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I. Executive Summary

Recent shifts in the global economy necessitate precise estimation of the natural interest rate (R*) given the importance of R* for determining monetary policy. This paper aims to construct effective Taylor Rules using US and international data to identify the most suitable value of R* for advanced economies. The firm aim is to determine the optimal R* value, which is achieved by inputting data from the United States, United Kingdom, European Union, and Japan into the Taylor Rule to extrapolate R* values over time. The second aim is to improve global applicability of the Taylor Rule by considering the significance of adding the real exchange rate. We employ simple and rolling regressions to gather estimates of R* and assess the significance of the real exchange rate. Our approach involves running these regressions in non-inertial and inertial settings upon gathering data on key economic indicators, such as interest rates, inflation, output gap, and exchange rates. We find that R* should be 2% for advanced economies (particularly the US, UK, and the EU) but countries facing deep-rooted economic problems (such as Japan) should adopt a lower R* value of 0.5%. We also find that adding the real exchange rate produces significant results for the UK and Japan thus suggesting it is a useful modification to the Taylor Rule. The international variances in R* could therefore be addressed using a modified version of the Taylor Rule which includes the real exchange rate. Although our paper faces limitations in data availability, our alternative uses of data as well as comprehensive methodology lends credibility to our results. Based on our findings, we recommend following the Taylor Rule to ensure policy stability, emphasizing the importance of maintaining a 2% inflation target. We also recommend incorporating the real exchange rate in the Taylor Rule when determining monetary policy as this holds promise for achieving improved global economic outcomes.

II. Introduction

In recent years, economists have been challenged by the task of accurately estimating the natural interest rate (denoted R^*) due to various exogenous factors, from increasing levels of globalized interconnectedness to severe economic disruptions such as the COVID-19 pandemic. The shifting dynamics of the global economy have affected the equilibrium interest rate thereby reigniting the debate on understanding the underlying drivers and precise values of R^* . This paper aims to contribute to the debate by addressing recent issues when estimating Taylor Rules using US and international evidence thereby improving globally-applicable methods for calculating R^* .

We divide the paper in two parts: (i) estimating a Taylor Rule to calculate a globally-applicable R* and (ii) assessing the significance of the real exchange rate when estimating the Taylor Rule on an international scale. The former section conducts multiple regressions to

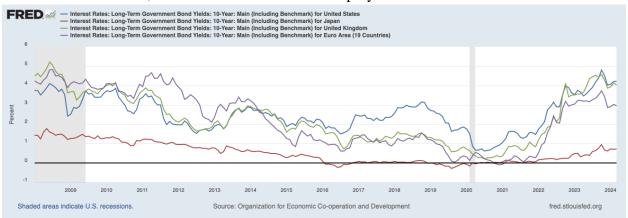
estimate Taylor Rules that are globally applicable and then infers the R* from these results. The latter section leverages international data for four countries – the US, UK, EU, Japan – to evaluate whether the real exchange rate is significant to the Taylor Rule. Our analysis allows us to estimate a more accurate R* by utilizing our improved estimation of the Taylor Rule, which in turn can provide valuable insights to policymakers striving to navigate the complexities of monetary policy in our increasingly globalized world. Our paper ultimately aspires to enhance the understanding of R* and its determinants in order to equip policymakers with improved tools when implementing monetary policy.

A. Background

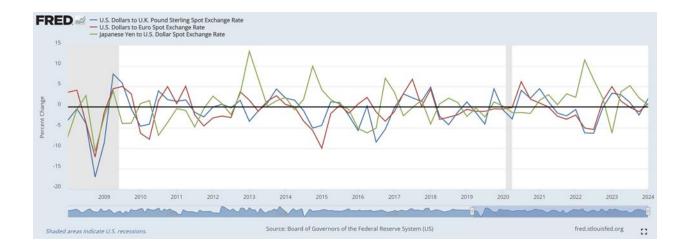
To develop a comprehensive understanding of shifting economic dynamics, we must consider recent trends across the following three variables: interest rate, exchange rate, and the unemployment rate.

Independent:

• Interest Rate - The central bank's interest rate policy can influence borrowing, spending, and investment, which in turn can affect unemployment rates.



 Exchange Rate - Exchange rate fluctuations can impact export competitiveness and demand for domestically produced goods, which can affect employment in exportoriented industries



Dependent:

Unemployment Rate



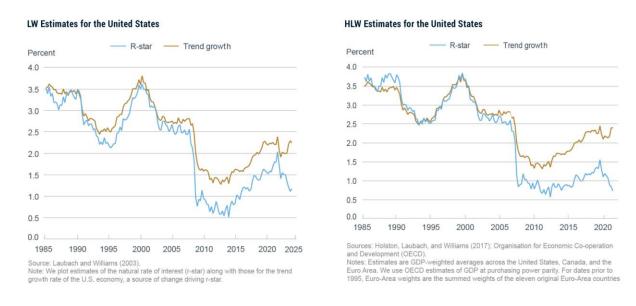
B. Literature Review

There is a vast amount of literature studying the impact of monetary policy on unemployment, both locally and on a global scale. This section explores the various dimensions covered by relevant literature in the field, focusing specifically on three key themes: (1) estimating R*; (2) the Taylor Rule and its modifications, (3) international applications of the Taylor Rule; (4) significance of the real exchange rate

Estimating R*

The natural rate of interest (R^*) is foundational in this field given its significance to monetary policy. R^* refers to the short-term interest rate that prevails when the economy is at full employment and stable inflation. Unlike the interest rate, which is set by the Federal Reserve, the natural rate of interest reflects the underlying characteristics of the economy. Recent

estimates for R^* vary from 0.5% to 2% – the FOMC's implied long-run equilibrium real interest rate is 0.5% whilst John Taylor maintains a rate of 2%. The current literature on estimating R^* is best represented through the most popular models being used today: the Laubach-Williams (LW) model and the Holston-Laubach-Williams (HLW) model.



The LW model estimates R* using data on real GDP, inflation, and FFR for the US whilst the HLW model extends the LW model by incorporating international data (US, Canada, and Euro Area) when estimating R*. The outcome of both these models can be seen in the graphs above. It is clear that both these methods of estimating R* produce different values for R*, which highlights the nature of the current debate on R*. Further, the Taylor Rule can also be used to estimate R* given its purpose of aligning actual interest rates with the natural interest rate. Specifically, the intercept of the Taylor Rule allows us to infer R*. However, the large variance in R* across these models suggests a gap that can be filled by improving the accuracy of the Taylor Rule.

It is also important to consider the impact of shocks when estimating R*. The San Francisco Fed defines the natural rate as "the real fed funds rate consistent with stable inflation absent shocks to demand and supply" so when accounting for shocks, the real fed funds rate cannot be used. This is because during times of crisis (e.g., COVID-19), the federal funds rate is often close to zero to help stimulate the economy, which makes it difficult to analyze monetary policy. Thus, instead of using the real fed funds rate, economists can use the Wu-Xia federal

¹ <u>https://www.frbsf.org/research-and-insights/publications/economic-letter/2003/10/the-natural-rate-of-interest/#subhead1</u>

funds rate. These shadow federal funds rates aim to fill in the gap by providing a measure of monetary policy even when the actual federal funds rate is near zero. The Wu-Xia shadow federal funds rate is an important measure for us to consider in our paper given recent times of economic distress. Notably, the shadow rates across countries internationally tend to follow the same path. This helped us standardize our selected four countries – US, EU, Japan, UK – because they all have data with the same rate.

Taylor Rule and its modifications

The Taylor Rule is central to the foundation of our paper given its significance for understanding, constructing, and applying monetary policy as a tool for addressing everchanging economic conditions. We focus on two key papers for this theme: "The Taylor principles" by Nikolsko-Rzhevskyy, Papell and Prodan, and "Taylor Rules and the Deutschmark–Dollar Real Exchange Rate" by Engel and West.

Nikolsko-Rzhevskyy, Papell and Prodan's paper "The Taylor Principles" provides an overview of the varying applications of the Taylor Rule as well as the resulting monetary policy eras. They note, in particular, the different modifications that are constantly being made. The modifications of the Taylor rule include increasing the coefficient on the output gap and allowing for a time-varying equilibrium real interest rate. The authors' economic analysis ultimately shows that violations of the Taylor principles led to significant deviations in different eras, affecting economic stability. The methodology used to achieve such modifications in Nikolsko-Rzhevskyy, Papell and Prodan's paper is immensely useful to our own project – The methodology employed by Nikolsko-Rzhevskyy, Papell, and Prodan is also crucial for our paper given its success in allowing the authors to conduct an in-depth analysis on how modifications of the Taylor Rule affect economic stability. We apply the R* equation for Taylor Rule estimates onto our data and use it to calculate our results – this is further elaborated upon in the methodology section.

Additionally, we are interested in adding real exchange rates to our estimates of the Taylor Rule given the relevance of real exchange rate in measuring the economic standings of different countries. "Taylor Rules and the Deutschmark–Dollar Real Exchange Rate" by Engel and West provides strong footing for understanding how to create further modifications. The authors add the deviation of the real exchange rate from its steady state value to a Taylor rule that also includes standard terms in inflation and output. This delivers a relation between current and expected real exchange rates on the one hand and inflation and output on the other. The model reproduces many significant features of the real and nominal dollar / DM exchange rates, but does not reproduce correlations of exchange rates with output and inflation. We intend to address this gap in our paper.

International applications of Taylor Rules

Whilst the Taylor Rule provides a strong foundation for our analysis, the global element of our paper necessitates a consideration of standardizing monetary policy rules across different countries. Nikolsko-Rzhevskyy, Papell and Prodan's paper leaves us with a number of international-oriented questions: How has the Taylor rule been implemented in different countries? What modifications to the Taylor rule have been made to suit specific economic conditions in various countries? How do the coefficients on inflation and output gap differ across countries? Our paper intends to answer these questions by leveraging existing literature on the international application of the Taylor Rule, specifically El-Shagi and Ma's "Taylor rules around the world: Mapping monetary policy" and Teryoshin's "Historical Performance of Rule-Like Monetary Policy".

The importance of Taylor Rules as a standardized framework of inflation targeting is emphasized by El-Shagi and Ma thus affirming our decision to use Taylor Rules as our central form of analysis. Existing literature further suggests that adherence to the Taylor Rule often produces the greatest success, as defined by economic stability. Teryoshin, for example, finds evidence that Taylor rule-like policies produce greater economic stability. This result is an outcome of constructing the differences between rule-based monetary policy for multiple interest rate rules and the actual interest rates for nine countries using real-time data available to policymakers at the time. Despite the substantiated success of adhering to the Taylor Rule, global applications of the Taylor Rule can be tricky. El-Shagi and Ma find that evidence on the global use of the Taylor rule is mixed, suggesting that strict adherence to it is more the exception than the rule. This highlights the variability in adherence to the Taylor Rule, especially in times of high inflation, which shows us the importance of modifying the Taylor Rule to suit countries' needs better.

Significance of real exchange rate

We hypothesize that the real exchange rate would be a useful modification to the Taylor Rule when considering international data. To develop this hypothesis, we explore existing literature on the matter, specifically "The Real Exchange Rate in Taylor Rules: A Re-Assessment" by Richard Froyen and Alfred Guender. This paper focuses on the role of the real exchange rate in Taylor Rules within the context of flexible inflation targeting strategies. The authors find that employing the real exchange rate in the Taylor Rule results in the inflation target strategies seeing a reduction of about 10%. The paper looks into a new Keynesian model of a small open economy to serve as the framework for evaluating Taylor-type rules. It was found that a small weight on the real exchange rate affects the performance of Taylor-type rules. This addition of the small weight of the real exchange rate leads to the optimal policy being less

aggressive. The less aggressive optimal approach leads to a more stable and predictive policy, which, in the realm of international policy, leads to the most favorable outcome. Much of this is because a less optimal policy will allow greater stability in the economy. The stability comes from the predictability of the policy so all countries can properly respond together, allowing for greater cooperation between nations. This reinforces our hypothesis of including the real exchange rate in our regressions. The real exchange rate allows the Taylor Rule to address international dimensions. Due to increased volatility in international markets, the real exchange rate is a necessary component to account for when looking at policy goals.

III. Data and Methodology

A. Descriptive Data

The inputs we gathered included shadow rate, inflation (CPI), output gap (real GDP-real potential gdp)/ potential real gdp), and exchange rate (real narrow exchange rate) across all the respective central bank regions. Most of our data was found from St. Louis FRED's website², and if not, we found it from the Bank of England and European Commission websites. Exchange rates were found from the IMF. Originally we were planning on looking at quarterly starting from 1985, but we were only able to find yearly data. Another important thing we did was look at shadow rates vs. federal funds rates. We began using federal funds rates but transitioned to shadow rates when the former turned negative, employing the Wu-Xia (2016) approach for calculating shadow rates. This approach estimates the shadow rate by modeling the term structure of interest rates in a way that accounts for the lower bound on nominal interest rates. This method allows for the calculation of shadow rates even when the policy rate is constrained by the zero lower bound, providing a more accurate representation of the monetary policy stance during periods of unconventional monetary policy measures.

B. Methodology

1. U.S. Methodology

To estimate R*, we follow the formula employed by Nikolsko-Rzhevskyy, Papell and Prodan's paper, "The Taylor Principles." Specifically, the equation is as follows:

$$R^* = \mu + \phi \pi^*$$

² https://fred.stlouisfed.org/

³ https://www.sciencedirect.com/science/article/pii/S016407041930028X?ref=pdf_download&fr=RR-2&rr=88f6233c98cc15f5

where R* is the equilibrium level of the real interest rate, μ is the intercept, ϕ represents the inflation coefficient - 1, and π represents the target inflation rate, in which we set at 2%, as is the standard. To find the relevant values, we run a series of linear regressions using the data described above for the U.S., U.K., European Union, and Japan. For the U.S., we run a simple regression in which *Inflation* represents the Core PCE and Output Gap is calculated by (Real GDP - Potential GDP) / Potential GDP. We run this simple regression for three time periods: (1) 1985-2019, (2) 1985-2007, and (3) 2009-2019, to understand how R* may change during different periods, especially before and after the Great Recession.

Federal Funds Shadow Rate = $\beta_1 + \beta_2 * Inflation + \beta_3 * Output Gap + \epsilon$

Further, to understand how R* changes continuously and through 2019-2023, we run a rolling regression using the above equation. This is characterized by utilizing a data fragment consisting of fifty quarters of data at once, and "rolling" the window over by one for the next regression. To illustrate this, take a dataset which consists of one-hundred datapoints. Using a window size of fifty observations, we are able to calculate fifty regressions: the first which takes observations one to fifty, the second which takes observations two to fifty-one, and so on. The last regression would be to use observations fifty-one to one-hundred. Using this method, we are able to supplement the low amount of post-2020 data with historic data to build our analysis of what affect the pandemic had on R*.

Finally, we run a regression employing inertial rules. Although the simple regression model previously outlined is expected to deliver promising and optimal R* estimates, it may predict a more variable path of interest rates than what is experienced in practice. This is because central banks display "interest rate smoothing" — a degree of dependence on the prior term's interest rate — in deciding the interest rate of the next quarter. The end result of this smoothing, as historically observed, is less volatility in the interest rate than what is expected under a research vacuum.

More explicitly, we modify our simple domestic regression model as so:

Federal Funds Shadow Rate_t = ρ * Federal Funds Shadow Rate_{t-1} + $(1 - \rho)$ [$\beta_1 + \beta_2$ * Inflation + β_3 * Output Gap] + ε_t

where ρ is the smoothing coefficient. That is, the next quarter's interest rate is the weighted average of the prior interest rate (given by the *Federal Funds Shadow Rate_{t-1}* term) and the state economy (given by the $[\beta_1 + \beta_2 * Inflation + \beta_3 * Output Gap]$ term), where the weights are dependent on the smoothing parameter, ρ . Prior research has shown that ρ is "typically on the

order of 0.8 or 0.9", which suggests very slow adjustment in practice⁴. Therefore, we decided to set ρ equal to the mean of these numbers, which is 0.85.

2. International Methodology

To estimate R* for the U.K., E.U., and Japan, we run a similar series of non-inertial and inertial regressions. For the non-inertial methodology, we run the following regressions:

Interest Rate_i =
$$\beta_1 + \beta_2 * CPI_i + \beta_3 * Output Gap_i + (\beta_4 * Real Exchange Rate_i) + \epsilon$$

with interest rates in region *i*, CPI as a measure for inflation in region *i*, output gaps using the same equation as for the U.S. section, and the real exchange rate representing the narrow real exchange rate. We ran two separate simple regressions, one with and one without the real exchange rate, to understand how influential the real exchange rate is in various markets' interest rates.

Akin to the smoothing parameter found in the domestic inertial equation, we derive an inertial equation for each international sector to estimate R*:

Interest Rate_t =
$$\rho$$
 * Interest Rate_{t-1} + (1 - ρ) [$\beta_1 + \beta_2$ * CPI + β_3 * Output Gap] + ε_t

keeping ρ , the smoothing coefficient, equal to 0.85 to mimic the smoothing experienced domestically. In this case, the next quarter's interest rate is the weighted average of the prior interest rate (given by the *Interest Rate_{t-1}* term) and the state economy (given by the $[\beta_1 + \beta_2 * CPI + \beta_3 * Output Gap]$ term), where the weights are dependent on the smoothing parameter, ρ .

IV. Discussion: Results and Analysis

A. US: Non-Inertial, Rolling, and Inertial

Our initial results for the simple domestic model can be found in Table 1; the calculated R^* values of these periods can be found in Table 2.

⁴ Clarida, R., Gali, J., & Gertler, M. (1999). *The Science of Monetary Policy: A New Keynesian Perspective*. https://doi.org/10.3386/w7147

Table 1: Regression Results

		$Dependent\ variable:$	
		FFR Shadow	
	1985-2019	1985-2007	2009-2019
Inflation	2.304***	2.006***	-0.386
	(0.138)	(0.132)	(0.692)
Output Gap	0.906***	0.895***	0.539***
	(0.073)	(0.090)	(0.120)
Constant	-0.908*	0.293	1.525
	(0.345)	(0.348)	(1.214)
Observations	156	92	44
R^2	0.436	0.739	0.358
Adjusted R ²	0.429	0.734	0.327
Residual Std. Error	2.396 (df = 153)	1.124 (df = 89)	1.243 (df = 41)
F Statistic	59.115*** (df = 2; 153)	126.265*** (df = 2; 89)	11.429*** (df = 2; 41)

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 1: Initial Domestic Simple Regression Results

Table 2: Summary of U.S. R* Values

Period	R_star
1985-2019	1.70
1985-2007	2.30
2009 - 2019	-1.25

Table 2: Initial Summary of Domestic R* Values

In Table 2, it is shown that the United States had an overall positive domestic R* value, reflecting the general upward trend of the economy over time. Furthermore, the larger R* value calculated from 1985 to 2007 could potentially reflect the strength of the United States economy in the years leading up to the financial crisis, which matches the conclusion found by Laubach and Willams where a boost to R* results from "relatively strong growth" during that period⁵. However, the post-Great Recession R* is a different story: from 2009 to 2019, the R* is negative. This again confirms the assumptions that Laubach and Williams derive: contrarily from a positive R*, a negative R* suggests that the "state of policy... [was not] as expansionary" following the Great Recession in comparison to pre-2008 levels⁶. This could have been due to an

⁵ Laubach and Williams, "Measuring the Natural Rate of Interest," *Review of Economics and Statistics* 85, no.4 (November 2003): 1063-70.

⁶ Laubach and Williams, "Measuring the Natural Rate of Interest," *Review of Economics and Statistics* 85, no.4 (November 2003): 1063-70.

overall desire by both the government to seek risk averse policies and investors to investment in low-yield, low-volatility portfolio strategies⁷.

Our initial plan was to analyze data up until 2019; however, our rolling regression results found in Table 3 caused us to slightly deviate from this original plan.

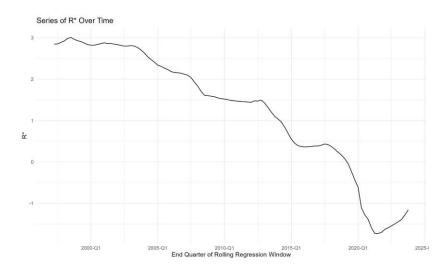


Table 3: Domestic Rolling Regression Results

As seen in the graph, R* plummeted following 2019 until 2021. However, the R* slope experienced a swift turnaround in 2022 (perhaps in response to the bounce-back of the global economy around this time), and has continued to rise since. Therefore, we decided to also include 2020 to 2023 data in a final regression displayed in Table 4, which derived an R* that can be seen in Table 5.

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⁷ Torga, E., Roma, C., Roma, P., & Ferreira, B. (2023). Ten years after the 2008 crisis: Has risk aversion won? *Brazilian Business Review*, 20(3), 323–338. https://doi.org/10.15728/bbr.2023.20.3.5.en

Table 4: Regression Results: 1985-2023

	$Dependent\ variable:$
	ffr_shadow
inflation	1.234***
	(0.180)
output_gap	0.726***
	(0.107)
Constant	0.918*
	(0.496)
Observations	156
\mathbb{R}^2	0.436
Adjusted R^2	0.429
Residual Std. Error	2.396 (df = 153)
F Statistic	$59.123^{***} (df = 2; 153)$
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 4: Domestic Regression Results of Years 1985 - 2023

Table 5: Summary of R* Values

Period	R_star
1985-2019	1.700
1985-2007	2.304
2009-2019	-1.246
1985-2023	1.387

Table 5: Summary of All Domestic R* Values, including 1985-2023

As expected, the 1985 to 2023 R* is slightly lower than the 1985 to 2019 R*. This is because, although the rolling regression shows a rising R* after 2022, it has still not yet reached pre-Covid levels.

Finally, our domestic regression results can be seen in Table 6, and its corresponding R* calculations can be observed in Table 7.

Table 6: Regression Results: U.S. Inertial

	$Dependent\ variable:$
	ffr_shadow
inertia	1.078***
	(0.016)
adjusted_inflation	1.145***
·	(0.243)
adjusted_output_gap	0.872***
	(0.143)
Constant	-0.036
	(0.092)
Observations	155
R^2	0.981
Adjusted R ²	0.981
Residual Std. Error	0.435 (df = 151)
F Statistic	$2,635.291^{***} \text{ (df} = 3; 151)$
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 6: Domestic Inertial Regression Results

Table 6: Summary of Domestic Inertial R* Value

Period	R_star
1985 Q1 - 2023 Q4	0.253

Table 7: Summary of Domestic Inertial R* Results from 1985 to 2019

As stated in the Methodology section, the role of utilizing an inertial regression model is to take into account a central bank's reliance on prior interest rates to determine the next interest rate. This results in less volatility in the R* values observed in the regressions. That is exactly the case in these results: although the simple model derived an R* of 1.387 from 1985 to 2023, the inertial model found an R* of .253.

B. International: Non-inertial and inertia

Non-Inertial

Our regression results for the simple international models on the U.K., EU, and Japan can be found in Table 8, Table 10, and Table 12, respectively. It is important to note that, for brevity, only the regression results from the years 1985 to 2023 are included in this paper. However, the calculated R* values of all observed periods for the U.K., EU, and Japan are seen in Table 9, Table 11, and Table 13, respectively.

Table 8: Regression Results: UK 1985-2023

	Dependent variable:
	$InterestRate_UK$
CPLUK	1.417***
	(0.276)
OutputGap_UK	0.710**
	(0.331)
Constant	0.789
	(0.959)
Observations	38
\mathbb{R}^2	0.456
Adjusted R^2	0.425
Residual Std. Error	3.188 (df = 35)
F Statistic	$14.677^{***} (df = 2; 35)$
Note:	*p<0.1; **p<0.05; ***p<0.05

Table 8: Regression Results for the U.K. for 1985 to 2023

Table 9: Summary of R* Values for UK

Period	R_star
1985-2019	1.758
1985-2007	3.412
2009-2019	-1.523
1985 - 2023	1.623

Table 9: Summary of R* Values for Every Period in U.K.

Table 10: Regression Results: EU 1985-2023

	$Dependent\ variable:$	
	$InterestRate_EU$	
CPLEU	0.765***	
	(0.233)	
OutputGap_EU	0.384	
	(0.252)	
Constant	1.970**	
	(0.744)	
Observations	38	
\mathbb{R}^2	0.342	
Adjusted R ²	0.305	
Residual Std. Error	2.251 (df = 35)	
F Statistic	$9.105^{***} (df = 2; 35)$	
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 10: Regression Results for the EU for 1985 to 2023

Table 11: Summary of R* Values for EU

Period	R_star
1985-2019	1.461
1985-2007	2.212
2009-2019	-1.783
1985-2023	1.499

Table 11: Summary of R* Values for Every Period in EU

Table 12: Regression Results: Japan 1985-2023

	$Dependent\ variable:$	
	InterestRate_JPN	
CPI_JPN	0.631**	
	(0.251)	
OutputGap_JPN	0.252	
	(0.172)	
Constant	0.855***	
	(0.253)	
bservations	38	
\mathbb{R}^2	0.449	
Adjusted R^2	0.418	
Residual Std. Error	1.214 (df = 35)	
F Statistic	$14.262^{***} \text{ (df} = 2; 35)$	
Note:	*p<0.1; **p<0.05; ***p<	

Table 12: Regression Results for Japan for 1985 to 2023

Table 13: Summary of R* Values for Japan

Period	R_star
1985-2019	0.526
1985-2007	1.612
2009-2019	-1.700
1985 - 2023	0.117

Table 13: Summary of R* Values for Every Period in Japan

Akin to the summary of domestic R* values, each of the three international banks—the U.K., EU, and Japan—all have positive R* values from 1985 to 2019, inflated R* values from 1985 to 2007, negative R* values from 2009 to 2019, and positive R* values from 1985 to 2023. This data reflects the effects of the Great Recession on an international scale: as was seen in the United States, these international banks sought risk-averse measures in an effort to stabilize their own economies post-2008. Furthermore, international investors also turned risk averse in the wake of the 2008 financial crisis: they reduced allocations to stocks but increased investments in long-term bonds, and investors repatriated capital from overseas investment destinations⁸. In all, these stabilization efforts led to lower returns on investments, resulting in lower R* levels.

Inertial

Our regression results for the inertial international models on the U.K., EU, and Japan can be found in Table 8, Table 10, and Table 12, respectively. It is important to note that, for brevity, only the regression results from the years 1985 to 2023 are included in this paper. However, the calculated R* values of all observed periods for the U.K., EU, and Japan are seen in Table 14, Table 15, and Table 16, respectively. The calculated R* results from these regressions can be found in Table 17.

⁸ Ogasawara, S., & Iwatsubo, K. (2016). International portfolio flows in the post-global financial crisis period. *Japanese Journal of Monetary and Financial Economics*, *4*(1), 18–37. https://doi.org/10.32184/jjmfe.4.1_18

Table 14: Regression Results: UK Inertial

	$Dependent\ variable:$
	InterestRate_UK
uk_inertia	0.968***
	(0.062)
adj_CPI_UK	2.150**
	(0.799)
adj_OutputGap_UK	3.108***
	(0.778)
Constant	-0.270
	(0.347)
Observations	37
\mathbb{R}^2	0.932
$Adjusted R^2$	0.925
Residual Std. Error	1.114 (df = 33)
F Statistic	$149.827^{***} (df = 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; $
Note:	*p<0.1; **p<0.05; ***p<

Table 14: Inertial Regression Results for the U.K. from 1985 to 2023

Table 15: Regression Results: EU Inertial

	Dependent variable:
	InterestRate_EU
eu_inertia	1.051***
	(0.041)
adj_CPI_EU	1.131***
	(0.389)
adj_OutputGap_EU	1.271***
	(0.385)
Constant	-0.147
	(0.190)
Observations	37
\mathbb{R}^2	0.968
Adjusted R ²	0.965
Residual Std. Error	0.503 (df = 33)
F Statistic	$331.501^{***} (df = 3; 3)$
Vote:	*p<0.1; **p<0.05; ***p<

Table 15: Inertial Regression Results for the EU from 1985 to 2023

Table 16: Regression Results: JPN Inertial

	$Dependent\ variable:$
	$InterestRate_JPN$
jpn_inertia	0.839***
	(0.067)
adj_CPI_JPN	0.689
	(0.706)
adj_OutputGap_JPN	1.371***
	(0.462)
Constant	0.189
	(0.113)
Observations	37
R^2	0.905
Adjusted R ²	0.897
Residual Std. Error	0.476 (df = 33)
F Statistic	$105.060^{***} (df = 3; 33)$
Note:	*p<0.1; **p<0.05; ***p<0.0

Table 16: Inertial Regression Results for Japan from 1985 to 2023

Table 17: Summary of International Inertial R* Values

Location	R_star
UK	2.031
EU	0.114
Japan	-0.433

Table 17: Summary of all International Inertial R* Values from the 1985 to 2023 Period

Interestingly, the repressed R^* pattern observed in the domestic inertial regression is observed in the EU and Japan, but not in the U.K.; rather, the U.K. inertial R^* (2.031) is greater than the U.K. simple R^* (1.499). This could suggest that the U.K.'s historic interest rates are too high in comparison to what is suggested by the simple regression model. Indeed, prior research indicates that internally appointed MPC members of the Bank of England have historically preferred setting higher interest rates than external appointees, suggesting that the Bank of England prefers higher interest rates than what would be expected.

⁹ Spencer, C. (2006). Reaction Functions of Bank of England MPC Members: Insiders versus Outsiders. *Discussion Papers in Economics*.

Real Exchange Rate

Our regression results for the effects of the real exchange rate on the Taylor rule are illustrated in Table 18, Table 19, and Table 20. While the real exchange rate is not considered in the Taylor Rule, we see that the real exchange rate is significant in estimating the interest rate in the U.K. and Japan, although not in the U.K. Given that the real exchange rate influences key economic variables that Central Banks consider in setting the interest rate, it follows that the real exchange rate is correlated with changes in the interest rate.

Table 7: Regression Results: UK with Real Exchange Rate

	$Dependent\ variable:$
	InterestRate
CPI	1.282***
	(0.212)
OutputGap	0.014
	(0.285)
ExchangeRate	0.249***
	(0.048)
Constant	-26.857***
	(5.405)
Observations	38
\mathbb{R}^2	0.695
$Adjusted R^2$	0.668
Residual Std. Error	2.422 (df = 34)
F Statistic	$25.835^{***} (df = 3; 34)$
Vote:	*p<0.1; **p<0.05; ***p<

Table 18: Regression Results: UK with Real Exchange Rate

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Table 8: Regression Results: EU with Exchange Rate

	$Dependent\ variable:$
	InterestRate
CPI_EU	0.764***
	(0.236)
OutputGap	0.398
	(0.264)
ExchangeRate	0.012
	(0.055)
Constant	0.782
	(5.595)
Observations	38
\mathbb{R}^2	0.343
Adjusted R ²	0.285
Residual Std. Error	2.282 (df = 34)
F Statistic	$5.920^{***} (df = 3; 34)$
Note:	*p<0.1; **p<0.05; ***p<0.05

Table 19: Regression Results: EU with Real Exchange Rate

Table 9: Regression Results: Japan with Exchange Rate

	$Dependent\ variable:$
	InterestRate
CPI_JPN	0.801***
	(0.233)
OutputGap	0.194
	(0.156)
ExchangeRate	0.028***
	(0.009)
Constant	-2.618**
	(1.166)
Observations	38
\mathbb{R}^2	0.567
Adjusted R ²	0.528
Residual Std. Error	1.092 (df = 34)
F Statistic	$14.821^{***} (df = 3; 34)$
Note:	*p<0.1; **p<0.05; ***p<0

Table 20: Regression Results: Japan with Real Exchange Rate

The Bank of England and the Bank of Japan may consider the real exchange rate as the U.K. is a significant financial sector with significant trade relationships and Japan is a highly export-oriented economy. These considerations influence both Central Banks, given the exchange rates' influence on inflationary and deflationary pressures. The effect of the real interest rate on the EU's interest rate may not be significant or diluted because the EU is composed of twenty-seven nations with diverse trade profiles; fluctuations in the real exchange rate would affect member countries differently, to maintain price stability across member states.

C. Constraints and Limitations

With the regression analysis, it is important to understand the strengths and weaknesses of the results. Even though our regression analysis found conclusive results in the United States and across the globe. It is important to note that the data sample found and used for the regression analysis for the European Union, United Kingdom, and Japan was taken annually. This was particularly a problem with data availability for the European Union. We standardized the data to be annual to stay consistent with the lack of data from the European Union. Although this is a distinct point of concern for delivering a clear-cut policy recommendation, we still believe calculating Euro Area data is much more impactful than arbitrarily selecting a single country within the European Union to focus on in our analysis.

V. Conclusion

Our paper aims to investigate monetary policy rules in four different countries (US, UK, EU, Japan) and understand its impact on economic stability during different periods. We do so by evaluating four different countries' monetary policy (US, UK, Japan, EU). We have addressed two key objectives: estimating a globally applicable R* using Taylor Rules and assessing the significance of the real exchange rate in international Taylor Rule regressions. To do this, we estimate Taylor Rules (both inertial and non-inertial) for each country using available data from 1985-2024 and use the results to estimate R* and modify the Taylor Rule to account for international data. Based on our findings, our recommendation is for countries to adhere to Taylor Rules given the significance of monetary policy on general economic well-being (as measured by inflation and output gap) as well as maintain John Taylor's recommendation of a 2% target inflation. This can be seen through our US simple regression. Further, for international locations, we find that the real exchange rate is indeed significant and it allows us to formulate improved policies across the globe. The LW model shows how R* fluctuated significantly over the past few years. While it seems R* was recovering in the first quarter of 2024, it is insufficient for a trend of increasing R*. Our policy recommendation would be that interest rates should stay

at the 5.25%-5.50% percent range. Through our research and data that we discovered, we believe that this is a strongly backed recommendation.

VI. Response to Comments

Describe the Taylor rule, each term and the data that you use for each variable. Also explain the inertial rule, what does the inertial coefficient stand for, why is it used and what are the values that are found in the literature (0.85 seems to be the value that is used)

This comment is addressed in the Methodology → US methodology section. Our response is pasted below but further detail can also be found in the body of the paper itself.

Describing the Taylor Rule and its variables:

"To estimate R*, we follow the formula employed by Nikolsko-Rzhevskyy, Papell and Prodan's paper, "The Taylor Principles." Specifically, the equation is as follows:

$$R^* = \mu + \phi \pi *$$

where R* is the equilibrium level of the real interest rate, μ is the intercept, ϕ represents the inflation coefficient - 1, and π represents the target inflation rate, in which we set at 2%, as is the standard."

Explaining inertial rule

"Finally, we run a regression employing inertial rules. Although the simple regression model previously outlined is expected to deliver promising and optimal R* estimates, it may predict a more variable path of interest rates than what is experienced in practice. This is because central banks display "interest rate smoothing" — a degree of dependence on the prior term's interest rate — in deciding the interest rate of the next quarter. The end result of this smoothing, as historically observed, is less volatility in the interest rate than what is expected under a research vacuum.

More explicitly, we modify our simple domestic regression model as so:

 $\textit{Federal Funds Shadow Rate}_t = \rho * \textit{Federal Funds Shadow Rate}_{t-1} + (1 - \rho) \left[\beta_1 + \beta_2 * \textit{Inflation} + \beta_3 * \textit{Output Gap} \right] + \varepsilon_t$

https://www.sciencedirect.com/science/article/pii/S016407041930028X?ref=pdf_download&fr=RR-2&rr=88f6233c98cc15f5

where ρ is the smoothing coefficient. That is, the next quarter's interest rate is the weighted average of the prior interest rate (given by the *Federal Funds Shadow Rate_{t-1}* term) and the state economy (given by the $[\beta_1 + \beta_2 * Inflation + \beta_3 * Output Gap]$ term), where the weights are dependent on the smoothing parameter, ρ . Prior research has shown that ρ is "typically on the order of 0.8 or 0.9", which suggests very slow adjustment in practice¹¹. Therefore, we decided to set ρ equal to the mean of these numbers, which is 0.85."

Before the results section you should have a section with the Data that you use. Explain data and sources.

We included a data and methodology section in our final draft, which includes a description of the data and its sources. Our direct response is pasted below.

"The inputs we gathered included shadow rate, inflation (CPI), output gap (real GDP-real potential gdp)/ potential real gdp), and exchange rate (real narrow exchange rate) across all the respective central bank regions. Most of our data was found from St. Louis FRED's website 12, and if not, we found it from the Bank of England and European Commission websites. Exchange rates were found from the IMF. Originally we were planning on looking at quarterly starting from 1985, but we were only able to find yearly data. Another important thing we did was look at shadow rates vs. federal funds rates. We began using federal funds rates but transitioned to shadow rates when the former turned negative, employing the Wu-Xia (2016) approach for calculating shadow rates. This approach estimates the shadow rate by modeling the term structure of interest rates in a way that accounts for the lower bound on nominal interest rates. This method allows for the calculation of shadow rates even when the policy rate is constrained by the zero lower bound, providing a more accurate representation of the monetary policy stance during periods of unconventional monetary policy measures."

Also explain where you get the $R^* = mu + phi^*pi$ _star. You can find that in my paper. We explain where we find the equation in the Data and Methodology section \rightarrow Methodology \rightarrow US Methodology. See below also.

"To estimate R*, we follow the formula employed by Nikolsko-Rzhevskyy, Papell and Prodan's paper, "The Taylor Principles." Specifically, the equation is as follows:

¹¹ https://www.nber.org/system/files/working_papers/w7147/w7147.pdf, p. 54

¹² https://fred.stlouisfed.org/

¹³ https://www.sciencedirect.com/science/article/pii/S016407041930028X?ref=pdf_download&fr=RR-2&rr=88f6233c98cc15f5

$$R^* = \mu + \phi \pi^*$$

I would not do the long samples 2009-2023 and 2009-2023. It is more meaningful to look at a series of 50 observations, a rolling window. You can split the sample but not from 2009 to 2023. The periods that make more sense are: 1985 - 2019, 1985 - 2007, 2009 - 2019 and full sample. Thank you for this comment. We have made these changes in our Results section. Please see above.

Do you have a series of R* over time? Please graph that so we can see how it is changing. Yes, please see the series of R* over time graphed in the Discussion section, specifically under the US: Non-Inertial and Rolling subsection.

Explain the regression results: what is significant, what do the coefficients mean, etc. The results of our regression is explained throughout the Discussion: Results and Analysis section; in particular, the regression results are used to calculate the R* values for both the simple and inertial regression models.

Have two sections for results: US: non-inertial and inertial; International non-inertial and inertial We restructured our paper to include two sections for results, within which we further divided it into subsections of non-inertial and inertial results. We appreciate the clarity such structure was able to provide to our paper!

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