Lee D N, 1976 "A theory of visual control of braking based on information about time-to-collision" *Perception* **5** 437 – 459. Original paper reprinted in the appendix.]

## Author's update

## General Tau Theory: evolution to date

### 1 Beginnings

General Tau Theory, as it was later to be called, was born at Ithaca Airport between 1969 and 1970. This was in Jimmy and Jackie Gibson's Perception Lab—the Airport Lab, as it was known. It was a most fitting place to ponder perception and action, for it was in another perception lab on another airfield during the Second World War that Jimmy Gibson gave birth to his ecological theory of perception (Gibson 1950, 1966, 1979)—a theory that has had profound influence in many fields (including airfields). I spent a most happy and fruitful post-doc year in the Airport Lab, discussing with Jimmy, Jackie, Bob Shaw, Jim Farber, John Kennedy, and other colleagues the basic epistemological problem of how the changing pattern of light at the eye the optic flow field—affords information for guiding movement. I learnt the seminal value of Jimmy's ecological approach to perception and action, and this inspired me to develop a mathematical theory of perception in action in the natural world. The first paper on tau (τ) was "Visual information during locomotion" (Lee 1974) a chapter, written mainly at the Airport Lab, for Jimmy's Festschrift. The second, "A theory of visual control of braking based on information about time to collision" (Lee 1976), was written after I moved to Edinburgh University in 1970. It appeared in Perception and the present article is an invited sequel to that paper. In it I shall give a brief account of the evolution of General Tau Theory.

I dedicate this article to the memory of Jimmy and Jackie Gibson, for their friend-ship and for teaching me so much about perception and action; to the memory of my maths teacher at school, S D Hays (Sid), who revealed to me the beauty of maths, which I later found so nicely mirroring the beauty of the natural world; and to all my other friends and colleagues with whom I have worked and who have inspired me over the years. Their names will appear as the story unfolds.

## 2 To stop or collide

The idea that  $\tau$  might play a central role in the perceptual guidance of action first occurred to me when puzzling over how drivers visually control their braking to stop at an obstacle. At first sight, it seemed they had to perceive the changing size, velocity, and deceleration of the gap to the obstacle, and then apply, albeit subconsciously, Newton's equations of motion to calculate how to adjust the pressure on the brake pedal. However, this seemed a highly implausible control procedure. Playing with the equations of motion of a decelerating gap, I came to realise that the only information a driver really needs to prospectively control ongoing braking is  $\dot{\tau}_X(t)$  the rate of change of  $\tau$  of the gap X(t) to the obstacle over time t.  $\tau$  of a gap, X, written  $\tau_X(t)$ , and conventionally taken to be negative when the gap is closing, is the time-to-closure of the gap at its current rate of closure. [In general,  $\tau_X(t)$  is not the actual timeto-closure of the gap, because the rate of closure may change, as when braking.] The equations showed that, if the current deceleration were maintained, the driver would stop before or at the obstacle if  $\dot{\tau}_X(t) \leq \frac{1}{2}$ ; but if  $\dot{\tau}_X(t) > \frac{1}{2}$  collision would ensue. Thus, there is a simple rule for braking without collision: if  $\dot{\tau}_X(t) > \frac{1}{2}$  brake harder, if  $\dot{\tau}_X(t) < \frac{1}{2}$  brake less hard. A more refined procedure is to continuously adjust

braking to follow the ' $\dot{\tau}_X(t)$  constant' braking equation

$$\dot{\tau}_X(t) = b\,,\tag{1}$$

where b is a constant during the braking. If  $b \le \frac{1}{2}$ , stopping will occur right at the obstacle; if  $b > \frac{1}{2}$ , collision will ensue. Other things being equal, the higher the value of b above  $\frac{1}{2}$  the higher will be the final deceleration and collision velocity (figure 1). Drivers, of course, usually aim to avoid collision and so need to keep b constant at less than or equal to  $\frac{1}{2}$ . A study of test drivers stopping at a designated point on the road indicated that on average they kept b = 0.42 (Spurr, reported in Lee 1976). Similar results were later obtained in a simulator, where the driver had no information about the size, velocity, or deceleration of the gap, X, to the stopping point, just information about  $\tau_X(t)$  and  $\dot{\tau}_X(t)$ . On average they kept b = 0.51 (Yilmaz and Warren 1995).

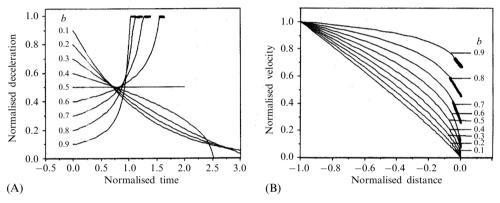
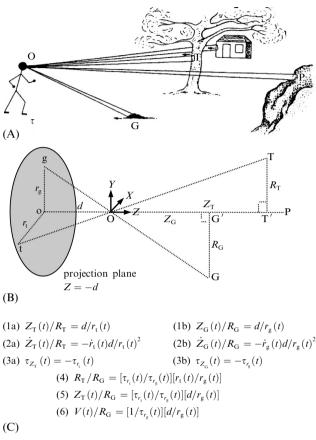


Figure 1. Kinematic profiles when approaching a destination and braking so that  $\dot{\tau}_X(t) = b$  at the different values of b shown. If  $0 < b < \frac{1}{2}$  deceleration monotonically decreases [curves for b = 0.1 - 0.4 in (A)] and the destination is just reached [corresponding curves in (B)]. If  $b = \frac{1}{2}$  deceleration is constant [horizontal line in (A)] and the destination is just reached [b = 0.5] line in (B)]. If  $\frac{1}{2} < b < 1$ , deceleration increases monotonically, as shown by the curves for b = 0.6 - 0.9 in (A) (the thick lines at the top of the curves correspond to reaching a deceleration ceiling). Corresponding curves in (B) show how velocity decreases (figure from Lee et al 1992a).

Closing a gap, X(t), by keeping  $\dot{\tau}_X(t) = b$ , with  $b > \frac{1}{2}$ , results in a controlled collision, which can be functional. Such has been recorded for the distance gap between a hummingbird's bill and the mouth of a feeder tube when aerially docking on it (mean b = 0.71; Lee et al 1991); for the angular gap between a somersaulter's body and the vertical when landing on a trampoline (mean b = 0.61; Lee et al 1992a); for the distance gap between an echolocating bat and a narrow aperture to be flown through (mean b = 0.75; Lee et al 1992b); and for the distance gap between a pigeon's feet and a perch when landing on it (mean b = 0.77; Lee et al 1993).

## 3 Seeing in time thence in space

How are τ and distance visually perceived? I shall present a brief summary of the theory (Lee 1974, 1976, 1980, 1998, 2005; Lee and Lishman 1977). When awake, an animal's head is moving most of the time, if only slightly. This means that the ecological stimulus at the eye is an optic flow field, not a time-frozen optic array or retinal image. To understand some of the visual information provided in an optic flow field, consider forward linear movement of an eye, O, through the environment. Figures 2A and 2B diagram the geometry of the optic flow field with regard to three environmental points: P, on the line of motion of O; G, on the flat ground directly beneath the line of motion; T, an arbitrary point on a tree. Figure 2C gives equations derived from the geometry of the optic flow field. The equations show the relations between



**Figure 2.** Monocular optic flow field resulting from linear movement. (A) Forward linear movement of the eye, O, produces an optic flow field consisting of an expanding bundle of narrow optic cones with their apices at O and their bases on surface texture elements in the environment. (B) A model projection plane perpendicular to the OZ axis is at a fixed distance, d, behind O. The projection plane and the rectangular Cartesian coordinate frame OXYZ move together with velocity V. The points t and g on the projection plane are the images of, respectively, a point T on the tree, and a point G on the ground vertically below the line of motion of O towards P. Lowercase letters designate instantaneous optical distances, uppercase letters designate instantaneous environmental distances. (C) Equations derived from (B), relating environmental measures (uppercase symbols) to optical measures (lowercase symbols). See section 3 for further details.

visually knowable optic distances and velocities (designated by lowercase letters) and visually to-be-known environmental distances and velocities (designated by uppercase letters). Equations (1a) and (1b) in figure 2C, derived from similar triangles, show that there is no information about distance in a time-frozen local region of the optic flow field, only information about the ratio of distances, ie about directions. However, by unfreezing the flow, by introducing time into the description, information emerges from the optic flow field. Equations (2a) and (2b) in figure 2C are the time derivatives of equations (1a) and (1b). Combining equations (1a) and (2a), and (1b) and (2b), yields the equations (3a) and (3b). These show that  $\tau_{z_{\tau}}(t)$ , the  $\tau$  of the environmental gap, OT', is equal to  $-\tau_{r_{\tau}}(t)$ , the negative of  $\tau$  of the optical gap ot [equation (3a)]; and, similarly for the point G [equation (3b)]. Combining equations (a) and (b) reveals how the environmental distances,  $R_{T}$  and  $Z_{T}(t)$ , and the person's velocity, V(t), are optically specified by  $\tau_{r_{\tau}}(t)$ ,  $\tau_{r_{g}}(t)$ ,  $\tau_{r_{g}}(t)$ , in units of the person's eye-height,  $R_{G}$ , above the ground.

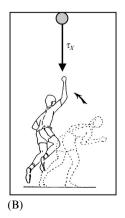
Thus, the primary, directly available information in the optic flow field is about directions and the  $\tau s$  of environmental gaps. There is much experimental evidence that information about  $\tau$  of a gap such as  $\tau_{Z_{\tau}}(t)$  [equation (3a)] can be directly picked up monocularly, independently of information about the size and velocity of closure of the motion-gap (eg Bootsma and Oudejans 1993; Gray and Regan 2004; Kaiser and Mowafy 1993; Kim et al 1993; Regan and Hamstra 1993). Sideways or vertical movement of O likewise generate optic flow fields at O that move with O. The geometry is similar to that in figure 2B, the only difference being that the environmental  $\tau s$  equal the corresponding optical  $\tau s$  rather than their negatives (Lee 1998). In binocular vision there are two optic flow fields, each containing much the same information as the other. In addition, there is information from the  $\tau$  of the binocular disparity gap (Regan and Gray 2004).

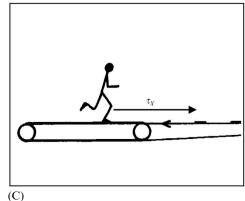
Spatial information about the sizes and velocities of closure of environmental gaps is indirect and second-order: it has to be derived by relating  $\tau$  and directional information from different locations in the optic flow field, as shown in equations (4), (5), and (6) in figure 2C. The natural units for the spatial information are body dimensions such as eye height or leg length, and action dimensions such as stride length or jumpable distance (Lee 1993; von Hofsten and Lee 1994). Squirrels never, and humans very rarely, usefully perceive distance in arbitrary units such as metres.

## 4 Spatio-timing actions

Moving in the world is a 4-D jigsaw puzzle. Actions have to be fitted to 3-D spatial constraints and a time constraint. The animal needs expropriospecific information about its changing relation to its surroundings (Lee 1978). And the information has to be prospective, for planning movement ahead. The theory summarised above suggests that animals may fit actions into 4-D slots largely by using only directional information and  $\tau$  information (which is both spatio-temporal and prospective). This idea has been examined empirically in several ways. For example, a film analysis of plunge-diving gannets (figure 3A) indicated that they started the action of streamlining their wings preparatory to entering the water when the  $\tau$  of the gap, X, to the water surface reached 820 ms on average (Lee and Reddish 1981). When attempting to replicate this study with human divers, no volunteers could be found! Therefore, an upside-down version was run. A football was dropped vertically down from different heights and the participant had to leap and punch it back up (figure 3B). Movement analysis indicated that they geared the flexion/extension of their punching arm and thrusting leg to the changing  $\tau$  of the gap, X, between the ball and the punching place (Lee et al 1983).







**Figure 3.** Fitting actions into spatio-temporal slots: (A) diving gannet, (B) leaping to punch a falling ball, (C) running on 'stepping stones' on a treadmill. See section 4 for details.

The long-jump run-up requires similar spatio-timing skill. Film analysis suggested that athletes visually regulated the last two or three steps to the take-off board by adjusting the durations of the steps to span the  $\tau$  of the gap to the board, much as dancers adjust the durations of their steps to synchronise with music (Lee et al 1982). We hypothesised that the long-jumpers achieved this by regulating the flight times of their steps, by adjusting the vertical impulses they applied to the ground, while keeping the contact times constant (flight time would then be proportional to vertical impulse). The hypothesis could not be adequately tested because the film frame rate (125 Hz) was too low. Therefore, another study was run where the movements of runners' limbs were measured at 312 Hz with Selspot motion capture equipment (Warren et al 1986). Participants ran at about 4 m s<sup>-1</sup> on a treadmill and had to adjust their steps to land on irregularly spaced patches, 'stepping-stones', on an extension to the treadmill belt (figure 3C). The runners principally regulated the flight times of their steps, keeping the contact times about constant, thus supporting our hypothesis. Ski-jumping involves similar skill: the jumper has to fit his jump-action (thrusting from crouch to upright) into a precise spatio-temporal slot ending at the lip of the jump. Jumpers were found to be remarkably accurate, starting the jump-action  $194 \pm 10$  ms before the lip, which is consistent with their use of visual information about the  $\tau$  of the gap to the lip (Lee et al 1982). At a pedestrian level, people often have to fit their crossing of a road into the spatio-temporal slot between moving vehicles. We set up a pretend road alongside a real road to give young children safe physical practice in road-crossing (Lee et al 1984; Young and Lee 1987). They learned quickly, suggesting that the pretend road would be valuable in training road skills.

Several studies indicate that  $\tau$  of the motion-gap between an effector and an object or surface can govern the timing of the initiation of a movement (eg Kaiser and Mowafy 1993; Lee and Reddish 1981; van der Meer et al 1994, 1995), or of neural activity (Wang and Frost 1992), or discrimination of time of arrival (Regan and Hamstra 1993). However, what determines the timing of the initiation of actions is often a complicated issue, which makes doing research on it difficult. For example, it is often not critical when a movement should start, providing it does not start too late. Also, when to start an action such as braking depends on a driver's skill and knowledge of the vehicle's braking potential, as well as accurate perception of the condition of road surface.  $\tau$  of the gap to the obstacle could be involved in such a task, but it is not the only information needed for safe braking. Drivers need to bring along to the task information picked up during previous experience. How a person acquires and uses such information needs to be researched.

## **5** General Tau Theory

Up to the late 1980s,  $\tau$  studies had addressed just visual guidance. I then became interested in echolocation (Griffin 1958). Also, Bernstein's (1967) writings on movement control increased their influence on me, as did the badgering of my long-time friend and colleague at Edinburgh University, Colwyn Trevarthen, to look inside the head! All this led me to wondering how  $\tau$  might generalise. Might  $\tau$  information be picked up by all perceptual systems? Might the closure of any kind of gap (distance, angle, force, etc) be guided by  $\tau$ ? Might movements be coordinated with  $\tau$ ? Might there be movement-guiding  $\tau$ s circulating around the nervous system? Pondering these questions led to the formulation of General Tau Theory, which I shall now briefly summarise.

## 5.1 Action-gaps

A fundamental tenet of General Tau Theory is that all purposeful movement entails controlling the closure of action-gaps. An action-gap is the separation between the current state an animal is in and the goal state to be achieved by action. (The gaps referred

to earlier are examples of action-gaps.) Action-gaps come in different dimensions. Here are some musical examples. Piano playing requires controlling the closure of the distance action-gap between the current position of the key and the aimed-for goal position (the hammer-release point). Looking at a note on the keyboard requires closing the angular action-gap between the current direction of the gaze and the direction of the note. Singing legato involves controlling the closure of the pitch action-gap between successive tones. Increasing the loudness of a tone on the violin involves controlling the closure of the pressure action-gap between the bow and a string. However, although action-gaps themselves come in different dimensions (distance, angle, pitch, pressure, etc), this does not mean that they are sensed, or measured by the person or animal in different dimensions. On the contrary, since movement frequently entails rapidly controlling the closure of many action-gaps of different dimensions at the same time, as when singing at the piano, it would be efficient, and make sense from an evolutionary point of view, if all action-gaps were measured in the same way.

## 5.2 $\tau$ —The measure of an action-gap

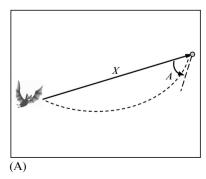
A second fundamental tenet of the theory is that the only measure used by animals or humans when controlling the closure of an action-gap of any kind is  $\tau$ , the time-to-closure of the action-gap at the current rate of closure. The theory shows that, in principle, controlling the closure of any action-gap only requires  $\tau$  information; information about, eg, the size and speed of closure of the action-gap is not needed. Furthermore,  $\tau$  of an action-gap is, in principle, directly perceptible in any sensory modality (see section 5.4). Thus, there is reason to assume that  $\tau$  is fundamental information underpinning movement control.

#### 5.3 $\tau$ -coupling

The theory posits that a principal way that  $\tau$  is used in guiding the closure of action-gaps is through the coordinating principle of  $\tau$ -coupling, whereby the  $\tau$ s of two action-gaps, X and Y, are kept in a constant ratio,  $k_{X,Y}$ , during the closure of the action-gaps. That is, the  $\tau$ -coupling equation

$$\tau_X(t) = k_{X,Y} \tau_Y(t) \tag{2}$$

is followed when closing the action-gaps. This ensures that the action-gaps, X and Y, reach closure simultaneously.  $\tau$ -coupling has been measured in the following two studies, for example. The first study analysed the movement of echolocating bats flying to land on a perch, and found that the distance action-gap, X, between the bat and the perch was  $\tau$ -coupled with the angular action-gap, A, between the current direction of the perch from the bat and the required direction of landing (figure 4A,



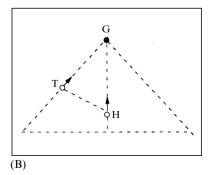


Figure 4.  $\tau$ -coupling action-gaps: (A) bat landing on a perch (action-gaps X and A were  $\tau$ -coupled); (B) interception task where the hand cursor, H, was moved vertically up the computer screen to stop in the goal zone, G, just as the target cursor, T, which moved in a straight line from the bottom left or bottom right of the screen with unpredictable constant acceleration, deceleration, or speed, reached G (motion-gaps HT and HG were  $\tau$ -coupled). See section 5.3 for details.

and Lee et al 1995). In the second study, humans intercepted with a hand cursor, H, a moving target, T, in a designated goal zone, G. The action-gap, HT, was found to be  $\tau$ -coupled with the action-gap, HG (figure 4B and Lee et al 2001).

### 5.4 Sensing through $\tau$ -coupling

Wherever there is a power law relation of the form  $S(t) = CX(t)^{k_{x,s}}$ , for constants C and  $k_{x,s}$ , between an action-gap, X, and a corresponding sensory-gap, S, in a sensory flow field (as is the case in all known sensory modalities—Lee 1998) it follows that

$$\tau_X(t) = k_{XS} \tau_S(t) \,. \tag{3}$$

This is the  $\tau$  sensory specification equation, which shows when the  $\tau$  of an action-gap is in principle directly perceptible and so offers a general method for studying perceptual systems.

## 5.5 Intrinsic $\tau_G$ -guidance of movement

In section 2 I showed how the deceleration phase of closing an action-gap can be intrinsically guided by following the equation  $\dot{\tau}_X(t) = b$  for a constant b. This procedure is appropriate when braking at the end of a protracted movement, as when approaching red traffic lights. However, for many action-gaps (eg when reaching), the movement starts at rest, accelerates to a peak velocity and then immediately decelerates to the end of the movement. General Tau Theory posits that such movements are intrinsically guided by  $\tau$ -coupling the action-gap, X(t), onto an intrinsically generated guiding action-gap, G(t), by following the  $\tau_G$ -guidance equation

$$\tau_X(t) = k_{X,G} \tau_G(t). \tag{4}$$

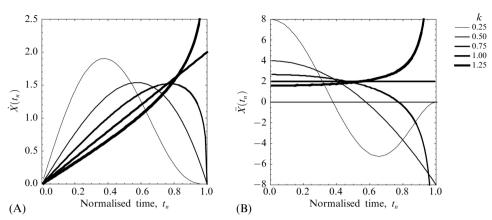
 $\tau_G(t)$  is a function of time t and specifies the  $\tau$  of a gap that closes under constant acceleration, as, eg, the gap between the proverbial falling apple and Newton's head. Since gravity is a major influence shaping animal movement,  $\tau_G(t)$  is a function rooted in the ecology of animals.  $\tau_G(t)$  is derived from Newton's equations of motion as

$$\tau_G(t) = \frac{1}{2}(t - T_G^2/t), \tag{5}$$

where  $T_G$  is the duration of the  $\tau_G$ -guidance, and time, t, runs from 0 to  $T_G$ . It is assumed that  $\tau_G$ -guides are expressed in the brain in the pattern of flow of electrical energy through ensembles of neurons. The theory posits that skilled closings of action-gaps from rest to rest follow equations (4) and (5). In equation (4),  $k_{X,G}$  is constant during a movement and its value for each movement is assumed to be set by the brain. The value of  $k_{X,G}$  determines kinematic aspects of the movement, including the shapes of the velocity and acceleration profiles (figures 5A and 5B). The higher the value of  $k_{X,G}$  the more delayed is the peak velocity, the shorter is the deceleration phase, the higher is the deceleration at the end, and so the more 'oomph' there is in the movement. (Imagine being driven to red traffic lights with a high  $k_{X,G}$ !) A number of experiments on skilled movements in both humans and animals have demonstrated  $\tau_G$ -guidance (Lee 2005). Some of these will be described later. Unskilled movements, on the other hand, are usually not  $\tau_G$ -guided; practice is apparently required to hone  $\tau_G$ -guidance of movement.

#### 5.6 Summary of General Tau Theory

Figure 6 presents a diagrammatic summary of some of the main points of General Tau Theory. The pianist is continuously picking up tau information (through kinaesthesis and touch, and vision, and/or hearing), about the action-gaps between her fingers and the hammer-release points of the keys. This sensory  $\tau$  information about her actual movements is continuously compared with intrinsically generated  $\tau$  information that prescribes how her fingers should be moving. On the basis of this comparison the movement of her fingers are continually adjusted by sending appropriate  $\tau$  information to her muscles.



**Figure 5.** Effect of varying the parameter  $k_{X,G}$  (labeled k in the figure) on: (A) the changing speed of closure, X(t), and (B) the changing acceleration of closure, X(t), of a movement gap, X(t), that is closing from rest and is governed by the  $\tau_G$ -guidance equation  $\tau_X(t) = k_{X,G} \tau_G(t)$ .

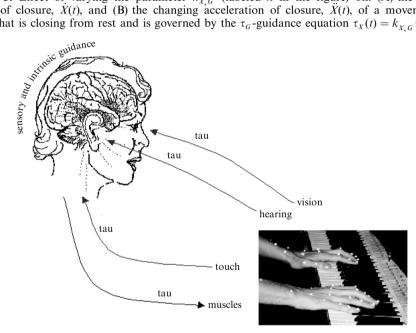
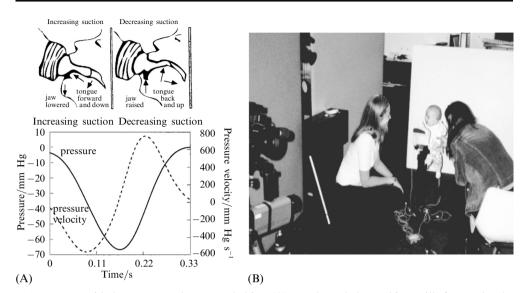


Figure 6. Diagrammatic summary of General Tau Theory. (The markers on the hands of the female pianist are for recording the movements.) See section 5.6.

## 6 $\tau$ in development

### 6.1 Suckling

Figure 7A illustrates a neonatal example of  $\tau_G$ -guidance—a full-term newborn baby sucking milk out of a bottle (Craig and Lee 1999). The graphs show how the intra-oral pressure (recorded via a catheter passing through the teat into the mouth) and its time derivative, pressure velocity, varied smoothly and systematically during a typical suck. As the baby drew in milk the pressure action-gap closed to the lowest pressure, and as the baby relaxed sucking the action-gap opened ready for the next suck. The pressure actiongap ( $\tau$  calculated as pressure/pressure-velocity) was tightly  $\tau_G$ -guided—as tightly as an adult's reaching (Lee et al 1999). Thus babies can be born already able to use  $\tau_G$ -guidance in suckling. This does not imply, of course, that they can at birth  $\tau_G$ -guide all their movements. They clearly cannot, and require practice to hone the ability. In the case of suckling, they might have practised  $\tau_G$ -guidance in the womb, as is suggested by ultrasound recordings of foetuses sucking their fists and the ambiotic fluid.



**Figure 7.**  $\tau_G$ -guided movements by young babies. (A) Newborn baby sucking milk from a bottle. From Craig and Lee (1999). See section 6.1 for details. (B) 10-week-old infant gesturing with arms and legs to music. From Lee (2005). See section 6.2 for details.

#### 6.2 Gesturing

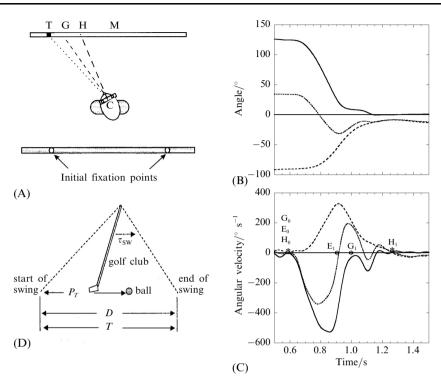
Gestural movements of arms and legs have also been observed to be  $\tau_G$ -guided at a young age. Figure 7B shows a 10-week-old baby gesturing with arms and legs when music was playing. The baby was supported in a special seat with its chest resting against the seat and its arms and legs free to move—like being supported against a shoulder. The movements of its arms and legs, recorded by motion-capture cameras, were found to be tightly  $\tau_G$ -guided (Lee 2005). This  $\tau_G$ -guidance of gestures contrasts with the imprecise and jerky guidance of the hands in infants, younger than about 18 weeks, when reaching for stationary and moving objects (von Hofsten 1983). This difference might be due to the different perceptual information used: the perceptual information for guiding gestures probably comes mainly through the receptors in the joints and muscles, whereas in reaching for a seen object visual information plays a significant role.

## 7 $\tau_G$ -guidance in adult skills

## 7.1 Guiding gaze

Action requires moving the perceptual systems to pick up information for guiding the action. In the case of vision, we see "with the eyes in the head on the shoulders of a body that gets about", as Gibson (1979) elegantly put it. Madeleine Grealy, Ben Schögler, and I (Lee 2005) measured the coordination of gaze, eye, and head movements in an experiment where participants looked over their shoulder and then, keeping their shoulders still, turned to look at a target (previously unseen) moving along a horizontal track in front of them (figure 8A). Figure 8B shows typical plots of the gaze–target, eye–head, and head–shoulders angles. Figure 8C shows the corresponding angular velocity plots, where  $G_0$ ,  $E_0$ ,  $H_0$ , and  $G_1$ ,  $E_1$ ,  $H_1$  mark, respectively, the starts and ends of the initial, principal gaze, eye, and head movement segments. (The starts and ends were when the angular velocity just exceeded 5% of the peak velocity on the trial.) Each segment was tightly  $\tau_G$ -guided (mean  $r^2 = 0.976$  over ten trials).

<sup>(1)</sup> Study carried out by K Hooker and J Perkins for their Honours Dissertations in Psychology, 2003, Edinburgh University, Edinburgh, Scotland, UK.



**Figure 8.**  $\tau_G$ -guidance of gaze, eye, and head when turning to look at a moving object. (A) Experimental setup. The participants first fixed their gaze over their shoulder and then turned to stabilise gaze on a horizontally moving target in front. CH is head direction, CG is gaze direction, and CT is target direction, relative to the shoulders. (B) Typical plots of gaze—target (solid line), eye—head (dotted line), and head—shoulders (dashed line) angles during the gaze shift. (C) Corresponding angular velocity graphs. From Lee (2005). See section 7.1 for details. (D)  $\tau_G$ -guidance of putting. From Craig et al (2000b). See section 7.2 for details.

The segments of the head, eye, and gaze movements overlap in time like the parts in contrapuntal music. The eye-head movement started  $7\pm 5$  ms before the gaze/target movement and  $43\pm 21$  ms before the head-shoulders movement, and the respective movement durations were progressively longer:  $274\pm 22$  ms for eye-head,  $331\pm 26$  ms for gaze-target, and  $535\pm 59$  ms for head-shoulders. The principal movements of eye-head, gaze-target, and head-shoulders were each followed by a series of stabilising movements (figures 8B and 8C), the first of which were again tightly  $\tau_G$ -guided, with a mean  $r^2$  of 0.974. (Subsequent stabilising movements were too small to measure accurately.) As with the principal movements, the first stabilising movements started at different times and were of different durations:  $214\pm 51$  ms for eye-head,  $89\pm 26$  ms for gaze-target, and  $308\pm 150$  ms for head-shoulders. In sum, the results suggest that gaze was  $\tau_G$ -guided to the target, while the synergistic movements of the eyes in the head and the head on the shoulders were independently  $\tau_G$ -guided over different time intervals to goal positions within bodily constraints.

## 7.2 Putting

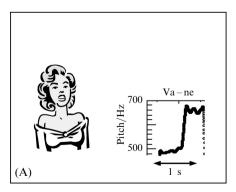
To investigate how golfers control the length of their putt, the club and ball movements of ten low-handicap players were analysed (Craig et al 2000b). They all tightly  $\tau_G$ -guided the movement of the club from the start to the end of the swing and increased the length of the putt mainly by increasing the length, D, of the swing and the  $k_{\chi,G}$  value in the  $\tau_G$ -guidance equation (4), while keeping the duration, T, of the swing and the relative position,  $P_T$ , of the ball in the swing quite constant (figure 8D).

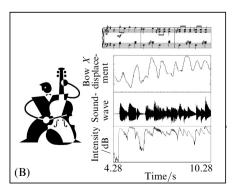
## 7.3 $\tau_G$ In musical expression

We thought  $\tau_G$ -guidance might figure in the control of musical expression, because expression is created by movements on an instrument or in the voice, and our studies had indicated that skilled movements are  $\tau_G$ -guided. The manner in which an actiongap, X, closes under  $\tau_G$ -guidance [equations (4) and (5)] is determined by the values of three parameters:  $k_{\chi G}$ , which specifies the shape of the velocity-of-closure profile of an action-gap (figure 5A);  $A_X$ , which specifies the initial amplitude, or size, of the action-gap; and  $T_G$ , which determines the duration of closure of the action-gap. Our working hypothesis was that (i) singers and players  $\tau_G$ -guide the closure of actiongaps, X, in their movements; (ii) they regulate the values of the parameters  $k_{XG}$ ,  $A_X$ , and  $T_G$  to convey expression; (iii) their movements generate  $\tau_G$ -guided sounds with related, though not necessarily identical,  $k_{X,G}$ ,  $A_X$ , and  $T_G$  parameter values; (iv) these  $k_{X,G}$ ,  $A_X$ , and  $T_G$  values in the sound can be perceived by a person who can then move to the music, making  $\tau_G$ -guided movements with related  $k_{X,G}$ ,  $A_X$ , and  $T_G$  values. In short,  $k_{XG}$ ,  $A_X$ , and  $T_G$  are expressive parameters used in musical interaction.  $k_{X,G}$  determines the shape of the velocity of flow of the movement or sound, and so, inter alia, the 'oomph' of the movement or sound (section 5.5).  $A_X$  determines the amplitude of the movement or sound, and so, inter alia, the loudness of the sound.  $T_G$  determines the duration of the movement or sound, and so, inter alia, pitch, rhythm, and timing (Craig et al 2005; Schögler et al 2008).

## 7.4 Singing 'pitch-glides' expressively

A singer's voice normally glides through the pitches separating adjacent notes of the song, unless they are singing staccato. These 'pitch-glides' were analysed in the separate voices in performances of a Pergolesi duet, 'Vanne, Vale, Dico Addio' (figure 9A and Schögler et al 2008). (2) After the performance, the singers marked on the score the pitch-glides they considered to be 'emotionally significant' or 'emotionally neutral'. It was subsequently found that all the pitch-glides (strictly speaking, fundamental frequency glides) were tightly  $\tau_G$ -guided (mean percentage of variance explained by  $\tau_G$ -guidance was greater than 98%); and the 'emotionally significant' pitch-glides had a statistically significantly higher mean value of  $k_{X,G}$ , than the 'emotionally neutral' ones. Thus the value of the  $\tau_G$ -guidance parameter,  $k_{X,G}$ , which describes the shape of the pitch-glide, was modulated in the direction of increased 'urgency' (section 5.5) to add emotional expression.





**Figure 9.**  $\tau_G$ -Guidance of movement and sound in musical expression. (A) Singing 'pitch-glides' (fundamental frequency glides) between adjacent tones. See section 7.4 for details. (B) Bowing 'intensity-glides' when 'attacking' tones. See section 7.5 for details.

<sup>&</sup>lt;sup>(2)</sup> Singing study carried out by P Biggs and E Ward for their Honours Dissertations in Psychology, 2004, Edinburgh University, Edinburgh, Scotland, UK.

## 7.5 Bowing 'intensity-glides' with feeling

How the bow is moved across the strings of a bass or other stringed instrument affects, inter alia, the 'attack' on the tone being played. The attack, the initial phase of the tone, includes an 'intensity-glide', a rapid continuous increase in sound intensity to a peak level (figure 9B). The attack affects the perceived character of the whole tone (Galembo et al 2001). Thus, a skilled musician varies how a tone is attacked to produce different acoustic effects. To investigate this, a professional bass player was asked to bow several times a key phrase from Tchaikovsky's *The Dance of the Sugar Plum Fairy* in two moods, 'happy' and 'sad' (Schögler et al 2008). The intensity-glides of the attacks on the tones of the piece, and the 'bow-glides' that produced them, were both significantly  $\tau_G$ -guided. Furthermore, both the shape parameter,  $k_{X,G}$ , and the duration parameter,  $T_G$ , of the  $\tau_G$ -guided intensity-glides and bow-glides were significantly higher in value in the 'sad' rendition of the tune than in the 'happy' rendition, indicating that the two parameters were being varied together to modulate the mood of the music.

## 8 Medical applications of General Tau Theory

In the recently established Perception – Movement – Action Research Centre (PMARC), which is based at Edinburgh University with an international membership, a group of us is working on applying General Tau Theory to medically related problems. Parkinson's disease is one area of potential application. A striking aspect of the disease is paradoxical movement: a person may suffer extreme slowness of movement (bradykinesia) and/or freezing (akinesia) when attempting to make self-initiated movements like stepping off across a clear floor or reaching for a cup, but these symptoms can largely disappear when the person is in a more dynamic, and apparently more demanding situation, such as walking downstairs or catching a ball. In the first case the persons are more reliant on intrinsic  $\tau_G$ -guidance of their limb movements, whereas in the latter case their limb movements can be  $\tau$ -coupled to extrinsic action-gaps such as the gap between themselves and the next stair or the gap between the approaching ball and the catching place. Therefore, if they could be 'injected' with a  $\tau_G$ -guide it might help alleviate their problems with self-initiated movements. We have devised a sonic  $\tau_G$  movement guide that produces a  $\tau_G$ -guided 'pitch-glide' (fundamental frequency glide) between two tones, following the formula  $\tau_P(t) = k_{P,G} \tau_G(t)$ , where P(t) stands for the changing pitch-gap. The duration  $T_G$  and the  $k_{P,G}$  shape parameter of the pitch-glide can be regulated to match the duration and style of the action to be aided—eg walking steps. Typically, the patients listen to the sonic guide, remember it and later play it back in their heads when performing an act. Results so far indicate that walking can be improved. Currently we are investigating whether the sonic  $\tau_G$  movement guide can also aid ataxic patients.

A further potential application of General Tau Theory that we have recently been pursuing (under an EU grant) is the diagnosis of neuro-developmental disorders in infants, which we hope will eventually lead to new methods of helping the babies. We are concentrating on intersubjective movements of infants with a parent, from birth onwards, including vocalising, gesturing, and suckling. Following from sections 6.1 and 6.2, we measure, in healthy infants, well-formed  $\tau_G$ -guided movements in suckling, gesturing, and vocalising that could be used diagnostically with babies considered neuro-logically at risk (Craig and Lee 1999; Craig et al 2000a). A variant of the sonic  $\tau_G$  movement guide may eventually, we hope, prove useful in helping afflicted babies improve their movement control.

<sup>&</sup>lt;sup>(3)</sup> Bass study carried out by B Harvey and J Scriven for their Honours Dissertations in Psychology, 2004, Edinburgh University, Edinburgh, Scotland, UK.

Another project (supported by a NESTA grant) was aimed at devising a musical instrument based on the  $\tau_G$  principle (sections 7.3–7.5) that would enable severely disabled children to create expressive musical sounds and thereby improve their movement control (see also Lee et al 1997).

At the neural level, we have developed a theory of neural direction of movement founded on the  $\tau_G$  principle (Lee et al 2009), based on behavioural and neural data. And at the cellular level we have found evidence for  $\tau$ -guidance of movement of cells (paramecia) in electric fields (Delafield-Butt et al 2009).

Results of the projects will be posted on the PMARC website: http://www.pmarc.ed.ac.uk.

**Acknowledgments.** The research reported here was supported by grants from the BBSRC, ESRC, MRC, NATO, SERC, SOHHD, SRC, University of Minnesota, and US Aerospace Medicine. The writing of the paper was supported by a Leverhulme fellowship.

#### David N Lee

Perception-inAction Laboratories, University of Edinburgh, Edinburgh EH8 8AQ, Scotland, UK

#### References

- Bernstein N A, 1967 The Co-ordination and Regulation of Movements (Oxford: Pergamon Press)
- Bootsma R J, Oudejans R R D, 1993 "Visual information about time-to-collision between two objects" Journal of Experimental Psychology: Human Perception and Performance 19 1041 – 1052
- Craig C M, Delay D, Grealy M A, Lee D N, 2000b "Guiding the swing in golf putting" *Nature* **405** 295 296
- Craig C M, Grealy M A, Lee D N, 2000a "Detecting motor abnormalities in pre-term infants" Experimental Brain Research 131 359 – 365
- Craig C M, Lee D N, 1999 "Neonatal control of nutritive sucking pressure: evidence for an intrinsic tau-guide" *Experimental Brain Research* 124 371 382
- Craig C M, Pepping G-J, Grealy M A, 2005 "Intercepting beats in pre-designated target zones" Experimental Brain Research 165 490 – 504
- Delafield-Butt T, Pepping G-J, McCaig C D, Lee D N, 2009 "How a cell prospectively guides its movement to a goal" (submitted)
- Galembo A, Askenfelt A, Cuddy L, Russo F, 2001 "Effects of relative phase on pitch and timbre in piano bass range" *Journal of the Acoustical Society of America* **110** 1649 1666
- Gibson J J, 1950 The Perception of the Visual World (Boston, MA: Houghton Mifflin)
- Gibson J J, 1966 The Senses Considered as Perceptual Systems (Boston, MA: Houghton Mifflin)
- Gibson J J, 1979 The Ecological Approach to Visual Perception (Hillsdale, NJ: Lawrence Erlbaum Associates)
- Gray R, Regan D, 2004 "The use of binocular time-to-contact information", in *Time to Contact* Eds H Hecht, G J P Savelsburgh (Amsterdam: Elsevier) pp 304-325
- Griffin D R, 1958 Listening in the Dark (New Haven, CT: Yale University Press)
- Hofsten C von, 1983 "Catching skills in infancy" Journal of Experimental Psychology: Human Perception and Performance 9 75-85
- Hofsten C von, Lee D N, 1994 "Measuring with the optic sphere", in *Perceiving Events and Objects* Eds G Jannson, S S Bergstrom, W Epstein (Hillsdale, NJ: Lawrence Erlbaum Associates) pp 455-467
- Kaiser M K, Mowafy L, 1993 "Optical specification of time-to-passage: Observers' sensitivity to global tau" Journal of Experimental Psychology: Human Perception and Performance 19 1028 – 1040
- Kim N G, Turvey M T, Carello C, 1993 "Optical information about the severity of upcoming contacts" *Journal of Experimental Psychology: Human Perception and Performance* **19** 179 193
- Lee D N, 1974 "Visual information during locomotion", in *Perception: Essays in Honor of James J. Gibson* Eds R B MacLeod, H L Pick Jr (Ithaca and London: Cornell University Press) pp 250-267
- Lee D N, 1976 "A theory of visual control of braking based on information about time to collision" Perception 5 437 – 459
- Lee D N, 1978 "The functions of vision", in *Modes of Perceiving and Processing Information* Eds H L Pick Jr, E Saltzman (Hillsdale, NJ: Lawrence Erlbaum Associates) pp 159-170
- Lee D N, 1980 "The optic flow-field: the foundation of vision" *Philosophical Transactions of the Royal Society of London, Series B* **290** 169 179
- Lee D N, 1993 "Body environment coupling", in *Ecological and Interpersonal Sources of Self-knowledge* Ed. U Neisser (Cambridge: Cambridge University Press) pp 43 67

- Lee D N, 1998 "Guiding movement by coupling taus" Ecological Psychology 10 221 250
- Lee D N, 2005 "Tau in action in development", in *Action as an Organizer of Learning and Development* Eds J J Rieser, J J Lockman, C A Nelson (Hillsdale, NJ: Lawrence Erlbaum Associates) pp 3-49
- Lee D N, Craig C M, Grealy M A, 1999 "Sensory and intrinsic coordination of movement" Proceedings of the Royal Society of London, Series B 266 2029 – 2035
- Lee D N, Davies M N O, Green P R, Weel F R van der, 1993 "Visual control of velocity of approach by pigeons when landing" *Journal of Experimental Biology* **180** 85–104
- Lee D N, Georgopoulos A P, Clark M J, Craig C M, Port N L, 2001 "Guiding contact by coupling the taus of gaps" *Experimental Brain Research* 139 151 159
- Lee D N, Georgopoulos A P, Lee T M, Pepping G-J, 2008 "A neural function directing movement" (submitted)
- Lee D N, Hofsten C von, Cotton E, 1997 "Perception in action approach to cerebral palsy", in *The Neurophysiology and Neuropsychology of Motor Development* Eds K Connelly, H Forrsberg (London: MacKeith Press) pp 257–285
- Lee D N, Lishman R, 1977 "Visual control of locomotion" Scandinavian Journal of Psychology 18 224-230
- Lee D N, Lishman J R, Thomson J A, 1982 "Regulation of gait in long jumping" *Journal of Experimental Psychology: Human Perception and Performance* **8** 448–459
- Lee D N, Reddish P E, 1981 "Plummeting gannets: a paradigm of ecological optics" *Nature* **293** 293 294
- Lee D N, Reddish P E, Rand D T, 1991 "Aerial docking by hummingbirds" *Naturwissenschaften* **78** 526 527
- Lee D N, Simmons J A, Saillant P A, Bouffard F, 1995 "Steering by echolocation: a paradigm of ecological acoustics" *Journal of Comparative Physiology A* **176** 347 354
- Lee D N, Weel F R van der, Matejowski E, Holmes P, Pettigrew J D, 1992b "Common principles of guidance by echolocation and vision" *Journal of Comparative Psychology A* **171** 563 571
- Lee D N, Young D S, McLaughlin C M, 1984 "A roadside simulation of road crossing for children" *Ergonomics* **27** 1271 1281
- Lee D N, Young D S, Reddish P E, Lough S, Clayton T M H, 1983 "Visual timing in hitting an accelerating ball" *Quarterly Journal of Experimental Psychology A* **35** 333 346
- Lee D N, Young D S, Rewt D, 1992a "How do somersaulters land on their feet?" Journal of Experimental Psychology: Human Perception and Performance 18 1195-1202
- Meer A L H van der, Weel F R van der, Lee D N, 1994 "Prospective control in catching in infants" *Perception* 23 287 302
- Meer A L H van der, Weel F R van der, Lee D N, Laing I A, Lin J P, 1995 "Development of prospective control of catching in premature, at risk infants" *Developmental Medicine and Child Neurology* 37 145 158
- Regan D, Gray R, 2004 "A step by step approach to research on time-to-contact and time-to-passage", in *Time to Contact* Eds H Hecht, G J P Savelsburgh (Amsterdam, Elsevier) pp 172 228
- Regan D, Hamstra S, 1993 "Dissociation of discrimination thresholds for time to contact and for rate of angular expansion" *Vision Research* 33 447 462
- Schögler B, Pepping G-J, Lee D N, 2008 "TauG-guidance of transients in expressive musical performance" *Experimental Brain Research* **198** 361 372
- Wang Y, Frost B, 1992 "Time to collision is signalled by neurons in the nucleus rotundus of pigeons" Nature 356 236 – 238
- Warren W H Jr, Young D S, Lee D N, 1986 "Visual control of step length during running over irregular terrain" *Journal of Experimental Psychology: Human Perception and Performance* 12 259 266
- Yilmaz E H, Warren W H, 1995 "Visual control of braking—A test of the tau dot hypothesis" Journal of Experimental Psychology: Human Perception and Performance 21 996 – 1014
- Young D S, Lee D N, 1987 "Training children in road-crossing skills using a roadside simulation" Accident Analysis and Prevention 19 327 – 341

#### Comments

## The (current) future is here!

Even though David Lee's contribution to *Perception* focused on the visual control of braking in driving, the formal description of 'information about time-to-collision' it provided has inspired work in many different areas. Indeed, according to ISI's Web of Science, between 1976 and 2006, theoretical and empirical articles citing this paper have been published in over 100 different scientific journals. Lee's 1976 contribution is truly seminal, in the sense that it posited that we directly perceive the time remaining until something will happen. In this commentary I will focus on some of the implications of this revolutionarily simple idea.

Useful visually detectable information for action is contained not in the static optic array but in the dynamically changing optic flow. Unfreezing the flow allows the emergence of information in the Gibsonian sense, where a property of the optic flow specifies a property of the environment – agent system (EAS). Detecting the specifying informational property allows perception of the specified EAS property without the need of calling on intermediary processes. Within this framework, in 1976 Lee formally demonstrated that the optic flow contains information about the time remaining until collision (the optic variable 'tau') and information about the sufficiency of current deceleration to avoid collision (the optic variable 'tau-dot', the rate of change of tau). While a large number of studies have since shown that human and other animals are indeed sensitive to these informational quantities, the debate continues on how these informational quantities are used in the regulation of action. Undoubtedly because of his background in mathematics, throughout his career Lee has demonstrated a natural penchant for the simplest possible solution (initiate an action at a constant value of tau, decelerate by maintaining a constant value of tau-dot, regulate action by maintaining a constant ratio of taus). While experimental evidence suggests that the mechanisms of information-movement coupling might be somewhat more complicated, the ongoing debate on the use of information in the regulation of action should not let us forget the profound contribution of Lee's analysis.

The optic variable tau specifies the time remaining until collision, if the current speed of approach is maintained. The optic variable tau-dot specifies whether a collision will occur if current deceleration is maintained. Thus, these optic variables, contained in the present pattern of change of the ambient optical structure, provide veridical information about the future: if things do not change, this is what is going to happen. Crucially, this information about the future is not inferential. It does not result from an inspired or educated guess. It is directly available in the optic flow.

What will happen if things do not change is what I refer to as the 'current future'. Information about the current future allows prospective control—control ahead of time. Of course, more often than not prospective control will entail that things do change during the course of action and the future will be different. Nevertheless, this does not imply that (information about) the current future is approximative or not veridical. Collision can be avoided precisely because the optic flow provides information about time to collision if the current speed of approach is maintained. This information about the current future establishes the required temporal relation that can subsequently be influenced so as to fulfil the action goal—attain the desired future. Revolutionarily simple and simply revolutionary!

## Reinoud J Bootsma

UMR 6233 Institut des Sciences du Mouvement E. J. Marey, Université de la Méditerranée, 163 avenue de Luminy—CP 910, Marseille, France; e-mail: reinoud.bootsma@univmed.fr

## Lee's tau operator

David Lee's research on the optical variables that specify time to collision (TTC), of which his 1976 *Perception* paper is but just one example, has been seminal to several different lines of research that range from control of insects landing on leaves (Wagner 1982), humans catching balls (Savelsbergh et al 1991), braking in driving simulators (Li and Milgram 2005a, 2005b, 2007), to the identification of neurons computing tau (Sun and Frost 1998; Wang and Frost 1992).

In studies directly involved with braking of vehicles Sun and Frost (1997), and Li and Milgram (2005a, 2005b, 2007) have shown that the tau formulation has not only great predictive value, but that expansion of leading car tail-lights or a barrier in the road, that is unnoticed by participants, nevertheless influences their braking behaviour in precisely the way predicted by Lee's tau equation. In the Sun and Frost (1997) study, participants riding a bicycle along a road in a virtual reality, observed through a helmet-mounted display, were required to brake to avoid hitting a barrier placed across the road. Subtle expansion or contraction of the barrier, in addition to that produced by the naturally correct rate, was completely unnoticed by the participants, yet resulted in them stopping further back, or closer to the barrier, respectively, than the control condition. In a similar vein, Li and Milgram (2007) have shown that expansion or contraction of the image of a leading car, or simply its tail-light displays, presented to subjects in a driving simulator, resulted in faster or slower braking decelerations.

In our own work on neurons in the pigeon's nucleus rotundus, prior to encountering David Lee's work and the equally influential program of Regan (Regan and Beverley 1978) and his colleagues, we had found neurons that specifically responded to monocularly presented looming (symmetrically expanding) stimuli. When we examined the response profile of these neurons, we (Wang and Frost 1992) were surprised to see that their response profile over time was identical even when different-sized objects or different velocities of approach were used, thus strongly implying that the tau operator was involved. These neurons did not respond when whole field moving textured patterns were loomed, to simulate self motion toward a stationary surface, thus suggesting that they were specifically signalling TTC for an approaching moving object, rather than the bird's approach to a stationary object. However, when the looming soccer ball stimulus we employed was presented to lightly anesthetised birds we found that, first, the tau neurons would fire, then the pectoralis flight muscles were activated, and then the heart rate increased 300%, indicating this neural pathway was most likely involved in escape from rapidly approaching objects (predators). Subsequently we (Sun and Frost 1998) showed that all looming sensitive neurons fell into one of these classes; tau neurons (50%) that were computing tau

$$\tau = \theta / \frac{\mathrm{d}\theta}{\mathrm{d}t} \,,$$

rho neurons (25%) computing the denominator, and the remaining ones (25%) computing

$$\eta = C \frac{\mathrm{d}\theta}{\mathrm{d}t} \mathrm{e}^{-\alpha\theta},$$

similar to those found in desert locusts (Hatsopoulos et al 1995).

Since we have found that the nucleus rotundus cells seem to process TTC only with approaching objects, and not surfaces, we suspect there will be yet another population of tau operator neurons found that process TTC with stationary surfaces (and not approaching objects) because the behavioural responses (affordances) to these two classes of event are quite distinct. Again, David Lee has shown in his work on gannets diving and pigeons landing, that this is a distinct possibility.

## Barrie J Frost

Department of Psychology, Queen's University, Kingston, Ontario K7L 3N6, Canada; e-mail: barrie.frost@queensu.ca

#### References

Hatsopoulos N, Gabbiani F, Laurent G, 1995 "Elementary computation of object approach by a wide-field visual neuron" *Science* **270** 1000 – 1003

- Li Z, Milgram P, 2005a "An investigation of the potential to influence braking behaviour through manipulation of optical looming cues in a simulated driving task", in *Proceedings of the 49th Annual Meeting of the Human Factors and Ergonomics Society, Orlando, Florida* (Santa Monica, CA: Human Factors and Ergonomic Society) pp 1540 1544
- Li Z, Milgram P, 2005b "An empirical investigation of a dynamic brake light concept for reduction of rear-end collision accidents during emergency braking", in *Proceedings of the 49th Annual Meeting of the Human Factors and Ergonomics Society, Orlando, Florida* (Santa Monica, CA: Human Factors and Ergonomic Society) pp 1940–1944
- Li Z, Milgram P, 2008 "An empirical investigation of a dynamic brake light concept for reduction of rear-end collisions through manipulation of optical looming" *International Journal of Human Computer Studies* **66** 158 172
- Regan D, Beverley K I, 1978 "Looming detectors in the human visual pathway" *Vision Research* 18 415-421
- Savelsbergh G J, Whiting H T, Bootsma R J, 1991 "Grasping tau" Journal of Experimental Psychology: Human Perception and Performance 17 315-322
- Sun H J, Frost B J, 1997 "Visual control of braking in humans: a test in a virtual environment", Paper presented at the IRIS/Precarn Conference
- Sun H, Frost B J, 1998 "Computation of different optical variables of looming objects in pigeon nucleus rotundus neurons" *Nature Neuroscience* 1 296 303
- Wagner H, 1982 "Flow-field variables trigger landing in flies" Nature 297 147 148
- Wang Y-C, Frost B J, 1992 "Time to collision is signaled by neurons in the nucleus rotundus of pigeons" *Nature* 356 236-238

Dave Lee's discovery that time-to-collision ( $\tau$ ) can be obtained from variables that can be measured directly by the eye—the ratio of the angular size of an approaching object to its rate of edge expansion—has had a huge effect on ideas about the control of all aspects of behaviour that involve gap closure. (Strictly, this was a rediscovery: unknown to Lee, Fred Hoyle had derived the time to contact of an asteroid in exactly the same way in his novel The Black Cloud in 1957.) In some ways  $\tau$  has been a victim of its own success; because of its conceptual elegance the use of  $\tau$  became the only serious explanation for behaviours such as braking and ball catching. Recently, however, doubts have crept in, not so much about the use of time-to-contact information, but in the way it is obtained. In the approach to an obstacle, for example, the timeto-contact is equal to distance/velocity, and both of these can be determined relatively simply from optic variables—distance from the angle to the base of an object viewed from a known height, and velocity from the optic flow velocity of the ground plane. These cues are perhaps not as neat as relative expansion rate, but, under some circumstances, they may be easier to obtain, and recent evidence (eg Rock and Harris 2006) suggests that this may be so. There is now much more acceptance of the idea that the brain will accept all valid cues in the performance of an action, and weight them according to their current reliability, rather than just using the one that seems intellectually most persuasive (eg Rushton and Wann 1999).

None of this detracts in any way from Lee's exemplary exposition of the role of time-to-contact in braking. Not only does he indicate how  $\tau$  may be obtained, he explains how its rate of change can be used to arrive at an appropriate deceleration, and discusses realistic ways that drivers might use  $\tau$  to control both safe braking and safe following in traffic. He also shows that existing data are consistent with a  $\tau$  strategy (see also Yilmaz and Warren 1995). To my mind, there is little doubt that  $\tau$  is of great importance in the control of braking. The only question is whether  $\tau$ , derived from object expansion, is supplemented by independently derived distance and velocity information.

I have much more difficulty with the "General  $\tau$  theory", which Lee outlines in his commentary. This is intended to be applicable to the closure of any kind of action gap, meaning the gap between an animal's current and intended states. It can apply to a bird landing, a singer changing pitch, or a bat flying through a gap. Central to the general theory (enough of capitals) is the idea of  $\tau_G$ -guidance, where  $\tau_G$  is an intrinsic (brain generated) function that specifies how  $\tau$  itself should change during gap closure. Without repeating Lee's equations [(4) and (5) in his commentary], the outcome of this argument is that the behaviour of  $\tau$  itself is the result of the interaction of a number of parameters. What Lee and his colleagues have shown is that many phenomena are consistent with this formulation, in that the dynamics fit the equations, if the parameters are suitably chosen. One worry is that any mechanism that works, ie that does not result in a crash, would probably fit such a formulation.

The other worry is that the theory implies a mechanism linked exclusively to the monitoring of time-to-closure. I will give one example (that I actually know something about). In figure 8 of Lee's commentary he shows the time course of eye, head, and gaze during a large gaze change. The eye moves first, with a fast trajectory whose time course is predictable, unmodifiable, and is the result of a brain-stem mechanism that gets the eye from A to B in the fastest time consistent with the kinematics of the eyeball. The head moves more slowly, because of its inertia, and obeys different rules: all such head movements take about half a second independent of size, and what changes with amplitude is the velocity. Gaze change is simply the sum of the two. Linking eye and head is the vestibulo-ocular reflex that counterrotates the eye as the head finishes its movement, and ensures that the gaze movement stops when the target position is achieved (Guitton et al 2003). After the turn in Lee's figure the eye picks up on a moving target, and the system switches to tracking which has different rules again. Lee's comment is that "Each segment was tightly  $\tau_G$ -guided. The segments of the head, eye, and gaze movements overlap in time like the parts in contrapuntal music". The second sentence is lyrical but not specially informative, but the first is misleading. The main part of the turn is specified by its end-point which is set at the outset, and there is no continuous guidance by  $\tau$  or any other externally monitored input, at least until tracking kicks in at the end of the turn. Push the button and off it goes. Saying that the turn was tightly  $\tau_G$ -guided seems to add nothing meaningful to the considerable amount that is actually known about the way this particular manoeuvre is conducted.

There are other more general ideas that are applicable to behaviour involving guidance. Ordinary systems theory has had an honourable history, and newer ideas involving internally generated 'forward models' seem particularly useful (Wolpert and Flanagan 2001). The crucial point is to identify the control variables actually used, and how they are used.  $\tau$  was spectacularly successful in helping to understand braking, and some sports involving interception, because it was a more readily obtainable variable than others. To what extent it has explanatory power in other contexts is debatable.

#### Mike Land

School of Life Sciences, University of Sussex, Brighton BN1 9QG, UK; e-mail: m.f.land@sussex.ac.uk

#### References

Guitton D, Bergeron A, Choi W Y, Matsuo S, 2003 "On the feedback control of orienting gaze shifts made with eye and head movements" *Progress in Brain Research* **142** 55-68

Rock P, Harris M G, 2006 "Tau-dot as a potential control variable for visually guided braking" Journal of Experimental Psychology: Human Perception and Performance 32 251 – 267

Rushton S K, Wann J P, 1999 "Weighted combination of size and disparity: a computational model for timing a ball catch" *Nature Neuroscience* **2** 186–190

Wolpert D M, Flanagan J R, 2001 "Motor prediction" Current Biology 11 R729 - R732

Yilmaz E H, Warren W H, 1995 "Visual control of braking: a test of the tau-dot hypothesis" Journal of Experimental Psychology: Human Perception and Performance 21 996 – 1014

In the context of a distant black cloud on a collision course with our planet, the astrophysicist Hoyle (1957) showed that the ratio  $\theta/(d\theta/dt)$ , generally known as tau, where  $\theta$  is the approaching object's angular subtense, and  $d\theta/dt$  is the rate of increase of the angular subtense, approximates the time to collision (TTC) with no requirement of knowing either the distance or speed of the approaching object. Lee hypothesised that drivers use this retinal image variable to control braking (Lee 1976), and other visually guided actions (eg Lee et al 1982). Certainly it seems that, if successful collision avoidance and interceptive actions are based on predictions of the future location of an object and its time of arrival at that location, then the prediction is better based on visual variables that reliably correlate with the quantities to be predicted rather than variables that do not (as seems to be implied by some studies, eg DeLucia 1991; DeLucia and Warren 1994; Harris and Drga 2005; Lopez-Moliner and Bonnet 2002; Smith et al 2001). Though subjected to some criticism (eg Tresilian 1999; Wann 1996), Lee's original hypothesis has proved to meet the classical definition of a fruitful hypothesis in that it sparked a great deal of research that led to considerable advances in our understanding of collision avoidance and interceptive action.

Evidence that TTC is judged on the basis of tau rather than the ratio (perceived distance)/(perceived absolute approach speed) was reported by Gray and Regan (2000a). Indeed, it has been proposed that the perceived speed of motion in depth is inversely proportional to TTC rather than being determined by the approaching object's absolute speed (Gray and Regan 2000a; Regan and Hamstra 1993).

For Lee's hypothesis to be valid it is necessary that tau be processed independently of other variables, and especially of correlated variables (Regan 1982). This requirement does seem to be satisfied. Psychophysical evidence has been reported that the detectors for tau function independently of those for motion within a frontoparallel plane (Regan and Beverley 1978, 1980), and that (in central vision) discriminations of tau, the rate of increase of angular subtense and the approaching object's angular subtense are independent of one another (Regan and Hamstra 1993; Regan and Vincent 1995). In principle, this provides a basis for judging TTC independently of the size of the approaching object. For example, Lee's hypothesis can explain why adaptation to local expansion causes overestimation of TTC and (in a non-stereo simulator) causes dangerous overtaking errors (Gray and Regan 1999, 2000b, 2005).

But laboratory psychophysics is a long step away from establishing what visual information is used to guide action in everyday life. Whether Lee's hypothesis accounts for performance in any given everyday situation can only be settled by a field study. However, in several field investigations of the role of tau, participants were not denied the use of binocular information, and it has been shown mathematically that approximations to TTC are provided by both  $I/D(d\delta/dt)$  (where I is the interpupillary separation, D the object's distance, and  $\delta$  is horizontal relative disparity) (Regan 1995), and  $2(d\delta/dt)/(d^2\delta/dt^2)$  (Regan 2002). (Like tau, the second equation does not involve distance.) Furthermore, estimates of TTC can be made by using binocular information only. And when binocular information is added to tau, TTC can be estimated to within 1.3% (Gray and Regan 1998). This percentage accuracy can account for the 2-3 ms accuracy demonstrated by top athletes (Bootsma and van Wieringen 1990; Regan 1992; Regan et al 1979). Binocular information dominates in some situations, eg when the width of the approaching object (W) is small [because  $(d\theta/dt)/(d\delta/dt)$  approximates W/I independently of distance (Regan and Beverley 1979)]. Again, if the expansion of the retinal image is accompanied by a shape change (anisotropic expansion), the motion-in-depth signal produced by tau can be strongly attenuated (Beverley and Regan 1979, 1980). In this situation tau is unreliable, but binocular information supports accurate estimates of TTC (Gray and Regan 2000c). Anisotropic expansion is by no means limited to such tasks as catching a tumbling rugby ball; when on a collision course with

a nonspherical object (eg a car), the retinal image expands approximately isotropically, but when passing close by, the expansion is nonisotropic.

### David Regan

Centre for Vision Research, Department of Psychology, York University, North York, Ontario M3J 1P3, Canada; e-mail: dregan@yorku.ca

#### Rob Gray

Department of Applied Psychology, Arizona State University, Sutton 340J, 7001 East Williams Field Road, Mesa, AZ 85212, USA

#### References

- Beverley K I, Regan D, 1979 "Separable aftereffects of changing-size and motion-in-depth: different neural mechanisms?" *Vision Research* **19** 727 732
- Beverley K I, Regan D, 1980 "Visual sensitivity to the shape and size of a moving object: implications for models of object perception" *Perception* 9 151 160
- Bootsma R J, Wieringen P C W van, 1990 "Timing an attacking forehand drive in table tennis" Journal of Experimental Psychology: Human Perception and Performance 16 21-22
- DeLucia P R, 1991 "Pictorial and motion-based information for depth perception" Journal of Experimental Psychology: Human Perception and Performance 17 738-748
- DeLucia P R, Warren R, 1994 "Pictorial and motion-based depth information during active control of self-motion: size-arrival effects on collision avoidance" *Journal of Experimental Psychology: Human Perception and Performance* **20** 783 798
- Gray R, Regan D, 1998 "Accuracy of estimating time to collision using binocular and monocular information" *Vision Research* **38** 499 512
- Gray R, Regan D, 1999 "Adapting to expansion increases perceived time to collision" *Vision Research* **39** 3602 3607
- Gray R, Regan D, 2000a "Do monocular time-to-collision estimates necessarily involve perceived distance?" *Perception* **28** 1257 1264
- Gray R, Regan D, 2000b "Risky driving behavior: A consequence of visual motion adaptation for visually guided motor action" *Journal of Experimental Psychology: Human Perception and Performance* **26** 1721 1732
- Gray R, Regan D, 2000c "Estimating time to collision with a rotating nonspherical object" *Vision Research* **40** 49 63
- Gray R, Regan D, 2005 "Perceptual processes used by drivers during overtaking in a driving simulator" *Human Factors* **47** 394 417
- Harris J M, Drga V F, 2005 "Using visual direction in three-dimensional motion perception" Nature Neuroscience 8 229 – 233
- Hoyle F, 1957 The Black Cloud (Harmondsworth: Penguin Books) pp 26-27
- Lee D N, 1976 "A theory of visual control of braking based on information about time to collision" Perception 5 437 – 459
- Lee D N, Lishman J R, Thomson J A, 1982 "Visual regulation of gait in long jumping" Journal of Experimental Psychology: Human Perception and Performance 8 448-459
- Lopez-Moliner J, Bonnet C, 2002 "Speed of response initiation in a time-to-contact discrimination task reflects the use of eta" Vision Research 42 2419-2430
- Regan D, 1982 "Visual information channeling in normal and disordered vision" *Psychological Review* **89** 407 444
- Regan D, 1992 "Visual judgements and misjudgements in cricket, and the art of flight" *Perception* **21** 91 115
- Regan D, 1995 "Spatial orientation in aviation: Visual contributions" *Journal of Vestibular Research* **5** 455 471
- Regan D, 2002 "Binocular information about time to collision and time to passage" *Vision Research* **42** 2479 2484
- Regan D, Beverley K I, 1978 "Looming detectors in the human visual pathways" *Vision Research* **18** 415 421
- Regan D, Beverley K I, 1979 "Binocular and monocular stimuli for motion in depth: changingdisparity and changing-size feed the same motion-in depth stage" *Vision Research* 19 1331 – 1342
- Regan D, Beverley K I, 1980 "Visual responses to changing-size and to sideways motion for different directions of motion in depth: linearization of visual responses" *Journal of the Optical Society of America* 11 1289–1296
- Regan D, Beverley K I, Cynader M, 1979 "The visual perception of motion in depth" *Scientific American* **241**(1) 136–151

Regan D, Hamstra S, 1993 "Dissociation of discrimination thresholds for time to contact and for rate of angular expansion" *Vision Research* 33 447-462

- Regan D, Vincent A, 1995 "Visual processing of looming and time to contact throughout the visual field" *Vision Research* **35** 1845 1857
- Smith M R, Flach J M, Dittman S M, Stanard T, 2001 "Monocular optical constraints on collision control" *Journal of Experimental Psychology: Human Perception and Performance* **27** 395 410
- Tresilian J R, 1999 "Visually timed action: time-out for tau?" *Trends in Cognitive Sciences* **3** 301 310 Wann J P, 1996 "Anticipating arrival: is the tau margin a specious theory?" *Journal of Experimental Psychology: Human Perception and Performance* **22** 1031 1048

## Author's response

I thank the commentators for their thoughtful remarks pointing to directions in which the theory needs to evolve. Two questions need to be addressed. First, how is multiple perceptual information about an action-gap composed into a single 'message'? David Regan's and Rob Gray's experiments on the synergistic use of binocular and monocular  $\tau$  information, and Barrie Frost's suggestion of two sets of 'tau operator neurons' concerned with  $\tau$  information from surfaces and from objects are pertinent here. Second, how are the closings of multiple temporally overlapping action-gaps prospectively organised to realise a coherent action, with  $\tau$  information about their 'current futures', to use Reinoud Bootsma's neat term? Running to catch a cricket ball provides a good example. It entails controlling the closure of action-gaps between feet and ground, between body orientation and goal orientation for catching, and between hand and ball. This requires guiding gaze, by controlling the head-shoulders, eyes-head, and gazeball action-gaps. In section 3 I showed that the τs of action-gaps and their directions comprise primary information that is necessary and sufficient for controlling purposive action. This primary information is directly perceptible; other information about, eg, the changing sizes and velocities of action-gaps, is secondary, being derived from the primary information (rather than the reverse, as Mike Land suggests in his commentary). The ubiquity of this primary  $\tau$  and direction information for controlling action indicates how the two questions might be addressed.

Consider shifting and stabilising gaze on a moving object when the eyes, head, and body are also moving. In section 7.1 I describe an experiment where the participant turned to fixate a previously unseen moving object. The gaze-target, eye-head, and head-shoulders action-gaps closely fitted the  $\tau_G$ -guidance equation (mean  $r^2 = 0.976$ ). The information for guiding gaze to target is visual. For guiding the eye-head and the head-shoulders action-gaps, which is important for avoiding injury to the eye muscles and neck, the information is bimodal—visual plus 'articular', involving muscle, joint, and skin receptors. (I exclude the vestibular system from consideration here because it monitors movement of the head relative to the inertial world, rather than to the body, and because we found no evidence in our data of direct vestibular - ocular reflexive movement; cf Mike Land's comments.) In the bimodal cases, the composite sensory  $\tau$  could be a weighted linear sum of the individual  $\tau$ s, since, if the individual  $\tau$ s are proportional to the  $\tau$  of an action-gap, their weighted linear sum will be too. The weightings could be adjusted according to the relative accuracy and reliability of the information. This hypothesis, relating to the first question, needs to be tested by manipulating the information in gaze shifting and other synergistic sensory scenarios.

Regarding the second question, in section 7.1 the eye-head, gaze-target, and head-shoulders action-gaps all fitted the  $\tau_G$ -guidance equation but were of different durations and overlapped in time. I suggested that each gap may be independently  $\tau_G$ -guided. But that leaves open the question how the three action-gap closures are organised in time. Alternatively, one action-gap may serve as a  $\tau_G$ -guided 'leader' which the other action-gaps  $\tau$ -couple onto. Careful experiments are needed to determine what is actually going on in shifting gaze and in numerous other cases of synergistic  $\tau$  control, from playing the piano to sprinting down a track.





www.perceptionweb.com

**Conditions of use.** This article may be downloaded from the Perception website for personal research by members of subscribing organisations. Authors are entitled to distribute their own article (in printed form or by e-mail) to up to 50 people. This PDF may not be placed on any website (or other online distribution system) without permission of the publisher.