Relativistic Quantum Information: Entanglement, Wormholes and Time Machines

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Abstract: The scope of this technical report is the review of multiple scientific papers regarding Quantum Information and its behavior in the context of relativistic systems that demonstrate unique mathematical properties. Such systems are certrain solutions of the Einstein Field Equations like Scwarzchild's metric, Gödel's metric, closed time-like curves, traversable and non-traversable wormholes, i.e. time machines.

I. INTRODUCTION

Quantum theory and general relativity, while highly successful in their respective domains, are ultimately incompatible. Their mathematical languages are very different as well as their conceptual bases. A unifying theory of quantum gravity is needed. However, due to the vastness of the technical and conceptual gap between general relativity and quantum theory, compounded by the scarcity of experimental evidence in the quantum gravity regime, the theory of quantum gravity is still, even after more than half a century, very much a work in progress. With the beginning of the 21st century, however, an at first seemingly unrelated development has begun to make an impact in the field of quantum gravity: the development of quantum information theory. Quantum information theory is the study of the encoding, transmission and processing of information stored in quantum systems [1]. While quantum information theory was originally discussed mostly in terms of non-relativistic quantum mechanics, recent years have seen increasing research interest in placing quantum information within the more fundamental framework of quantum field theory. The field of Relativistic Quantum Information (RQI) is concerned with the next logical step: to place quantum information theory within the yet more fundamental framework of quantum field theory in curved spacetime. From this still relatively safe ground, explorations are then made into black hole physics, cosmology and into various approaches of quantum gravity. Vice versa, RQI scope lies in applications of the newfound techniques, such as new ways to control entanglement, in the more traditional application areas of quantum information theory: quantum communication, quantum simulation, quantum computing and quantum metrology. On the one hand, RQI aims to extend the applicability of quantum information to regimes in which relativistic effects become relevant. On the other hand, it uses information-related ideas to investigate the fundamental structure of spacetime. RQI is a multidisciplinary research field with far-reaching objectives, going from future large-scale quantum technologies to the understanding of gravity and spacetime at the quantum level. To that extend the necessity of the extensive study of entanglement as part of our relativistic universe is crucial in order to understand better the very fabric of our reality but also to unlock the full potential of quantum information processing and effectively engineer new techniques of computations.

In this review initially the relativistic quantum information framework is being examined. The review is being sectioned in two main chapters: One for relativistic quantum information, how the relativistic motion affects quantum information processing in terms of inertial and non-inertial observers. Also, the properties of quantum entanglement are being discussed in the context of the quantum fluctuated Minkowski vacuum and how entanglement behaves nearby a black hole spacetime. The other chapter refers to the time travel interpretations of quantum mechanics and what properties quantum entanglement displays in curved spacetime. The term time-machines is being used here as solutions of the main mathematical tool of general relativity, the Einstein's Field Equations. Furthermore, two distinct ways of quantum state manipulation with closed time-like curves are being discussed [2] and lastly the conclusions and future work.

II. RELATIVISTIC QUANTUM INFORMATION

Almost every quantum information system studied up to date consists an approximation. The argument for that is the non-relativistic approach of quantum mechanics. As the space applications of quantum technology such as satellites using quantum key distribution (QKD) for communication and quantum metrology protocols are emerging and the technology's realization is imminent, the distances of quantum information protocols' deployments are increasing, and time scales of the quantum operations are shortening [3]. As an outcome, the aforementioned approximation eventually will start to fail [2]. A simple and everyday example that is affecting by gravitational field forces and is operating in space is the Global Positioning System (GPS), a technology based on satellite information transmission. If the under-development system is being ruled by quantum physics, the information theory principles framework must be modified. Further modification is required if the relativity theory is being taken in consideration. The effectiveness of the redefinition of quantum information science in a fully relativistic setting will change the structure of the quantum information protocols by restricting the current limits of their applications and enabling new ones with more profound understanding of the very fabric our universe.

Inertial and accelerated observers

One very fundamental question when it comes to relativity is the reference framework of motion. As the question was set for the quantum theory at its early stage, it arises once more with the study of quantum information theory. A well-defined coordinate system with high consistency is required. Different observers traveling with different constant speeds, i.e. inertial observers, are going to observe different quantum states of the same system of qubits such as entanglement and entropy or purity [4], [5]. At this point it should be mentioned that entropy can be used to quantify the information content of qubit [1]. That being said, a state with zero entropy is a pure state, i.e. there is no extra information we could gain about it. A pure state can be considered one that is maximally entangled [6]. One more property of entanglement when relativistic effects are taken into consideration is that the degree of Bell inequality violation depends on the velocity of the particles. A coupling, that depends on the observer, between the discrete quantum state degree of freedom, such as spin, and the linear momentum degree of freedom [2]. Based on Lorentz transformation the inertial observer will see a different discrete qubit from the one that the stationary observer does. This difference will be analogous to the particle's linear momentum. Furthermore, the Copenhagen interpretation states that the wavefunction before the measurement, i.e. before it collapses, is in a superposition of quantum states or else energy/momentum states. Thus, linear momentum and spin are correlated, and this mixture of these two under the Lorentz transformation is causing different intensity on the entanglement between two discrete quantum states of the system [5]. It is important to mention that if one chooses not to measure the momentum degree of freedom then the strength of the entanglement will have an effect on the reduced density matrix (tracing out the momentum dof) and therefore the purity will not be the same for different observers. This means that the entropy of a single quantum state is not a Lorentz invariant concept [7]. It is possible to encode quantum information which is invariant under Lorentz transformation [8].

On the other hand, for the case of accelerated observers one needs to consider the Unruh-Davies effect [9], [10], i.e. the prediction that an accelerating observer will observe blackbody radiation where an inertial observer would observe none. The Unruh effect is an observer-dependent effect as is the quantum theory of a field, for example an inertial observer would see the flat spacetime vacuum state as completely empty of particles whereas a uniformly accelerating one would observe a thermalisation with particles at the Unruh temperature:

$$T = \frac{a\hbar}{2\pi kc} \tag{1}$$

The Unruh effect is happening because of the entanglement of a quantum field when it is in a vacuum state and due to the inability of quantum information exchange between the uniformly accelerated observer and a prospective observer out of his causal horizon leading to a huge amount of uncertainty which can be translated s the Unruh thermalisation particles, i.e. the entire state vacuum is in pure entangled state. The interaction of accelerating atomic qubits with thermalized Unruh particles will cause decoherence analogous to the acceleration rate [11]. For infinite acceleration we have a completely mixed state. The decoherence can cause a low quantum information protocol performance, i.e. the quantum state transmission of an inertial system would slow down due to the thermal bathing. This renders also the fidelity of the teleportation process reduced. Further, studies have shown that the behavior of the entanglement between the modes of a free field differs when it comes to a fermionic Dirac field and a bosonic scalar one. The outcome of the studies conducted on a electromagnetic field concluded to the preservation of entanglement against the acceleration radiation without affecting it by the Unruh effect [12]. The final conclusion of the uniformly accelerating observer is an unpredicted behavior of entanglement when the Unruh effect is occurring, in many cases the effect degrades entanglement but there are also cases where the sudden revival of entanglement occurs. For non-uniform acceleration the creation and annihilation of particles due to the Unruh effect is requiring the study of symmetries. The geometry of the spacetime that needs to be taken in consideration is the spacelike hypersurfaces where the required symmetries exist. Therefore, for non-uniform acceleration the entanglement is degrading to a finite amount even for bosonic fields [13].

Entanglement in vacuum and black-holes

Entanglement in vacuum

In Minkowski spacetime the quantum vacuum can preserve the entanglement state even for spacelike separated regions intervals, i.e. the spacetime interval is positive (faster than the speed of light travel between two events). We can derive that from Minkowski vacuum in terms of Rindler modes, i.e. quantum state in Rindler coordinates:

$$|vac\rangle_M = \prod_i \sum_{n_i=0}^{\infty} c_i \exp(-\pi n_i \omega_i / a) |n_i\rangle_R |n_i\rangle_L \tag{2}$$

With $|n_i\rangle_R$ is symbolized the right wedge of the Rindler mode and similarly for the left one, and thus the states is entangled since the wedges are non-separable. The requirements of *locality* and *unitarity* in QFT imply that the vacuum must have a non-zero energy associated with it. Locality requires that the quantum fields of a given QFT must satisfy equal-time commutation relations of the form:

$$[\phi(t,x), \psi(t,y)] = i\ \delta^{(3)}(x-y) \tag{3}$$

which naturally introduces an infinite constant term $\sim \delta^{(3)}(0)$ upon calculation (if ϕ and ψ are expressed in terms of their Fourier modes). Furthermore, unitarity requires that the time evolution of any quantum system is such that the sum of the probabilities of all possible outcomes always sums to one. Intuitively this ensures that if one considers all possible outcomes at a given instant, one of them will definitely occur. This implies that operators that evolve quantum systems, which in the case of QFT is the scattering matrix S, are themselves unitary, i.e. they satisfy $S^\dagger S = 1$ [14]. All states of bounded energy demonstrate entanglement between two spacelike regions in the context of quantum field theory [15]. The results in [16] show a distance dependence of entanglement too. If the ratio of the tested probes' distance in specific time interval was less than 1.1 then the entanglement state was disappearing. The aforementioned examined model showed that when two atomic qubits were interacting with the quantum vacuum field by violating the Bell inequalities, a local hidden variable model couldn't be taken in consideration for entanglement states in a vacuum state [17] and that the Feynman propagator has non-zero values in spacelike regions without leading to information propagation faster than the speed of light [18]. In [19], it was shown that an entanglement state can also exist in timelike spacetime intervals for massless quantum fields between future and past in a vacuum state.

Entanglement and black-holes

A black-hole is a region of spacetime surrounded by an event horizon due to an extremely strong gravitational field, i.e. strong spacetime curvature [20]. The metric of Minkowski spacetime is:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \tag{4}$$

with $\mathrm{d}t$ and $\mathrm{d}s$ the infinitesimal time and space interval respectively, where spherical spatial coordinates and units where were used. The amount of matter affects the curvature of spacetime and it produces gravitational forces. That is the case in general relativity when a single particle is examined traveling under the influence of gravity while the distance between two spacetime points is an invariant quantity. This particle will follow the shortest path, geodesic of the spacetime, determined by the metric of the energy-momentum tensor $T_{\mu\nu}$ of a certain matter-energy distribution by solving the Einstein field equations. The Schwarzschild metric is such a solution of the Einstein field equations:

$$\begin{aligned} \mathrm{d}s^2 = (1 - \frac{2M}{r})\mathrm{d}t^2 - (1 - \frac{2M}{r})^{-1}\mathrm{d}r^2, \\ -r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \end{aligned} \tag{5}$$

Based on the current scientific knowledge, anything that passes the black hole's event horizon is unable to escape the gravitational forces, not even light. Likewise, the Unruh effect in non-inertial observer situation that examined in previous chapter the Hawking effect shows that the quantum vacuum that is trapped behind the event horizon of the black hole is entangled with the outside. Thus, based on quantum field theory the black hole should emit thermalised particles at the Hawking temperature:

$$T_H = \frac{\hbar c^3}{8\pi G M k} \tag{6}$$

A mixed thermal state can be observed from those standing behind the event horizon of the black hole, who have no access on the inside and the quantum information that enters the black hole is irreversibly lost [2]. Some very interesting results derived from various studies on the field such as the fact that entangled observers will face decoherence if they travel too close to a black hole [21], [22]. Furthermore, by means of projective measurement an entanglement state can also emerge between two qubits, one close to a black hole and one far away [23], [24]. Another major role on the study of entanglement in the context of Hawking effect plays the recovery of the unitarity of Hawking radiation. The issue can be addressed by means of two major premises in the context of semi-classical approximation of black hole evaporation:

1. The quantum information is encoded in correlations. Quantum correlations are also called entanglement [25]. The quantum information lost in the form of Hawking radiation may be recovered in the entanglement which exists in the quantum field [2], [26], [27], [28].

2. The black holes remnants and the final state projection models of the evaporation evolution. The hypothesis of a remaining part of the black hole survives of the evaporation process and the missing information is contained there [29]. This is similar to the final state projection models where the solution is to place boundary conditions on the final state of the black-hole in order to recover unitarity [2], [30], [31].

A different point of view examined in [32] and [33], is that the loss of information due to the evaporation of the black hole described as quantum mechanical feature of gravitational forces and that a quantum gravity theory would explain better the loss of quantum information inside a black hole. A proposed solution from Hawking himself is also given in [34], proving that at late evaporation stage only the unitary information is preserving path integrals over metrics with trivial topology and that elementary quantum gravity interactions do not lose information or quantum coherence.

III. TIME MACHINES

Time machines in general relativity and quantum mechanics

General relativity and time machines

The mathematical epitome of the Genera Relativity theory is the Einstein field equations. They are ten equations contained in the following tensor equation:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor, or trace-reversed Ricci tensor and is used to mathematically express the curvature of a pseudo-Riemannian manifold, with $R_{\mu\nu}$ the traverse curvature tensor, R the scalar curvature and $g_{\mu\nu}$ the metric tensor which expresses the geometric and causal relation between the points of the Lorentzian manifold, i.e. spacetime, and it is being used to define notions such as time, distance, volume, curvature, angle, and separation of the future and the past. Where Λ is the cosmological constant, i.e. the energy density of vacuum closely related to dark energy and G, c are Newton's gravitational constant and the speed of light respectively. Lastly $T_{\mu\nu}$ is the stress-energy-momentum tensor as a generalization of the stress tensor of Newtonian physics describing the density and flux of energy and momentum. The solutions to these equations are the components of the metric tensor, which specifies the spacetime geometry. The inertial trajectories of particles can be found using the geodesic equation.

In general relativity it is mathematically possible to create time machines that would render time travel into one's own past, or future, feasible in theory. That way an acausal loop would be created referred to as closed timelike curve, i.e. a curvature with negative spacetime interval where one can travel between two events slower than the speed of light. The question remains: how to resolve the paradoxes that come out of this scenario in case of its hypothetical realization.

One major solution of the Einstein field equations is the famous Gödel metric [35]. The term of the spacetime's regions referred to as closed timelike curves is the common name for the aforementioned metric. Its main property is that it enables traveling into one's own past through the trajectory of previous travels. Essentially this effect renders the CTC a time machine. When one considers quantum field theory, certain restrictions will derive which is due to the lack of mathematical compatibility of general relativity with quantum mechanics. The only known natural process that is theoretically predicted to form a wormhole in the context of general relativity and quantum mechanics was put forth by Leonard Susskind in his ER=EPR conjecture [36]. Once more the need for a quantum gravity theory emerges in order to provide a consistent explanation.

Another metric that poses a solution of the Einstein's field equations is the one proposed by Morris and Thorne which is a traversable version of a wormhole [37]. Wormholes as a solution of Einstein's equations existed long before, known as Einstein Rosen Bridges or as abbreviation ER. The first introduction was referring to the wormholes as non-traversable bridges between disparate separated points in spacetime, due to their extreme time evolution and the existence of a horizon in the throat of the wormhole [38], [39]. The metric of Morris and Thorne is static without a horizon and it is the following:

$$\begin{aligned} &\mathrm{d}s^2 = -\exp(2\Phi)\mathrm{d}t^2 - \mathrm{d}r^2 \frac{1}{1 - b(r)/r} + \\ &r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \end{aligned} \tag{7}$$

Where t is the time coordinate of a static observer r is the radial coordinate and θ and ϕ are the polar coordinates, $b(r)$ is the shape function of the wormhole and $\Phi(r)$ is the red-shift function.

This metric also determines what energy-matter distribution is required for generating the wormhole. The result of the estimation is a massive negative energy-matter density within the throat of the wormhole. At this point it should be mentioned that quantum fields can have negative energy densities without any known macroscopic effect though.

The separated regions that are connected with the wormhole are spacelike separated and therefore any potential information transmission would be enabled to faster than light speed. The respective result for timelike (CTC) separated regions would be also inconsistent with the laws of classical physics, i.e. the information would emerge from the other end of the wormhole in a short distance where it started but also at an earlier time rendering effectively the wormhole a time machine.

As a logical outcome of the previous premises regarding time machines, paradoxes arise that are unsolvable with any classical mean. A famous example would be the grandfather paradox where a man travels into the past killing his grandfather thus rendering rationally impossible to be born and preventing his own existence. A different approach would be the assumption of the CTCs existence and then the examination of the decision's consequences would justify in deciding the argument against it. Whereas classical theory might not be enough to justify the results, quantum mechanics might pose to solve this paradoxical behavior.

A quantum mechanical time machine

Various interpretations of quantum mechanics exist in order to provide some explanation on counter-intuitive phenomena [2]. Two major examples, the Feynman electron-positron interpretation [40] or the transactional interpretation of quantum mechanics [41]. In the first, Richard Feynman try to interpret by means of non-relativistic quantum mechanics the electron anti-particle, i.e. positron, where he found that the contribution of all possible paths that it can follow satisfies the Schrödinger's equation which includes the probability for a positron to be an electron traveling backwards in time. The transactional interpretation is explicitly non-local and thereby consistent with tests of the Bell inequality, yet is relativistically invariant and fully causal [41]. The collapse of the state vector, i.e. with probability $P = \psi\psi^*$, is the formation of a transaction which occurs by an exchange of retarded and advanced wave functions. Advanced waves functions have characteristic eigenvalues of negative energy and frequency, and they propagate in the negative time direction, whereas retarded ones propagate forward in time.

There is also the Everett's Relative State Formulation interpretation where attempts to remedy the Copenhagen interpretation are being made [36]. Instead of a wavefunction collapses occurrence, interactions take place that entangled the subsystems, i.e. observers. There is only one system where all observers, i.e. subsystems are part of it and are subjected to the unitarity of quantum mechanics. This complicated network of entanglements is the universe and reversibility is in principle possible for state reformulation [36] and time manipulation as well.

No matter which interpretation one studies, the key element of quantum mechanics is the entanglement. Foe spacelike separated qubits we assume:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1)(|1\rangle_2 + (|1\rangle_1)(|0\rangle_2) \tag{8}$$

For 1, 2 spatial separated points in space where the qubits are. Measuring the first qubit in state *zero* will cause the second qubit to collapse to the state one. To that end faster than light communication would be needed which means that time traveling scenarios could be enabled. Quantum teleportation protocols could be alternatively interpreted as an implicit time machines [42].

In the original postulation of quantum teleportation [43] long range purely non classical Einstein-Podolski-Rosen correlations can contribute to the teleportation occurrence even if superluminal motions are prohibited. Let us assume that Alice wants to share her quantum state with Bob, she creates a qubit state:

$$|a\rangle = \mu|0\rangle + \nu|1\rangle \tag{9}$$

at time t_p, she makes a Bell measurement at time t_m which is projected into one of the four Bell states. Bob is waiting for Alice's projection measurement, his state collapses into the one Alice sends him at time t_s. Alice has already created a two-mode Bell measurement which enables the entanglement between the two halves of the Bell state so Bob receives the on arm Bell state $|\phi^+\rangle$. Depending on the measurement Bob's state collapses into bit flip and/or phase flip operations. The bit flip operation follows$|0\rangle \rightarrow |1\rangle$ and $|1\rangle \rightarrow |0\rangle$ whereas a phase flip operation takes$|0\rangle \rightarrow |0\rangle$ and $|1\rangle \rightarrow -|1\rangle$. Then Bob can receive a classical message from Alice in order to make the necessary corrections and obtain the original state. There is a special scenario/hypothesis that would have Bob producing the same entangle state as the projected Bell measurement which wouldn't require any correction. That scenario is invoking a retrodictive aspect of quantum mechanics [44], which assumes that the state Bob has, is a time-retarded version of Alice's state before she created the Bell pairs [2]. The time-retarded state of Alice's qubit after some time interval τ is $|a\rangle = \mu|0\rangle + \exp(-i\omega\tau)\nu|1\rangle$, whereas the retrodicted entangled state projection qubit state is:

$$\langle a|\phi^+\rangle = \mu^*|1\rangle + \nu^*|0\rangle \tag{10}$$

As a retarded quantum state it is propagating backwards in time and thus we get:

$$\langle a|\phi^+\rangle = \mu^* \exp[-i\omega(t_s - t_p)]|1\rangle + \nu^*|0\rangle \tag{11}$$

The projection of the Equation (11) is the time retarder version that Bob sees on his mode before it has been created by Alice. In that notion teleportation could be assumed as a time machine. A series of well-founded questions arise where one could argue on what would b the outcome of providing Alice with an inconsistent qubit to teleport [42], [2]. A bit flipping prior to the Bell pair creation from Alice could cause for example inconsistent history due to the fact that the bit flipped qubit that Bob would receive in that case would be orthogonal to the one he actually received in the original timeline. Of course the paradox is self-resolved because the probability of receiving the measurement result that corresponds to $|\phi^+\rangle$ is zero. Renormalizing to the self-consistent solutions of the quantum circuit could pose a basis for resolving such paradoxes [2]. An illustration of the examined system can be seen in Figure 1.

Quantum mechanics on closed time-like curves (CTC)

A thought experiment is elaborated in this chapter. The assumption is made that there exist time machines

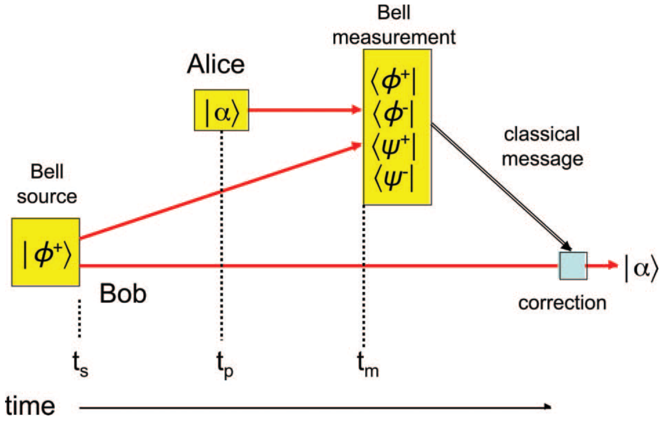

FIG. 1. The aforementioned system of qubit teleportation in discrete times [2].

metrics that could describe such hypothetical systems in our Universe. We examine the behavior of a single quantum state system when is placed in such a CTC and thus rendering it capable of interacting with its own past. The system is a two level circuit of two qubits with a unitary operation U described as an elastic collision between the two qubits, apparently on the same system. Scattered by the collision the first qubit enters a wormhole/time-machine which is also part of the system. The wormhole entrance is the future point (F) in time when the qubit is supposed to be. The time machine however, leads the first qubit to the past (P) when paradoxically it becomes the qubit with which its past self collided elastically, thus creating a close timelike curve. Afterwards the scattered qubit is propagating away from the wormhole and it can finally be measured. The assumption is made that the traversing time is negligible. The most important issue is the consistency of the timeline that is created for every single input state and all unitary operations that can be considered, i.e. its universal consistency. An representation of the quantum circuit is being shown in Figure 2.

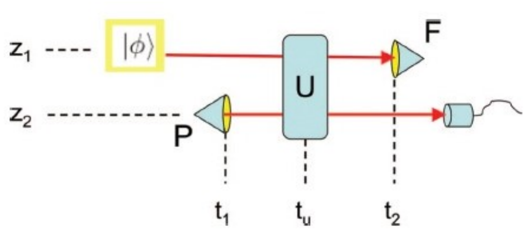

FIG. 2. The quantum circuit of the closed timelike curve formation and the time travel [2].

For resolving this paradoxical behavior two methods are proposed [45]:

1. The standard Feynman path-integral approach of quantum mechanics is applied to the circuit for calculating the probabilities of every possible outcome with a boundary condition defined as the field at (z_1, t_2) matches the field at (z_2, t_1) for $t_1 < t_2$. The peculiar property of the probability summation via integration that is practiced in this method, is that will not add up to one. Thus, the final state must be renormalized in order for the Copenhagen interpretation to be preserved. The final output will be given by the coherent superposition of the two paths $|\phi\rangle = 1/\sqrt{2}(\alpha + \beta)|0\rangle$ [2]. A restriction that arises and seems problematic is due to the renormalization that yields always the ground state $|0\rangle$, which suppresses the paradoxical nature of the experiment. The restriction is the loss of linearity of quantum mechanics because of the state dependency of the renormalization. Also, there is a possibility that the events in the far future would affect the present experiments. In general, it turns out that the outcome of the renormalization for the examined circuit is equivalent to the teleportation time machine circuit examined in previous chapter [46].

2. The other approach is based on Deutsch's solution [47]. Deutsch's method requires that the entire state defined by the reduced density matrix operator ρat (z_1, t_2) matches that at (z_2, t_1) which is always true. The solution steps is to determine initially the state of the second qubit by means of density operator ρ via: $\rho = \mathrm{Tr}_2[U(\rho_{in} \otimes \rho)U^\dagger]$ for the trace being over the Hilbert space of the second qubit. For a solution of ρ we have the output: $\rho_{out} = \mathrm{Tr}_1[U(\rho_{in} \otimes \rho)U^\dagger]$ where now the trace is over the Hilbert space of the first qubit. Since the solution is non-linear function of the input state and non-unitary, to apply the method we need to define first the density operator: $\rho = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$ which satisfies the consistency equation and the same applies for the output state: $\rho_{out} = (|\alpha|^4 + |\beta|^4)|0\rangle\langle 0| + 2|\alpha\beta|^2|1\rangle\langle 1|$, which is also a nonlinear function of the input state and a non-unitary solution since ρ_{out} is diagonal and complete decoherence occurs. The difference with the first proposed method is that the input state is not renormalized which prevents the future from affecting the past outside of the CTC.

The quantum information processing empowerment that is derived from the proposed methods of CTC exploitation is examined by some authors. In [48] is proved that a quantum computer using an adaptive quantum algorithm and by exploiting the non-linearities introduced by the Deutsch solution, would be able to solve NP-complete problems, i.e. cannot be solved by classical computers [2]. Furthermore, in [49] a refined version of the CTC/time-machine circuit is shown to potentially distinguish any finite set of non-orthogonal quantum states, thus violating the uncertainty principle [2].

In the following Figure is illustrated the Deutsch's solution equivalent model of the quantum circuit explained at the beginning of the chapter. The qubit interacts with the CTC created by a wormhole and it is propagating either forward along the path of the initial qubit or back where the qubit from the detector leads to infinite strings of identical interactions with infinite copies of the qubit. Any multiple unitary identical interactions may occur is due to the scattering of the qubit through the wormhole. By solving this equivalent model one can come to a unique solution to the CTC interaction [2].

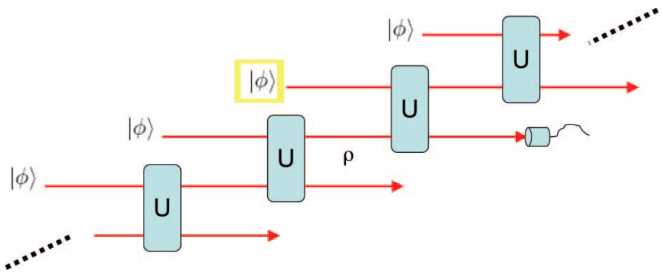

FIG. 3. The equivalent quantum circuit of the time traveling qubit based on Deutsch's solution [2] .

This equivalent derived circuit is addressing successfully two issues: the requirement for consistency of Deutsch's solution is resolved by the circuit's unique solution property due to the conjecture of maximum entropy that is made in his solution [50]. The second one is the mixed input states problem [51] which is resolved by the equivalent circuit [52].

Entanglement and time machines

A more profound examination of the two methods that was analyzed in the previous chapter, concludes that the Deutsch solution could be considered as more self-consistent comparing to the path-integral solution [2]. As an argument to substantiate the aforementioned premise, one would simply have to answer the question of what happens when an entangled pair of qubits are used in the CTC quantum circuit. More precisely, we assume that the state $|\phi^+\rangle$ is being created by Alice at time t_A and she sends one arm to Bob while she keeps the other. Afterwards, at time t_B Bob interacts with the wormhole and sends his entangled qubit into the past through the quantum time-machine circuit with the unitary operation, where in this case it is a CNOT gate. For this case the path integral renormalized state is:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_A|0\rangle_B \tag{12}$$

Alice always measures her state to be diagonal regardless the fact that it's entangled with Bob's qubit, which respectively always found to zero state. In the standard quantum mechanics interpretation of entanglement where relativistic metrics are not taken into consideration, Alice would always measure a maximally mixed state no matter of what Bob holds [2]. The most intrigued and peculiar property of the system is the consequence of the renormalization on the initial state: Alice can predict the future actions of Bob, i.e. if Bob will encounter the CTC or not, only by measuring her state immediately after t_A but long before t_B. The event's causality in relativistic systems, also renders a regressive explanation (which was also examined in the respective subsection): Bob can affect Alice's past [53], [2]. One can conclude that through entanglement, any effort of resolution of acausal encounters with the past trough the CTC would end up to acausal effects without any time consistency.

On Deutsch's solution approach, the computations yields different results when the respective boundary conditions are taken into consideration. The density operator shared by the two parties is $I_A I_B$, indicating a completely decohered initial entangled state and thus loss of unitarity, which is not predicted by any quantum mechanics interpretation. The explanation to that is the loss of quantum information into unobserved modes in the equivalent circuit (Figure 3). In the case of Deutsch's approach, the acausal effects remain inside the CTC epoch due to the fact that Bob's action does not affecting Alice's past anymore, who holds a maximally mixed state, i.e. standard quantum mechanics prediction, when she doesn't have any information about Bob's actions [2].

IV. CONCLUSION

Although there is some valid work on modelling techniques for localised quantum systems in the relativistic framework [54] for a more practical approach, the nature of the concerning research field is purely theoretical and doesn't explicitly affect any practical advancement on quantum computing technologies. That is substantiated by the fact that all the examined scenarios concerning entanglement were using global modes that cannot be understood as physical entities being able to manipulate their state or to measure them. The examined paper with its general reviewing characteristics is showing a need for the formulation of more sophisticated techniques that would enable us engaging with more realistic scenarios [2]. For example, we didn't discuss cases like the existence of absorbing/reflecting boundaries in finite distances or time dependencies on the Unruh and Hawking effects, which were investigated in the context of free fields extending to infinity. The quintessence of any endeavors in that field of study is the finding of an experimental quantum information system in the framework of relativistic effects.

Furthermore, while studying quantum communication protocols, i.e quantum teleportation, and the quantum limits to parameter estimation in curved spacetime with timelike characteristics, it was clear that the path integral form is a quantum field theory based approach with an initial state renormalization dependency, and its efficient is limited by incompatibility of any CTC metric restrictions in the context of standard field theory. On the other hand, the Deutsch's boundary condition approach combined with an adaptive field theory would require various modifications in terms of the spacetime's field commutativity in order to successfully incorporate the density operator boundary condition, although an related approach only makes sense in the context of curved spacetime and close timelike curves [55].

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