Stochastic Simulation 2020
Assessed Coursework

Deadline: 4pm, 11th December 2020.

Consider the following density,

$$f_X(x) \propto \begin{cases} \frac{1}{(x-a)(b-x)} \exp\left\{-\frac{1}{c}\left(d + \log\left(\frac{x-a}{b-x}\right)\right)^2\right\}, & a < x < b; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Where a, b, c and d are parameters. Instructions on how to find the values for your assigned parameters can be found in the file `Params2020.pdf` on Blackboard. If you cannot find your parameters on the list let me know!

You should create an `R Markdown` which contains:

1. Implementation of either a rejection or ratio-of-uniform scheme to generate random variates from the probability density function $f_X(\cdot)$ given in Equation (1). You should include both descriptions of your methodology and the commented `R` code, you should include a discussion of the acceptance rate of your scheme and of the computational time.

2. Verification of your scheme using at least two diagnostic plots and at least two statistical tests. You should include a discussion of the methods and interpret your results.

3. Implementation of a Monte Carlo procedure to estimate the normalising constant associated with $f_X(\cdot)$. You should include a description of the approach that you have taken and compare your procedure to hit-or-miss Monte Carlo.

You should submit both the `Rmd` file and the knitted `pdf` file to Blackboard.

Notes:

1. Rejection and Ratio of Uniforms methods require maximisations:

   Rejection: e.g. $M = \sup_x \frac{f_X(x)}{g_X(x)}$.

   R of U: a, b, c of bounding rectangle.

   These maximizations may not be available theoretically and may need to be approximated numerically. You are permitted to use the `R` function `optimize`, which can be found in the stats package; if you do, be sure to explain carefully what it is being used for.

2. You are permitted to use the `R` functions `proc.time` and `system.time`, both of which can be found in the base package.

3. You are permitted to use `runif`.

4. The Monte Carlo estimate of the integral can be used to approximate your acceptance probability and verified using sample estimates (*i.e.* noting the proportion accepted during your algorithm).

5. Think about using squeezing methods and/or variance reduction techniques.

6. Your final pdf should be no more that 20 A4 pages inclusive.

7. The project will be marked out of 50: part 1: 25; part 2: 10; part 3: 10; presentation: 5.