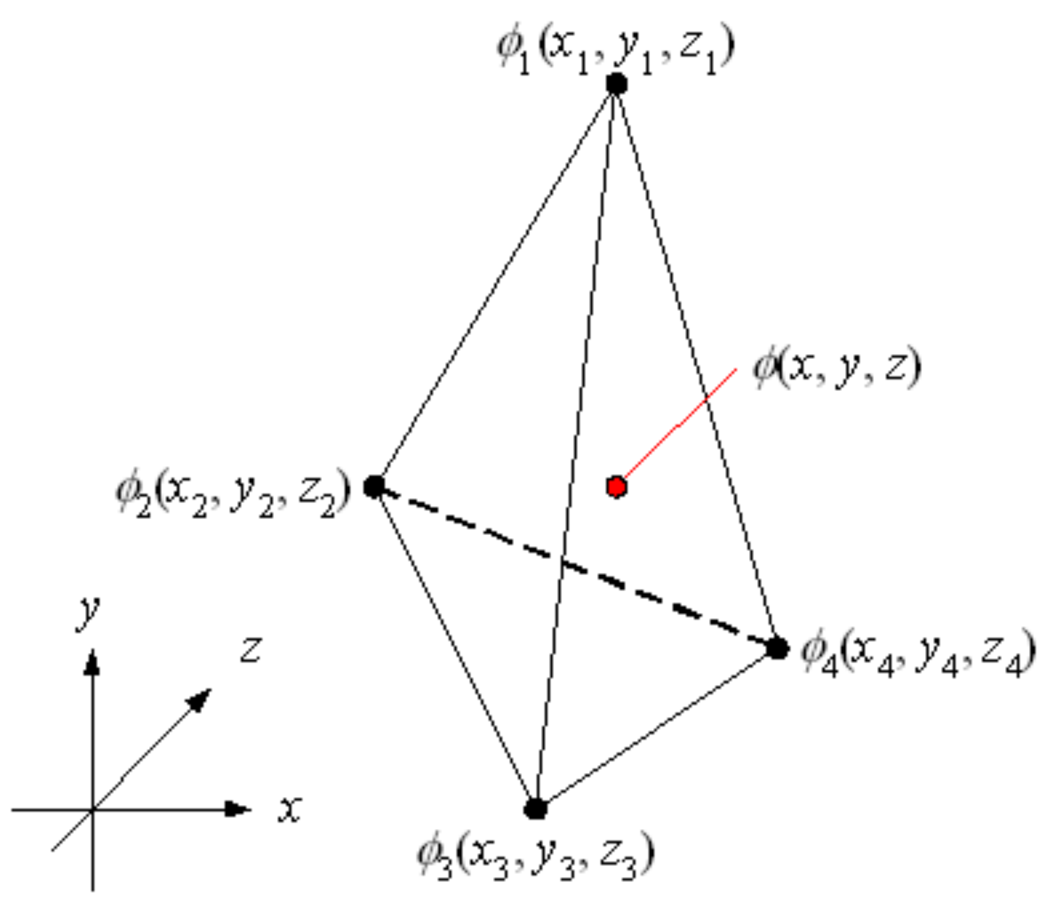
・4面体1次要素

4面体1次要素の場合も同様に定義されます。



上図の様に*xyz*座標に1 つの4面体要素を考える。1 次要素の場合は、物理量*f*は次式となります。

$$\phi(x,y,z) = \alpha + \beta x + \gamma y + \xi z$$

ここで、 a, b, g, hは未知数です。 これら4つの未知数を 4つの節点の物理量から計算します。

$$\begin{cases} \phi_1 = \alpha + \beta x_1 + \gamma y_1 + \xi z_1 \\ \phi_2 = \alpha + \beta x_2 + \gamma y_2 + \xi z_2 \\ \phi_3 = \alpha + \beta x_3 + \gamma y_3 + \xi z_3 \\ \phi_4 = \alpha + \beta x_4 + \gamma y_4 + \xi z_4 \end{cases}$$

この連立方程式の解を物理量*f* の式に代入すると次式が得られます。

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$$\phi(x,y) = N_1\phi_1 + N_2\phi_2 + N_3\phi_3 + N_4\phi_4$$
$$= L_1\phi_1 + L_2\phi_2 + L_3\phi_3 + L_4\phi_4$$

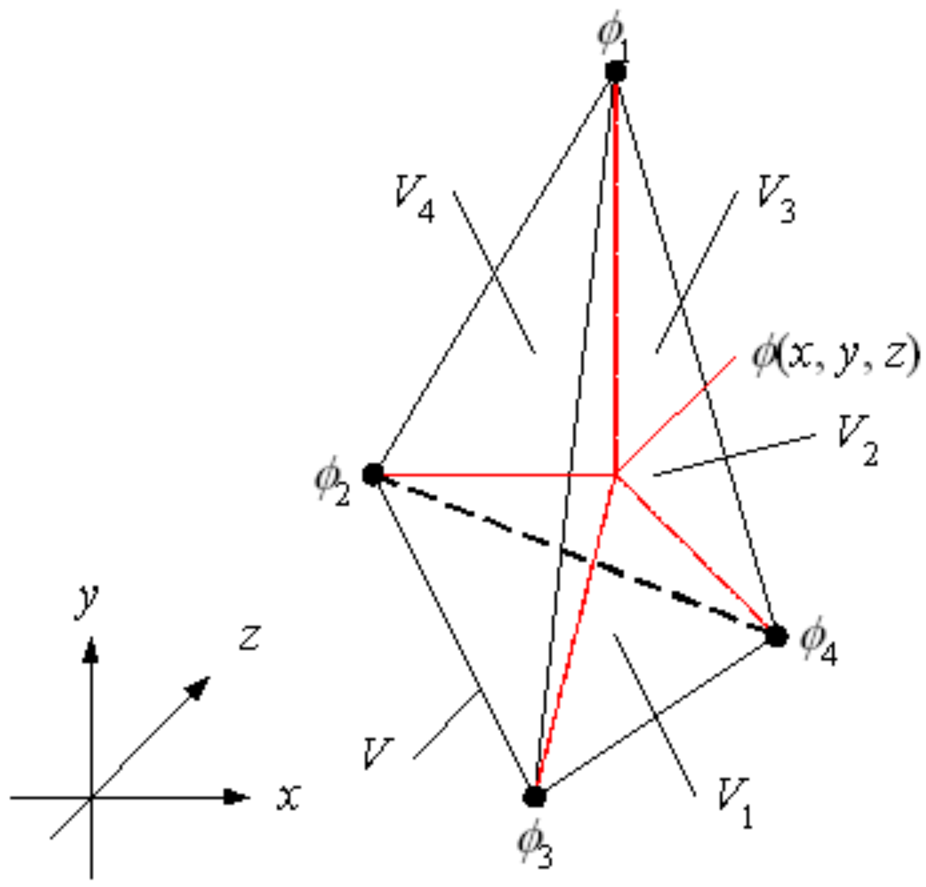
ここで、 L_1, L_2, L_3[-] は形状関数と呼ばれ次式で定義されます。

$$L_1 = \frac{V_1}{V}, L_2 = \frac{V_2}{V}, L_3 = \frac{V_3}{V}, L_4 = \frac{V_4}{V}$$

また、内挿関数は次式で定義されます。

$$N_1 = L_1, \quad N_2 = L_2, \quad N_3 = L_3, \quad N_4 = L_4$$
$$N_1 + N_2 + N_3 + N_4 = 1$$

つまり、物理量は体積比で決まっていることがわかります。



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・要素の積分

要素の積分には下記の公式が用いられます。

3角形要素

$$\int_S L_1^p L_2^q L_3^r dS = \frac{p!q!r!}{(p+q+r+2)!}2S$$

ここで、Sは要素の面積です。

4面体要素

$$\int_V L_1^p L_2^q L_3^r L_4^s dV = \frac{p!q!r!s!}{(p+q+r+s+3)!}6V$$

ここで、Vは要素の体積です。

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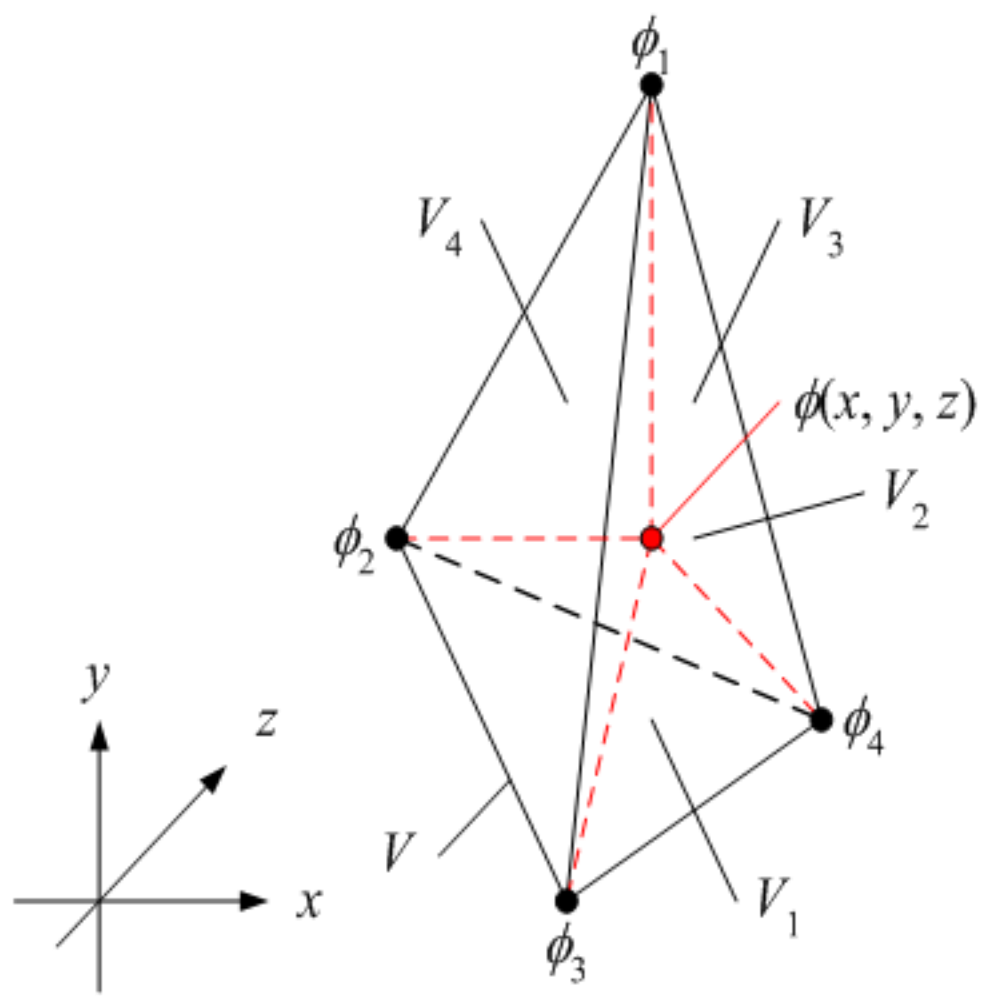
・4面体要素の体積



4面体要素の体積を示します。

$$V = \frac{1}{6}\det\begin{pmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{pmatrix}$$

$$\begin{aligned} &= \frac{1}{6}(x_2y_3z_4 - x_2z_3y_4 - y_2x_3z_4 + y_2z_3x_4 + z_2x_3y_4 - z_2y_3x_4 \\ &- x_1y_3z_4 + x_1z_3y_4 + x_1y_2z_4 - x_1y_2z_3 - x_1z_2y_4 + x_1z_2y_3 \\ &+ y_1x_3z_4 - y_1z_3x_4 - y_1x_2z_4 + y_1x_2z_3 + y_1z_2x_4 - y_1z_2x_3 \\ &- z_1x_3y_4 + z_1y_3x_4 + z_1x_2y_4 - z_1x_2y_3 - z_1y_2x_4 + z_1y_2x_3) \end{aligned}$$

ここで、

$$
\begin{aligned}
&a_{11}=1, a_{12}=x_1, a_{13}=y_1, a_{14}=z_1 \\
&a_{21}=1, a_{22}=x_2, a_{23}=y_2, a_{24}=z_2 \\
&a_{31}=1, a_{32}=x_3, a_{33}=y_3, a_{34}=z_3 \\
&a_{41}=1, a_{42}=x_4, a_{43}=y_4, a_{44}=z_4
\end{aligned}
$$

同様に

$$
V_1=\frac{1}{6}\det\begin{pmatrix} 1 & x & y & z \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{pmatrix}
$$

$$
\begin{aligned}
&=\frac{1}{6}(x_2y_3z_4-x_2z_3y_4-y_2x_3z_4+y_2z_3x_4+z_2x_3y_4-z_2y_3x_4 \\
&-xy_3z_4+xz_3y_4+xy_2z_4-xy_2z_3-xz_2y_4+xz_2y_3 \\
&+yx_3z_4-yz_3x_4-yx_2z_4+yx_2z_3+yz_2x_4-yz_2x_3 \\
&-zx_3y_4+zy_3x_4+zx_2y_4-zx_2y_3-zy_2x_4+zy_2x_3)
\end{aligned}
$$

$$
V_2=\frac{1}{6}\det\begin{pmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x & y & z \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{pmatrix}
$$

$$
\begin{aligned}
&=\frac{1}{6}(xy_3z_4-xz_3y_4-yx_3z_4+yz_3x_4+zx_3y_4-zy_3x_4 \\
&-x_1y_3z_4+x_1z_3y_4+x_1yz_4-x_1yz_3-x_1zy_4+x_1zy_3 \\
&+y_1x_3z_4-y_1z_3x_4-y_1xz_4+y_1xz_3+y_1zx_4-y_1zx_3 \\
&-z_1x_3y_4+z_1y_3x_4+z_1xy_4-z_1xy_3-z_1yx_4+z_1yx_3)
\end{aligned}
$$

$$
V_3=\frac{1}{6}\det\begin{pmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x & y & z \\ 1 & x_4 & y_4 & z_4 \end{pmatrix}
$$

$$
\begin{aligned}
&=\frac{1}{6}(x_2yz_4-x_2zy_4-y_2xz_4+y_2zx_4+z_2xy_4-z_2yx_4 \\
&-x_1yz_4+x_1zy_4+x_1y_2z_4-x_1y_2z-x_1z_2y_4+x_1z_2y \\
&+y_1xz_4-y_1zx_4-y_1x_2z_4+y_1x_2z+y_1z_2x_4-y_1z_2x \\
&-z_1xy_4+z_1yx_4+z_1x_2y_4-z_1x_2y-z_1y_2x_4+z_1y_2x)
\end{aligned}
$$

$$V_4 = \frac{1}{6}\det\begin{pmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x & y & z \end{pmatrix}$$

$$\begin{aligned}
&= \frac{1}{6}(x_2 y_3 z - x_2 z_3 y - y_2 x_3 z + y_2 z_3 x + z_2 x_3 y - z_2 y_3 x \\
&- x_1 y_3 z + x_1 z_3 y + x_1 y_2 z - x_1 y_2 z_3 - x_1 z_2 y + x_1 z_2 y_3 \\
&+ y_1 x_3 z - y_1 z_3 x - y_1 x_2 z + y_1 x_2 z_3 + y_1 z_2 x - y_1 z_2 x_3 \\
&- z_1 x_3 y + z_1 y_3 x + z_1 x_2 y - z_1 x_2 y_3 - z_1 y_2 x + z_1 y_2 x_3)
\end{aligned}$$

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・内挿関数N_1の微分

4面体1次要素 の内挿関数N_1の微分値を示します。

x微分

$$
\begin{aligned}
\frac{\partial N_1}{\partial x} &= \frac{\partial L_1}{\partial x} = \frac{\partial (V_1/V)}{\partial x} = \frac{1}{V}\frac{\partial V_1}{\partial x} \\
&= \frac{1}{V}\frac{\partial}{\partial x}\frac{1}{6}(x_2y_3z_4 - x_2z_3y_4 - y_2x_3z_4 + y_2z_3x_4 + z_2x_3y_4 - z_2y_3x_4 \\
&\qquad - xy_3z_4 + xz_3y_4 + xy_2z_4 - xy_2z_3 - xz_2y_4 + xz_2y_3 \\
&\qquad + yx_3z_4 - yz_3x_4 - yx_2z_4 + yx_2z_3 + yz_2x_4 - yz_2x_3 \\
&\qquad - zx_3y_4 + zy_3x_4 + zx_2y_4 - zx_2y_3 - zy_2x_4 + zy_2x_3) \\
&= \frac{1}{6V}(-y_3z_4 + z_3y_4 + y_2z_4 - y_2z_3 - z_2y_4 + z_2y_3)
\end{aligned}
$$

y微分

$$
\begin{aligned}
\frac{\partial N_1}{\partial y} &= \frac{\partial L_1}{\partial y} = \frac{\partial (V_1/V)}{\partial y} = \frac{1}{V}\frac{\partial V_1}{\partial y} \\
&= \frac{1}{V}\frac{\partial}{\partial y}\frac{1}{6}(x_2y_3z_4 - x_2z_3y_4 - y_2x_3z_4 + y_2z_3x_4 + z_2x_3y_4 - z_2y_3x_4 \\
&\qquad - xy_3z_4 + xz_3y_4 + xy_2z_4 - xy_2z_3 - xz_2y_4 + xz_2y_3 \\
&\qquad + yx_3z_4 - yz_3x_4 - yx_2z_4 + yx_2z_3 + yz_2x_4 - yz_2x_3 \\
&\qquad - zx_3y_4 + zy_3x_4 + zx_2y_4 - zx_2y_3 - zy_2x_4 + zy_2x_3) \\
&= \frac{1}{6V}(x_3z_4 - z_3x_4 - x_2z_4 + x_2z_3 + z_2x_4 - z_2x_3)
\end{aligned}
$$

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z 微分

$$
\begin{aligned}
&\frac{\partial N_1}{\partial z} = \frac{\partial L_1}{\partial z} = \frac{\partial (V_1/V)}{\partial z} = \frac{1}{V}\frac{\partial V_1}{\partial z} \\
&= \frac{1}{V}\frac{\partial}{\partial z}\frac{1}{6}(x_2y_3z_4 - x_2z_3y_4 - y_2x_3z_4 + y_2z_3x_4 + z_2x_3y_4 - z_2y_3x_4 \\
&\qquad - xy_3z_4 + xz_3y_4 + xy_2z_4 - xy_2z_3 - xz_2y_4 + xz_2y_3 \\
&\qquad + yx_3z_4 - yz_3x_4 - yx_2z_4 + yx_2z_3 + yz_2x_4 - yz_2x_3 \\
&\qquad - zx_3y_4 + zy_3x_4 + zx_2y_4 - zx_2y_3 - zy_2x_4 + zy_2x_3) \\
&= \frac{1}{6V}(-x_3y_4 + y_3x_4 + x_2y_4 - x_2y_3 - y_2x_4 + y_2x_3)
\end{aligned}
$$

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・運動量収支式の離散化

x, y, z軸方向の無次元化した運動量収支式は次式となります。

$$\phi_x = \frac{\partial V_x}{\partial \tau} + V_x\frac{\partial V_x}{\partial X} + V_y\frac{\partial V_x}{\partial Y} + V_z\frac{\partial V_x}{\partial Z} - \frac{\partial \sigma^*_{xx}}{\partial X} - \frac{\partial \sigma^*_{yx}}{\partial Y} - \frac{\partial \sigma^*_{zx}}{\partial Z} - g^*_x = 0$$

$$\phi_y = \frac{\partial V_y}{\partial \tau} + V_x\frac{\partial V_y}{\partial X} + V_y\frac{\partial V_y}{\partial Y} + V_z\frac{\partial V_y}{\partial Z} - \frac{\partial \sigma^*_{xy}}{\partial X} - \frac{\partial \sigma^*_{yy}}{\partial Y} - \frac{\partial \sigma^*_{zy}}{\partial Z} - g^*_y = 0$$

$$\phi_z = \frac{\partial V_z}{\partial \tau} + V_x\frac{\partial V_z}{\partial X} + V_y\frac{\partial V_z}{\partial Y} + V_z\frac{\partial V_z}{\partial Z} - \frac{\partial \sigma^*_{xz}}{\partial X} - \frac{\partial \sigma^*_{yz}}{\partial Y} - \frac{\partial \sigma^*_{zz}}{\partial Z} - g^*_z = 0$$

内挿関数N_iを重み関数に使用すると離散化式は次式となります。

$$\int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \phi_i dV = \int_V \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} dV = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

速度、圧力は、内挿関数N_iを用いてそれぞれ次式で表されます。

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$$V_x = N_1 V_{x,1} + N_2 V_{x,2} + N_3 V_{x,3} + N_4 V_{x,4} = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{Bmatrix} V_{x,1} \\ V_{x,2} \\ V_{x,3} \\ V_{x,4} \end{Bmatrix} = [N]\{V_x\}$$

$$V_y = N_1 V_{y,1} + N_2 V_{y,2} + N_3 V_{y,3} + N_4 V_{y,4}$$
$$V_z = N_1 V_{z,1} + N_2 V_{z,2} + N_3 V_{z,3} + N_4 V_{z,4}$$
$$P = N_1 P_{z,1} + N_2 P_{z,2} + N_3 P_{z,3} + N_4 P_{z,4}$$

i軸方向の離散化式は、次式となります。

$$\int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \phi_i dV$$
$$= \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \left(\frac{\partial V_i}{\partial \tau} + V_x \frac{\partial V_i}{\partial X} + V_y \frac{\partial V_i}{\partial Y} + V_z \frac{\partial V_i}{\partial Z} - \frac{\partial \sigma^*_{xi}}{\partial X} - \frac{\partial \sigma^*_{yi}}{\partial Y} - \frac{\partial \sigma^*_{zi}}{\partial Z} - g^*_i \right) dV$$

展開すると

$$= \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \frac{\partial V_i}{\partial \tau} dV$$
$$+ V_x \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \frac{\partial V_i}{\partial X} dV + V_y \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \frac{\partial V_i}{\partial Y} dV + V_Z \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \frac{\partial V_i}{\partial Z} dV$$
$$- \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \frac{\partial \sigma^*_{xi}}{\partial X} dV - \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \frac{\partial \sigma^*_{yi}}{\partial Y} dV - \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \frac{\partial \sigma^*_{zi}}{\partial Z} dV$$
$$- \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} g^*_i dV$$

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$$
\begin{aligned}
&= \int_V [N]^T \frac{\partial V_i}{\partial \tau} dV \\
&+ V_x \int_V [N]^T \frac{\partial V_i}{\partial X} dV + V_y \int_V [N]^T \frac{\partial V_i}{\partial Y} dV + V_Z \int_V [N]^T \frac{\partial V_i}{\partial Z} dV \\
&- \int_V [N]^T \frac{\partial \sigma_{xi}^*}{\partial X} dV - \int_V [N]^T \frac{\partial \sigma_{yi}^*}{\partial Y} dV - \int_V [N]^T \frac{\partial \sigma_{zi}^*}{\partial Z} dV \\
&- \int_V [N]^T g_i^* dV
\end{aligned}
$$

グリーン・ガウスの定理より

$$
\begin{aligned}
&= \int_V [N]^{\mathrm{T}} \frac{[N](\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau})}{\Delta\tau} dV \\
&+ V_x \int_V [N]^{\mathrm{T}} \frac{\partial [N]\{V_i\}}{\partial X} dV + V_y \int_V [N]^{\mathrm{T}} \frac{\partial [N]\{V_i\}}{\partial Y} dV + V_Z \int_V [N]^{\mathrm{T}} \frac{\partial [N]\{V_i\}}{\partial Z} dV \\
&- \int_S [N]^{\mathrm{T}} \sigma_{xi}^* n_x dS + \int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \sigma_{xi}^* dV \\
&- \int_S [N]^{\mathrm{T}} \sigma_{yi}^* n_y dS + \int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \sigma_{yi}^* dV \\
&- \int_S [N]^{\mathrm{T}} \sigma_{zi}^* n_z dS + \int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \sigma_{zi}^* dV \\
&- \int_V [N]^{\mathrm{T}} g_i^* dV
\end{aligned}
$$

界面に作用する応力をまとめると

$$
\begin{aligned}
&= \int_V [N]^{\mathrm{T}} [N] dV \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau} \\
&+ V_x \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial X}\right] dV \{V_i\} + V_y \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial Y}\right] dV \{V_i\} + V_Z \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial Z}\right] dV \{V_i\} \\
&+ \int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \sigma_{xi}^* dV + \int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \sigma_{yi}^* dV + \int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \sigma_{zi}^* dV \\
&- \int_S [N]^{\mathrm{T}} (\sigma_{xi}^* n_x + \sigma_{yi}^* n_y + \sigma_{zi}^* n_z) dS \\
&- \int_V [N]^{\mathrm{T}} g_i^* dV
\end{aligned}
$$

応力を展開して

$$= \int_V [N]^{\mathrm{T}} [N] dV \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau}$$

$$+ V_x \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial X}\right] dV \{V_i\} + V_y \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial Y}\right] dV \{V_i\} + V_Z \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial Z}\right] dV \{V_i\}$$

$$+ \int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \{-\delta_{xi} P + \frac{1}{Re}(\frac{\partial V_i}{\partial X} + \frac{\partial V_x}{\partial X_i})\} dV$$

$$+ \int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \{-\delta_{yi} P + \frac{1}{Re}(\frac{\partial V_i}{\partial Y} + \frac{\partial V_y}{\partial X_i})\} dV$$

$$+ \int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \{-\delta_{zi} P + \frac{1}{Re}(\frac{\partial V_i}{\partial Z} + \frac{\partial V_z}{\partial X_i})\} dV$$

$$- \frac{-2K^*}{We} n_i \int_S [N]^{\mathrm{T}} dS$$

$$- \int_V [N]^{\mathrm{T}} g_i^* dV$$

$$= \int_V [N]^{\mathrm{T}} [N] dV \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau}$$

$$+ V_x \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial X}\right] dV \{V_i\} + V_y \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial Y}\right] dV \{V_i\} + V_Z \int_V [N]^{\mathrm{T}} \left[\frac{\partial N}{\partial Z}\right] dV \{V_i\}$$

$$+ \int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \{-\delta_{xi} P + \frac{1}{Re}(\frac{\partial V_i}{\partial X} + \frac{\partial V_x}{\partial X_i})\} dV$$

$$+ \int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \{-\delta_{yi} P + \frac{1}{Re}(\frac{\partial V_i}{\partial Y} + \frac{\partial V_y}{\partial X_i})\} dV$$

$$+ \int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \{-\delta_{zi} P + \frac{1}{Re}(\frac{\partial V_i}{\partial Z} + \frac{\partial V_z}{\partial X_i})\} dV$$

$$- \frac{-2K^*}{We} n_i \int_S [N]^{\mathrm{T}} dS$$

$$- \int_V [N]^{\mathrm{T}} g_i^* dV$$

項ごとに分解して

$$
\begin{aligned}
&= \int_V [N]^{\mathrm{T}}[N]dV \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau} \\
&+ V_x \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial X}\right]dV\{V_i\} + V_y \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial Y}\right]dV\{V_i\} + V_Z \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial Z}\right]dV\{V_i\} \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \frac{\partial V_i}{\partial X}dV + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \frac{\partial V_x}{\partial X_i}dV - \int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \delta_{xi} P dV \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \frac{\partial V_i}{\partial Y}dV + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \frac{\partial V_y}{\partial X_i}dV - \int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \delta_{yi} P dV \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \frac{\partial V_i}{\partial Z}dV + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \frac{\partial V_z}{\partial X_i}dV - \int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \delta_{zi} P dV \\
&+ \frac{2K^*}{We} n_i \int_S [N]^{\mathrm{T}} dS \\
&- g_i^* \int_V [N]^{\mathrm{T}} dV
\end{aligned}
$$

速度、圧力を内挿関数で表示して

$$
\begin{aligned}
&= \int_V [N]^{\mathrm{T}}[N]dV \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau} \\
&+ V_x \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial X}\right]dV\{V_i\} + V_y \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial Y}\right]dV\{V_i\} + V_Z \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial Z}\right]dV\{V_i\} \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \frac{\partial [N]\{V_i\}}{\partial X}dV + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} \frac{\partial [N]\{V_x\}}{\partial X_i}dV - \delta_{xi}\int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}} [N]\{P\} dV \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \frac{\partial [N]\{V_i\}}{\partial Y}dV + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} \frac{\partial [N]\{V_y\}}{\partial X_i}dV - \delta_{yi}\int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}} [N]\{P\} dV \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \frac{\partial [N]\{V_i\}}{\partial Z}dV + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} \frac{\partial [N]\{V_z\}}{\partial X_i}dV - \delta_{zi}\int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}} [N]\{P\} dV \\
&+ \frac{2K^*}{We} n_i \int_S [N]^{\mathrm{T}} dS \\
&- g_i^* \int_V [N]^{\mathrm{T}} dV
\end{aligned}
$$

速度、圧力は定数なので積分の外に出す

$$
\begin{aligned}
&= \int_V [N]^{\mathrm{T}}[N] dV \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau} \\
&+ V_x \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial X}\right] dV\{V_i\} + V_y \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial Y}\right] dV\{V_i\} + V_Z \int_V [N]^{\mathrm{T}}\left[\frac{\partial N}{\partial Z}\right] dV\{V_i\} \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial X}\right] dV\{V_i\} + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial X_i}\right] dV\{V_x\} - \delta_{xi}\int_V \left[\frac{\partial N}{\partial X}\right]^{\mathrm{T}}[N] dV\{P\} \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial Y}\right] dV\{V_i\} + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial X_i}\right] dV\{V_y\} - \delta_{yi}\int_V \left[\frac{\partial N}{\partial Y}\right]^{\mathrm{T}}[N] dV\{P\} \\
&+ \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial Z}\right] dV\{V_i\} + \frac{1}{Re}\int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial X_i}\right] dV\{V_z\} - \delta_{zi}\int_V \left[\frac{\partial N}{\partial Z}\right]^{\mathrm{T}}[N] dV\{P\} \\
&+ \frac{2K^*}{We} n_i \int_S [N]^{\mathrm{T}} dS \\
&- g_i^* \int_V [N]^{\mathrm{T}} dV \qquad (i = 1,2,3)
\end{aligned}
$$

行列式で表す

蓄積量

$$
= \int_V \begin{bmatrix} N_1N_1 & N_1N_2 & N_1N_3 & N_1N_4 \\ N_2N_1 & N_2N_2 & N_2N_3 & N_2N_4 \\ N_3N_1 & N_3N_2 & N_3N_3 & N_3N_4 \\ N_4N_1 & N_4N_2 & N_4N_3 & N_4N_4 \end{bmatrix} dV \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau}
$$

対流項

$$
+V_x \int_V \begin{bmatrix} N_1 \frac{\partial N_1}{\partial X} & N_1 \frac{\partial N_2}{\partial X} & N_1 \frac{\partial N_3}{\partial X} & N_1 \frac{\partial N_4}{\partial X} \\ N_2 \frac{\partial N_1}{\partial X} & N_2 \frac{\partial N_2}{\partial X} & N_2 \frac{\partial N_3}{\partial X} & N_2 \frac{\partial N_4}{\partial X} \\ N_3 \frac{\partial N_1}{\partial X} & N_3 \frac{\partial N_2}{\partial X} & N_3 \frac{\partial N_3}{\partial X} & N_3 \frac{\partial N_4}{\partial X} \\ N_4 \frac{\partial N_1}{\partial X} & N_4 \frac{\partial N_2}{\partial X} & N_4 \frac{\partial N_3}{\partial X} & N_4 \frac{\partial N_4}{\partial X} \end{bmatrix} dV \{V_i\}
$$

$\boxed{\text{対流項}}$

$$
+V_y \int_V \begin{bmatrix} N_1 \frac{\partial N_1}{\partial Y} & N_1 \frac{\partial N_2}{\partial Y} & N_1 \frac{\partial N_3}{\partial Y} & N_1 \frac{\partial N_4}{\partial Y} \\ N_2 \frac{\partial N_1}{\partial Y} & N_2 \frac{\partial N_2}{\partial Y} & N_2 \frac{\partial N_3}{\partial Y} & N_2 \frac{\partial N_4}{\partial Y} \\ N_3 \frac{\partial N_1}{\partial Y} & N_3 \frac{\partial N_2}{\partial Y} & N_3 \frac{\partial N_3}{\partial Y} & N_3 \frac{\partial N_4}{\partial Y} \\ N_4 \frac{\partial N_1}{\partial Y} & N_4 \frac{\partial N_2}{\partial Y} & N_4 \frac{\partial N_3}{\partial Y} & N_4 \frac{\partial N_4}{\partial Y} \end{bmatrix} dV \{V_i\}
$$

$$
+V_Z \int_V \begin{bmatrix} N_1 \frac{\partial N_1}{\partial Z} & N_1 \frac{\partial N_2}{\partial Z} & N_1 \frac{\partial N_3}{\partial Z} & N_1 \frac{\partial N_4}{\partial Z} \\ N_2 \frac{\partial N_1}{\partial Z} & N_2 \frac{\partial N_2}{\partial Z} & N_2 \frac{\partial N_3}{\partial Z} & N_2 \frac{\partial N_4}{\partial Z} \\ N_3 \frac{\partial N_1}{\partial Z} & N_3 \frac{\partial N_2}{\partial Z} & N_3 \frac{\partial N_3}{\partial Z} & N_3 \frac{\partial N_4}{\partial Z} \\ N_4 \frac{\partial N_1}{\partial Z} & N_4 \frac{\partial N_2}{\partial Z} & N_4 \frac{\partial N_3}{\partial Z} & N_4 \frac{\partial N_4}{\partial Z} \end{bmatrix} dV \{V_i\}
$$

x軸方向の粘性項と圧力項

$$
+\frac{1}{Re}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial X}\frac{\partial N_1}{\partial X} & \frac{\partial N_1}{\partial X}\frac{\partial N_2}{\partial X} & \frac{\partial N_1}{\partial X}\frac{\partial N_3}{\partial X} & \frac{\partial N_1}{\partial X}\frac{\partial N_4}{\partial X} \\ \frac{\partial N_2}{\partial X}\frac{\partial N_1}{\partial X} & \frac{\partial N_2}{\partial X}\frac{\partial N_2}{\partial X} & \frac{\partial N_2}{\partial X}\frac{\partial N_3}{\partial X} & \frac{\partial N_2}{\partial X}\frac{\partial N_4}{\partial X} \\ \frac{\partial N_3}{\partial X}\frac{\partial N_1}{\partial X} & \frac{\partial N_3}{\partial X}\frac{\partial N_2}{\partial X} & \frac{\partial N_3}{\partial X}\frac{\partial N_3}{\partial X} & \frac{\partial N_3}{\partial X}\frac{\partial N_4}{\partial X} \\ \frac{\partial N_4}{\partial X}\frac{\partial N_1}{\partial X} & \frac{\partial N_4}{\partial X}\frac{\partial N_2}{\partial X} & \frac{\partial N_4}{\partial X}\frac{\partial N_3}{\partial X} & \frac{\partial N_4}{\partial X}\frac{\partial N_4}{\partial X} \end{bmatrix} dV\{V_i\}
$$

$\boxed{\text{粘性項の } x \text{ 成分}}$

$$
+\frac{1}{Re}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial X}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_1}{\partial X}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_1}{\partial X}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_1}{\partial X}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_2}{\partial X}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_2}{\partial X}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_2}{\partial X}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_2}{\partial X}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_3}{\partial X}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_3}{\partial X}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_3}{\partial X}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_3}{\partial X}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_4}{\partial X}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_4}{\partial X}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_4}{\partial X}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_4}{\partial X}\frac{\partial N_4}{\partial X_i} \end{bmatrix} dV\{V_x\}
$$

$$
-\delta_{xi}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial X}N_1 & \frac{\partial N_1}{\partial X}N_2 & \frac{\partial N_1}{\partial X}N_3 & \frac{\partial N_1}{\partial X}N_4 \\ \frac{\partial N_2}{\partial X}N_1 & \frac{\partial N_2}{\partial X}N_2 & \frac{\partial N_2}{\partial X}N_3 & \frac{\partial N_2}{\partial X}N_4 \\ \frac{\partial N_3}{\partial X}N_1 & \frac{\partial N_3}{\partial X}N_2 & \frac{\partial N_3}{\partial X}N_3 & \frac{\partial N_3}{\partial X}N_4 \\ \frac{\partial N_4}{\partial X}N_1 & \frac{\partial N_4}{\partial X}N_2 & \frac{\partial N_4}{\partial X}N_3 & \frac{\partial N_4}{\partial X}N_4 \end{bmatrix} dV\{P\}
$$

$\boxed{\text{圧力項の } x \text{ 成分}}$

y軸方向の粘性項と圧力項

$$
+\frac{1}{Re}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial Y}\frac{\partial N_1}{\partial Y} & \frac{\partial N_1}{\partial Y}\frac{\partial N_2}{\partial Y} & \frac{\partial N_1}{\partial Y}\frac{\partial N_3}{\partial Y} & \frac{\partial N_1}{\partial Y}\frac{\partial N_4}{\partial Y} \\ \frac{\partial N_2}{\partial Y}\frac{\partial N_1}{\partial Y} & \frac{\partial N_2}{\partial Y}\frac{\partial N_2}{\partial Y} & \frac{\partial N_2}{\partial Y}\frac{\partial N_3}{\partial Y} & \frac{\partial N_2}{\partial Y}\frac{\partial N_4}{\partial Y} \\ \frac{\partial N_3}{\partial Y}\frac{\partial N_1}{\partial Y} & \frac{\partial N_3}{\partial Y}\frac{\partial N_2}{\partial Y} & \frac{\partial N_3}{\partial Y}\frac{\partial N_3}{\partial Y} & \frac{\partial N_3}{\partial Y}\frac{\partial N_4}{\partial Y} \\ \frac{\partial N_4}{\partial Y}\frac{\partial N_1}{\partial Y} & \frac{\partial N_4}{\partial Y}\frac{\partial N_2}{\partial Y} & \frac{\partial N_4}{\partial Y}\frac{\partial N_3}{\partial Y} & \frac{\partial N_4}{\partial Y}\frac{\partial N_4}{\partial Y} \end{bmatrix} dV\{V_i\}
$$

粘性項の y 成分

$$
+\frac{1}{Re}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial Y}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_1}{\partial Y}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_1}{\partial Y}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_1}{\partial Y}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_2}{\partial Y}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_2}{\partial Y}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_2}{\partial Y}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_2}{\partial Y}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_3}{\partial Y}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_3}{\partial Y}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_3}{\partial Y}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_3}{\partial Y}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_4}{\partial Y}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_4}{\partial Y}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_4}{\partial Y}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_4}{\partial Y}\frac{\partial N_4}{\partial X_i} \end{bmatrix} dV\{V_y\}
$$

$$
-\delta_{yi}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial Y}N_1 & \frac{\partial N_1}{\partial Y}N_2 & \frac{\partial N_1}{\partial Y}N_3 & \frac{\partial N_1}{\partial Y}N_4 \\ \frac{\partial N_2}{\partial Y}N_1 & \frac{\partial N_2}{\partial Y}N_2 & \frac{\partial N_2}{\partial Y}N_3 & \frac{\partial N_2}{\partial Y}N_4 \\ \frac{\partial N_3}{\partial Y}N_1 & \frac{\partial N_3}{\partial Y}N_2 & \frac{\partial N_3}{\partial Y}N_3 & \frac{\partial N_3}{\partial Y}N_4 \\ \frac{\partial N_4}{\partial Y}N_1 & \frac{\partial N_4}{\partial Y}N_2 & \frac{\partial N_4}{\partial Y}N_3 & \frac{\partial N_4}{\partial Y}N_4 \end{bmatrix} dV\{P\}
$$

圧力項の y 成分

z軸方向の粘性項と圧力項

粘性項の z 成分

$$+\frac{1}{Re}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial Z}\frac{\partial N_1}{\partial Z} & \frac{\partial N_1}{\partial Z}\frac{\partial N_2}{\partial Z} & \frac{\partial N_1}{\partial Z}\frac{\partial N_3}{\partial Z} & \frac{\partial N_1}{\partial Z}\frac{\partial N_4}{\partial Z} \\ \frac{\partial N_2}{\partial Z}\frac{\partial N_1}{\partial Z} & \frac{\partial N_2}{\partial Z}\frac{\partial N_2}{\partial Z} & \frac{\partial N_2}{\partial Z}\frac{\partial N_3}{\partial Z} & \frac{\partial N_2}{\partial Z}\frac{\partial N_4}{\partial Z} \\ \frac{\partial N_3}{\partial Z}\frac{\partial N_1}{\partial Z} & \frac{\partial N_3}{\partial Z}\frac{\partial N_2}{\partial Z} & \frac{\partial N_3}{\partial Z}\frac{\partial N_3}{\partial Z} & \frac{\partial N_3}{\partial Z}\frac{\partial N_4}{\partial Z} \\ \frac{\partial N_4}{\partial Z}\frac{\partial N_1}{\partial Z} & \frac{\partial N_4}{\partial Z}\frac{\partial N_2}{\partial Z} & \frac{\partial N_4}{\partial Z}\frac{\partial N_3}{\partial Z} & \frac{\partial N_4}{\partial Z}\frac{\partial N_4}{\partial Z} \end{bmatrix} dV\{V_i\}$$

$$+\frac{1}{Re}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial Z}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_1}{\partial Z}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_1}{\partial Z}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_1}{\partial Z}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_2}{\partial Z}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_2}{\partial Z}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_2}{\partial Z}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_2}{\partial Z}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_3}{\partial Z}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_3}{\partial Z}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_3}{\partial Z}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_3}{\partial Z}\frac{\partial N_4}{\partial X_i} \\ \frac{\partial N_4}{\partial Z}\frac{\partial N_1}{\partial X_i} & \frac{\partial N_4}{\partial Z}\frac{\partial N_2}{\partial X_i} & \frac{\partial N_4}{\partial Z}\frac{\partial N_3}{\partial X_i} & \frac{\partial N_4}{\partial Z}\frac{\partial N_4}{\partial X_i} \end{bmatrix} dV\{V_z\}$$

圧力項の z 成分

$$-\delta_{zi}\int_V \begin{bmatrix} \frac{\partial N_1}{\partial Z}N_1 & \frac{\partial N_1}{\partial Z}N_2 & \frac{\partial N_1}{\partial Z}N_3 & \frac{\partial N_1}{\partial Z}N_4 \\ \frac{\partial N_2}{\partial Z}N_1 & \frac{\partial N_2}{\partial Z}N_2 & \frac{\partial N_2}{\partial Z}N_3 & \frac{\partial N_2}{\partial Z}N_4 \\ \frac{\partial N_3}{\partial Z}N_1 & \frac{\partial N_3}{\partial Z}N_2 & \frac{\partial N_3}{\partial Z}N_3 & \frac{\partial N_3}{\partial Z}N_4 \\ \frac{\partial N_4}{\partial Z}N_1 & \frac{\partial N_4}{\partial Z}N_2 & \frac{\partial N_4}{\partial Z}N_3 & \frac{\partial N_4}{\partial Z}N_4 \end{bmatrix} dV\{P\}$$

表面張力項

表面張力項

$$+\frac{2K^*}{We}n_i\int_S \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} dS$$

重力項

重力項

$$-g_i^*\int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} dV \qquad (i=1,2,3)$$

内挿関数を形状関数に直す

$$=\int_V \begin{bmatrix} L_1L_1 & L_1L_2 & L_1L_3 & L_1L_4 \\ L_2L_1 & L_2L_2 & L_2L_3 & L_2L_4 \\ L_3L_1 & L_3L_2 & L_3L_3 & L_3L_4 \\ L_4L_1 & L_4L_2 & L_4L_3 & L_4L_4 \end{bmatrix} dV \frac{\{V_i\}^{\tau+\Delta\tau}-\{V_i\}^{\tau}}{\Delta\tau}$$

対流項

$$+V_x\int_V \frac{1}{6V}\begin{bmatrix} L_1c_{1x} & L_1c_{2x} & L_1c_{3x} & L_1c_{4x} \\ L_2c_{1x} & L_2c_{2x} & L_2c_{3x} & L_2c_{4x} \\ L_3c_{1x} & L_3c_{2x} & L_3c_{3x} & L_3c_{4x} \\ L_4c_{1x} & L_4c_{2x} & L_4c_{3x} & L_4c_{4x} \end{bmatrix} dV\{V_i\}$$

$$+V_y\int_V \frac{1}{6V}\begin{bmatrix} L_1c_{1y} & L_1c_{2y} & L_1c_{3y} & L_1c_{4y} \\ L_2c_{1y} & L_2c_{2y} & L_2c_{3y} & L_2c_{4y} \\ L_3c_{1y} & L_3c_{2y} & L_3c_{3y} & L_3c_{4y} \\ L_4c_{1y} & L_4c_{2y} & L_4c_{3y} & L_4c_{4y} \end{bmatrix} dV\{V_i\}$$

$$+V_Z\int_V \frac{1}{6V}\begin{bmatrix} L_1c_{1z} & L_1c_{2z} & L_1c_{3z} & L_1c_{4z} \\ L_2c_{1z} & L_2c_{2z} & L_2c_{3z} & L_2c_{4z} \\ L_3c_{1z} & L_3c_{2z} & L_3c_{3z} & L_3c_{4z} \\ L_4c_{1z} & L_4c_{2z} & L_4c_{3z} & L_4c_{4z} \end{bmatrix} dV\{V_i\}$$

x軸方向の粘性項と圧力項

$$+\frac{1}{Re}\int_V \frac{1}{36V^2}\begin{bmatrix} c_{1x}c_{1x} & c_{1x}c_{2x} & c_{1x}c_{3x} & c_{1x}c_{4x} \\ c_{2x}c_{1x} & c_{2x}c_{2x} & c_{2x}c_{3x} & c_{2x}c_{4x} \\ c_{3x}c_{1x} & c_{3x}c_{2x} & c_{3x}c_{3x} & c_{3x}c_{4x} \\ c_{4x}c_{1x} & c_{4x}c_{2x} & c_{4x}c_{3x} & c_{4x}c_{4x} \end{bmatrix} dV\{V_i\}$$

$$+\frac{1}{Re}\int_V \frac{1}{36V^2}\begin{bmatrix} c_{1x}c_{1i} & c_{1x}c_{2i} & c_{1x}c_{3i} & c_{1x}c_{4i} \\ c_{2x}c_{1i} & c_{2x}c_{2i} & c_{2x}c_{3i} & c_{2x}c_{4i} \\ c_{3x}c_{1i} & c_{3x}c_{2i} & c_{3x}c_{3i} & c_{3x}c_{4i} \\ c_{4x}c_{1i} & c_{4x}c_{2i} & c_{4x}c_{3i} & c_{4x}c_{4i} \end{bmatrix} dV\{V_x\}$$

$$-\delta_{xi}\int_V \frac{1}{6V}\begin{bmatrix} c_{1x}L_1 & c_{1x}L_2 & c_{1x}L_3 & c_{1x}L_4 \\ c_{2x}L_1 & c_{2x}L_2 & c_{2x}L_3 & c_{2x}L_4 \\ c_{3x}L_1 & c_{3x}L_2 & c_{3x}L_3 & c_{3x}L_4 \\ c_{4x}L_1 & c_{4x}L_2 & c_{4x}L_3 & c_{4x}L_4 \end{bmatrix} dV\{P\}$$

y軸方向の粘性項と圧力項

$$+\frac{1}{Re}\int_V \frac{1}{36V^2}\begin{bmatrix} c_{1y}c_{1y} & c_{1y}c_{2y} & c_{1y}c_{3y} & c_{1y}c_{4y} \\ c_{2y}c_{1y} & c_{2y}c_{2y} & c_{2y}c_{3y} & c_{2y}c_{4y} \\ c_{3y}c_{1y} & c_{3y}c_{2y} & c_{3y}c_{3y} & c_{3y}c_{4y} \\ c_{4y}c_{1y} & c_{4y}c_{2y} & c_{4y}c_{3y} & c_{4y}c_{4y} \end{bmatrix} dV\{V_i\}$$

$$+\frac{1}{Re}\int_V \frac{1}{36V^2}\begin{bmatrix} c_{1y}c_{1i} & c_{1y}c_{2i} & c_{1y}c_{3i} & c_{1y}c_{4i} \\ c_{2y}c_{1i} & c_{2y}c_{2i} & c_{2y}c_{3i} & c_{2y}c_{4i} \\ c_{3y}c_{1i} & c_{3y}c_{2i} & c_{3y}c_{3i} & c_{3y}c_{4i} \\ c_{4y}c_{1i} & c_{4y}c_{2i} & c_{4y}c_{3i} & c_{4y}c_{4i} \end{bmatrix} dV\{V_y\}$$

$$-\delta_{yi}\int_V \frac{1}{6V}\begin{bmatrix} c_{1y}L_1 & c_{1y}L_2 & c_{1y}L_3 & c_{1y}L_4 \\ c_{2y}L_1 & c_{2y}L_2 & c_{2y}L_3 & c_{2y}L_4 \\ c_{3y}L_1 & c_{3y}L_2 & c_{3y}L_3 & c_{3y}L_4 \\ c_{4y}L_1 & c_{4y}L_2 & c_{4y}L_3 & c_{4y}L_4 \end{bmatrix} dV\{P\}$$

z軸方向の粘性項と圧力項

$$+\frac{1}{Re}\int_V \frac{1}{36V^2}\begin{bmatrix} c_{1z}c_{1z} & c_{1z}c_{2z} & c_{1z}c_{3z} & c_{1z}c_{4z} \\ c_{2z}c_{1z} & c_{2z}c_{2z} & c_{2z}c_{3z} & c_{2z}c_{4z} \\ c_{3z}c_{1z} & c_{3z}c_{2z} & c_{3z}c_{3z} & c_{3z}c_{4z} \\ c_{4z}c_{1z} & c_{4z}c_{2z} & c_{4z}c_{3z} & c_{4z}c_{4z} \end{bmatrix} dV\{V_i\}$$

$$+\frac{1}{Re}\int_V \frac{1}{36V^2}\begin{bmatrix} c_{1z}c_{1i} & c_{1z}c_{2i} & c_{1z}c_{3i} & c_{1z}c_{4i} \\ c_{2z}c_{1i} & c_{2z}c_{2i} & c_{2z}c_{3i} & c_{2z}c_{4i} \\ c_{3z}c_{1i} & c_{3z}c_{2i} & c_{3z}c_{3i} & c_{3z}c_{4i} \\ c_{4z}c_{1i} & c_{4z}c_{2i} & c_{4z}c_{3i} & c_{4z}c_{4i} \end{bmatrix} dV\{V_z\}$$

$$-\delta_{zi}\int_V \frac{1}{6V}\begin{bmatrix} c_{1z}L_1 & c_{1z}L_2 & c_{1z}L_3 & c_{1z}L_4 \\ c_{2z}L_1 & c_{2z}L_2 & c_{2z}L_3 & c_{2z}L_4 \\ c_{3z}L_1 & c_{3z}L_2 & c_{3z}L_3 & c_{3z}L_4 \\ c_{4z}L_1 & c_{4z}L_2 & c_{4z}L_3 & c_{4z}L_4 \end{bmatrix} dV\{P\}$$

表面張力項

$$+\frac{2K^*}{We}n_i\int_S \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} dS$$

重力項

$$-g_i^*\int_V \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix}^{\mathrm{T}} dV \qquad (i=1,2,3)$$

ここで、面積積分、体積積分の公式より

$$\int_V L_1^p L_2^q L_3^r L_4^s dV = \frac{p!q!r!s!}{(p+q+r+s+3)!}6V$$

$$\int_V L_i L_j dV = \begin{cases} \dfrac{1!1!}{(1+1+3)!}6V = \dfrac{6}{5!}V = \dfrac{1}{20}V & (i \neq j) \\ \dfrac{2!}{(1+1+3)!}6V = \dfrac{12}{5!}V = \dfrac{1}{10}V & (i = j) \end{cases}$$

$$\int_V L_i dV = \frac{1!}{(1+3)!}6V = \frac{6}{4!}V = \frac{1}{4}V$$

$$\int_S L_1^p L_2^q L_3^r dS = \frac{p!q!r!}{(p+q+r+2)!}2S$$

$$\int_S L_i dS = \frac{1}{(1+2)!}2S = \frac{1}{3}S$$

形状関数を積分すると

$$
= \frac{1}{20} V \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau}
$$

対流項

$$
+ V_x \frac{1}{6V} \frac{V}{4} \begin{bmatrix} c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \end{bmatrix} \{V_i\}
$$

$$
+ V_y \frac{1}{6V} \frac{V}{4} \begin{bmatrix} c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \end{bmatrix} \{V_i\}
$$

$$
+ V_Z \frac{1}{6V} \frac{V}{4} \begin{bmatrix} c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \end{bmatrix} \{V_i\}
$$

x軸方向の粘性項と圧力項

$$
+ \frac{1}{Re} \frac{1}{36V^2} V \begin{bmatrix} c_{1x}c_{1x} & c_{1x}c_{2x} & c_{1x}c_{3x} & c_{1x}c_{4x} \\ c_{2x}c_{1x} & c_{2x}c_{2x} & c_{2x}c_{3x} & c_{2x}c_{4x} \\ c_{3x}c_{1x} & c_{3x}c_{2x} & c_{3x}c_{3x} & c_{3x}c_{4x} \\ c_{4x}c_{1x} & c_{4x}c_{2x} & c_{4x}c_{3x} & c_{4x}c_{4x} \end{bmatrix} \{V_i\}
$$

$$
+ \frac{1}{Re} \frac{1}{36V^2} V \begin{bmatrix} c_{1x}c_{1i} & c_{1x}c_{2i} & c_{1x}c_{3i} & c_{1x}c_{4i} \\ c_{2x}c_{1i} & c_{2x}c_{2i} & c_{2x}c_{3i} & c_{2x}c_{4i} \\ c_{3x}c_{1i} & c_{3x}c_{2i} & c_{3x}c_{3i} & c_{3x}c_{4i} \\ c_{4x}c_{1i} & c_{4x}c_{2i} & c_{4x}c_{3i} & c_{4x}c_{4i} \end{bmatrix} \{V_x\}
$$

$$
- \delta_{xi} \frac{1}{6V} \frac{V}{4} \begin{bmatrix} c_{1x} & c_{1x} & c_{1x} & c_{1x} \\ c_{2x} & c_{2x} & c_{2x} & c_{2x} \\ c_{3x} & c_{3x} & c_{3x} & c_{3x} \\ c_{4x} & c_{4x} & c_{4x} & c_{4x} \end{bmatrix} \{P\}
$$

y軸方向の粘性項と圧力項

$$+\frac{1}{Re}\frac{1}{36V^2}V\begin{bmatrix} c_{1y}c_{1y} & c_{1y}c_{2y} & c_{1y}c_{3y} & c_{1y}c_{4y} \\ c_{2y}c_{1y} & c_{2y}c_{2y} & c_{2y}c_{3y} & c_{2y}c_{4y} \\ c_{3y}c_{1y} & c_{3y}c_{2y} & c_{3y}c_{3y} & c_{3y}c_{4y} \\ c_{4y}c_{1y} & c_{4y}c_{2y} & c_{4y}c_{3y} & c_{4y}c_{4y} \end{bmatrix}\{V_i\}$$

$$+\frac{1}{Re}\frac{1}{36V^2}V\begin{bmatrix} c_{1y}c_{1i} & c_{1y}c_{2i} & c_{1y}c_{3i} & c_{1y}c_{4i} \\ c_{2y}c_{1i} & c_{2y}c_{2i} & c_{2y}c_{3i} & c_{2y}c_{4i} \\ c_{3y}c_{1i} & c_{3y}c_{2i} & c_{3y}c_{3i} & c_{3y}c_{4i} \\ c_{4y}c_{1i} & c_{4y}c_{2i} & c_{4y}c_{3i} & c_{4y}c_{4i} \end{bmatrix}\{V_y\}$$

$$-\delta_{yi}\frac{1}{6V}\frac{V}{4}\begin{bmatrix} c_{1y} & c_{1y} & c_{1y} & c_{1y} \\ c_{2y} & c_{2y} & c_{2y} & c_{2y} \\ c_{3y} & c_{3y} & c_{3y} & c_{3y} \\ c_{4y} & c_{4y} & c_{4y} & c_{4y} \end{bmatrix}\{P\}$$

z軸方向の粘性項と圧力項

$$+\frac{1}{Re}\frac{1}{36V^2}V\begin{bmatrix} c_{1z}c_{1z} & c_{1z}c_{2z} & c_{1z}c_{3z} & c_{1z}c_{4z} \\ c_{2z}c_{1z} & c_{2z}c_{2z} & c_{2z}c_{3z} & c_{2z}c_{4z} \\ c_{3z}c_{1z} & c_{3z}c_{2z} & c_{3z}c_{3z} & c_{3z}c_{4z} \\ c_{4z}c_{1z} & c_{4z}c_{2z} & c_{4z}c_{3z} & c_{4z}c_{4z} \end{bmatrix}\{V_i\}$$

$$+\frac{1}{Re}\frac{1}{36V^2}V\begin{bmatrix} c_{1z}c_{1i} & c_{1z}c_{2i} & c_{1z}c_{3i} & c_{1z}c_{4i} \\ c_{2z}c_{1i} & c_{2z}c_{2i} & c_{2z}c_{3i} & c_{2z}c_{4i} \\ c_{3z}c_{1i} & c_{3z}c_{2i} & c_{3z}c_{3i} & c_{3z}c_{4i} \\ c_{4z}c_{1i} & c_{4z}c_{2i} & c_{4z}c_{3i} & c_{4z}c_{4i} \end{bmatrix}\{V_z\}$$

$$-\delta_{zi}\frac{1}{6V}\frac{V}{4}\begin{bmatrix} c_{1z} & c_{1z} & c_{1z} & c_{1z} \\ c_{2z} & c_{2z} & c_{2z} & c_{2z} \\ c_{3z} & c_{3z} & c_{3z} & c_{3z} \\ c_{4z} & c_{4z} & c_{4z} & c_{4z} \end{bmatrix}\{P\}$$

表面張力項

$$+\frac{2K^*}{We}n_i\frac{S}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

重力項

$$-g_i^*\frac{V}{4}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (i=1,2,3)$$

係数をまとめて

$$= \frac{1}{20} V \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau}$$

対流項

$$+ V_x \frac{1}{24} \begin{bmatrix} c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \end{bmatrix} \{V_i\}$$

$$+ V_y \frac{1}{24} \begin{bmatrix} c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \end{bmatrix} \{V_i\}$$

$$+ V_Z \frac{1}{24} \begin{bmatrix} c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \end{bmatrix} \{V_i\}$$

x軸方向の粘性項と圧力項

$$+ \frac{1}{Re} \frac{1}{36V} \begin{bmatrix} c_{1x}c_{1x} & c_{1x}c_{2x} & c_{1x}c_{3x} & c_{1x}c_{4x} \\ c_{2x}c_{1x} & c_{2x}c_{2x} & c_{2x}c_{3x} & c_{2x}c_{4x} \\ c_{3x}c_{1x} & c_{3x}c_{2x} & c_{3x}c_{3x} & c_{3x}c_{4x} \\ c_{4x}c_{1x} & c_{4x}c_{2x} & c_{4x}c_{3x} & c_{4x}c_{4x} \end{bmatrix} \{V_i\}$$

$$+ \frac{1}{Re} \frac{1}{36V} \begin{bmatrix} c_{1x}c_{1i} & c_{1x}c_{2i} & c_{1x}c_{3i} & c_{1x}c_{4i} \\ c_{2x}c_{1i} & c_{2x}c_{2i} & c_{2x}c_{3i} & c_{2x}c_{4i} \\ c_{3x}c_{1i} & c_{3x}c_{2i} & c_{3x}c_{3i} & c_{3x}c_{4i} \\ c_{4x}c_{1i} & c_{4x}c_{2i} & c_{4x}c_{3i} & c_{4x}c_{4i} \end{bmatrix} \{V_x\}$$

$$- \delta_{xi} \frac{1}{24} \begin{bmatrix} c_{1x} & c_{1x} & c_{1x} & c_{1x} \\ c_{2x} & c_{2x} & c_{2x} & c_{2x} \\ c_{3x} & c_{3x} & c_{3x} & c_{3x} \\ c_{4x} & c_{4x} & c_{4x} & c_{4x} \end{bmatrix} \{P\}$$

y軸方向の粘性項と圧力項

$$+\frac{1}{Re}\frac{1}{36V}\begin{bmatrix} c_{1y}c_{1y} & c_{1y}c_{2y} & c_{1y}c_{3y} & c_{1y}c_{4y} \\ c_{2y}c_{1y} & c_{2y}c_{2y} & c_{2y}c_{3y} & c_{2y}c_{4y} \\ c_{3y}c_{1y} & c_{3y}c_{2y} & c_{3y}c_{3y} & c_{3y}c_{4y} \\ c_{4y}c_{1y} & c_{4y}c_{2y} & c_{4y}c_{3y} & c_{4y}c_{4y} \end{bmatrix}\{V_i\}$$

$$+\frac{1}{Re}\frac{1}{36V}\begin{bmatrix} c_{1y}c_{1i} & c_{1y}c_{2i} & c_{1y}c_{3i} & c_{1y}c_{4i} \\ c_{2y}c_{1i} & c_{2y}c_{2i} & c_{2y}c_{3i} & c_{2y}c_{4i} \\ c_{3y}c_{1i} & c_{3y}c_{2i} & c_{3y}c_{3i} & c_{3y}c_{4i} \\ c_{4y}c_{1i} & c_{4y}c_{2i} & c_{4y}c_{3i} & c_{4y}c_{4i} \end{bmatrix}\{V_y\}$$

$$-\delta_{yi}\frac{1}{24}\begin{bmatrix} c_{1y} & c_{1y} & c_{1y} & c_{1y} \\ c_{2y} & c_{2y} & c_{2y} & c_{2y} \\ c_{3y} & c_{3y} & c_{3y} & c_{3y} \\ c_{4y} & c_{4y} & c_{4y} & c_{4y} \end{bmatrix}\{P\}$$

z軸方向の粘性項と圧力項

$$+\frac{1}{Re}\frac{1}{36V}\begin{bmatrix} c_{1z}c_{1z} & c_{1z}c_{2z} & c_{1z}c_{3z} & c_{1z}c_{4z} \\ c_{2z}c_{1z} & c_{2z}c_{2z} & c_{2z}c_{3z} & c_{2z}c_{4z} \\ c_{3z}c_{1z} & c_{3z}c_{2z} & c_{3z}c_{3z} & c_{3z}c_{4z} \\ c_{4z}c_{1z} & c_{4z}c_{2z} & c_{4z}c_{3z} & c_{4z}c_{4z} \end{bmatrix}\{V_i\}$$

$$+\frac{1}{Re}\frac{1}{36V}\begin{bmatrix} c_{1z}c_{1i} & c_{1z}c_{2i} & c_{1z}c_{3i} & c_{1z}c_{4i} \\ c_{2z}c_{1i} & c_{2z}c_{2i} & c_{2z}c_{3i} & c_{2z}c_{4i} \\ c_{3z}c_{1i} & c_{3z}c_{2i} & c_{3z}c_{3i} & c_{3z}c_{4i} \\ c_{4z}c_{1i} & c_{4z}c_{2i} & c_{4z}c_{3i} & c_{4z}c_{4i} \end{bmatrix}\{V_z\}$$

$$-\delta_{zi}\frac{1}{24}\begin{bmatrix} c_{1z} & c_{1z} & c_{1z} & c_{1z} \\ c_{2z} & c_{2z} & c_{2z} & c_{2z} \\ c_{3z} & c_{3z} & c_{3z} & c_{3z} \\ c_{4z} & c_{4z} & c_{4z} & c_{4z} \end{bmatrix}\{P\}$$

表面張力項

$$+\frac{2}{3}\frac{K^*}{We}n_i S\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

重力項

$$-g_i^*\frac{V}{4}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (i=1,2,3)$$

行列式をまとめると

$$
\begin{aligned}
&= [C]\frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau} + V_x[C_x]\{V_i\} + V_y[C_y]\{V_i\} + V_Z[C_z]\{V_i\} \\
&+ \frac{1}{Re}[S_{xx}]\{V_i\} + \frac{1}{Re}[S_{xi}]\{V_x\} - \delta_{xi}[H_x]\{P\} \\
&+ \frac{1}{Re}[S_{yy}]\{V_i\} + \frac{1}{Re}[S_{yi}]\{V_y\} - \delta_{yi}[H_y]\{P\} \\
&+ \frac{1}{Re}[S_{zz}]\{V_i\} + \frac{1}{Re}[S_{zi}]\{V_z\} - \delta_{zi}[H_z]\{P\} \\
&+ \frac{2}{3}\frac{K^*}{We} n_i S \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
&- g_i^* \frac{V}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (i = 1,2,3)
\end{aligned}
$$

行列ごとにまとめると

$$
\begin{aligned}
&= [C]\frac{\{V_i\}^{\tau+\Delta\tau} - \{V_i\}^{\tau}}{\Delta\tau} \\
&+ (V_x[C_x] + V_y[C_y] + V_Z[C_z])\{V_i\} \\
&+ \frac{1}{Re}([S_{xx}] + [S_{yy}] + [S_{zz}])\{V_i\} \\
&+ \frac{1}{Re}([S_{xi}]\{V_x\} + [S_{yi}]\{V_y\} + [S_{zi}]\{V_z\}) \\
&- (\delta_{xi}[H_x] + \delta_{yi}[H_y] + \delta_{zi}[H_z])\{P\} \\
&+ \frac{2}{3}\frac{K^*}{We} n_i S \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
&- g_i^* \frac{V}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (i = 1,2,3) \\
&= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{aligned}
$$

最終的に次式が導出されます。

$$
\begin{aligned}
&[C]\frac{\{V_i\}^{\tau+\Delta\tau}-\{V_i\}^{\tau}}{\Delta\tau}+(V_x[C_x]+V_y[C_y]+V_Z[C_z])\{V_i\}\\
&+\frac{1}{Re}([S_{xx}]+[S_{yy}]+[S_{zz}])\{V_i\}\\
&+\frac{1}{Re}([S_{xi}]\{V_x\}+[S_{yi}]\{V_y\}+[S_{zi}]\{V_z\})\\
&-(\delta_{xi}[H_x]+\delta_{yi}[H_y]+\delta_{zi}[H_z])\{P\}\\
&+\frac{2}{3}\frac{K^*}{We}n_iS\begin{bmatrix}1\\1\\1\end{bmatrix}-g_i^*\frac{V}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \qquad (i=1,2,3)
\end{aligned}
$$

既知の項を右辺に移項します。

$$
\begin{aligned}
&\frac{[C]}{\Delta\tau}\{V_i\}^{\tau+\Delta\tau}+(V_x[C_x]+V_y[C_y]+V_Z[C_z])\{V_i\}^{\tau+\Delta\tau}\\
&+\frac{1}{Re}([S_{xx}]+[S_{yy}]+[S_{zz}])\{V_i\}^{\tau+\Delta\tau}\\
&+\frac{1}{Re}([S_{xi}]\{V_x\}^{\tau+\Delta\tau}+[S_{yi}]\{V_y\}^{\tau+\Delta\tau}+[S_{zi}]\{V_z\}^{\tau+\Delta\tau})\\
&-(\delta_{xi}[H_x]+\delta_{yi}[H_y]+\delta_{zi}[H_z])\{P\}^{\tau+\Delta\tau}\\
&=\frac{[C]}{\Delta\tau}\{V_i\}^{\tau}-\frac{2}{3}\frac{K^*}{We}n_iS\begin{bmatrix}1\\1\\1\end{bmatrix}+g_i^*\frac{V}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \qquad (i=1,2,3)
\end{aligned}
$$

x軸方向の成分は次式となります。

$$
\begin{aligned}
&\frac{[C]}{\Delta\tau}\{V_x\}^{\tau+\Delta\tau}+(V_x[C_x]+V_y[C_y]+V_Z[C_z])\{V_x\}^{\tau+\Delta\tau}\\
&+\frac{1}{Re}\{(2[S_{xx}]+[S_{yy}]+[S_{zz}])\{V_x\}^{\tau+\Delta\tau}+[S_{yx}]\{V_y\}^{\tau+\Delta\tau}+[S_{zx}]\{V_z\}^{\tau+\Delta\tau}\}-[H_x]\{P\}^{\tau+\Delta\tau}\\
&=\frac{[C]}{\Delta\tau}\{V_x\}^{\tau}-\frac{2}{3}\frac{K^*}{We}n_xS\begin{bmatrix}1\\1\\1\end{bmatrix}+g_x^*\frac{V}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}
\end{aligned}
$$

y軸方向の成分は次式となります。

$$
\frac{[C]}{\Delta\tau}\{V_y\}^{\tau+\Delta\tau}+(V_x[C_x]+V_y[C_y]+V_Z[C_z])\{V_y\}^{\tau+\Delta\tau}
$$

$$
+\frac{1}{Re}\{[S_{xy}]\{V_x\}^{\tau+\Delta\tau}+([S_{xx}]+2[S_{yy}]+[S_{zz}])\{V_y\}^{\tau+\Delta\tau}+[S_{zy}]\{V_z\}^{\tau+\Delta\tau}\}-[H_y]\{P\}^{\tau+\Delta\tau}
$$

$$
=\frac{[C]}{\Delta\tau}\{V_y\}^{\tau}-\frac{2}{3}\frac{K^*}{We}n_y S\begin{bmatrix}1\\1\\1\end{bmatrix}+g_y^*\frac{V}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}
$$

z軸方向の成分は次式となります。

$$
\frac{[C]}{\Delta\tau}\{V_z\}^{\tau+\Delta\tau}+(V_x[C_x]+V_y[C_y]+V_Z[C_z])\{V_z\}^{\tau+\Delta\tau}
$$

$$
+\frac{1}{Re}\{[S_{xz}]\{V_x\}^{\tau+\Delta\tau}+[S_{yz}]\{V_y\}^{\tau+\Delta\tau}+([S_{xx}]+[S_{yy}]+2[S_{zz}])\{V_z\}^{\tau+\Delta\tau}\}-[H_z]\{P\}^{\tau+\Delta\tau}
$$

$$
=\frac{[C]}{\Delta\tau}\{V_z\}^{\tau}-\frac{2}{3}\frac{K^*}{We}n_z S\begin{bmatrix}1\\1\\1\end{bmatrix}+g_z^*\frac{V}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\qquad(i=1,2,3)
$$

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忍者ホームページ 【アクセスUP!】忍者アクセスランキング カイロ・整体・各種療法・健... 最短5分であなたのサイトに広告配信【忍者AdMax】 忍者ツールズ 多くの人が実感した！スカルプケ...多くの人が実感した！スカルプケ...

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メッシュ
SIMPLE法

・質量収支式の離散化

無次元化した質量収支式は次式となります。

$$\phi = \frac{\partial P}{\partial \tau} + V_x \frac{\partial P}{\partial x} + V_y \frac{\partial P}{\partial y} + V_z \frac{\partial P}{\partial z} + \frac{1}{Ma^2}\left(\frac{\partial V_x}{\partial X} + \frac{\partial V_y}{\partial Y} + \frac{\partial V_z}{\partial Z}\right) = 0$$

内挿関数N_iを重み関数に使用すると離散化式は次式となります。

$$\int_V [N]^T \phi dV = \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \phi dV$$

$$= \int_V \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} dV$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

速度、圧力は、内挿関数N_iを用いてそれぞれ次式で表されます。

SIMPLE法
Appendix
コラム
サブコンテンツ
伝熱工学
Fortran
ソフトライブラリ
書籍紹介
PDF資料
English

$$
\begin{aligned}
V_x &= N_1 V_{x1} + N_2 V_{x2} + N_3 V_{x3} + N_4 V_{x4} \\
&= \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \{ V_{x1} \quad V_{x2} \quad V_{x3} \quad V_{x4} \} \\
&= [N]^T \{V_x\} \\
V_y &= [N]^T \{V_y\} \\
V_z &= [N]^T \{V_z\} \\
P &= [N]^T \{P\}
\end{aligned}
$$

離散化式は、次式となります。

$$
\int_V [N]^T \phi dV
$$

$$
= \int_V [N]^T \{ \frac{\partial P}{\partial \tau} + V_x \frac{\partial P}{\partial x} + V_y \frac{\partial P}{\partial y} + V_z \frac{\partial P}{\partial z} + \frac{1}{Ma^2} (\frac{\partial V_x}{\partial X} + \frac{\partial V_y}{\partial Y} + \frac{\partial V_z}{\partial Z}) \} dV
$$

項ごとに分解して

$$
= \int_V [N]^T \frac{\partial P}{\partial \tau} dV + \int_V [N]^T V_x \frac{\partial P}{\partial X} dV + \int_V [N]^T V_y \frac{\partial P}{\partial Y} dV + \int_V [N]^T V_z \frac{\partial P}{\partial Z} dV
$$

$$
+ \frac{1}{Ma^2} \int_V [N]^T \frac{\partial V_x}{\partial X} dV + \frac{1}{Ma^2} \int_V [N]^T \frac{\partial V_y}{\partial Y} dV + \frac{1}{Ma^2} \int_V [N]^T \frac{\partial V_z}{\partial Z} dV
$$

速度、圧力を内挿関数で表示して

$$
= \int_V [N]^T [N] dV \frac{\{P\}^{\Delta\tau+\tau} - \{P\}^{\tau}}{\Delta \tau}
$$

$$
+ V_x \int_V [N]^T \frac{\partial [N] \{P\}^{\Delta\tau+\tau}}{\partial X} dV + V_y \int_V [N]^T \frac{\partial [N] \{P\}^{\Delta\tau+\tau}}{\partial Y} dV
$$

$$
+ V_z \int_V [N]^T \frac{\partial [N] \{P\}^{\Delta\tau+\tau}}{\partial Z} dV
$$

$$
+ \frac{1}{Ma^2} \int_V [N]^T \frac{\partial [N] \{V_x\}^{\Delta\tau+\tau}}{\partial X} dV + \frac{1}{Ma^2} \int_V [N]^T \frac{\partial [N] \{V_y\}^{\Delta\tau+\tau}}{\partial Y} dV
$$

$$
+ \frac{1}{Ma^2} \int_V [N]^T \frac{\partial [N] \{V_z\}^{\Delta\tau+\tau}}{\partial Z} dV
$$

速度、圧力は定数なので積分の外に出す

カウンタ

$$
\begin{aligned}
&= \int_V [N]^T [N] dV \frac{\{P\}^{\Delta\tau+\tau} - \{P\}^{\tau}}{\Delta\tau} \\
&+ V_x \int_V [N]^T \frac{\partial [N]}{\partial X} dV \{P\}^{\Delta\tau+\tau} + V_y \int_V [N]^T \frac{\partial [N]}{\partial Y} dV \{P\}^{\Delta\tau+\tau} \\
&+ V_z \int_V [N]^T \frac{\partial [N]}{\partial Z} dV \{P\}^{\Delta\tau+\tau} \\
&+ \frac{1}{Ma^2} \int_V [N]^T \frac{\partial [N]}{\partial X} dV \{V_x\}^{\Delta\tau+\tau} + \frac{1}{Ma^2} \int_V [N]^T \frac{\partial [N]}{\partial Y} dV \{V_y\}^{\Delta\tau+\tau} \\
&+ \frac{1}{Ma^2} \int_V [N]^T \frac{\partial [N]}{\partial Z} dV \{V_z\}^{\Delta\tau+\tau}
\end{aligned}
$$

行列式で表す

$$
\begin{aligned}
&= \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} dV \frac{\{P\}^{\Delta\tau+\tau} - \{P\}^{\tau}}{\Delta\tau} \\
&+ V_x \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial X} & \frac{\partial N_2}{\partial X} & \frac{\partial N_3}{\partial X} & \frac{\partial N_4}{\partial X} \end{bmatrix} dV \{P\}^{\Delta\tau+\tau} \\
&+ V_y \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial Y} & \frac{\partial N_2}{\partial Y} & \frac{\partial N_3}{\partial Y} & \frac{\partial N_4}{\partial Y} \end{bmatrix} dV \{P\}^{\Delta\tau+\tau} \\
&+ V_z \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial Z} & \frac{\partial N_2}{\partial Z} & \frac{\partial N_3}{\partial Z} & \frac{\partial N_4}{\partial Z} \end{bmatrix} dV \{P\}^{\Delta\tau+\tau} \\
&+ \frac{1}{Ma^2} \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial X} & \frac{\partial N_2}{\partial X} & \frac{\partial N_3}{\partial X} & \frac{\partial N_4}{\partial X} \end{bmatrix} dV \{V_x\}^{\Delta\tau+\tau} \\
&+ \frac{1}{Ma^2} \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial Y} & \frac{\partial N_2}{\partial Y} & \frac{\partial N_3}{\partial Y} & \frac{\partial N_4}{\partial Y} \end{bmatrix} dV \{V_y\}^{\Delta\tau+\tau} \\
&+ \frac{1}{Ma^2} \int_V \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial Z} & \frac{\partial N_2}{\partial Z} & \frac{\partial N_3}{\partial Z} & \frac{\partial N_4}{\partial Z} \end{bmatrix} dV \{V_z\}^{\Delta\tau+\tau}
\end{aligned}
$$

行列を計算する

$$= \int_V \begin{bmatrix} N_1N_1 & N_1N_2 & N_1N_3 & N_1N_4 \\ N_2N_1 & N_2N_2 & N_2N_3 & N_2N_4 \\ N_3N_1 & N_3N_2 & N_3N_3 & N_3N_4 \\ N_4N_1 & N_4N_2 & N_4N_3 & N_4N_4 \end{bmatrix} dV \frac{\{P\}^{\Delta\tau+\tau} - \{P\}^{\tau}}{\Delta\tau}$$

$$+ V_x \int_V \begin{bmatrix} N_1\frac{\partial N_1}{\partial X} & N_1\frac{\partial N_2}{\partial X} & N_1\frac{\partial N_3}{\partial X} & N_1\frac{\partial N_4}{\partial X} \\ N_2\frac{\partial N_1}{\partial X} & N_2\frac{\partial N_2}{\partial X} & N_2\frac{\partial N_3}{\partial X} & N_2\frac{\partial N_4}{\partial X} \\ N_3\frac{\partial N_1}{\partial X} & N_3\frac{\partial N_2}{\partial X} & N_3\frac{\partial N_3}{\partial X} & N_3\frac{\partial N_4}{\partial X} \\ N_4\frac{\partial N_1}{\partial X} & N_4\frac{\partial N_2}{\partial X} & N_4\frac{\partial N_3}{\partial X} & N_4\frac{\partial N_4}{\partial X} \end{bmatrix} dV \{V_x\}^{\Delta\tau+\tau}$$

$$+ V_y \int_V \begin{bmatrix} N_1\frac{\partial N_1}{\partial Y} & N_1\frac{\partial N_2}{\partial Y} & N_1\frac{\partial N_3}{\partial Y} & N_1\frac{\partial N_4}{\partial Y} \\ N_2\frac{\partial N_1}{\partial Y} & N_2\frac{\partial N_2}{\partial Y} & N_2\frac{\partial N_3}{\partial Y} & N_2\frac{\partial N_4}{\partial Y} \\ N_3\frac{\partial N_1}{\partial Y} & N_3\frac{\partial N_2}{\partial Y} & N_3\frac{\partial N_3}{\partial Y} & N_3\frac{\partial N_4}{\partial Y} \\ N_4\frac{\partial N_1}{\partial Y} & N_4\frac{\partial N_2}{\partial Y} & N_4\frac{\partial N_3}{\partial Y} & N_4\frac{\partial N_4}{\partial Y} \end{bmatrix} dV \{V_y\}^{\Delta\tau+\tau}$$

$$+ V_z \int_V \begin{bmatrix} N_1\frac{\partial N_1}{\partial Z} & N_1\frac{\partial N_2}{\partial Z} & N_1\frac{\partial N_3}{\partial Z} & N_1\frac{\partial N_4}{\partial Z} \\ N_2\frac{\partial N_1}{\partial Z} & N_2\frac{\partial N_2}{\partial Z} & N_2\frac{\partial N_3}{\partial Z} & N_2\frac{\partial N_4}{\partial Z} \\ N_3\frac{\partial N_1}{\partial Z} & N_3\frac{\partial N_2}{\partial Z} & N_3\frac{\partial N_3}{\partial Z} & N_3\frac{\partial N_4}{\partial Z} \\ N_4\frac{\partial N_1}{\partial Z} & N_4\frac{\partial N_2}{\partial Z} & N_4\frac{\partial N_3}{\partial Z} & N_4\frac{\partial N_4}{\partial Z} \end{bmatrix} dV \{V_z\}^{\Delta\tau+\tau}$$

$$
+\frac{1}{Ma^2}\int_V \begin{bmatrix} N_1\frac{\partial N_1}{\partial X} & N_1\frac{\partial N_2}{\partial X} & N_1\frac{\partial N_3}{\partial X} & N_1\frac{\partial N_4}{\partial X} \\ N_2\frac{\partial N_1}{\partial X} & N_2\frac{\partial N_2}{\partial X} & N_2\frac{\partial N_3}{\partial X} & N_2\frac{\partial N_4}{\partial X} \\ N_3\frac{\partial N_1}{\partial X} & N_3\frac{\partial N_2}{\partial X} & N_3\frac{\partial N_3}{\partial X} & N_3\frac{\partial N_4}{\partial X} \\ N_4\frac{\partial N_1}{\partial X} & N_4\frac{\partial N_2}{\partial X} & N_4\frac{\partial N_3}{\partial X} & N_4\frac{\partial N_4}{\partial X} \end{bmatrix} dV\{V_x\}^{\Delta\tau+\tau}
$$

$$
+\frac{1}{Ma^2}\int_V \begin{bmatrix} N_1\frac{\partial N_1}{\partial Y} & N_1\frac{\partial N_2}{\partial Y} & N_1\frac{\partial N_3}{\partial Y} & N_1\frac{\partial N_4}{\partial Y} \\ N_2\frac{\partial N_1}{\partial Y} & N_2\frac{\partial N_2}{\partial Y} & N_2\frac{\partial N_3}{\partial Y} & N_2\frac{\partial N_4}{\partial Y} \\ N_3\frac{\partial N_1}{\partial Y} & N_3\frac{\partial N_2}{\partial Y} & N_3\frac{\partial N_3}{\partial Y} & N_3\frac{\partial N_4}{\partial Y} \\ N_4\frac{\partial N_1}{\partial Y} & N_4\frac{\partial N_2}{\partial Y} & N_4\frac{\partial N_3}{\partial Y} & N_4\frac{\partial N_4}{\partial Y} \end{bmatrix} dV\{V_y\}^{\Delta\tau+\tau}
$$

$$
+\frac{1}{Ma^2}\int_V \begin{bmatrix} N_1\frac{\partial N_1}{\partial Z} & N_1\frac{\partial N_2}{\partial Z} & N_1\frac{\partial N_3}{\partial Z} & N_1\frac{\partial N_4}{\partial Z} \\ N_2\frac{\partial N_1}{\partial Z} & N_2\frac{\partial N_2}{\partial Z} & N_2\frac{\partial N_3}{\partial Z} & N_2\frac{\partial N_4}{\partial Z} \\ N_3\frac{\partial N_1}{\partial Z} & N_3\frac{\partial N_2}{\partial Z} & N_3\frac{\partial N_3}{\partial Z} & N_3\frac{\partial N_4}{\partial Z} \\ N_4\frac{\partial N_1}{\partial Z} & N_4\frac{\partial N_2}{\partial Z} & N_4\frac{\partial N_3}{\partial Z} & N_4\frac{\partial N_4}{\partial Z} \end{bmatrix} dV\{V_z\}^{\Delta\tau+\tau}
$$

内挿関数を形状関数に直す

$$
=\int_V \begin{bmatrix} L_1L_1 & L_1L_2 & L_1L_3 & L_1L_4 \\ L_2L_1 & L_2L_2 & L_2L_3 & L_2L_4 \\ L_3L_1 & L_3L_2 & L_3L_3 & L_3L_4 \\ L_4L_1 & L_4L_2 & L_4L_3 & L_4L_4 \end{bmatrix} dV \frac{\{P\}^{\Delta\tau+\tau}-\{P\}^{\tau}}{\Delta\tau}
$$

$$
+V_x\int_V \begin{bmatrix} L_1\frac{\partial L_1}{\partial X} & L_1\frac{\partial L_2}{\partial X} & L_1\frac{\partial L_3}{\partial X} & L_1\frac{\partial L_4}{\partial X} \\ L_2\frac{\partial L_1}{\partial X} & L_2\frac{\partial L_2}{\partial X} & L_2\frac{\partial L_3}{\partial X} & L_2\frac{\partial L_4}{\partial X} \\ L_3\frac{\partial L_1}{\partial X} & L_3\frac{\partial L_2}{\partial X} & L_3\frac{\partial L_3}{\partial X} & L_3\frac{\partial L_4}{\partial X} \\ L_4\frac{\partial L_1}{\partial X} & L_4\frac{\partial L_2}{\partial X} & L_4\frac{\partial L_3}{\partial X} & L_4\frac{\partial L_4}{\partial X} \end{bmatrix} dV\{P\}^{\Delta\tau+\tau}
$$

$$
+V_y\int_V \begin{bmatrix} L_1\frac{\partial L_1}{\partial Y} & L_1\frac{\partial L_2}{\partial Y} & L_1\frac{\partial L_3}{\partial Y} & L_1\frac{\partial L_4}{\partial Y} \\ L_2\frac{\partial L_1}{\partial Y} & L_2\frac{\partial L_2}{\partial Y} & L_2\frac{\partial L_3}{\partial Y} & L_2\frac{\partial L_4}{\partial Y} \\ L_3\frac{\partial L_1}{\partial Y} & L_3\frac{\partial L_2}{\partial Y} & L_3\frac{\partial L_3}{\partial Y} & L_3\frac{\partial L_4}{\partial Y} \\ L_4\frac{\partial L_1}{\partial Y} & L_4\frac{\partial L_2}{\partial Y} & L_4\frac{\partial L_3}{\partial Y} & L_4\frac{\partial L_4}{\partial Y} \end{bmatrix} dV\{P\}^{\Delta\tau+\tau}
$$

$$
+V_z\int_V \begin{bmatrix} L_1\frac{\partial L_1}{\partial Z} & L_1\frac{\partial L_2}{\partial Z} & L_1\frac{\partial L_3}{\partial Z} & L_1\frac{\partial L_4}{\partial Z} \\ L_2\frac{\partial L_1}{\partial Z} & L_2\frac{\partial L_2}{\partial Z} & L_2\frac{\partial L_3}{\partial Z} & L_2\frac{\partial L_4}{\partial Z} \\ L_3\frac{\partial L_1}{\partial Z} & L_3\frac{\partial L_2}{\partial Z} & L_3\frac{\partial L_3}{\partial Z} & L_3\frac{\partial L_4}{\partial Z} \\ L_4\frac{\partial L_1}{\partial Z} & L_4\frac{\partial L_2}{\partial Z} & L_4\frac{\partial L_3}{\partial Z} & L_4\frac{\partial L_4}{\partial Z} \end{bmatrix} dV\{P\}^{\Delta\tau+\tau}
$$

$$
+\frac{1}{Ma^2}\int_V \begin{bmatrix} L_1\frac{\partial L_1}{\partial X} & L_1\frac{\partial L_2}{\partial X} & L_1\frac{\partial L_3}{\partial X} & L_1\frac{\partial L_4}{\partial X} \\ L_2\frac{\partial L_1}{\partial X} & L_2\frac{\partial L_2}{\partial X} & L_2\frac{\partial L_3}{\partial X} & L_2\frac{\partial L_4}{\partial X} \\ L_3\frac{\partial L_1}{\partial X} & L_3\frac{\partial L_2}{\partial X} & L_3\frac{\partial L_3}{\partial X} & L_3\frac{\partial L_4}{\partial X} \\ L_4\frac{\partial L_1}{\partial X} & L_4\frac{\partial L_2}{\partial X} & L_4\frac{\partial L_3}{\partial X} & L_4\frac{\partial L_4}{\partial X} \end{bmatrix} dV\{V_x\}^{\Delta\tau+\tau}
$$

$$
+\frac{1}{Ma^2}\int_V \begin{bmatrix} L_1\frac{\partial L_1}{\partial Y} & L_1\frac{\partial L_2}{\partial Y} & L_1\frac{\partial L_3}{\partial Y} & L_1\frac{\partial L_4}{\partial Y} \\ L_2\frac{\partial L_1}{\partial Y} & L_2\frac{\partial L_2}{\partial Y} & L_2\frac{\partial L_3}{\partial Y} & L_2\frac{\partial L_4}{\partial Y} \\ L_3\frac{\partial L_1}{\partial Y} & L_3\frac{\partial L_2}{\partial Y} & L_3\frac{\partial L_3}{\partial Y} & L_3\frac{\partial L_4}{\partial Y} \\ L_4\frac{\partial L_1}{\partial Y} & L_4\frac{\partial L_2}{\partial Y} & L_4\frac{\partial L_3}{\partial Y} & L_4\frac{\partial L_4}{\partial Y} \end{bmatrix} dV\{V_y\}^{\Delta\tau+\tau}
$$

$$
+\frac{1}{Ma^2}\int_V \begin{bmatrix} L_1\frac{\partial L_1}{\partial Z} & L_1\frac{\partial L_2}{\partial Z} & L_1\frac{\partial L_3}{\partial Z} & L_1\frac{\partial L_4}{\partial Z} \\ L_2\frac{\partial L_1}{\partial Z} & L_2\frac{\partial L_2}{\partial Z} & L_2\frac{\partial L_3}{\partial Z} & L_2\frac{\partial L_4}{\partial Z} \\ L_3\frac{\partial L_1}{\partial Z} & L_3\frac{\partial L_2}{\partial Z} & L_3\frac{\partial L_3}{\partial Z} & L_3\frac{\partial L_4}{\partial Z} \\ L_4\frac{\partial L_1}{\partial Z} & L_4\frac{\partial L_2}{\partial Z} & L_4\frac{\partial L_3}{\partial Z} & L_4\frac{\partial L_4}{\partial Z} \end{bmatrix} dV\{V_z\}^{\Delta\tau+\tau}
$$

形状関数の微分値を計算する

$$= \int_V \begin{bmatrix} L_1L_1 & L_1L_2 & L_1L_3 & L_1L_4 \\ L_2L_1 & L_2L_2 & L_2L_3 & L_2L_4 \\ L_3L_1 & L_3L_2 & L_3L_3 & L_3L_4 \\ L_4L_1 & L_4L_2 & L_4L_3 & L_4L_4 \end{bmatrix} dV \frac{\{P\}^{\Delta\tau+\tau} - \{P\}^{\tau}}{\Delta\tau}$$

$$+ V_x \int_V \frac{1}{6V} \begin{bmatrix} L_1c_{1x} & L_1c_{2x} & L_1c_{3x} & L_1c_{4x} \\ L_2c_{1x} & L_2c_{2x} & L_2c_{3x} & L_2c_{4x} \\ L_3c_{1x} & L_3c_{2x} & L_3c_{3x} & L_3c_{4x} \\ L_4c_{1x} & L_4c_{2x} & L_4c_{3x} & L_4c_{4x} \end{bmatrix} dV \{P\}^{\Delta\tau+\tau}$$

$$+ V_y \int_V \frac{1}{6V} \begin{bmatrix} L_1c_{1y} & L_1c_{2y} & L_1c_{3y} & L_1c_{4y} \\ L_2c_{1y} & L_2c_{2y} & L_2c_{3y} & L_2c_{4y} \\ L_3c_{1y} & L_3c_{2y} & L_3c_{3y} & L_3c_{4y} \\ L_4c_{1y} & L_4c_{2y} & L_4c_{3y} & L_4c_{4y} \end{bmatrix} dV \{P\}^{\Delta\tau+\tau}$$

$$+ V_z \int_V \frac{1}{6V} \begin{bmatrix} L_1c_{1z} & L_1c_{2z} & L_1c_{3z} & L_1c_{4z} \\ L_2c_{1z} & L_2c_{2z} & L_2c_{3z} & L_2c_{4z} \\ L_3c_{1z} & L_3c_{2z} & L_3c_{3z} & L_3c_{4z} \\ L_4c_{1z} & L_4c_{2z} & L_4c_{3z} & L_4c_{4z} \end{bmatrix} dV \{P\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2} \int_V \frac{1}{6V} \begin{bmatrix} L_1c_{1x} & L_1c_{2x} & L_1c_{3x} & L_1c_{4x} \\ L_2c_{1x} & L_2c_{2x} & L_2c_{3x} & L_2c_{4x} \\ L_3c_{1x} & L_3c_{2x} & L_3c_{3x} & L_3c_{4x} \\ L_4c_{1x} & L_4c_{2x} & L_4c_{3x} & L_4c_{4x} \end{bmatrix} dV \{V_x\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2} \int_V \frac{1}{6V} \begin{bmatrix} L_1c_{1y} & L_1c_{2y} & L_1c_{3y} & L_1c_{4y} \\ L_2c_{1y} & L_2c_{2y} & L_2c_{3y} & L_2c_{4y} \\ L_3c_{1y} & L_3c_{2y} & L_3c_{3y} & L_3c_{4y} \\ L_4c_{1y} & L_4c_{2y} & L_4c_{3y} & L_4c_{4y} \end{bmatrix} dV \{V_y\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2} \int_V \frac{1}{6V} \begin{bmatrix} L_1c_{1z} & L_1c_{2z} & L_1c_{3z} & L_1c_{4z} \\ L_2c_{1z} & L_2c_{2z} & L_2c_{3z} & L_2c_{4z} \\ L_3c_{1z} & L_3c_{2z} & L_3c_{3z} & L_3c_{4z} \\ L_4c_{1z} & L_4c_{2z} & L_4c_{3z} & L_4c_{4z} \end{bmatrix} dV \{V_z\}^{\Delta\tau+\tau}$$

ここで、体積積分の公式より

$$\int_V L_1^p L_2^q L_3^r L_4^s dV = \frac{p!q!r!s!}{(p+q+r+s+3)!} 6V$$

$$\int_V L_i L_j dV = \begin{cases} \dfrac{1!1!}{(1+1+3)!} 6V = \dfrac{6}{5!} V = \dfrac{1}{20} V & (i \neq j) \\ \dfrac{2!}{(1+1+3)!} 6V = \dfrac{12}{5!} V = \dfrac{1}{10} V & (i = j) \end{cases}$$

$$\int_V L_i dV = \frac{1!}{(1+3)!} 6V = \frac{6}{4!} V = \frac{1}{4} V$$

形状関数を積分すると

$$= \frac{V}{20}\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \frac{\{P\}^{\Delta\tau+\tau} - \{P\}^{\tau}}{\Delta\tau}$$

$$+ \frac{1}{6V}\frac{V}{4}\begin{bmatrix} c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \end{bmatrix} \{P\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{6V}\frac{V}{4}\begin{bmatrix} c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \end{bmatrix} \{P\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{6V}\frac{V}{4}\begin{bmatrix} c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \end{bmatrix} \{P\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2}\frac{1}{6V}\frac{V}{4}\begin{bmatrix} c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \end{bmatrix} \{V_x\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2}\frac{1}{6V}\frac{V}{4}\begin{bmatrix} c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \end{bmatrix} \{V_y\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2}\frac{1}{6V}\frac{V}{4}\begin{bmatrix} c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \end{bmatrix} \{V_z\}^{\Delta\tau+\tau}$$

係数をまとめて

$$= \frac{V}{20}\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \frac{\{P\}^{\Delta\tau+\tau} - \{P\}^{\tau}}{\Delta\tau}$$

$$+ \frac{1}{24}\begin{bmatrix} c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \end{bmatrix}\{P\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{24}\begin{bmatrix} c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \end{bmatrix}\{P\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{24}\begin{bmatrix} c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \end{bmatrix}\{P\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2}\frac{1}{24}\begin{bmatrix} c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \\ c_{1x} & c_{2x} & c_{3x} & c_{4x} \end{bmatrix}\{V_x\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2}\frac{1}{24}\begin{bmatrix} c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \\ c_{1y} & c_{2y} & c_{3y} & c_{4y} \end{bmatrix}\{V_y\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2}\frac{1}{24}\begin{bmatrix} c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \\ c_{1z} & c_{2z} & c_{3z} & c_{4z} \end{bmatrix}\{V_z\}^{\Delta\tau+\tau}$$

行列式をまとめると

$$= [C]\frac{\{P\}^{\Delta\tau+\tau} - \{P\}^{\tau}}{\Delta\tau}$$

$$+ V_x[C_x]\{P\}^{\Delta\tau+\tau} + V_y[C_y]\{P\}^{\Delta\tau+\tau} + V_z[C_Z]\{P\}^{\Delta\tau+\tau}$$

$$+ \frac{1}{Ma^2}[C_x]\{V_x\}^{\Delta\tau+\tau} + \frac{1}{Ma^2}[C_y]\{V_y\}^{\Delta\tau+\tau} + \frac{1}{Ma^2}[C_Z]\{V_z\}^{\Delta\tau+\tau}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

最終的に次式が導出されます。

$$[C]\frac{\{P\}^{\Delta\tau+\tau}-\{P\}^{\tau}}{\Delta\tau}+V_x[C_x]\{P\}^{\Delta\tau+\tau}+V_y[C_y]\{P\}^{\Delta\tau+\tau}+V_z[C_Z]\{P\}^{\Delta\tau+\tau}$$
$$+\frac{1}{Ma^2}[C_x]\{V_x\}^{\Delta\tau+\tau}+\frac{1}{Ma^2}[C_y]\{V_y\}^{\Delta\tau+\tau}+\frac{1}{Ma^2}[C_Z]\{V_z\}^{\Delta\tau+\tau}=0$$

既知の項を右辺に移項します。

$$\frac{[C]}{\Delta\tau}\{P\}^{\Delta\tau+\tau}$$
$$+V_x[C_x]\{P\}^{\Delta\tau+\tau}+V_y[C_y]\{P\}^{\Delta\tau+\tau}+V_z[C_Z]\{P\}^{\Delta\tau+\tau}$$
$$+\frac{1}{Ma^2}[C_x]\{V_x\}^{\Delta\tau+\tau}+\frac{1}{Ma^2}[C_y]\{V_y\}^{\Delta\tau+\tau}+\frac{1}{Ma^2}[C_Z]\{V_z\}^{\Delta\tau+\tau}=\frac{[C]}{\Delta\tau}\{P\}^{\tau}$$

行列ごとにまとめると

$$(\frac{[C]}{\Delta\tau}+V_x[C_x]+V_y[C_y]+V_z[C_Z])\{P\}^{\Delta\tau+\tau}$$
$$+\frac{1}{Ma^2}([C_x]\{V_x\}^{\Delta\tau+\tau}+[C_y]\{V_y\}^{\Delta\tau+\tau}+[C_Z]\{V_z\}^{\Delta\tau+\tau})=\frac{[C]}{\Delta\tau}\{P\}^{\tau}$$