DISSERTATION TITLE

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SUPERVISOR(S), MARKERS/CHECKER AND ORGANISER

The undersigned hereby certify that the markers have independently marked the dissertation entitled "**Dissertation Title**" by **John Smith**, and the external examiner has checked the marking, in accordance with the marking criteria and the requirements for the degree of **Master of Science**.

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DISSERTATION INFORMATION AND STATEMENT

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STATEMENT:

Unless otherwise noted or referenced in the text, the work described in this dissertation is, to the best of my knowledge and belief, my own work. It has not been submitted, either in whole or in part for any degree at this or any other academic or professional institution.

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Abstract

Write your abstract here(no more than one page).

Usually the first paragraph describes briefly background and motivation of your dissertation work/research, and then aim and objectives.

The second paragraph may describe the methods and experiments. $z = \frac{x1}{y2}$.

Third paragraph gives the results and conclusions.

Acknowledgements

I would like to thank my supervisor, Dr. xxx xxx, for his/her many suggestions and constant support during this research, bla bla, bla.
I would also like to express my gratitude to whoever sponsored my course in full or part.

Finally, I am very grateful to my family (parents, etc.) for their patience and *love*.I ......

Your name here

at Norwich, UK.

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List of Abbreviations

API Application Programming Interface. 5

CMP School of Computing Sciences. 1

KDD Knowledge Discovery form Database. 5

LOA List of Abbreviations. 4, 5

SVM Support Vector Machine. 5

UEA University of East Anglia. 1

UML Unified Modelling Language. 5

Chapter 1

Notes on how to use the Latex Dissertation template

This LATEX template was created by Dr. Wenjia Wang[1] with aim of helping Master students at the School of Computing Sciences (CMP), the University of East Anglia (UEA), to write their dissertation with Latex.

This section gives some brief instructions on how to use it and you should read it carefully before attempting it.

NOTE: you should use *TexStudio* as your text editor and Latex compiler to make sure this Latex Template working properly, although it may work with other text editors.

Please let me know if you find any bugs or problems, although I may not have time to resolve them in time.

1.1 How to use this Latex Template

Brief Instructions:

1.1.1 Preparations: P1 to P4

P1. Download the template package from the blackboard of Dissertation Module and Unzipped it to an intended working folder on your U drive, e.g. Dissertation.

[1]Created in 2005 based on a thesis style file from Stanford University.
Previous versions: created in 2005 (v0), updated in 2010(v1) and 2012(v1.1). Major Revision on 06/08/2015-17/08/2015(v2): Added the function of generating the list of Abbreviations (you must follow the instructions carefully and exactly in order to produce a list of Abbreviations). Major revision on 22/11/2016(v3), added notes for how to use the Template. Latest update on 15/04/2019: Just updated the Instruction notes.

P2. Start TexStudio and open "DissertationTemplate4.tex"

Note: it is a tex file that (1) uses, i.e. includes all the other files, such as Abstract, Acknowledgement, and chapter files, which are written or edited separately with Latex, (2) generates a pdf file of your dissertation as a whole.

P3. Change/replace/fill few places in this file to suit your need: such as, Your course, Year, Dissertation title, your name, markers, etc.

P4. Save it with your new file name, e.g. "Wang_Dissertation2019.tex"

1.1.2 Work on each latex file

Then following steps below to work on each file to write your dissertation.

1. Write your abstract in a separate tex file and name it as Abstract.tex 2. Write your acknowledgement in a separate tex file and name it as Acknowledgement.tex Note: Both Abstract.tex and Acknowledgement.tex files are already included in the style file. So you must not change their names but only the contents.

3. write each chapter in a separate tex file and name them as, e.g. Ch1, Ch2, etc. and then use "\include{...}" to include them as shown in this example.

New notes added on 06/08/2015

4. If you wish to produce a list of abbreviations/acronyms that are used in your dissertation, you must read notes below.

5. Using footnote. (wjw added this note on 11/09/2015)

If you want to use footnotes in any chapters of your dissertation, you can use command \foodnote{foodnote text} in where you want, for example [2]

The footnotes are numbered automatically and continuously within a CHAPTER.

1.2 Making Citations and Citation Styles

You are required to use the Harvard style for citing references.

Specifically, there are two sub-styles to be used in different situations.

1. Use command $\backslash citep\{...\}$.

[2]your footnote text: If you want to generate a list of Abbreviations, you must follow the instructions given here carefully and exactly, particularly using Command "Makeglossaries" in "Tools" before Compiling.

If the authors of a reference are NOT part of your sentence, e.g. "A study (Wang, 2008) has been done to investigate the influence of some factors on the accuracy of an ensemble.", then use \citep{...} in your Latex file, such as "A study \citep{Wang08} has been done...", it then produces the text as: "A study (Wang, 2008) has been done......"

2 Use command *citet*{...}.

If the authors of a reference are part of your sentence, e.g. "Wang (2008) studied the factors that can affect the performance of a machine learning ensemble.", then use \citet{Wang08} studied ... . It then produces the text as: " Wang (2008) studied ......"

You can press function key "F8" in TexStudio to compile bibliography, i.e. to pull all the cited references out from your Bibtex file and generate a bib file. The message shows if there is any error in this process.

1.3 Creating Equations

You can write an equation by using \begin{equation} write equation here \ end{equation}. For example,

$$y = a + b_1 x_1 + b_2 x_2 \tag{1.3.1}$$

If your equation is too long for a single line, instead of using the above environment, use "\begin{align}" command to align an equation of multiple lines at a specified point. Use \\\\ to specify a line break, and & to indicate where the lines should be aligned.

For example, the following equation is aligned at "=".

$$\begin{aligned} f(x) &= (x+a)(x+b) \\ &= x^2 + (a+b)x + ab \end{aligned} \tag{1.3.2}$$

The following equation is aligned at the left brace.

$$
\begin{aligned}
f(x) = \pi \left\{ a + b_1 x_1 + b_2 x_2 + b_3 x_3^4 + b_4 {x_4}^3 + b_5 x_5^2 \right. \\
\left. + b_6 x_6^5 + b_7 x_7^2 + b_8 x_8^3 + b_9 {x_9}^3 \right\}
\end{aligned} \tag{1.3.3}
$$

Note: "{*align*}" must not be nested within "{*equation*}", it replaces "{*equation*}".

If you do not want to automatically number an equation, use {equation*} or {align*}. For example, the following equation will not be numbered.

$$
y = a + b_1 x_1 + b_2 x_2^2 + b_3 x_3^3
$$

1.4 How to define and generate a list of Abbreviations

In the first paragraph I will show you how to use the acronyms defined in file "acronyms.tex", which will be then listed in the List of Abbreviations (LOA) if they are used in your text.

1.4.1 Define abbreviations/acronyms

(Notes and sample files:

"acronymNotes.tex": brief introduction on how to make a LOA.

"acronyms.tex": a sample file where you define abbreviations.)

To define an abbreviation or acronym, open "acronym.tex" file in any text editor, e.g. TeXstudio, you can see some abbreviations (or acronyms) already defined in it.

You can simple use teh following command *newacronym* to define an abbreviation/acronym in the format: \newacronym{label}{name}{description}

For example: \newacronym{api}{API}{Application Programming Interface}}

1.4.2 Use the defined abbreviations/Acronyms

You can use \gls, or \Gls, Capital, to insert the abbreviation to any where you want in your tex file. Or use \glspl, or \Glspl for using their plural forms.

In the first time you use it, it will produce the full text of the abbreviation, followed by its abbreviation in (). After that, it will only produce the abbreviation.

For example, \Gls{api} will be shown as Application Programming Interface (API), i.e. 'Application Programming Interface (API)' (without the quotation marks), and will add a linked page number to where it uis used, e.g. '1' in this case, and will be shown in the LOA.

After that, \Gls{api} will produce only the abbreviation, i.e. API.

Unified Modelling Language (UML), Support Vector Machine (SVM), Knowledge Discovery form Database (KDD) are some other abbreviation examples I defined in "acrynom.tex" file. Their plural format can be produced by using command: \glspl{}. e.g. \glspl{uml}, \glspl{svm}, \glspl{kdd}, which produce: UMLs, SVMs, KDDs.

1.5 Compiling/Building your tex file

After you have defined your abbreviations or acronyms in file "acronyms.tex", and use some of them in your text file of other Chapters, such as in this note file, by using the commands given above, you need to compile and build your integrating tex file (e.g. DissertationSample1.tex) to produce the intended files, e.g. pdf file, with following steps in TeXstudio.

1. Run "Compile" or "Build/View" by clicking their icon. (note: you may see a pdf file with your text, but it won't have the list of abbreviations.)

2. Run "Makeglossaries": Click "Tools" and then "Commands", and choose "Makeglossaries" to run it.

Ignore any warning message.

Note: whenever you make any new entry to your "acronym.tex" file, and/or use any abbreviation/acronym in your other tex file, you must do this step to update your generated .gls file.

3. Run "Build and View" again. This time the pdf file should contain the actual

list of abbreviations after the list of Figures and teh title appears in the Table of Content(TOC).

Please note:

(1) Only the used acronyms will appear in the list of Abbreviations.

(2) notice the difference in using "gls" and "glspl"

Chapter 2

The Space of Lomonosov Functions

This chapter gives a constructive proof of an abstract approximation theorem, inspired by the celebrated result of V.I. Lomonosov (Lomonosov, 1973).

Abramovich et al. (1995) gave an alternative proof of some recent characterizations of the invariant subspace problem. We also establish density of non–cyclic vectors for certain convex sets of compact quasinilpotent operators, and conclude with a related open question. In Chapter 2 we extend the techniques introduced in this chapter to non–compact operators acting on a Hilbert space.

2.1 Introduction

Lomonosov (1991) conjectured that the adjoint of a bounded operator on a Banach space has a non–trivial closed invariant subspace. In view of the known examples of operators without an invariant subspace (Enflo, 1987; Read, 1985), this is the strongest version of the invariant subspace problem that can possibly have an affirmative answer. In particular, if the Lomonosov conjecture is true, then every operator on a reflexive Banach space has a non–trivial invariant subspace(as shown in Figure 2.1).

Considering the strong influence of Lomonosov's results on the theory of invariant subspaces, it is not surprising that both the conjecture and the techniques developed in the interesting paper (Lomonosov, 1991) received further attention. L. de Branges used this result to obtain a characterization of the invariant subspace problem in terms of density of certain functions. This stimulated another characterization of the invariant subspace problem given by Y.A. Abramovich, C.D. Aliprantis, and O. Burkinshaw in (Abramovich et al., 1995). Section 1.4 presents a more detailed account of this

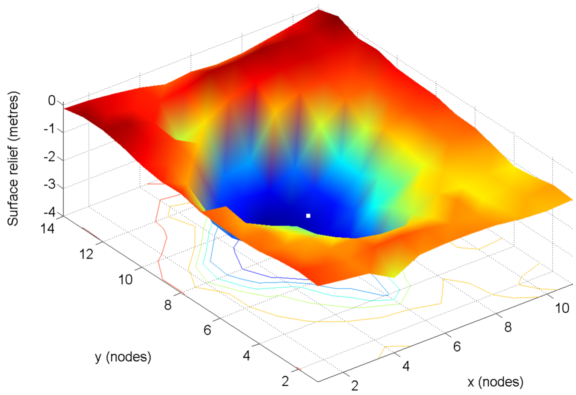

Figure 2.1: A figure sample that is not associated with the text.

Table 2.1: A sample Table

Day	Min Temp	Max Temp	Summary
Monday	11C	22C	A clear day with lots of sunshine. However, the strong breeze will bring down the temperatures.
Tuesday	9C	19C	Cloudy with rain, across many northern regions. Clear spells across most of Scotland and Northern Ireland, but rain reaching the far northwest.
Wednesday	10C	21C	Rain will still linger for the morning. Conditions will improve by early afternoon and continue throughout the evening.

work.

We take a slightly different approach. First we give a constructive proof of the approximation theorem, inspired by the well known Lomonosov construction used in (Lomonosov, 1973; Radjavi and Rosenthal, 1973). This theorem is then applied to give an alternative proof of the main result in (Abramovich et al., 1995). Our proof applies to both real and complex Banach spaces, while the original result was established for complex Banach spaces only. The alternative proof somehow explains the role of compact operators that appear in the characterizations of the invariant subspace problem (Abramovich et al., 1995).

One may notice that the weak*–compactness of the unit ball in dual Banach spaces plays an important role in (Abramovich et al., 1995; de Branges, 1959, 1993; Lomonosov, 1991), as well as in the applications given in this chapter. In other words, if the Lomonosov conjecture is true, then the compactness of the unit ball, with respect to the weak* topology, is likely to be an important ingredient of its proof.

In the last section we put this observation to the test. A straightforward application of the approximation theorem obtained in Section 1.3, together with the Schauder–Tychonoff Fixed Point Theorem, yields density of non–cyclic vectors for the dual of a convex set of compact quasinilpotent operators. We end with the open problem of obtaining a similar result for the original set, rather than its dual.

This work is more or less self–contained and the notation and terminology used in it is (supposed to be) standard. However, here are a few conventions that hold throughout this chapter:

2.2 Reflexive Topological Spaces and Continuous Indicator Functions

This section introduces some topological preliminaries that lead to a fairly general treatment of the approximation theory in the next section, where an important role is played by the partition of unity and the "continuous indicator functions" associated with a basis for the topology on a compact domain of certain functions. The existence of continuous indicator functions can be characterized by a purely topological property of the underlying space, which is defined as "reflexivity" of the topological space. In this section we introduce both concepts and establish the connection between them.

Definition 2.2.1. Let $S = (S, \tau)$ be a topological space and denote by $C(S, \mathbb{R})$ the space of all continuous real–valued functions on S. A topological space S is called *reflexive* if the topology τ coincides with the weakest topology τ_w on S for which all the functions in $C(S, \mathbb{R})$ are continuous.

Remark 2.2.1. The reflexivity of topological spaces is not to be confused with the corresponding concept of the reflexivity of Banach spaces. Indeed, we conclude this section by showing that every subset of a locally convex space is reflexive.

Proposition 2.2.1. *Reflexivity is a hereditary property; i.e. a subspace S of a reflexive topological space X is reflexive with the relative topology.*

Proof. Consider the restrictions of the functions in $C(X, \mathbb{R})$ to the subset S, and observe that they induce the relative topology on S, whenever X is reflexive. □

Definition 2.2.2. Suppose U is an open subset of a topological space S. A continuous function $\Gamma\colon S \longrightarrow [0, \infty)$ is called a *continuous indicator function* of U in S if

$$U = \{s \in S \mid \Gamma(s) > 0\}.$$

Remark 2.2.2. If X is a metric space then every open ball

$$U = U(x_0, r) = \{x \in X \mid d(x, x_0) < r\},$$

admits a continuous indicator function $\Gamma_U\colon X \longrightarrow [0, \infty)$, defined by

$$\Gamma_U(x) = \max\{0,\, r - d(x, x_0)\}.$$

Furthermore, suppose $f \in C(S, X)$. Then the open set $V = f^{-1}(U) \subset S$ "inherits" an indicator function from U by setting: $\Gamma_V(s) = \Gamma_U(f(s))$.

2.3 Lomonosov Functions

The proof of the celebrated result of V.I. Lomonosov (Lomonosov, 1973; Radjavi and Rosenthal, 1973) was based on the ingenious idea of defining a continuous function with compact domain in a Banach space, assuming that certain local conditions are met. In this section we generalize this idea in the form of an approximation theorem. Since our construction was greatly inspired by the proof of Lomonosov's Lemma (Lomonosov, 1973; Radjavi and Rosenthal, 1973), we suggest the following definition.

Definition 2.3.1. Let $\mathcal{A} \subset C(S, X)$ be a subset of the space of continuous functions from a topological space S to a locally convex space X. The convex subset $\mathcal{L}(\mathcal{A}) \subset$

$C(S, X)$, defined by

$$\mathcal{L}(\mathcal{A}) = \left\{ \sum_{k=1}^{n} \alpha_k A_k \;\middle|\; A_k \in \mathcal{A}, \alpha_k \in C(S, [0,1]) \text{ and } \sum_{k=1}^{n} \alpha_k \equiv 1;\; n < \infty \right\}.$$

is called the *Lomonosov space* associated with the set $\mathcal{A}$, and a function $\Lambda \in \mathcal{L}(\mathcal{A})$ is called a *Lomonosov function.*

$$y = a + b_1 x_1 + b_2 x_2 \tag{2.3.1}$$

Recall that the *uniform topology* on $C(S, X)$ is induced by the topology on a linear space X. If $\mathcal{B}$ is a local basis for the topology on X then the sets

$$\widehat{U} = \{f \in C(S, X) \mid f(S) \subset U \in \mathcal{B}\}$$

define a local basis for the uniform topology on $C(S, X)$. If X is a locally convex space then so is $C(S, X)$. In particular, if X is a Banach space then $C(S, X)$ with the uniform topology is a Banach space, as well.

We are now ready to give a construction of the Lomonosov function that uniformly approximates a continuous function within a given neighborhood.

2.4 Subspace Problem

We introduce some basic concepts and notation that is consistent with (Abramovich et al., 1995). However, for more details and further references on the *invariant subspace problem*, the reader is advised to consult the nicely written and comprehensible original (Abramovich et al., 1995).

Chapter 3

Discussion and Conclusion

In this chapter we firstly evaluate our experimental results using statistical significance tests, then discuss the strengths and limits of our methods, and finally draw conclusions and make some suggestions for further work.

3.1 Evaluation and Discussion

Normally you should have a chapter before conclusion chapter, particularly for describing evaluation and discussion on your work as a whole. If you do not have such a chapter, you should have at least a Section dedicated to this, before giving conclusion and suggestion for further work.

3.2 Conclusions

write your conclusions in this section. Every point of your conclusions must have been evaluated and discussed in the previous Chapter or Section

3.3 Suggestion for Further Work

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