$13 - \frac{8}{6} - 12$

HW#2. Matroid Intersection

- $G = (V, E)$, $c: E \to \mathbb{Z}$ 정수, $b: U \to \mathbb{N}$ 자연수 $(U \subseteq V)$

  (U is independent. (every $u, v \in U$, $\nexists (u,v) \in E$))

- Find a minimum cost spanning tree / $\deg(v) \leq b(v)$ for $\forall v \in V$ and $v \in U$.

  $M_1 = (S, \mathcal{I}_1) = (E, \mathcal{I}_1)$ where $\mathcal{I}_1 = \{I \mid I$ is acyclic, $I \subseteq E\}$ Graphic Matroid.

  $M_2 = (S, \mathcal{I}_2) = (E, \mathcal{I}_2)$ where $\mathcal{I}_2 = \{I \mid (\#$ of $e)$ adj to $u \in U \leq b(u)$ where U is independent$\}$

  → Find Minimum weight common independent set $I \in \mathcal{I}_1 \cap \mathcal{I}_2$ with cardinality $(k+1)$

- Given I_k

M_1 For $x = (u, v) \in (S \setminus I_k) = (E \setminus I_k)$

  If ($\exists$ uv-path $\subseteq I_k$)

    foreach edge $y \in$ (uv-path) : $I_k - y + x \in \mathcal{I}_1 \Rightarrow A \leftarrow A + \langle \overrightarrow{y, x} \rangle$

  else ($\nexists$ uv-path $\subseteq I_k$) $w = c_1(y) - c_1(x)$

    : $I_k + x \in \mathcal{I}_1 \Rightarrow A \leftarrow A + \langle \overrightarrow{s, x} \rangle$ $w = M_1 - c_1(x)$

M_2 For $x = (u, v) \in (E \setminus I_k)$

  If ($u \notin U$ and $v \notin U$) : $I_k + x \in \mathcal{I}_2 \Rightarrow A \leftarrow A + \langle \overrightarrow{x, t} \rangle$ $w = M_2 - c_2(x)$

  else ($u \in U$ then $v \notin U$) if($\deg(u) < b(u)$) then : $I_k + x \in \mathcal{I}_2 \Rightarrow A \leftarrow A + \langle \overrightarrow{x, t} \rangle$

    else for $\forall y \in \delta(u)$: $I_k - y + x \in \mathcal{I}_2 \Rightarrow A \leftarrow A + \langle \overrightarrow{x, y} \rangle$

    $c_2(y) - c_2(x)$

$\Rightarrow \boxed{G_{M_1, M_2}(I_k)}$

Repeat

  BFS weight($=$) ($=$)

  $R \leftarrow$ reachable from s in $\bar{G}(I_k)$: Set of Edges $R \subseteq E$

  ① $\exists$ s-t path.

    $\bar{G}$에서 Shortest s-t path → $w = 0$인 경로에서

    $I_{k+1} \leftarrow I_k \triangle (P \setminus \{s, t\})$

    $k \leftarrow k+1$ (if (k) $== |V|-1$ break)

    $\varepsilon \leftarrow 0$

  else $\varepsilon \leftarrow \infty$

    in exchange Graph G

    $\varepsilon \leftarrow \min\{w(z_1, z_2) \mid z_1 \in R, z_2 \notin R\}$

    if ε finite

      then each $z \in R \cap S$

        $c_1(z) \leftarrow c_1(z) - \varepsilon$

        $c_2(z) \leftarrow c_2(z) + \varepsilon$.

until $\varepsilon = \infty$.

if $k \neq |V| - 1 \Rightarrow \nexists$

o/w