0.0S3  :=  0.0S4  :=
  0.0S5  :=  0.0S6  :=  0.0
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S2 x1 x3 x2
Current Objective Value:  21.130434782608692
x1  :=  0.043478260869565 2x2  :=  0.2826086956521739x3  :=  0.6739130434782608x4  :=  0.0x5  :=  0.0S1  :=  0.
0S2  :=  0.0S3  :=  0.0S4  :=  0.0S5  :=  0.0S6  :=  0.0
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S2 x1 x3 x4
Current Objective Value:  20.812499999999996
x1  :=  0.34374999999999994x2  :=  0.0x3  :=  0.24999999999999978x4  :=  0.40625x5  :=  0.0S1  :=  0.0S2  :=
0.0S3  :=  0.0S4  :=  0.0S5  :=  0.0S6  :=  0.0
- Terminate Procedure: Optimal Value =  20.812499999999996
------------------------------------------------------------------
Current Basic Variables:  S1 S3 S2 x1 x3 x4
Current Objective Value:  20.812499999999996
x1  :=  0.34374999999999994x2  :=  0.0x3  :=  0.24999999999999978x4  :=  0.40625x5  :=  0.0S1  :=  0.0S2  :=
0.0S3  :=  0.0S4  :=  0.0S5  :=  0.0S6  :=  0.0
```

Run model12_cplex.py

```
Geonsikui-MacBook-Pro-2:IE535_Simplex geonsik$ python model12_cplex.py
CPXPARAM_Read_DataCheck                          1
Tried aggregator 1 time.
No LP presolve or aggregator reductions.
Presolve time = 0.00 sec. (0.00 ticks)

Iteration log . . .
Iteration:     1   Dual objective     =            12.666667
Solution status = 1: 'optimal'
Solution value  = 20.812500000000004
Variable x1: Value = 0.34374999999999994
Variable x2: Value = 0.0
Variable x3: Value = 0.2500000000000001
Variable x4: Value = 0.40625
Variable x5: Value = 0.0
Constraint Tin(%): Shadow Price = 0.21874999999999994
Constraint Zinc(%): Shadow Price = 0.26875
Constraint Lead(%): Shadow Price = 0.10625000000000001
```

Run model27_cplex_Q1.py

```
Geonsikui-MacBook-Pro-2:IE535_Simplex geonsik$ python model27_mySimplex_Q1.py
Initialize the given LP problem:
Initialize the zero-th row:
-----------------------------------------------------------------
Current Basic Variables:  A3 A8 A9 A1 S4 S6 A4 A5 A6 A7 S8 S18 A10 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11 A
14
Current Objective Value:  35.50000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  0.0x7  :=  0.0x8  :=  0.0x9  :=  0.0
x10  :=  0.0x11  :=  0.0x12  :=  0.0S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  4.4S13  :=  0.0S14  :=  4.3S15  :=  0.0S16  :=  4.2S
17  :=  0.0S18  :=  3.6S19  :=  0.0S20  :=  2.7S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  4.4A7  :=  4.3A8  :=  4.2A9  :=  3.6A10  :=  2.7A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
TWO STEP METHOD CALLED:
-----------------------------------------------------------------
Current Basic Variables:  A3 A8 A9 A1 S4 S6 A4 A5 A6 A7 S8 S18 A10 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11 A
14
Current Objective Value:  35.50000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  0.0x7  :=  0.0x8  :=  0.0x9  :=  0.0
x10  :=  0.0x11  :=  0.0x12  :=  0.0S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  4.4S13  :=  0.0S14  :=  4.3S15  :=  0.0S16  :=  4.2S
17  :=  0.0S18  :=  3.6S19  :=  0.0S20  :=  2.7S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  4.4A7  :=  4.3A8  :=  4.2A9  :=  3.6A10  :=  2.7A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
Run the 1st step of Two Step Method
-----------------------------------------------------------------
Current Basic Variables:  A3 A8 A9 A1 S4 S6 A4 A5 A6 A7 S8 S18 x12 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11 A
14
Current Objective Value:  32.80000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  0.0x7  :=  0.0x8  :=  0.0x9  :=  0.0
x10  :=  0.0x11  :=  0.0x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  4.4S13  :=  0.0S14  :=  4.3S15  :=  0.0S16  :=  4.2S
17  :=  0.0S18  :=  3.6S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  4.4A7  :=  4.3A8  :=  4.2A9  :=  3.6A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
-----------------------------------------------------------------
Current Basic Variables:  A3 A8 x11 A1 S4 S6 A4 A5 A6 A7 S8 S18 x12 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11
A14
Current Objective Value:  29.20000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  0.0x7  :=  0.0x8  :=  0.0x9  :=  0.0
x10  :=  0.0x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  4.4S13  :=  0.0S14  :=  4.3S15  :=  0.0S16  :=  4.2S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  4.4A7  :=  4.3A8  :=  4.2A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
-----------------------------------------------------------------
Current Basic Variables:  A3 x10 x11 A1 S4 S6 A4 A5 A6 A7 S8 S18 x12 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11
 A14
Current Objective Value:  25.00000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  0.0x7  :=  0.0x8  :=  0.0x9  :=  0.0
x10  :=  4.2x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  4.4S13  :=  0.0S14  :=  4.3S15  :=  0.0S16  :=  8.4S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  4.4A7  :=  4.3A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
-----------------------------------------------------------------
```

```
----------------------------------------------------------------------
Current Basic Variables:  A3 x10 x11 A1 S4 S6 A4 A5 A6 x9 S8 S18 x12 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11
 A14
Current Objective Value:  20.70000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  0.0x7  :=  0.0x8  :=  0.0x9  :=  4.3
x10  :=  4.2x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  4.4S13  :=  0.0S14  :=  8.6S15  :=  0.0S16  :=  8.4S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  4.4A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
----------------------------------------------------------------------
Current Basic Variables:  A3 x10 x11 A1 S4 S6 A4 A5 x8 x9 S8 S18 x12 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11
 A14
Current Objective Value:  16.30000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  0.0x7  :=  0.0x8  :=  4.4x9  :=  4.3
x10  :=  4.2x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  8.8S13  :=  0.0S14  :=  8.6S15  :=  0.0S16  :=  8.4S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
----------------------------------------------------------------------
Current Basic Variables:  A3 x10 x11 A1 S4 S6 A4 x7 x8 x9 S8 S18 x12 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11
 A14
Current Objective Value:  12.00000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  0.0x7  :=  4.3x8  :=  4.4x9  :=  4.3
x10  :=  4.2x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  4.2S9  :=  0.0S10  :=  8.6S11  :=  0.0S12  :=  8.8S13  :=  0.0S14  :=  8.6S15  :=  0.0S16  :=  8.4S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
----------------------------------------------------------------------
Current Basic Variables:  A3 x10 x11 A1 S4 S6 x6 x7 x8 x9 S8 S18 x12 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11
 A14
Current Objective Value:  7.800000000003009
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  0.0x6  :=  4.2x7  :=  4.3x8  :=  4.4x9  :=  4.3
x10  :=  4.2x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7S7  :=
0.0S8  :=  8.4S9  :=  0.0S10  :=  8.6S11  :=  0.0S12  :=  8.8S13  :=  0.0S14  :=  8.6S15  :=  0.0S16  :=  8.4S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
----------------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 A1 S4 S6 x6 x7 x8 x9 S8 S18 x12 S2 A2 S12 S16 S10 S14 A13 A12 S20 S24 A11
 A14
Current Objective Value:  4.100000000003009
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  0.0x5  :=  3.7x6  :=  4.2x7  :=  4.3x8  :=  4.4x9  :=  4.3
x10  :=  4.2x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  7.4S7  :=
0.0S8  :=  8.4S9  :=  0.0S10  :=  8.6S11  :=  0.0S12  :=  8.8S13  :=  0.0S14  :=  8.6S15  :=  0.0S16  :=  8.4S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  2.1A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
----------------------------------------------------------------------
```

```
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 A1 S4 S6 x6 x7 x8 x9 S8 S18 x12 S2 x4 S12 S16 S10 S14 A13 A12 S20 S24 A11
 A14
Current Objective Value:  2.0000000000030087
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0x4  :=  2.1x5  :=  3.7x6  :=  4.2x7  :=  4.3x8  :=  4.4x9  :=  4.3
x10  :=  4.2x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  4.2S5  :=  0.0S6  :=  7.4S7  :=
0.0S8  :=  8.4S9  :=  0.0S10  :=  8.6S11  :=  0.0S12  :=  8.8S13  :=  0.0S14  :=  8.6S15  :=  0.0S16  :=  8.4S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  1.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S4 S6 x6 x7 x8 x9 S8 S18 x12 S2 x4 S12 S16 S10 S14 A13 A12 S20 S24 A11
 A14
Current Objective Value:  1.0000000000030087
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  1.0x4  :=  2.1x5  :=  3.7x6  :=  4.2x7  :=  4.3x8  :=  4.4x9  :=  4.3
x10  :=  4.2x11  :=  3.6x12  :=  2.7S1  :=  0.0S2  :=  2.0S3  :=  0.0S4  :=  4.2S5  :=  0.0S6  :=  7.4S7  :=
0.0S8  :=  8.4S9  :=  0.0S10  :=  8.6S11  :=  0.0S12  :=  8.8S13  :=  0.0S14  :=  8.6S15  :=  0.0S16  :=  8.4S
17  :=  0.0S18  :=  7.2S19  :=  0.0S20  :=  5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1
 :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S4 S6 x6 x7 x8 x9 S8 S18 x12 S2 x4 S12 S16 S10 S14 A13 A12 S20 S24 b2
A14
Current Objective Value:  1.0000000000020086
b0  :=  0.0b1  :=  0.0b2  :=  1e-12x3  :=  0.9999999999962x4  :=  2.0999999999956x5  :=  3.6999999999943003x6
 :=  4.1999999999948x7  :=  4.2999999999923x8  :=  4.399999999994001x9  :=  4.2999999999946x10  :=  4.19999999
99943x11  :=  3.5999999999945x12  :=  2.6999999999975004S1  :=  0.0S2  :=  1.9999999999924S3  :=  0.0S4  :=  4
.1999999999912S5  :=  0.0S6  :=  7.3999999999886015S7  :=  0.0S8  :=  8.3999999999896S9  :=  0.0S10  :=  8.5999
999999846S11  :=  0.0S12  :=  8.799999999988001S13  :=  0.0S14  :=  8.5999999999892S15  :=  0.0S16  :=  8.3999
999999886S17  :=  0.0S18  :=  7.199999999989S19  :=  0.0S20  :=  5.399999999995001S21  :=  0.0S22  :=  0.0S23
 :=  0.0S24  :=  0.999999999999S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A
7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  0.0A12  :=  2e-12A13  :=  1e-12A14  :=  0.999999999999
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S4 S6 x6 x7 x8 x9 S8 S18 x12 S2 x4 S12 S16 S10 S14 A13 b1 S20 S24 b2 A
14
Current Objective Value:  1.0000000000000087
b0  :=  0.0b1  :=  2e-12b2  :=  1e-12x3  :=  0.9999999999874001x4  :=  2.0999999999842x5  :=  3.6999999999839x
6  :=  4.1999999999794x7  :=  4.2999999999803x8  :=  4.3999999999832005x9  :=  4.2999999999832x10  :=  4.19999
99999833x11  :=  3.5999999999895x12  :=  2.6999999999959003S1  :=  0.0S2  :=  1.9999999999748002S3  :=  0.0S4
 :=  4.1999999999684S5  :=  0.0S6  :=  7.3999999999678S7  :=  0.0S8  :=  8.3999999999588S9  :=  0.0S10  :=  8.
5999999999606S11  :=  0.0S12  :=  8.799999999966401S13  :=  0.0S14  :=  8.5999999999664S15  :=  0.0S16  :=  8.
3999999999666S17  :=  0.0S18  :=  7.1999999999979S19  :=  0.0S20  :=  5.399999999991801S21  :=  0.0S22  :=  0.0
S23  :=  0.0S24  :=  0.9999999999970001S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6
:=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  0.0A12  :=  0.0A13  :=  3e-12A14  :=  0.999999999
9970001
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S4 S6 x6 x7 x8 x9 S8 S18 x12 S21 x4 S12 S16 S10 S14 A13 b1 S20 S24 b2
A14
Current Objective Value:  0.8780487804893501
b0  :=  0.0b1  :=  0.12195121951265855b2  :=  0.12195121951165856x3  :=  0.0x4  :=  0.8682926829265483x5  :=
2.3707317073177722x6  :=  2.626829268291904x7  :=  2.629268292684278x8  :=  3.009756097561693x9  :=  2.94634146
341489x10  :=  2.8341463414639243x11  :=  2.6243902439042315x12  :=  2.2975609756107275S1  :=  0.0S2  :=  0.0S3
  :=  0.0S4  :=  1.7365853658530965S5  :=  0.0S6  :=  4.741463414635444S7  :=  0.0S8  :=  5.253658536583808S9
 :=  0.0S10  :=  5.258536585368556S11  :=  0.0S12  :=  6.019512195123386S13  :=  0.0S14  :=  5.89268292682978S
15  :=  0.0S16  :=  5.668292682927849S17  :=  0.0S18  :=  5.248780487808463S19  :=  0.0S20  :=  4.595121951221
454S21  :=  0.12195121951065856S22  :=  0.0S23  :=  0.0S24  :=  0.7560975609756829S25  :=  0.0A1  :=  0.0A2  :
=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  0.0A12
  :=  0.0A13  :=  0.12195121951365856A14  :=  0.7560975609756829
-----------------------------------------------------------------
```

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-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S4 S6 x6 x7 x8 x9 S8 S18 x12 S21 S1 S12 S16 S10 S14 A13 b1 S20 S24 b2
A14
Current Objective Value:  0.7920792079223651
b0  :=  0.0b1  :=  0.20792079207964353b2  :=  0.20792079207864353x3  :=  0.7049504950492766x4  :=  0.0x5  :=
1.4336633663375857x6  :=  1.517821782177798x7  :=  1.4514851485165838x8  :=  2.0297029702980645x9  :=  1.99207
92079213565x10  :=  1.8712871287136927x11  :=  1.9366336633683519x12  :=  2.013861386139676651  :=  1.40990099
00985533S2  :=  0.0S3  :=  0.0S4  :=  0.0S5  :=  0.0S6  :=  2.8673267326751715S7  :=  0.0S8  :=  3.03564356435
5596S9  :=  0.0S10  :=  2.9029702970331677S11  :=  0.0S12  :=  4.059405940596129S13  :=  0.0S14  :=  3.9841584
15842713S15  :=  0.0S16  :=  3.7425742574273855S17  :=  0.0S18  :=  3.8732673267367037S19  :=  0.0S20  :=  4.0
2772277227935321  :=  0.20792079207764352S22  :=  0.0S23  :=  0.0S24  :=  0.584158415841713S25  :=  0.0A1  :=
  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11
:=  0.0A12  :=  0.0A13  :=  0.2079207920806435A14  :=  0.584158415841713
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S3 S6 x6 x7 x8 x9 S8 S18 x12 S21 S1 S12 S16 S10 S14 A13 b1 S20 S24 b2
A14
Current Objective Value:  0.7920792079223651
b0  :=  0.0b1  :=  0.20792079207964353b2  :=  0.20792079207864353x3  :=  0.7049504950492766x4  :=  0.0x5  :=
1.4336633663375857x6  :=  1.517821782177798x7  :=  1.4514851485165838x8  :=  2.0297029702980645x9  :=  1.99207
92079213565x10  :=  1.8712871287136927x11  :=  1.9366336633683519x12  :=  2.013861386139676651  :=  1.40990099
00985533S2  :=  0.0S3  :=  0.0S4  :=  0.0S5  :=  0.0S6  :=  2.8673267326751715S7  :=  0.0S8  :=  3.03564356435
5596S9  :=  0.0S10  :=  2.9029702970331677S11  :=  0.0S12  :=  4.059405940596129S13  :=  0.0S14  :=  3.9841584
15842713S15  :=  0.0S16  :=  3.7425742574273855S17  :=  0.0S18  :=  3.8732673267367037S19  :=  0.0S20  :=  4.0
2772277227935321  :=  0.20792079207764352S22  :=  0.0S23  :=  0.0S24  :=  0.584158415841713S25  :=  0.0A1  :=
  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11
:=  0.0A12  :=  0.0A13  :=  0.2079207920806435A14  :=  0.584158415841713
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S3 S6 x6 x7 x8 x9 S8 S18 x12 S21 S1 S12 S16 x4 S14 A13 b1 S20 S24 b2 A
14
Current Objective Value:  0.6861313868627604
b0  :=  0.0b1  :=  0.3138686131392482b2  :=  0.3138686131382482x3  :=  1.5737226277380347x4  :=  1.07007299270
20072x5  :=  0.2788321167878949x6  :=  0.15109489050889757x7  :=  0.0x8  :=  0.8218978102185712x9  :=  0.81605
83941597446x10  :=  0.6846715328461206x11  :=  1.0890510948915146x12  :=  1.66423357664298125 1  :=  3.14744525
54760695S2  :=  0.0S3  :=  2.1401459854040144S4  :=  0.0S5  :=  0.0S6  :=  0.5576642335757898S7  :=  0.0S8  :=
  0.30218978101779514S9  :=  0.0S10  :=  0.0S11  :=  0.0S12  :=  1.6437956204371424S13  :=  0.0S14  :=  1.6321
167883194891S15  :=  0.0S16  :=  1.3693430656922412S17  :=  0.0S18  :=  2.1781021897830293S19  :=  0.0S20  :=
 3.3284671532859624S21  :=  0.3138686131372482S22  :=  0.0S23  :=  0.0S24  :=  0.3722627737225037S25  :=  0.0A
1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0
A11  :=  0.0A12  :=  0.0A13  :=  0.31386861314024816A14  :=  0.3722627737225037
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S3 S6 x6 x7 x8 x9 S9 S18 x12 S21 S1 S12 S16 x4 S14 A13 b1 S20 S24 b2 A
14
Current Objective Value:  0.6744186046527684
b0  :=  0.0b1  :=  0.32558139534924024b2  :=  0.32558139534824027x3  :=  1.6697674418599695x4  :=  1.188372093
022927x5  :=  0.1511627906989814 5x6  :=  0.0x7  :=  0.16046511627689 12x8  :=  0.6883720930246617x9  :=  0.6860
465116288327x10  :=  0.5534883720942095x11  :=  0.9953488372115782x12  :=  1.62558139535000735 1  :=  3.3395348
83719939S2  :=  0.0S3  :=  2.3767441860458545 4  :=  0.0S5  :=  0.0S6  :=  0.302325581397962957  :=  0.0S8  :=
 0.0S9  :=  0.32093023255378245 10  :=  0.0S11  :=  0.0S12  :=  1.3767441860493235S13  :=  0.0S14  :=  1.372093
0232576654S15  :=  0.0S16  :=  1.106976744188419S17  :=  0.0S18  :=  1.9906976744231564S19  :=  0.0S20  :=  3.
2511627907000147S21  :=  0.32558139534724023S22  :=  0.0S23  :=  0.0S24  :=  0.3488372093025196S25  :=  0.0A1
 :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A1
1  :=  0.0A12  :=  0.0A13  :=  0.3255813953502402A14  :=  0.3488372093025196
```

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-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S3 S7 x6 x7 x8 x9 S9 S18 x12 S21 S1 S12 S16 x4 S14 A13 b1 S20 S24 b2 A
14
Current Objective Value:  0.660550458717082
b0  :=  0.0b1  :=  0.3394495412849266b2  :=  0.3394495412839266x3  :=  1.7834862385325978x4  :=  1.32844036697
33593x5  :=  0.0x6  :=  0.1788990825703542x7  :=  0.3504587155957944 5x8  :=  0.5302752293578371x9  :=  0.53211
0091742714x10  :=  0.3981651376145222x11  :=  0.8844036697260872x12  :=  1.5798165137622424S1  :=  3.566972477
0651956S2  :=  0.0S3  :=  2.6568807339467186S4  :=  0.0S5  :=  0.0S6  :=  0.0S7  :=  0.3577981651407084S8  :=
 0.0S9  :=  0.7009174311915889S10  :=  0.0S11  :=  0.0S12  :=  1.0605504587156742S13  :=  0.0S14  :=  1.064220
183485428S15  :=  0.0S16  :=  0.7963302752290444S17  :=  0.0S18  :=  1.7688073394521744S19  :=  0.0S20  :=  3.
1596330275244 85S21  :=  0.3394495412829266S22  :=  0.0S23  :=  0.0S24  :=  0.3211009174311469S25  :=  0.0A1  :
=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11
 :=  0.0A12  :=  0.0A13  :=  0.3394495412859266A14  :=  0.3211009174311469
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S3 S7 x6 x7 x8 x9 S9 S18 x12 S21 S1 S12 S5 x4 S14 A13 b1 S20 S24 b2 A1
4
Current Objective Value:  0.6250000000014996
b0  :=  0.0b1  :=  0.3750000000005089b2  :=  0.3749999999950895x3  :=  2.075000000000373x4  :=  1.68750000000
07412x5  :=  0.3874999999998476x6  :=  0.6375000000013664x7  :=  0.8374999999992725x8  :=  0.1250000000001984x
9  :=  0.1374999999974988x10  :=  0.0x11  :=  0.6000000000014284x12  :=  1.4625000000008206S1  :=  4.15000000
0000746S2  :=  0.0S3  :=  3.375000000014824S4  :=  0.0S5  :=  0.7749999999996952S6  :=  0.0S7  :=  1.27500000
00027328S8  :=  0.0S9  :=  1.674999999998545S10  :=  0.0S11  :=  0.0S12  :=  0.2500000000003968S13  :=  0.0S14
  :=  0.2749999999949976S15  :=  0.0S16  :=  0.0S17  :=  0.0S18  :=  1.2000000000028568S19  :=  0.0S20  :=  2
.925000000001641S21  :=  0.3749999999850897S22  :=  0.0S23  :=  0.0S24  :=  0.2499999999998218S25  :=  0.0A1
  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A
11  :=  0.0A12  :=  0.0A13  :=  0.3750000000015089A14  :=  0.2499999999998218
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S3 S7 x6 x7 x8 x9 S9 S18 x12 S21 S1 S15 S5 x4 S14 A13 b1 S20 S24 b2 A1
4
Current Objective Value:  0.6140350877207804
b0  :=  0.0b1  :=  0.3859649122812281b2  :=  0.3859649122802281x3  :=  2.1649122807022705x4  :=  1.79824561403
60048x5  :=  0.5070175438596864x6  :=  0.7789473684226436x7  :=  0.9877192982451249x8  :=  0.0x9  :=  0.015789
473683767208x10  :=  0.1228070175440545 4x11  :=  0.5122807017556751x12  :=  1.4263157894744474S1  :=  4.329824
561404541S2  :=  0.0S3  :=  3.5964912280720096S4  :=  0.0S5  :=  1.0140350877193 73S6  :=  0.0S7  :=  1.5578947
368452871S8  :=  0.0S9  :=  1.9754385964902499S10  :=  0.0S11  :=  0.0S12  :=  0.0S13  :=  0.0S14  :=  0.03157
8947367534416S15  :=  0.2456140350881090 8S16  :=  0.0S17  :=  0.0S18  :=  1.0245614035113502S19  :=  0.0S20  :
=  2.8526315789488947S21  :=  0.3859649122792281 3S22  :=  0.0S23  :=  0.0S24  :=  0.2280701754385438 5S25  :=
0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=
 0.0A11  :=  0.0A12  :=  0.0A13  :=  0.3859649122822280 7A14  :=  0.22807017543854385
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S3 S7 x6 x7 x8 x9 S9 S18 x12 S21 S1 S15 S5 x4 S11 A13 b1 S20 S24 b2 A1
4
Current Objective Value:  0.6126126126141347
b0  :=  0.0b1  :=  0.3873873873878737 6b2  :=  0.3873873873868738x3  :=  2.1765765765767653x4  :=  1.8126126126
131263x5  :=  0.5225225225221245x6  :=  0.7972972972983731x7  :=  1.0072072072061709x8  :=  0.0162162162157609
15x9  :=  0.0x10  :=  0.1387387387384863x11  :=  0.5009009009025095x12  :=  1.4216216216225166S1  :=  4.353153
153153531S2  :=  0.0S3  :=  3.6252252252262527S4  :=  0.0S5  :=  1.0450450450442490S6  :=  0.0S7  :=  1.5945945
945967461S8  :=  0.0S9  :=  2.0144144144123417S10  :=  0.0S11  :=  0.0324324324315218 3S12  :=  0.0S13  :=  0.0
S14  :=  0.0S15  :=  0.2774774774769726S16  :=  0.0S17  :=  0.0S18  :=  1.0018018018050190S19  :=  0.0S20  :=
2.843243243245033S21  :=  0.3873873873858738S22  :=  0.0S23  :=  0.0S24  :=  0.2252252252252524 6S25  :=  0.0A1
  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A
11  :=  0.0A12  :=  0.0A13  :=  0.3873873873888737 4A14  :=  0.22522522522525246
-----------------------------------------------------------------
```

```
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 x11 x3 S3 S7 x6 x7 x8 x9 S9 S13 x12 S21 S1 S15 S5 x4 S11 A13 b1 S20 S24 b2 S1
7
Current Objective Value:  0.5000000000015085
b0  :=  0.0b1  :=  0.5000000000005b2  :=  0.4999999999995x3  :=  3.1000000000003x4  :=  2.95000000000651x5  :
=  1.7499999999997504x6  :=  2.2500000000012514x7  :=  2.549999999999915x8  :=  1.2999999999996996x9  :=  1.250
000000000151x10  :=  1.3999999999998995x11  :=  0.39999999999850033x12  :=  1.0500000000000085S1  :=  6.20000000
00006S2  :=  0.0S3  :=  5.900000000001302S4  :=  0.0S5  :=  3.4999999999999501S6  :=  0.0S7  :=  4.500000000002
503S8  :=  0.0S9  :=  5.0999999999983S10  :=  0.0S11  :=  2.5999999999993992S12  :=  0.0S13  :=  2.50000000000
0302S14  :=  0.0S15  :=  2.799999999999799S16  :=  0.0S17  :=  0.7999999999970007S18  :=  0.0S19  :=  0.0S20
:=  2.1000000000017S21  :=  0.49999999999850003S22  :=  0.0S23  :=  0.0S24  :=  0.0S25  :=  0.0A1  :=  0.0A2
:=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  0.0A1
2  :=  0.0A13  :=  0.5000000000015A14  :=  0.0
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 b0 x3 S3 S7 x6 x7 x8 x9 S9 S13 x12 S21 S1 S15 S5 x4 S11 A13 b1 S20 S24 b2 S17
Current Objective Value:  0.3125000000022113
b0  :=  0.12499999999953132b1  :=  0.4375000000007344b2  :=  0.4374999999997343x3  :=  3.299999999999955x4  :=
 2.968750000000581x5  :=  2.0312499999986957x6  :=  2.193750000001462x7  :=  2.3687499999998294x8  :=  1.29999
99999996996x9  :=  1.2437500000001742x10  :=  1.0125000000013524x11  :=  0.0x12  :=  1.2062500000002643S1  :=
 6.5999999999991S2  :=  0.0S3  :=  5.937500000001162S4  :=  0.0S5  :=  4.0624999999997391S6  :=  0.0S7  :=  4.3
8750000000002924S8  :=  0.0S9  :=  4.737499999999659S10  :=  0.0S11  :=  2.5999999999993992S12  :=  0.0S13  :=
2.4875000000003484S14  :=  0.0S15  :=  2.0250000000002705S16  :=  0.0S17  :=  1.1102230246251565e-16S18  :=  0.
0S19  :=  0.0S20  :=  2.4125000000005286S21  :=  0.43749999999873435S22  :=  0.0S23  :=  0.0S24  :=  0.0S25  :
=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10
:=  0.0A11  :=  0.0A12  :=  0.0A13  :=  0.312500000002203A14  :=  0.0
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 b0 x3 S3 S7 x6 x7 x8 x9 S9 S13 x12 S21 S1 S15 S5 x4 S11 A12 b1 S20 S24 b2 S17
Current Objective Value:  0.2597402597420992
b0  :=  0.24675324675363636b1  :=  0.24675324675263646b2  :=  0.5064935064937273x3  :=  3.4168831168834903x4
:=  2.818181818181813374x5  :=  2.3701298701309543x6  :=  1.8142857142845008x7  :=  2.4129870129871542x8  :=  1.
3779220779223267x9  :=  1.1987012987011554x10  :=  0.6610389610378358x11  :=  0.0x12  :=  1.1376623376621184S1
  :=  6.8337662337669 81S2  :=  0.0S3  :=  5.636363636362675S4  :=  0.0S5  :=  4.740259740261909S6  :=  0.0S7
:=  3.6285714285690016S8  :=  0.0S9  :=  4.8259740259743085S10  :=  0.0S11  :=  2.7558441558446534S12  :=  0.0
S13  :=  2.3974025974023108S14  :=  0.0S15  :=  1.3220779220756715S16  :=  0.0S17  :=  3.4171799589274597e-16S
18  :=  0.0S19  :=  0.0S20  :=  2.275324675324237S21  :=  0.5064935064927273S22  :=  0.0S23  :=  0.0S24  :=  0
.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=
0.0A10  :=  0.0A11  :=  0.0A12  :=  0.25974025974209086A13  :=  0.0A14  :=  0.0
-----------------------------------------------------------------
Current Basic Variables:  x5 x10 b0 x3 S3 S7 x6 x7 x8 x9 S9 S13 x12 S21 S1 S15 S5 x4 S11 A12 b1 S20 S25 b2 S17
Current Objective Value:  0.2597402597420992
b0  :=  0.24675324675363636b1  :=  0.24675324675263646b2  :=  0.5064935064937273x3  :=  3.4168831168834903x4
:=  2.818181818181813374x5  :=  2.3701298701309543x6  :=  1.8142857142845008x7  :=  2.4129870129871542x8  :=  1.
3779220779223267x9  :=  1.1987012987011554x10  :=  0.6610389610378358x11  :=  0.0x12  :=  1.1376623376621184S1
  :=  6.833766233766981S2  :=  0.0S3  :=  5.636363636362675S4  :=  0.0S5  :=  4.740259740261909S6  :=  0.0S7
:=  3.6285714285690016S8  :=  0.0S9  :=  4.8259740259743085S10  :=  0.0S11  :=  2.7558441558446534S12  :=  0.0
S13  :=  2.3974025974023108S14  :=  0.0S15  :=  1.3220779220756715S16  :=  0.0S17  :=  3.4171799589274597e-16S
18  :=  0.0S19  :=  0.0S20  :=  2.275324675324237S21  :=  0.5064935064927273S22  :=  0.0S23  :=  0.0S24  :=  0
.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=
0.0A10  :=  0.0A11  :=  0.0A12  :=  0.25974025974209086A13  :=  0.0A14  :=  0.0
```

```
---------------------------------------------------------------------
Current Basic Variables:  x5 x10 b0 x3 S3 S7 x6 x7 x8 x9 S9 S13 x12 S21 S1 S15 S5 x4 S11 x11 b1 S20 S25 b2 S18
Current Objective Value:  7.93809462606987e-15
b0  :=  0.33333333333433335b1  :=  0.33333333333333335b2  :=  0.3333333333323333x3  :=  3.633333333335233x4  :=
  3.0000000000008007x5  :=  2.5000000000019997x6  :=  2.100000000000801x7  :=  2.0666666666664367x8  :=  1.2999
999999996996x9  :=  1.2333333333333434x10  :=  0.36666666666346637x11  :=  0.6666666666713668x12  :=  1.4666666
66668767S1  :=  7.2666666666670466S2  :=  0.0S3  :=  6.000000000001601S4  :=  0.0S5  :=  5.000000000039995S6
:=  0.0S7  :=  4.2000000000001602S8  :=  0.0S9  :=  4.133333333328734S10  :=  0.0S11  :=  2.5999999999993992S12
  :=  0.0S13  :=  2.4666666666666868S14  :=  0.0S15  :=  0.7333333333269327S16  :=  0.0S17  :=  0.0S18  :=  1.3
333333333427337S19  :=  0.0S20  :=  2.933333333337534S21  :=  0.3333333333313333S22  :=  0.0S23  :=  0.0S24  :
=  0.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9
:=  0.0A10  :=  0.0A11  :=  0.0A12  :=  0.0A13  :=  0.0A14  :=  0.0
Run the Vanilla Simplex Method or the 2st step of Two Step Method
Initialize the zero-th row:
---------------------------------------------------------------------
Current Basic Variables:  x5 x10 b0 x3 S3 S7 x6 x7 x8 x9 S9 S13 x12 S21 S1 S15 S5 x4 S11 x11 b1 S20 S25 b2 S18
Current Objective Value:  18.333333333339937
b0  :=  0.33333333333433335b1  :=  0.33333333333333335b2  :=  0.3333333333323333x3  :=  3.633333333335233x4  :=
  3.0000000000008007x5  :=  2.5000000000019997x6  :=  2.100000000000801x7  :=  2.0666666666664367x8  :=  1.2999
999999996996x9  :=  1.2333333333333434x10  :=  0.36666666666346637x11  :=  0.6666666666713668x12  :=  1.4666666
66668767S1  :=  7.2666666666670466S2  :=  0.0S3  :=  6.000000000001601S4  :=  0.0S5  :=  5.000000000039995S6
:=  0.0S7  :=  4.2000000000001602S8  :=  0.0S9  :=  4.133333333328734S10  :=  0.0S11  :=  2.5999999999993992S12
  :=  0.0S13  :=  2.4666666666666868S14  :=  0.0S15  :=  0.7333333333269327S16  :=  0.0S17  :=  0.0S18  :=  1.3
333333333427337S19  :=  0.0S20  :=  2.933333333337534S21  :=  0.3333333333313333S22  :=  0.0S23  :=  0.0S24  :
=  0.0S25  :=  0.0
---------------------------------------------------------------------
Current Basic Variables:  x5 x10 b0 x3 S3 S7 x6 x7 x8 x9 S9 S13 x12 S21 S1 S15 S5 x4 S11 x11 b1 S20 S24 b2 S18
Current Objective Value:  18.333333333339937
b0  :=  0.33333333333433335b1  :=  0.33333333333333335b2  :=  0.3333333333323333x3  :=  3.633333333335233x4  :=
  3.0000000000008007x5  :=  2.5000000000019997x6  :=  2.100000000000801x7  :=  2.0666666666664367x8  :=  1.2999
999999996996x9  :=  1.2333333333333434x10  :=  0.36666666666346637x11  :=  0.6666666666713668x12  :=  1.4666666
66668767S1  :=  7.2666666666670466S2  :=  0.0S3  :=  6.000000000001601S4  :=  0.0S5  :=  5.000000000039995S6
:=  0.0S7  :=  4.2000000000001602S8  :=  0.0S9  :=  4.133333333328734S10  :=  0.0S11  :=  2.5999999999993992S12
  :=  0.0S13  :=  2.4666666666666868S14  :=  0.0S15  :=  0.7333333333269327S16  :=  0.0S17  :=  0.0S18  :=  1.3
333333333427337S19  :=  0.0S20  :=  2.933333333337534S21  :=  0.3333333333313333S22  :=  0.0S23  :=  0.0S24  :
=  0.0S25  :=  0.0
- Terminate Procedure: Optimal Value =  18.333333333339937
---------------------------------------------------------------------
Current Basic Variables:  x5 x10 b0 x3 S3 S7 x6 x7 x8 x9 S9 S13 x12 S21 S1 S15 S5 x4 S11 x11 b1 S20 S24 b2 S18
Current Objective Value:  18.333333333339937
b0  :=  0.33333333333433335b1  :=  0.33333333333333335b2  :=  0.3333333333323333x3  :=  3.633333333335233x4  :=
  3.0000000000008007x5  :=  2.5000000000019997x6  :=  2.100000000000801x7  :=  2.0666666666664367x8  :=  1.2999
999999996996x9  :=  1.2333333333333434x10  :=  0.36666666666346637x11  :=  0.6666666666713668x12  :=  1.4666666
66668767S1  :=  7.2666666666670466S2  :=  0.0S3  :=  6.000000000001601S4  :=  0.0S5  :=  5.000000000039995S6
:=  0.0S7  :=  4.2000000000001602S8  :=  0.0S9  :=  4.133333333328734S10  :=  0.0S11  :=  2.5999999999993992S12
  :=  0.0S13  :=  2.4666666666666868S14  :=  0.0S15  :=  0.7333333333269327S16  :=  0.0S17  :=  0.0S18  :=  1.3
333333333427337S19  :=  0.0S20  :=  2.933333333337534S21  :=  0.3333333333313333S22  :=  0.0S23  :=  0.0S24  :
=  0.0S25  :=  0.0
```

Run model27_cplex_Q1.py

```
CPXPARAM_Read_DataCheck                                 1
Tried aggregator 1 time.
LP Presolve eliminated 1 rows and 0 columns.
Reduced LP has 23 rows, 13 columns, and 87 nonzeros.
Presolve time = 0.00 sec. (0.03 ticks)
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Scaled dual infeas =            72.399999
Iteration:    13   Dual objective     =            -7.828571
Solution status = 1: 'optimal'
Solution value  = 18.333333333339937
Variable b0: Value = 0.3333333333343333
Variable b1: Value = 0.3333333333333333
Variable b2: Value = 0.33333333333233334
Variable A3: Value = 3.633333333335233
Variable A4: Value = 3.0000000000008
Variable A5: Value = 2.5000000000019997
Variable A6: Value = 2.100000000000799
Variable A7: Value = 2.066666666664367
Variable A8: Value = 1.2999999999996992
Variable A9: Value = 1.2333333333334333
Variable A10: Value = 0.36666666666346615
Variable A11: Value = 0.6666666666713663
Variable A12: Value = 1.4666666666687669
Constraint Period 3-(1): Shadow Price = -0.0
Constraint Period 3-(2): Shadow Price = 1.0
Constraint Period 4-(1): Shadow Price = -0.0
Constraint Period 4-(2): Shadow Price = 1.0
Constraint Period 5-(1): Shadow Price = -0.0
Constraint Period 5-(2): Shadow Price = 1.0
Constraint Period 6-(1): Shadow Price = -0.0
Constraint Period 6-(2): Shadow Price = 1.0
Constraint Period 7-(1): Shadow Price = -0.0
Constraint Period 7-(2): Shadow Price = 1.0
Constraint Period 8-(1): Shadow Price = -0.0
Constraint Period 8-(2): Shadow Price = 1.0
Constraint Period 9-(1): Shadow Price = -0.0
Constraint Period 9-(2): Shadow Price = 1.0
Constraint Period 10-(1): Shadow Price = -0.0
Constraint Period 10-(2): Shadow Price = 1.0
Constraint Period 11-(1): Shadow Price = 1.0
Constraint Period 11-(2): Shadow Price = -0.0
Constraint Period 12-(1): Shadow Price = 1.0
Constraint Period 12-(2): Shadow Price = -0.0
Constraint b2: Shadow Price = -0.0
Constraint b1 - b2: Shadow Price = 4.333333333333336
Constraint b0 - b1: Shadow Price = 2.266666666666673
Constraint b0+b1+b2: Shadow Price = 40.233333333333334
```

Run model27_mySimplex_Q2.py

```
Geonsikui-MacBook-Pro-2:IE535_Simplex geonsik$ python model27_mySimplex_Q2.py
Initialize the given LP problem:
Initialize the zero-th row:
------------------------------------------------------------------
Current Basic Variables:  A3 A8 A9 S10 A1 S2 A2 S4 A6 S6 A4 S8 A5 S16 S12 A7 S14 A13 S18 A12 A10 S20 S24 A11 A
14
Current Objective Value:  35.50000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7
S7  :=  0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  4.4S13  :=  0.0S14  :=  4.3S15  :=  0.0S16
:=  4.2S17  :=  0.0S18  :=  3.6S19  :=  0.0S20  :=  2.7S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=
  0.0A1  :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  4.4A7  :=  4.3A8  :=  4.2A9  :=  3.6A10  :
=  2.7A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
TWO STEP METHOD CALLED:
------------------------------------------------------------------
Current Basic Variables:  A3 A8 A9 S10 A1 S2 A2 S4 A6 S6 A4 S8 A5 S16 S12 A7 S14 A13 S18 A12 A10 S20 S24 A11 A
14
Current Objective Value:  35.50000000000301
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  0.0S1  :=  0.0S2  :=  1.0S3  :=  0.0S4  :=  2.1S5  :=  0.0S6  :=  3.7
S7  :=  0.0S8  :=  4.2S9  :=  0.0S10  :=  4.3S11  :=  0.0S12  :=  4.4S13  :=  0.0S14  :=  4.3S15  :=  0.0S16
:=  4.2S17  :=  0.0S18  :=  3.6S19  :=  0.0S20  :=  2.7S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=
  0.0A1  :=  1.0A2  :=  2.1A3  :=  3.7A4  :=  4.2A5  :=  4.3A6  :=  4.4A7  :=  4.3A8  :=  4.2A9  :=  3.6A10  :
=  2.7A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
Run the 1st step of Two Step Method
------------------------------------------------------------------
Current Basic Variables:  A3 A8 A9 S10 x3 S2 A2 S4 A6 S6 A4 S8 A5 S16 S12 A7 S14 A13 S18 A12 A10 S20 S24 A11 A
14
Current Objective Value:  25.500000000003013
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  1.0S1  :=  0.0S2  :=  2.0S3  :=  0.0S4  :=  3.1S5  :=  0.0S6  :=  4.7
S7  :=  0.0S8  :=  5.2S9  :=  0.0S10  :=  5.3S11  :=  0.0S12  :=  5.4S13  :=  0.0S14  :=  5.3S15  :=  0.0S16
:=  5.2S17  :=  0.0S18  :=  4.6S19  :=  0.0S20  :=  3.7S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=
  0.0A1  :=  0.0A2  :=  1.1A3  :=  2.7A4  :=  3.2A5  :=  3.3A6  :=  3.4000000000000004A7  :=  3.3A8  :=  3.2A9
  :=  2.6A10  :=  1.7000000000000002A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
------------------------------------------------------------------
Current Basic Variables:  A3 A8 A9 S10 x3 S2 S1 S4 A6 S6 A4 S8 A5 S16 S12 A7 S14 A13 S18 A12 A10 S20 S24 A11 A
14
Current Objective Value:  15.600000000003012
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  2.1S1  :=  1.1S2  :=  3.1S3  :=  0.0S4  :=  4.2S5  :=  0.0S6  :=  5.8
00000000000001S7  :=  0.0S8  :=  6.300000000000001S9  :=  0.0S10  :=  6.4S11  :=  0.0S12  :=  6.5S13  :=  0.0S
14  :=  6.4S15  :=  0.0S16  :=  6.300000000000001S17  :=  0.0S18  :=  5.699999999999999S19  :=  0.0S20  :=  4.
800000000000001S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  1.6A4
  :=  2.1A5  :=  2.1999999999999997A6  :=  2.3000000000000003A7  :=  2.1999999999999997A8  :=  2.1A9  :=  1.5A
10  :=  0.6000000000000001A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
------------------------------------------------------------------
Current Basic Variables:  A3 A8 A9 S10 x3 S2 S1 S4 A6 S6 A4 S8 A5 S16 S12 A7 S14 A13 S18 A12 S3 S20 S24 A11 A1
4
Current Objective Value:  10.800000000003012
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  2.7S1  :=  1.7000000000000002S2  :=  3.7S3  :=  0.6000000000000001S4
 :=  4.800000000000001S5  :=  0.0S6  :=  6.4S7  :=  0.0S8  :=  6.9S9  :=  0.0S10  :=  7.0S11  :=  0.0S12  :=
7.1S13  :=  0.0S14  :=  7.0S15  :=  0.0S16  :=  6.9S17  :=  0.0S18  :=  6.299999999999999S19  :=  0.0S20  :=
5.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  1.0A4  :=  1.5A5
 :=  1.5999999999999996A6  :=  1.7000000000000002A7  :=  1.5999999999999996A8  :=  1.5A9  :=  0.899999999999999
99A10  :=  0.0A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
```

```
------------------------------------------------------------------
Current Basic Variables:  A3 A8 S19 S10 x3 S2 S1 S4 A6 S6 A4 S8 A5 S16 S12 A7 S14 A13 S18 A12 S3 S20 S24 A11 A
14
Current Objective Value:  4.500000000003013
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  3.6S1  :=  2.6S2  :=  4.6S3  :=  1.5S4  :=  5.700000000000001S5  :=
0.0S6  :=  7.300000000000001S7  :=  0.0S8  :=  7.800000000000001S9  :=  0.0S10  :=  7.9S11  :=  0.0S12  :=  8.
0S13  :=  0.0S14  :=  7.9S15  :=  0.0S16  :=  7.800000000000001S17  :=  0.0S18  :=  7.199999999999999S19  :=
0.8999999999999999S20  :=  6.300000000000001S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1  :=
  0.0A2  :=  0.0A3  :=  0.10000000000000009A4  :=  0.6000000000000001A5  :=  0.6999999999999997A6  :=  0.80000
00000000003A7  :=  0.6999999999999997A8  :=  0.6000000000000001A9  :=  0.0A10  :=  0.0A11  :=  1e-12A12  :=  1
e-12A13  :=  1e-12A14  :=  1.0
------------------------------------------------------------------
Current Basic Variables:  S17 A8 S19 S10 x3 S2 S1 S4 A6 S6 A4 S8 A5 S16 S12 A7 S14 A13 S18 A12 S3 S20 S24 A11
A14
Current Objective Value:  3.900000000003012
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  3.7S1  :=  2.7S2  :=  4.699999999999999S3  :=  1.6S4  :=  5.800000000
000001S5  :=  0.0S6  :=  7.4S7  :=  0.0S8  :=  7.9S9  :=  0.0S10  :=  8.0S11  :=  0.0S12  :=  8.1S13  :=  0.0S
14  :=  8.0S15  :=  0.0S16  :=  7.9S17  :=  0.10000000000000009S18  :=  7.299999999999999S19  :=  1.0S20  :=
6.4S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.5A5
 :=  0.5999999999999996A6  :=  0.7000000000000002A7  :=  0.5999999999999996A8  :=  0.5A9  :=  0.0A10  :=  0.0A
11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
------------------------------------------------------------------
Current Basic Variables:  S17 A8 S19 S10 x3 S2 S1 S4 A6 S6 S5 S8 A5 S16 S12 A7 S14 A13 S18 A12 S3 S20 S24 A11
A14
Current Objective Value:  1.4000000000030122
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  4.2S1  :=  3.2S2  :=  5.199999999999999S3  :=  2.1S4  :=  6.300000000
000001S5  :=  0.5S6  :=  7.9S7  :=  0.0S8  :=  8.4S9  :=  0.0S10  :=  8.5S11  :=  0.0S12  :=  8.6S13  :=  0.0S
14  :=  8.5S15  :=  0.0S16  :=  8.4S17  :=  0.6000000000000001S18  :=  7.799999999999999S19  :=  1.5S20  :=  6
.9S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5
:=  0.09999999999999964A6  :=  0.20000000000000018A7  :=  0.09999999999999964A8  :=  0.0A9  :=  0.0A10  :=  0.
0A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
------------------------------------------------------------------
Current Basic Variables:  S17 S7 S19 S10 x3 S2 S1 S4 A6 S6 S5 S8 A5 S16 S12 A7 S14 A13 S18 A12 S3 S20 S24 A11
A14
Current Objective Value:  1.4000000000030122
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  4.2S1  :=  3.2S2  :=  5.199999999999999S3  :=  2.1S4  :=  6.300000000
000001S5  :=  0.5S6  :=  7.9S7  :=  0.0S8  :=  8.4S9  :=  0.0S10  :=  8.5S11  :=  0.0S12  :=  8.6S13  :=  0.0S
14  :=  8.5S15  :=  0.0S16  :=  8.4S17  :=  0.6000000000000001S18  :=  7.799999999999999S19  :=  1.5S20  :=  6
.9S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  1.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5
:=  0.09999999999999964A6  :=  0.20000000000000018A7  :=  0.09999999999999964A8  :=  0.0A9  :=  0.0A10  :=  0.
0A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
------------------------------------------------------------------
Current Basic Variables:  S17 S7 S19 S10 x3 S2 S1 S4 A6 S6 S5 S8 A5 S16 S12 S15 S14 A13 S18 A12 S3 S20 S24 A11
 A14
Current Objective Value:  1.1000000000030132
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  4.3S1  :=  3.3S2  :=  5.299999999999999S3  :=  2.1999999999999997S4
:=  6.4S5  :=  0.5999999999999996S6  :=  8.0S7  :=  0.09999999999999964S8  :=  8.5S9  :=  0.0S10  :=  8.6S11
:=  0.0S12  :=  8.7S13  :=  0.0S14  :=  8.6S15  :=  0.09999999999999964S16  :=  8.5S17  :=  0.6999999999999997
S18  :=  7.899999999999999S19  :=  1.5999999999999996S20  :=  7.0S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=
1.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.10000000000000053A7  :=  0.0A8
  :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
------------------------------------------------------------------
```

```
Current Basic Variables:  S17 S7 S19 S10 x3 S2 S1 S4 A6 S6 S5 S8 S13 S16 S12 S15 S14 A13 S18 A12 S3 S20 S24 A1
1 A14
Current Objective Value:  1.1000000000030132
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  4.3S1  :=  3.3S2  :=  5.299999999999999S3  :=  2.1999999999999997S4
:=  6.4S5  :=  0.5999999999999996S6  :=  8.0S7  :=  0.09999999999999964S8  :=  8.5S9  :=  0.0S10  :=  8.6S11
:=  0.0S12  :=  8.7S13  :=  0.0S14  :=  8.6S15  :=  0.09999999999999964S16  :=  8.5S17  :=  0.6999999999999997
S18  :=  7.899999999999999S19  :=  1.5999999999999996S20  :=  7.0S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=
1.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.10000000000000053A7  :=  0.0A8
  :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S19 S10 x3 S2 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 A13 S18 A12 S3 S20 S24 A1
1 A14
Current Objective Value:  1.0000000000030127
b0  :=  0.0b1  :=  0.0b2  :=  0.0x3  :=  4.4S1  :=  3.4000000000000004S2  :=  5.3999999999999995S3  :=  2.3000
000000000003S4  :=  6.500000000000001S5  :=  0.7000000000000002S6  :=  8.100000000000001S7  :=  0.200000000000
00018S8  :=  8.600000000000001S9  :=  0.10000000000000053S10  :=  8.7S11  :=  0.0S12  :=  8.8S13  :=  0.100000
00000000053S14  :=  8.7S15  :=  0.20000000000000018S16  :=  8.600000000000001S17  :=  0.8000000000000003S18  :
=  7.999999999999999S19  :=  1.7000000000000002S20  :=  7.1000000000000005S21  :=  0.0S22  :=  0.0S23  :=  0.0
S24  :=  1.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0
.0A9  :=  0.0A10  :=  0.0A11  :=  1e-12A12  :=  1e-12A13  :=  1e-12A14  :=  1.0
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S19 S10 x3 S2 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 A13 S18 A12 S3 S20 S24 b2
 A14
Current Objective Value:  1.0000000000020126
b0  :=  0.0b1  :=  0.0b2  :=  1e-12x3  :=  4.399999999994001S1  :=  3.399999999978003S2  :=  5.39999999999019
9S3  :=  2.29999999999984S4  :=  6.499999999989601S5  :=  0.6999999999997002S6  :=  8.09999999999883S7  :=  0.19
9999999992001 8S8  :=  8.599999999988801S9  :=  0.1000000000170053S10  :=  8.69999999999863S11  :=  0.0S12  :=
  8.799999999988001S13  :=  0.09999999999940053S14  :=  8.699999999988599S15  :=  0.19999999999970017S16  :=
8.5999999999883S17  :=  0.7999999999995002S18  :=  7.999999999988499S19  :=  1.69999999999965S20  :=  7.0999999
99991501S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  0.999999999999S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0
.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  0.0A12  :=  2e-12A13
 :=  1e-12A14  :=  0.999999999999
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S19 S10 x3 S2 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 A13 S18 b1 S3 S20 S24 b2
A14
Current Objective Value:  1.0000000000000127
b0  :=  0.0b1  :=  2e-12b2  :=  1e-12x3  :=  4.399999999832005S1  :=  3.399999999958S2  :=  5.39999999997059
9S3  :=  2.29999999999S4  :=  6.499999999967401S5  :=  0.6999999999993002S6  :=  8.099999999671S7  :=  0.200
000000380017S8  :=  8.599999999962602S9  :=  0.1000000000290053S10  :=  8.699999999635S11  :=  0.0S12  :=
 8.799999999966401S13  :=  0.10000000000000053S14  :=  8.699999999664S15  :=  0.1999999999990017S16  :=  8.5
99999999966501S17  :=  0.7999999999937002S18  :=  7.999999999972699S19  :=  1.699999999873001S20  :=  7.09999
9999979101S21  :=  0.0S22  :=  0.0S23  :=  0.0S24  :=  0.9999999999970001S25  :=  0.0A1  :=  0.0A2  :=  0.0A3
 :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  0.0A12  :=  0.0
A13  :=  3e-12A14  :=  0.9999999999970001
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S21 S10 x3 S2 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 A13 S18 b1 S3 S20 S24 b2
A14
Current Objective Value:  0.790123456791705
b0  :=  0.0b1  :=  0.20987654321030863b2  :=  0.20987654320930862x3  :=  2.007407407408482 2S1  :=  2.728395061
7292128S2  :=  1.28641975308775S3  :=  2.02716049382819 9S4  :=  1.98765432098876 45S5  :=  0.5950617283951458S6
  :=  3.419753086421816 4S7  :=  0.5148148148162631S8  :=  3.5000000000007017S9  :=  0.5827160493820103S10  :=
 3.4320987654349535S11  :=  0.0S12  :=  4.014814814816964S13  :=  0.03703703703750798S14  :=  3.97777777777945
4S15  :=  0.1580246913582384S16  :=  3.856790123458726S17  :=  0.08641975308545 08S18  :=  3.928395061731510 7S1
9  :=  0.0S20  :=  4.0148148148169645S21  :=  0.20987654320830862S22  :=  0.0S23  :=  0.0S24  :=  0.5802469135
803828S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9
:=  0.0A10  :=  0.0A11  :=  0.0A12  :=  0.0A13  :=  0.2098765432113086A14  :=  0.5802469135803828
```

```
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S21 S10 x3 S11 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 A13 S18 b1 S3 S20 S24 b2
 A14
Current Objective Value:  0.6782608695666839
b0  :=  0.0b1  :=  0.3217391304353303 7b2  :=  0.32173913043433033x3  :=  1.6382608695659107S1  :=  3.276521739
1318195S2  :=  0.0S3  :=  2.7878260869583 47S4  :=  0.4886956521734 7266S5  :=  1.4452173913053112S6  :=  1.8313
04347826507 2S7  :=  1.588695652176472S8  :=  1.687826086955349 2S9  :=  1.7460869565222366S10  :=  1.5304347826
09584S11  :=  0.9060869565226762S12  :=  2.3704347826091436S13  :=  0.9095652173926777S14  :=  2.3669565217391
4S15  :=  1.0417391304359103S16  :=  2.234782608695911S17  :=  0.6121739130430531S18  :=  2.6643478260887643S1
9  :=  0.0S20  :=  3.2765217391318213S21  :=  0.32173913043333036S22  :=  0.0S23  :=  0.0S24  :=  0.3565217391
3033935S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9
 :=  0.0A10  :=  0.0A11  :=  0.0A12  :=  0.0A13  :=  0.32173913043633035A14  :=  0.35652173913033935
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S21 S10 x3 S11 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 A13 S18 b1 S3 S20 S24 b2
 S19
Current Objective Value:  0.500000000001515
b0  :=  0.0b1  :=  0.5000000000005b2  :=  0.4999999999995x3  :=  3.1000000000000302S1  :=  6.2000000000000602S2
 :=  0.0S3  :=  6.0500000000000953S4  :=  0.14999999999964958S5  :=  4.8500000000000053S6  :=  1.350000000000548
3S7  :=  5.35000000000001552S8  :=  0.8499999999990512S9  :=  5.6499999999994515S10  :=  0.550000000011512S11
 :=  4.4000000000000001S12  :=  1.8000000000006005S13  :=  4.3500000000000452S14  :=  1.8500000000001473S15  :=
4.5000000000002025S16  :=  1.7000000000004025S17  :=  3.4999999999988014S18  :=  2.700000000001797S19  :=  2.0
4999999999999451S20  :=  4.150000000001153S21  :=  0.49999999999850003S22  :=  0.0S23  :=  0.0S24  :=  0.0S25  :
=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8  :=  0.0A9  :=  0.0A10
 :=  0.0A11  :=  0.0A12  :=  0.0A13  :=  0.5000000000015A14  :=  0.0
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S21 S10 x3 S11 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 b0 S18 b1 S3 S20 S24 b2
S19
Current Objective Value:  1.63202784619898e-14
b0  :=  0.33333333333433335b1  :=  0.33333333333333337b2  :=  0.33333333333233334x3  :=  3.6333333333335235S1
 :=  7.2666666666704068S2  :=  0.0S3  :=  6.6333333333336036S4  :=  0.6333333333344339S5  :=  6.1333333333372355S
6  :=  1.1333333333332318S7  :=  5.7333333333336034S8  :=  1.5333333333344346S9  :=  5.6999999999996005S10  :=
 1.5666666666708675S11  :=  4.9333333333334934S12  :=  2.3333333333355344S13  :=  4.8666666666668667S14  :=  2.4
000000000001798S15  :=  3.9999999999987015S16  :=  3.2666666666717687S17  :=  2.9666666666638672S18  :=  4.3000
00000006597S19  :=  2.1666666666666467S20  :=  5.1000000000004002S21  :=  0.33333333333133336S22  :=  0.0S23  :=
  0.0S24  :=  0.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8
 :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  0.0A12  :=  0.0A13  :=  0.0A14  :=  0.0
- Terminate Procedure: Optimal Value =  1.63202784619898e-14
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S21 S10 x3 S11 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 b0 S18 b1 S3 S20 S24 b2
S19
Current Objective Value:  1.63202784619898e-14
b0  :=  0.33333333333433335b1  :=  0.33333333333333337b2  :=  0.33333333333233334x3  :=  3.6333333333335235S1
 :=  7.2666666666704068S2  :=  0.0S3  :=  6.6333333333336036S4  :=  0.6333333333344339S5  :=  6.1333333333372355S
6  :=  1.1333333333332318S7  :=  5.7333333333336034S8  :=  1.5333333333344346S9  :=  5.6999999999996005S10  :=
 1.5666666666708675S11  :=  4.9333333333334934S12  :=  2.3333333333355344S13  :=  4.8666666666668667S14  :=  2.4
000000000001798S15  :=  3.9999999999987015S16  :=  3.2666666666717687S17  :=  2.9666666666638672S18  :=  4.3000
00000006597S19  :=  2.1666666666666467S20  :=  5.1000000000004002S21  :=  0.33333333333133336S22  :=  0.0S23  :=
  0.0S24  :=  0.0S25  :=  0.0A1  :=  0.0A2  :=  0.0A3  :=  0.0A4  :=  0.0A5  :=  0.0A6  :=  0.0A7  :=  0.0A8
 :=  0.0A9  :=  0.0A10  :=  0.0A11  :=  0.0A12  :=  0.0A13  :=  0.0A14  :=  0.0
```

```
Run the Vanilla Simplex Method or the 2st step of Two Step Method
Initialize the zero-th row:
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S21 S10 x3 S11 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 b0 S18 b1 S3 S20 S24 b2
S19
Current Objective Value:  3.6333333333335235
b0  :=  0.33333333333433335b1  :=  0.33333333333333337b2  :=  0.33333333333233334x3  :=  3.6333333333335235S1
 :=  7.2666666666704068S2  :=  0.0S3  :=  6.6333333333336036S4  :=  0.6333333333344339S5  :=  6.1333333333372355S
6  :=  1.1333333333332318S7  :=  5.7333333333336034S8  :=  1.5333333333344346S9  :=  5.6999999999996005S10  :=
 1.5666666666708675S11  :=  4.9333333333334934S12  :=  2.3333333333355344S13  :=  4.8666666666668667S14  :=  2.4
000000000001798S15  :=  3.9999999999987015S16  :=  3.2666666666717687S17  :=  2.9666666666638672S18  :=  4.3000
00000006597S19  :=  2.1666666666666467S20  :=  5.1000000000004002S21  :=  0.33333333333133336S22  :=  0.0S23  :=
  0.0S24  :=  0.0S25  :=  0.0
- Terminate Procedure: Optimal Value =  3.6333333333335235
-----------------------------------------------------------------
Current Basic Variables:  S17 S7 S21 S10 x3 S11 S1 S4 S9 S6 S5 S8 S13 S16 S12 S15 S14 b0 S18 b1 S3 S20 S24 b2
S19
Current Objective Value:  3.6333333333335235
b0  :=  0.33333333333433335b1  :=  0.33333333333333337b2  :=  0.33333333333233334x3  :=  3.6333333333335235S1
 :=  7.2666666666704068S2  :=  0.0S3  :=  6.6333333333336036S4  :=  0.6333333333344339S5  :=  6.1333333333372355S
6  :=  1.1333333333332318S7  :=  5.7333333333336034S8  :=  1.5333333333344346S9  :=  5.6999999999996005S10  :=
 1.5666666666708675S11  :=  4.9333333333334934S12  :=  2.3333333333355344S13  :=  4.8666666666668667S14  :=  2.4
000000000001798S15  :=  3.9999999999987015S16  :=  3.2666666666717687S17  :=  2.9666666666638672S18  :=  4.3000
00000006597S19  :=  2.1666666666666467S20  :=  5.1000000000004002S21  :=  0.33333333333133336S22  :=  0.0S23  :=
  0.0S24  :=  0.0S25  :=  0.0
```

Run model27_cplex_Q2.py

```
CPXPARAM_Read_DataCheck                                 1
Tried aggregator 1 time.
LP Presolve eliminated 1 rows and 0 columns.
Reduced LP has 23 rows, 4 columns, and 87 nonzeros.
Presolve time = 0.00 sec. (0.02 ticks)
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Scaled dual infeas =             2.000000
Iteration:     3   Dual objective     =             0.000000
Solution status = 1: 'optimal'
Solution value  = 3.6333333333352327
Variable b0: Value = 0.3333333333343333
Variable b1: Value = 0.3333333333333333
Variable b2: Value = 0.33333333333233334
Variable A: Value = 3.633333333335233
Constraint Period 3-(1): Shadow Price = -0.0
Constraint Period 3-(2): Shadow Price = 1.0
Constraint Period 4-(1): Shadow Price = -0.0
Constraint Period 4-(2): Shadow Price = -0.0
Constraint Period 5-(1): Shadow Price = -0.0
Constraint Period 5-(2): Shadow Price = -0.0
Constraint Period 6-(1): Shadow Price = -0.0
Constraint Period 6-(2): Shadow Price = -0.0
Constraint Period 7-(1): Shadow Price = -0.0
Constraint Period 7-(2): Shadow Price = -0.0
Constraint Period 8-(1): Shadow Price = -0.0
Constraint Period 8-(2): Shadow Price = -0.0
Constraint Period 9-(1): Shadow Price = -0.0
Constraint Period 9-(2): Shadow Price = -0.0
Constraint Period 10-(1): Shadow Price = -0.0
Constraint Period 10-(2): Shadow Price = -0.0
Constraint Period 11-(1): Shadow Price = -0.0
Constraint Period 11-(2): Shadow Price = -0.0
Constraint Period 12-(1): Shadow Price = -0.0
Constraint Period 12-(2): Shadow Price = -0.0
Constraint b2: Shadow Price = -0.0
Constraint b1 - b2: Shadow Price = 0.8333333333333333
Constraint b0 - b1: Shadow Price = 1.0666666666666673
Constraint b0+b1+b2: Shadow Price = 4.633333333333333
```