An introduction to survival analysis

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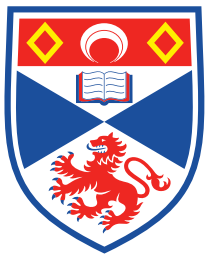

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TTE data consists of $(time, \overbrace{event}^{\text{yes/no}})$ tuples.

Time-to-event (TTE) data

TTE analysis is also known as:

- survival analysis
- failure time analysis
- reliability theory (engineering)
- duration modelling (economics)
- event history analysis (sociology)

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Use cases for TTE analysis:

- TODO

Example: Covid-19 treatment trial

A randomised controlled trial ($n = 4$) was conducted to assess the efficacy of drug ABC in treating Covid-19. This is what happened to the patients:

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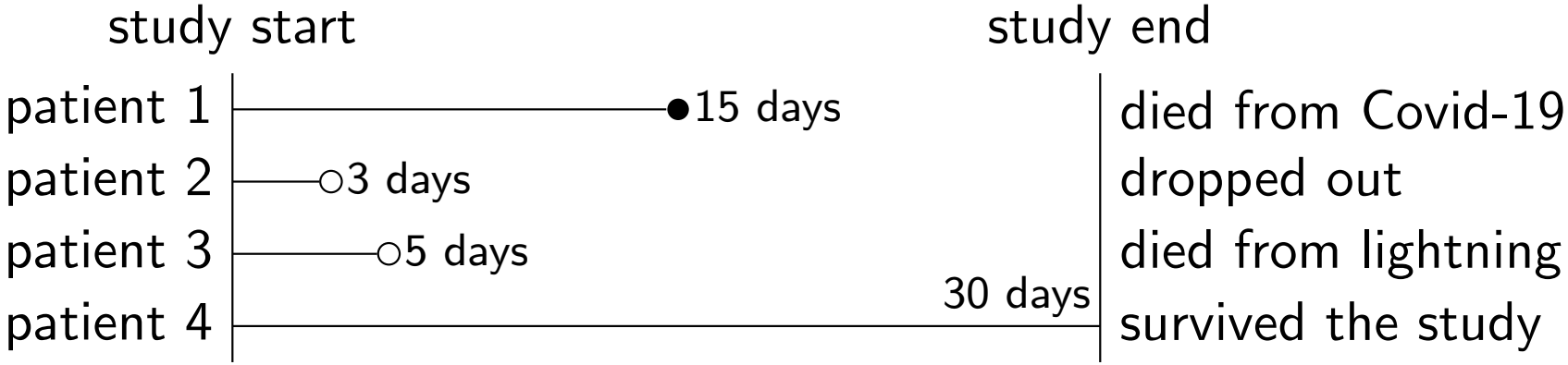
patient	received ABC?	outcome
1	yes	died from Covid-19 on day 15
2	no	dropped out of the study after day 3
3	yes	died by a lightning stroke on day 5
4	no	survived the study (30 days)

Example: Covid-19 treatment trial



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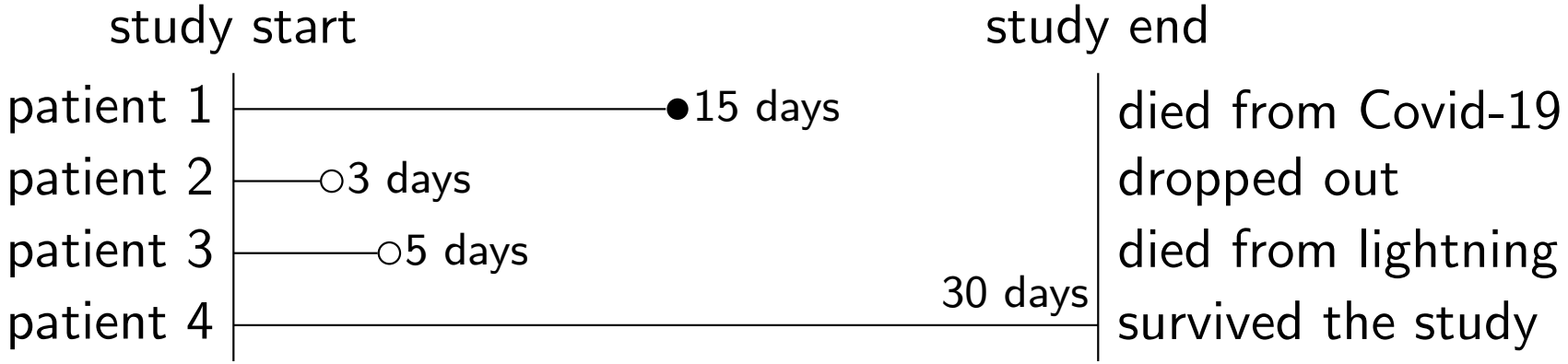

The **time** is the number of days since testing positive for Covid-19.
The **event** is whether the patient died due to Covid-19.

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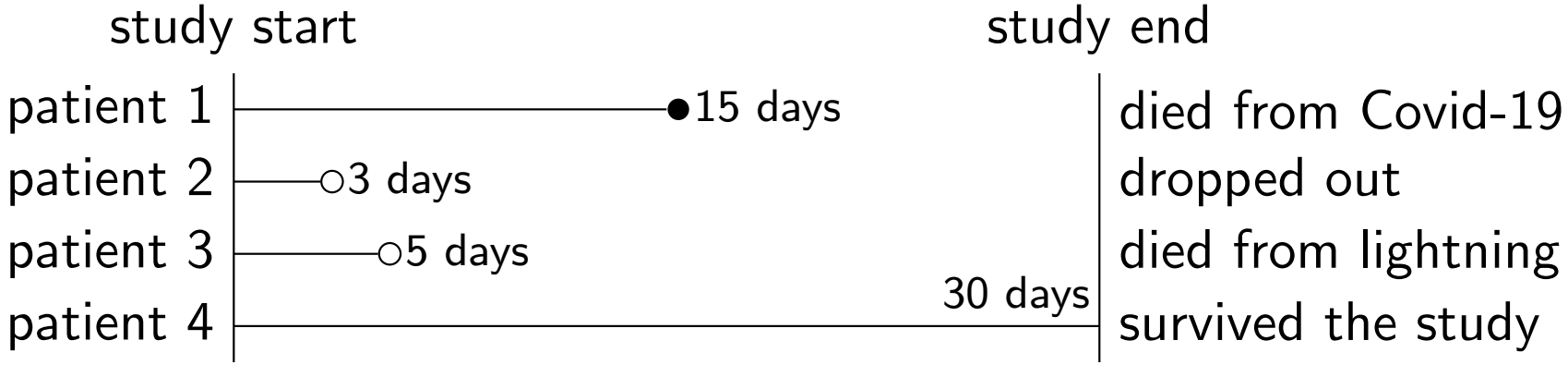

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Time-to-event data

patient	time	event
1	15	yes
2	?	?
3	?	?
4	?	no

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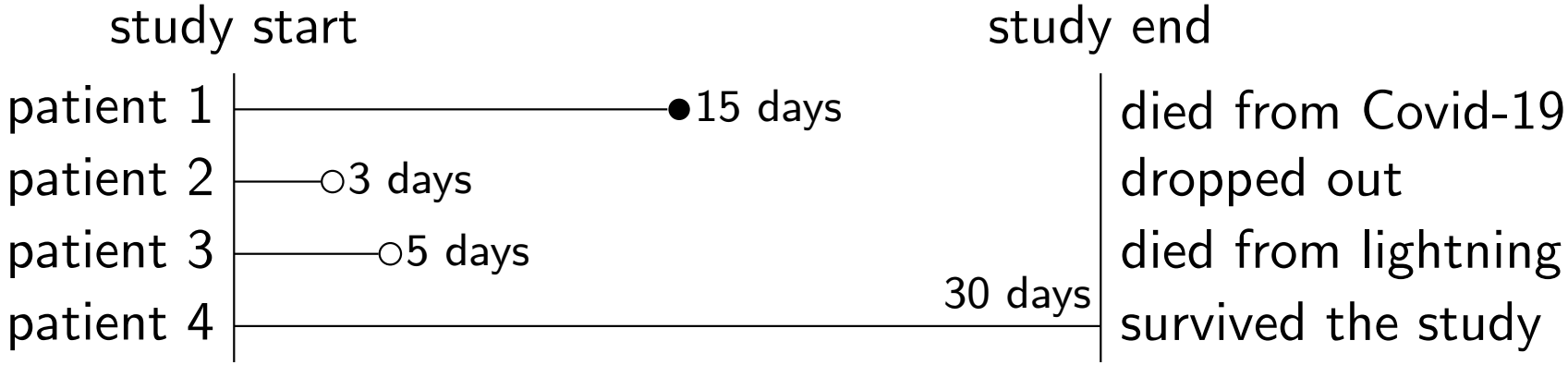

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Time-to-event data

patient	time	event
1	15	yes
2	$[0, 3]$	no
3	$[0, 5)$	no
4	$[0, 30]$	no

Censoring

We just saw an example of **right-censored** data.

Survival function

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$$S(t) = \Pr(T > t)$$

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