

For given discrete signal $x[n]$, its FFT, $X[k]$ is given by

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{2i\pi nk}{N}} \quad (1)$$

Now let the *Fourier* spectrum of the *Hilbert* transform of $x[n]$ is $Z[k]$,

$$Z[k] = \begin{cases} X[k], & k = 0 \\ 2X[k], & k = 1, 2, 3, \dots, \frac{N}{2} - 1 \\ 0, & k = \frac{N}{2}, \frac{N+1}{2}, \dots, N - 1 \end{cases} \quad (2)$$

Doing *IFFT* on $Z[k]$, the imaginary part of $IFFT[Z[k]]$ is the *Hilbert* transform of $x[n]$, $h[n]$,

$$h[n] = \mathcal{I} \left[\sum_{k=0}^{N-1} Z[k] e^{\frac{2i\pi nk}{N}} \right] \quad (3)$$