June 8, 2022

The results below are generated from an R script.

```
## Install a package manager and packages
if (!require("pacman")) {
  install.packages("pacman")
}
pacman::p_load(Rfast, foreach, doParallel, mvnfast, rstudioapi)
current_path = rstudioapi::getActiveDocumentContext()$path
setwd(dirname(current_path))
pacman::p_load_gh("pkimes/sigclust2")
shc = get("shc", env = environment(shc))
source("sequential_function.R")
\# k = 3 \# number of clusters (3 or 10)
# uneven = FALSE #whether or not to have uneven weights
# distribution = 't' # t distribution or normal distribution?
# iterations = 50 # number of iterations
n = 500 \# total number of samples
alpha = 0.05
if (distribution=='t'){
  distribution_name = 'True distribution components: t-distribution (df=3) mixture distribution'
  samplefunc <- function(n, mu, sigma, w){</pre>
    rmixt(n = n,mu = mu,sigma = sigma,w = w,df = 3)
  }
}else{
  distribution_name = 'True distribution: Normal mixture distribution'
 samplefunc <- function(n, mu, sigma, w){</pre>
 rmixn(n=n, mu=mu, sigma=sigma, w=w)
}
}
# formulating d, delta (dimension and distance between clusters)
if (k == 10){
  a = c(2, 20, 2, 40, 2, 60, 2, 80, 2, 100, 2, 150, 2, 200) # dim2
  b =c(8, 20, 8, 40, 8, 60, 8, 80, 8, 100, 8, 150, 8, 200) # dim8
  d_delta = matrix(c(a, b) , ncol = 2, byrow = T)
} else if (k == 3){}
 a = c(2, 1, 2, 2, 2, 3, 2, 4, 2, 5, 2, 6, 2, 7, 2, 8, 2, 9) # dim2
```

```
b = c(8, 1, 8, 2, 8, 3, 8, 4, 8, 5, 8, 6, 8, 7, 8, 8, 8, 9) # dim8
 d_delta = matrix(c(a, b) , ncol = 2, byrow = T)
} else {
 stop("k != 3 or 10")
#weights
w = rep.int(1, k)
if (uneven){
 w[1] = 1 / 4
 w[2] = 1 / 2
w = w / sum(w)
K = floor(sqrt(n / 2)) #num clusters to test
K = min(K, 14L) # to ensure not estimating too many clusters
coresToUse = floor(detectCores() / 2) # cores to use
# function which creates data and performs one iteration
simulation <- function(iteration) {</pre>
  mu = matrix(runif(k*d, min = 0, max = delta), nrow = k)
  # simulate data
  set.seed(18 + iteration)
  data = samplefunc(n=n, mu=mu, sigma=sigma, w=w)
  D1 = data[1:floor(n / 2), ]
  D2 = data[(floor(n / 2) + 1):n,]
  # Estimate no.clusters
  Cluster_numbers = estimate.cluster.all(D1, D2, alpha, K)
  sigclust_splits = sum(shc(data, alpha = alpha)$nd_type == "sig")
 return(c(unlist(Cluster_numbers, use.names = F), sigclust_splits + 1L))
meanEstimate = matrix(nrow = nrow(d_delta), ncol = iterations)
medianEstimate = meanEstimate
meanEstimate12 = meanEstimate
medianEstimate12 = meanEstimate
AICEstimate = meanEstimate
BICEstimate = meanEstimate
sigclustEstimate = meanEstimate
RIFThierEstimate = meanEstimate
# For parallel computing
cl <- makeCluster(coresToUse) #not to overload computer</pre>
registerDoParallel(cl)
```

```
for (j in 1:nrow(d_delta)) {
 d = d_{delta[j, 1]}
  delta = d_delta[j, 2]
  \#sigma = lapply(c(3,1,1), function(x) diag(x, nrow=d))
  sigma = lapply(rep.int(1, k), function(x)
   diag(x, nrow = d))
  estimates <-
   foreach(
     i = 1:iterations,
      .combine = cbind,
      .inorder = F,
      .packages = c("mclust", "Rfast", "mvnfast", "MASS"),
      .verbose = F
    ) %dopar% {
     simulation(i)
   }
  # format data into table
  meanEstimate[j, ] = estimates[1, ]
  medianEstimate[j, ] = estimates[2, ]
  meanEstimatel2[j, ] = estimates[3, ]
  medianEstimatel2[j, ] = estimates[4, ]
  BICEstimate[j, ] = estimates[5, ]
  AICEstimate[j, ] = estimates[6, ]
  RIFThierEstimate[j, ] = estimates[7, ]
  sigclustEstimate[j, ] = estimates[8, ]
  df = stack(data.frame(
    cbind(
      "Mean" = meanEstimate[j, ],
      "Mean12" = meanEstimatel2[j, ],
      "Median" = medianEstimate[j, ],
      "Median12" = medianEstimate12[j, ],
      "AIC" = AICEstimate[j, ],
     "BIC" = BICEstimate[j, ],
     "RIFT.hc" = RIFThierEstimate[j, ],
      "shc" = sigclustEstimate[j, ]
   )
  ))
  print(pasteO("(dimension, delta) = (", d, ",", delta, ")"))
  colnames(df) = c("ESTIMATE" , "METHOD")
 tableEstimates = with(df, table(METHOD, ESTIMATE))
 print(tableEstimates)
## [1] "(dimension, delta) = (2,20)"
## ESTIMATE
```

```
## METHOD 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
            0 0 1 6 10 10 17 20 22 11 3 0 0 0
    Mean
##
    Meanl2
            0 0 1 6 10 10 17 20 22 11
                                     3
                                        0 0 0 0
                                     8
##
    Median
            0 0 1 2 4 8 15 19 19 17
                                        3
                                          1
                                             3
                                               0
##
    Medianl2 0 0 1 2 6 7 15 18 20 17
                                     8 3 0 3 0
##
    AIC
            0 0 1 1 2 2 10 21 34 21
                                     6
                                       1
                                          0
                                             1
##
    BIC
            0 0 1 1 2 4 20 28 29 12
                                     3
                                        0
                                           0
                                             0
                                               0
                                                  0
##
    RIFT.hc 6 1 5 13 19 29 15 9 2 1 0 0 0 0
##
          2 0 0 0 1 1 6 10 15 27 17 12 4 3 1
##
  [1] "(dimension, delta) = (2,40)"
##
         ESTIMATE
## METHOD
           1 2 3 4 5 6 7 8 9 10 11 12 13 14
##
    Mean
            0 0 0 0 0 5 12 30 18 31 3 1 0
##
          0 0 0 0 0 5 12 30 18 31 3 1
    Meanl2
    Median
            0 0 0 0 2 12 12 20 18 14
                                     4
##
##
    Median12 0 0 0 0 2 12 15 17 19 13 3 3 9 7
            0 0 0 0 0 0 6 25 27 37 4 1 0
##
    AIC
##
    BTC.
            0 0 0 0 0 1 5 28 27 34 4 1
##
    RIFT.hc 10 1 1 3 3 6 15 21 28 12 0
                                        0
                                           \cap
##
           2 0 0 0 0 0 0 3 20 60 5 6 3 1
  [1] "(dimension, delta) = (2,60)"
##
          ESTIMATE
## METHOD
            1 2 3 4 5 6 7 8 9 10 11 12 13 14
##
    Mean
            0 0 0 0 1 3 15 30 35 16 0 0 0
##
            0 0 0 0 1 3 15 30 35 16 0 0 0
    Meanl2
            0 0 0 0 1 10 28 29 13 7
                                     3
                                        2
##
    Median
                                          1
##
    Medianl2 0 0 0 0 2 10 28 30 11 7
                                     4
                                        1
                                          1
            0 0 0 0 1 2 9 28 42 18 0
##
    AIC
                                        0
##
    BTC
            0 0 0 0 1 2 9 26 39 23 0 0
                                             0
##
    RIFT.hc
            7 3 2 5 2 4 7 10 25 31 4
                                        0
                                           0
            0 0 0 0 0 0 0 0 14 71 12 1 2 0
##
    shc
  [1] "(dimension, delta) = (2,80)"
##
         ESTIMATE
## METHOD
           1 2 3 4 5 6 7 8 9 10 11 12 13 14
##
            0 0 0 0 0 10 30 38 22 0 0 0 0
    Mean
    Meanl2
            0 0 0 0 0 10 29 38 22 0
            0 0 0 0 2 10 21 27 19 8 3
##
                                             7
    Median
                                        \cap
    Medianl2 0 0 0 0 3 10 21 31 14 8
##
                                     4
                                        0
##
    AIC
            0 0 0 0 0 10 25 42 23 0 0 0
##
    BIC
            0 0 0 0 0 10 25 39 26 0 0 0 0
##
    RIFT.hc 11 1 4 5 3 3 0 7 12 51 3 0 0
          3 0 0 1 0 0 0 0 6 70 18 1 1
##
##
  [1] "(dimension, delta) = (2,100)"
##
         ESTIMATE
## METHOD
           1 2 3 4 5 6 7 8 9 10 11 12 13 14
            0 0 0 0 0 1 9 25 42 23 0 0
##
    Mean
##
    Mean12
            0 0 0 0 0 1 8 26 42 23 0
                                        0
            0 0 0 0 1 10 18 31 18 9 2 1
##
                                             7
    Median
    Medianl2 0 0 0 0 4 9 20 32 13 9
                                        1
##
##
    ATC
            0 0 0 0 0 1 7 24 43 25
                                     0
                                        0
                                          \cap
            0 0 0 0 0 1 7 24 43 25
##
    RIFT.hc 13 2 2 6 2 3 1 2 17 49 3 0
##
                                             0
##
    shc
            2 0 0 0 0 0 1 0 2 81 12 0
## [1] "(dimension, delta) = (2,150)"
```

```
ESTIMATE
##
## METHOD 1 2 3 4 5 6 7 8 9 10 11 13 14
##
           0 0 0 0 0 0 8 30 42 19 1 0 0
    Mean
    Meanl2 0 0 0 0 0 0 8 30 42 18 2
##
    Median 0 0 0 1 2 15 13 21 28 12 2 2
##
    Medianl2 0 0 0 1 6 15 16 25 17 12 2 3
##
##
    AIC
           0 0 0 0
                     0 0 8 29 43 19
                                   1
##
    BIC
           0 0 0 0 0 0 8 28 44 19 1 0
    RIFT.hc 16 3 8 2 1 3 0 2 8 57 0 0 0
##
##
          0 0 0 0 1 0 0 0 4 81 13 1 0
    shc
  [1] "(dimension, delta) = (2,200)"
##
##
    ESTIMATE
## METHOD 1 2 3 4 5 6 7 8 9 10 11 12 13 14
##
          0 0 0 0 0 1 6 21 46 26 0 0 0 0
    Mean
         0 0 0 0 0 1 6 21 46 26 0
##
    Mean12
                                     0
   Median 0 0 0 0 1 12 9 29 20 14 1 2 1 11
##
##
   Median12 0 0 0 0 4 14 17 22 14 14 0 2 2 11
           0 0 0 0 0 1 6 21 45 27 0 0 0 0
##
    AIC
##
    BIC
           0 0 0 0 0 1 6 21 44 28 0
##
   RIFT.hc 12 2 6 6 6 2 2 0 8 56 0 0 0 0
    shc 0 0 0 0 0 0 0 0 3 81 15 0 1 0
  [1] "(dimension, delta) = (8,20)"
##
##
        ESTIMATE
## METHOD
        8 9 10 11 12
##
           0 0 100
                     0 0
   Mean
   Meanl2 0
              0 100
                     0 0
##
##
   Median
            0 1 99
                     0 0
   Median12 0 1 99
##
##
   ATC
           0 0 100
                     0 0
##
    BIC
            0
              0 100
                      0 0
##
   RIFT.hc 1 0 99
                     0 0
           0 0 91
## [1] "(dimension, delta) = (8,40)"
    ESTIMATE
## METHOD
          10 11 12
##
   Mean
         100 0 0
   Meanl2 100
##
              0
                   0
##
   Median 98
               2
                 0
##
   Median12 98 2 0
##
   AIC 100 0 0
##
    BIC
          100
              0
                   0
##
   RIFT.hc 100
              0
                   0
          90
##
              7
                   3
## [1] "(dimension, delta) = (8,60)"
##
         ESTIMATE
## METHOD
          10 11 12
##
   Mean
          100 0
   Meanl2 100
##
              0
                   \cap
##
    Median
           97
               3
                   0
##
   Medianl2 97
               3
                   0
##
   AIC 100
                   0
    BIC 100
##
                   0
               0
##
    RIFT.hc 100
               0
                   0
##
   shc 93 6 1
```

```
## [1] "(dimension, delta) = (8,80)"
##
         ESTIMATE
## METHOD
           10 11 12 13
          100 0 0
##
                      0
    Mean
##
    Meanl2 100 0 0
##
    Median 97 2 0 1
##
    Median12 97 2 0 1
##
    AIC 100 0 0 0
##
    BIC
           100 0 0 0
    RIFT.hc 100 0 0 0
##
          90 9
##
    shc
                  1 0
## [1] "(dimension, delta) = (8,100)"
##
          ESTIMATE
## METHOD
          10 11 12 13 14
           100 0
                  0 0
##
    Mean
                         0
    Meanl2 100 0
                  0 0 0
##
##
    Median 93 5 0 1 1
##
    Medianl2 93 5 0 1
    AIC 100 0 0 0 0
##
##
    BIC
         100 0 0 0 0
    RIFT.hc 100 0 0 0 0
##
        89 10 1 0 0
##
    shc
## [1] "(dimension, delta) = (8,150)"
##
        ESTIMATE
           10 11 12
## METHOD
         100 0
##
    Mean
    Meanl2 100 0 0
##
##
    Median 97 3 0
    Median12 97 3 0
##
##
    AIC 100 0 0
           100 0 0
##
    BIC
    RIFT.hc 100 0 0
##
          89 9
##
    shc
                  2
## [1] "(dimension, delta) = (8,200)"
##
         ESTIMATE
## METHOD
           10 11 12 13
##
    Mean
           100
               0
                   0
                      0
    Meanl2 100 0 0 0
##
##
    Median 95 2 2 1
    Median12 95 2 2 1
##
##
    AIC 100 0 0 0
##
    BIC
         100 0 0 0
##
    RIFT.hc 100 0 0 0
##
    shc 91 8 1
                      0
#stop cluster (parallel computing)
stopCluster(cl)
print(distribution_name )
## [1] "True distribution: Normal mixture distribution"
print(paste(k, 'true clusters:'))
## [1] "10 true clusters:"
```

```
print('Cluster weights:')
## [1] "Cluster weights:"
print(w)
## [1] 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1
```

The R session information (including the OS info, R version and all packages used):

```
sessionInfo()
## R version 4.1.2 (2021-11-01)
## Platform: x86 64-apple-darwin17.0 (64-bit)
## Running under: macOS Monterey 12.0.1
## Matrix products: default
## LAPACK: /Library/Frameworks/R.framework/Versions/4.1/Resources/lib/libRlapack.dylib
##
## [1] en_GB.UTF-8/en_GB.UTF-8/en_GB.UTF-8/C/en_GB.UTF-8/en_GB.UTF-8
## attached base packages:
## [1] grid
                 parallel stats
                                     graphics grDevices utils
                                                                   datasets methods
## [9] base
##
## other attached packages:
## [1] knitr_1.37
                           sigclust_1.1.0
                                              mixtools_1.2.0
                                                                 gridExtra_2.3
## [5] ggplot2_3.3.5
                          MASS 7.3-54
                                              pracma 2.3.6
                                                                 mclust 5.4.9
## [9] sigclust2 1.2.4
                          rstudioapi 0.13
                                              mvnfast 0.2.7
                                                                 doParallel 1.0.16
## [13] iterators_1.0.13 foreach_1.5.1
                                              Rfast_2.0.6
                                                                 RcppZiggurat_0.1.6
## [17] Rcpp_1.0.8
                           pacman_0.5.1
##
## loaded via a namespace (and not attached):
     [1] colorspace_2.0-2
                                                       dynamicTreeCut_1.63-1
##
                               ellipsis_0.3.2
##
     [4] rprojroot_2.0.3
                               htmlTable_2.4.0
                                                       XVector_0.34.0
                                                       fs_1.5.2
##
   [7] base64enc_0.1-3
                                ggdendro_0.1.23
   [10] remotes_2.4.2
                                bit64_4.0.5
                                                       AnnotationDbi_1.56.2
##
##
   [13] fansi_0.5.0
                                codetools_0.2-18
                                                       splines_4.1.2
## [16] cachem_1.0.6
                                impute_1.68.0
                                                       pkgload_1.2.4
## [19] Formula 1.2-4
                                WGCNA 1.70-3
                                                       cluster 2.1.2
## [22] kernlab_0.9-29
                                GO.db_3.14.0
                                                       png_0.1-7
##
   [25] compiler 4.1.2
                                httr 1.4.2
                                                       backports 1.4.1
## [28] Matrix_1.3-4
                                                       cli_3.3.0
                                fastmap_1.1.0
## [31] htmltools 0.5.2
                                prettyunits_1.1.1
                                                       tools 4.1.2
##
   [34] gtable_0.3.0
                                glue_1.6.1
                                                       GenomeInfoDbData_1.2.7
##
   [37] dplyr_1.0.7
                                ggthemes 4.2.4
                                                       Biobase 2.54.0
##
   [40] vctrs_0.4.1
                                Biostrings_2.62.0
                                                       preprocessCore_1.56.0
   [43] xfun_0.30
                                fastcluster_1.2.3
                                                       stringr_1.4.0
   [46] ps_1.7.0
                                brio_1.1.3
##
                                                       testthat_3.1.4
##
   [49] lifecycle_1.0.1
                                devtools_2.4.3
                                                       zlibbioc_1.40.0
## [52] scales_1.1.1
                                RColorBrewer_1.1-2
                                                       memoise_2.0.1
## [55] rpart_4.1-15
                                segmented_1.3-4
                                                       latticeExtra_0.6-29
## [58] stringi_1.7.6
                                RSQLite_2.2.10
                                                       highr_0.9
## [61] S4Vectors_0.32.3
                              desc_1.4.1
                                                       checkmate_2.0.0
```

```
## [64] BiocGenerics_0.40.0
                               pkgbuild_1.3.1
                                                       GenomeInfoDb_1.30.1
## [67] rlang_1.0.2
                               pkgconfig_2.0.3
                                                      matrixStats_0.61.0
## [70] bitops 1.0-7
                                evaluate 0.15
                                                      lattice_0.20-45
## [73] purrr_0.3.4
                               htmlwidgets_1.5.4
                                                      bit_4.0.4
## [76] tidyselect_1.1.1
                               processx_3.5.3
                                                      magrittr_2.0.2
## [79] R6_2.5.1
                               IRanges_2.28.0
                                                       generics_0.1.1
## [82] Hmisc_4.6-0
                               DBI_1.1.2
                                                      pillar_1.6.4
## [85] foreign_0.8-81
                               withr_2.4.3
                                                       survival_3.2-13
## [88] KEGGREST_1.34.0
                               RCurl_1.98-1.6
                                                      nnet_7.3-16
## [91] tibble_3.1.6
                                crayon_1.4.2
                                                      utf8_1.2.2
## [94] jpeg_0.1-9
                               usethis_2.1.6
                                                       data.table_1.14.2
## [97] blob_1.2.2
                                callr_3.7.0
                                                       digest_0.6.29
## [100] stats4_4.1.2
                               munsell_0.5.0
                                                       {\tt sessioninfo\_1.2.2}
Sys.time()
## [1] "2022-06-08 17:30:03 BST"
```