

# Modelling and Verification of Cyber Physical System Using Timed Petri Net

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**Abstract:** Development processes, networking and the evaluation of models are becoming more demanding, especially in the areas requiring, high verification, safety and security. These are being studied in many scientific and technological fields and with the blooming of Cyber Physical Systems (CPSs), distributed systems are becoming more reliable and the usage of a simple and more definitive modelling strategy is becoming increasingly crucial. In this paper, a timed Petri nets based strategy enabling behavioral modeling and performance analysis of Cyber-Physical-Social Systems (CPSSs) is presented. This also addresses uncertain scenarios where the social aspect is also having a significant impact on the functioning of these systems. Petri nets models, included with dynamic time dependencies that associated with transitions, are applied in a case study, and corroborated as a potential tool for modelling and analysis of these kind of systems.

## 1 Motivation

The notion of Cyber-Physical Systems emerged in recent years as a result of the integration of computer or cybernetic systems with physical systems (CPSs). CPSs are heterogeneous entities that span the hardware and software, sensors and actuators, and other domains. Such systems must be extremely adaptive and flexible to respond in non-deterministic and changing situations with acceptable performance because they are typically used for dynamically changing purposes. Eventually, the human studying perspective as a whole and crucial component must be necessary to carefully design such systems. Thus, these systems must be carefully evaluated because they are mostly deployed in security-critical applications where their failures can have serious consequences. Public services, smart factories, smart healthcare, and smart cities can all be categorized as a few common CPSS applications. Their modeling is more challenging due to the CPSSs' inherent complexity as well as the connections and interactions between system components. Concurrency, synchronization, distributed, real-time, discrete, and continuous aspects are among the characteristics of these kinds of systems. With regard to smart cities, a wide range of connected topics should be taken into account and examined in light of the high system performance, including intelligent traffic management and intelligent transportation systems, to name a few. In order to represent CPSSs in uncertain environments, this study introduces a correct formalization, utilizing an intelligent traffic management system as a validation example. Petri nets (PN) are well-suited to deal with the challenges of CPSSs among modeling formalisms suitable for use in the specification, analysis, and implementation of CPSSs. They support a model-based development strategy, including component design, orchestration of components, as well as component and overall performance evaluation. In this study, non-autonomous Petri net modeling is utilized for the definition, analysis, and implementation of CPSSs. The PN model's inherent properties were enhanced by the addition of dynamic time related with the development of the model, as suggested in the section below. The proposals are validated using an application example from a traffic light management system for intersections, where the arrival rate of automobiles are taken into consideration to constraint the behavior affecting the system's performance.

## 2 Systems Overview

### 2.1 Cyber Physical Systems:

A cyber physical system (CPS) is an integration of computing, communication with monitoring or/and control of things in the physical environment, according to a popular description given for the concept. A key component of the theory combining computing, communication, and control is information. Information may come from several sources, including societies, human beings, and physical objects like sensors and actuators, or several sources, including networks that monitor and regulate the physical processes, typically with feedback loops where the physical processes influence computation, human operators, and embedded computers as depicted in Fig. 1. CPS are frequently operated by human operators, so human factors need to be incorporated into the design of such systems. They are heterogeneous, concurrent, and time sensitive, so modeling them is challenging. In order to model CPSs, both qualitative and quantitative models are needed since they are discrete and dynamic in nature. Petri nets are a popular formalisation of CPS.

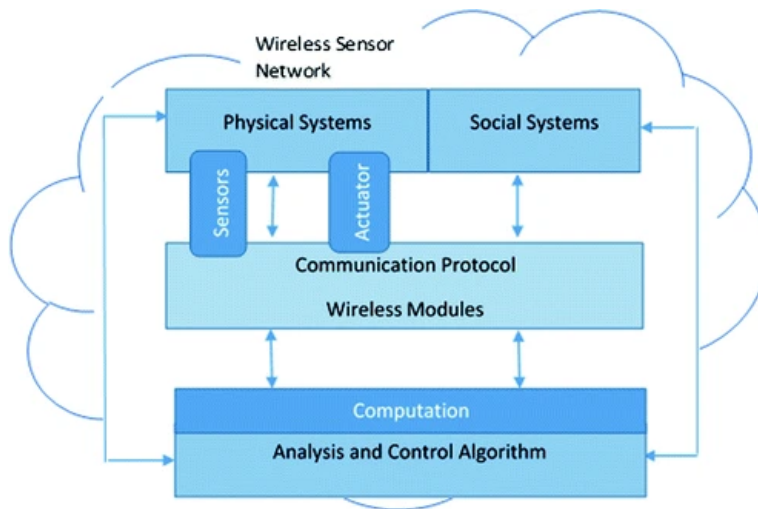


Fig. 1: Cyber Physical System.

By applying modeling formalisms, like Petri nets, to different phases of development, such as specification, and implementation, as well as allowing a-priori verification of properties and anticipating the impact of failures or misconduct, systems' resilience can be improved. The adoption of a model-based development attitude, such as the one proposed in this paper, can contribute to improving confidence in the functioning correctness of real-time operations, survivable during attacks, and fault tolerance.

## 2.2 Petri Nets:

A Petri nets are a specific category of bipartite directed graphs that are made up of three different kinds of objects. The items in question are places, transitions, and directed arcs that connect both transitions and places to one another. Places are represented visually by circles, and transitions are represented graphically by bars or boxes. as seen in figure 2. A Petri net can be represented in its most basic form by a transition along with its input and output locations. Different facets of the modeled systems can be represented by this simple net. Each spot may possibly hold either none or a positive number of tokens, represented by little solid dots, in order to examine the dynamic behavior of the modeled system in terms of its states and their changes. Whether a condition related to a location is true or false can be determined by the existence or absence of a token in that location.

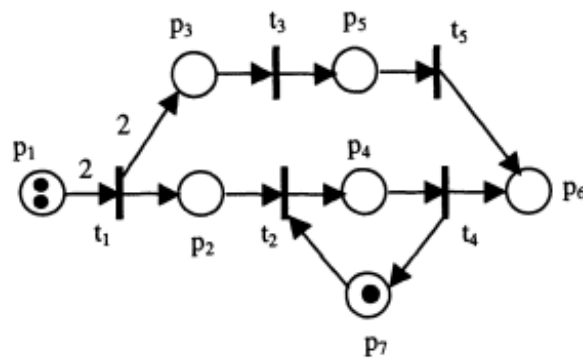


Fig. 2: A Simple Petri Net

The Petri net (PN) provides a clear means of presenting simulation and control logic for a wide variety of discrete event systems. The definition, basic terminologies, transition firing rules, attributes, and analytic methods of Petri nets are all introduced in this chapter. The book uses the fundamental ideas presented in this chapter.

### 2.2.1 Transition Firing

The quantity and distribution of tokens in a Petri net determine how it will operate. Tokens are objects that live in specific locations and govern how the net's transitions are carried out. One can examine the dynamic behavior of the modeled system by altering the distribution of tokens in locations that might, for example, reflect the occurrence of events or the execution of processes. By firing transitions, a Petri net operates. The transition's enabling rule and firing rule are now discussed. We now discuss the transition's enabling rule and firing rule, which control the flow of tokens. [Zh95]

1. *Enabling Rule:* A transition  $t$  is said to be enabled if each input place  $P$  of  $t$  contains at least the number of tokens equal to the weight of the directed arc connecting  $p$  to  $t$ , i.e.,  $M(P) \geq J(t,p)$  for any place  $P$ . [Zh95]
2. *Firing Rule:*
  - (a) An enabled transition  $t$  may or may not fire depending on the additional interpretation, and
  - (b) A firing of an enabled transition  $t$  removes from each input place  $P$  the number of tokens equal to the weight of the directed arc connecting  $P$  to  $t$ . [Zh95]

Keep in mind that the number of tokens in each place is never negative when a transition fires because only enabled transitions may occur. A token that isn't there can never be attempted to be removed by firing transition.

Priorities are not represented in the traditional Petri nets that have been addressed so far. An inhibitor arc can be used to attain such modeling power. The inhibitor arc, which connects an input location to a transition, is visually depicted as an arc with a little circle at its end. The transition enabling circumstances are altered by the presence of an inhibitor arc linking an input location to a transition. A transition is considered enabled in the presence of the inhibitor arc if each input place connected to it by a normal arc (an arc ended with an arrow) has at least as many tokens as the weight of the arc and if there are no tokens on each input place connected to it by the inhibitor arc. However, the marking in the locations related to the inhibitor arc is unaffected by its firing. Figure 3 depicts a Petri net with an inhibitor arc (h).

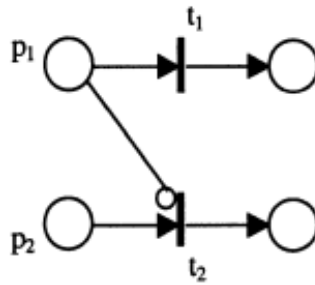


Fig. 3: Petri Net with Inhibitor arc

As shown in [HCO8] and [Hu12], they are therefore suitable for designing traffic control systems. Despite this, they cannot determine the exact time of transition firing without extending the time dimension. Thus, they can only be used to analyze the functional or qualitative behavior of the systems. Time Petri nets are deployed in this case to enhance the capability. In recent years, TPNs have been successfully used to model railway level

crossings and urban traffic control systems. Additionally, timed coloured Petri nets (TCPNs) are used to model complex systems visually.

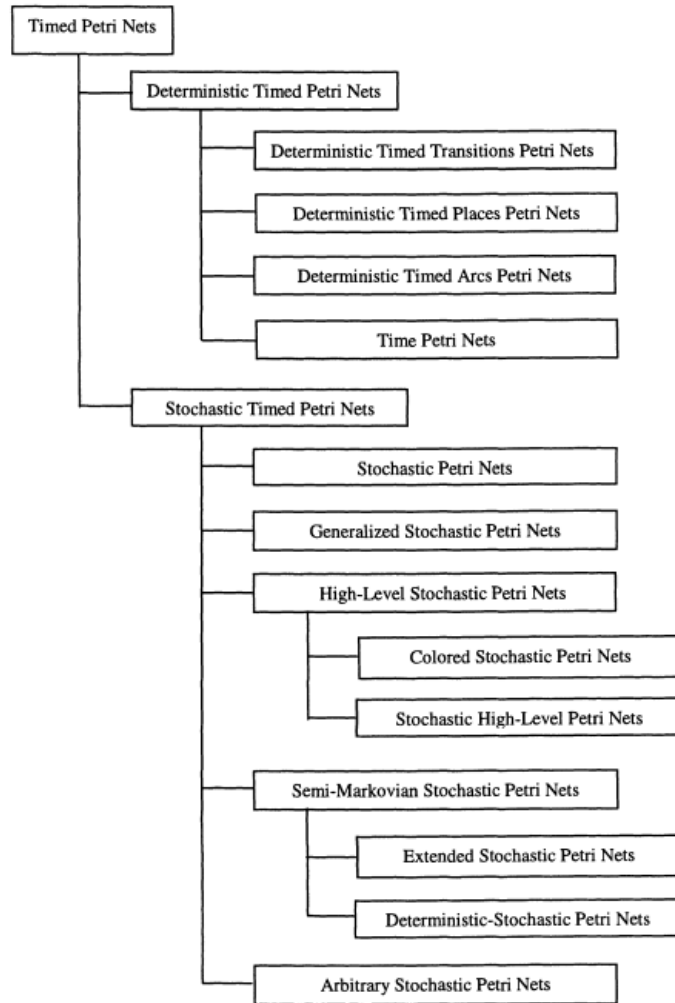


Fig. 4: Classification of timed Petri nets.

### 2.3 Timed Petri Net:

A Time Petri net can be obtained from a Petri Net by associating two dates with each transition. Assuming that  $t$  became active for the end time at date , then  $t$  cannot fire (cannot be taken) before the date + min and must fire no later than the date + max, except if firing another transition disables  $t$  before then. As time Petri nets naturally express specifications in delays,"by making explicit when an action begins and ends, they can also express specifications in durations."They have broad applicability. A timed Petri Net can be used as both a logical and a quantitative model. These models allow the same language to be used for specification, validation, and estimation of functional/logical properties (e.g., the number of deadlocks) and performance properties (e.g., average waiting times). As seen from Figure 4, There are two types of timed Petri Nets: deterministic timed petri nets, in which every transition, place, or directed arc has a deterministic firing time or time interval, and stochastic timed petri Nets, in which each transition has a random firing time. In discrete event dynamic systems performance evaluation, both deterministic and stochastic timed Petri nets have been widely used. In particular, deterministic timed Petri nets can be used to estimate production cycle time, identify bottleneck workstations, verify timing constraints, etc. Meanwhile, a stochastic timed Petri net can be used to determine production rates, throughput, average delays, critical resource utilization, reliability measures, etc. In this study, we will focus more on the deterministic timed Petri Net family. It consists of Deterministic Timed Transitions Petri Nets that have each transition associated with a specific firing time, deterministic Timed Places Petri Nets that have a specific firing time for each place, deterministic Timed Arcs Petri Nets whose directed arcs are associated with deterministic firing times, and Time Petri Nets whose transitions are associated with deterministic firing times.

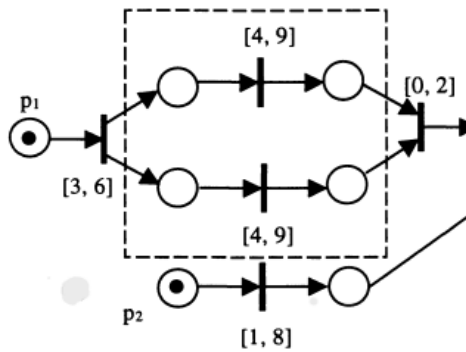


Fig. 5: A Timed Transition Petri Net



## 2.4 Methods of verifying Properties of Timed Petri Nets

A Timed Petri net is a mathematical tool that has a number of properties. System designers can identify the presence or absence of application domain-specific functional properties in the modeled system by interpreting these properties in the context of the modeled system. Property types can be divided into behavioral and structural types. A Timed Petri net's behavioral properties depend on its initial state or marking. However, the structural properties of a Time Petri net are not affected by its initial marking. The structure of a Timed Petri net determines their topology. Our objective here is to provide an overview of some of the most important behavioral properties from a practical perspective. These properties are reachability, boundedness, conservativeness, and liveness.

### 2.4.1 Reachability

The question of whether a system can achieve a particular state or display a particular functional behavior is crucial when building a Cyber Physical System. In general, the inquiry is whether the Petri net-modeled system exhibits all desirable qualities. Finding a sequence of transition firings that would change a marking  $M_0$  to  $M_i$ , where  $M_i$  stands in for the specific state and the sequence of firings represents the required functional behavior, is necessary to determine whether the modeled system can reach the desired state as a result of the required functional behavior. It should be highlighted that a genuine system might exhibit several functional behavior patterns that are acceptable in order to arrive at a specific state, changing  $M_0$  to the necessary  $M_i$ . The Petri net model may not accurately represent the structure and dynamics of the actual system because it contains additional sequences of transition firings that convert  $M_0$  to  $M_i$ . If the Petri net model truly matches the underlying system requirement specification, this may also be a sign of unexpected aspects of the real system's functional behavior. If there is a series of transition firings that changes a marking  $M_0$  into a marking  $M_i$ , then the marking  $M_i$  is said to be reachable from the marking  $M_0$ . If firing an enabled transition in  $M_0$  results in  $M_1$ , then a marking  $M_1$  is said to be immediately reachable from  $M_0$ .

### 2.4.2 Boundedness and Safeness

Places are frequently used to represent information storage spaces in computer and communication systems, product and tool storage areas in manufacturing systems, etc. in a Timed Petri net. It's crucial to be able to tell whether the planned control measures would stop these storage areas from overflowing. The idea of boundedness is a Petri net characteristic that aids in locating overflows in the represented system.

1. *Definition: A place  $P$  is said to be  $k$ -bounded if the number of tokens in  $P$  is always less or equal to  $k$  ( $k$  is a nonnegative integer number) for every marking  $M$  reachable from the initial marking  $M_0$  i.e.,  $M \in R(M_0)$  It is said to be  $1$ -bounded. [Zh95]*
2. *Definition: A Petri net  $N = (P, T, \text{pre}, \text{post}, M_0)$  is  $k$ -bounded (safe) if each place in  $P$  is  $k$ -bounded (safe). [Zh95]*

### 2.4.3 Conservativeness

In a Petri net, tokens could stand in for resources. A Petri net model of this system should maintain the same number of tokens regardless of the marking the net adopts because the number of which in a real system is often fixed. Conservation is a crucial characteristic when resource allocation systems are represented by Timed Petri nets.

Strict conservation has a solid connection. It means that every reachable marking of a Petri net has precisely the same number of tokens. This is only possible from the perspective of net structure when the quantity of input arcs to each transition is equal to the quantity of output arcs. However, in actual systems, resources are regularly mixed to enable the completion of certain tasks. After the assignment is over, they are separated. Weights may be connected to locations in order to get around this issue and maintain a constant weighted sum of tokens in a net.

### 2.4.4 Liveness

The concept of liveness and the deadlock issue, which has been extensively discussed in the context of computer operating systems, are closely related. There must be an active Petri net simulating a deadlock-free system. According to this, it is finally conceivable to fire any transition in the net for each reachable marking  $M$  by proceeding through some firing sequence. However, this criterion might be too stringent to accurately describe some real systems or scenarios that don't deadlock. For instance, a transition (or series of transitions) that fire a certain number of times can be used to simulate how a system initializes. [Da08]

To verify and model a Cyber Physical System, a traffic light case study was implemented in this study using deterministic timed petri net.

## 3 Introducing a Traffic Light Control System

Taking a look at traffic management, a common problem that affects millions of people on a daily basis, its relation to time, was the focus of this study. An intersection traffic light system consists of a physical controller and a computation section. Furthermore, the performance of the system is also affected by social factors. Modeling of intelligent traffic light control

systems involves considering several variables that constrain the time management of the system. Here are some of the considered variables; Number of queued vehicles, pedestrians and etc. It is the goal of an intelligent traffic system to reduce queuing times and improve traffic flow by controlling the volume of traffic. For this reason, it is essential to design a model that will yield efficient and safe operations under the current conditions. There are several operation modes that have previously been studied and are considered (night, day, rush hours, etc.), each with predefined sequences and temporal behavior (fixed or nearly fixed periods of time). This paper proposes a modelled system that controls vehicles at the intersection based on time periods.

### 3.1 Modelling Crossroad Traffic Light Control Systems using Timed Petri Net

On the basis of the above discussion, this section illustrates a crossroad traffic light control system using TPNs. A general traffic system with two-phase traffic lights can be seen in Figure 6(a). Some significant regulations are required when taking vehicle safety into account.

- If all of the traffic signal lights are in the red condition, a traffic light control system can be activated.
- There cannot be more than one green light on at a time, and
- The traffic signal alternates between red, green, and yellow.

The TPNs in Fig. 8 accurately simulates the overall operation of the system. The system consists of three traffic signal lights, namely red (R), yellow (Y), and green (G). Northward, southward, westward, and eastward directions are symbolized as sn, ns, ew, and we, respectively. The TPNs model is built using two different sorts of locations. Places of Type I consist of Rns, Yns and Gns, corresponding to R, Y, and G traffic lights. The type II ones are Rwe, Ywe, and Gwe. It is possible to derive the TPN transitions of the system based on the above described two-phase.

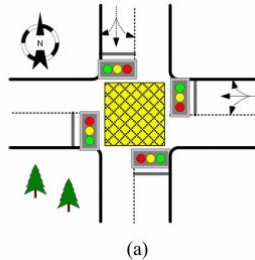


Fig. 6: (a) A two-phase traffic light control system.

### 3.2 Analysis and Simulation

In this section, the analysis and simulation of the system was made, which were true in accordance with the design specification, as seen in the figures below. Figure 7. shows the system state machine diagram with the supposed assigned transition time intervals.

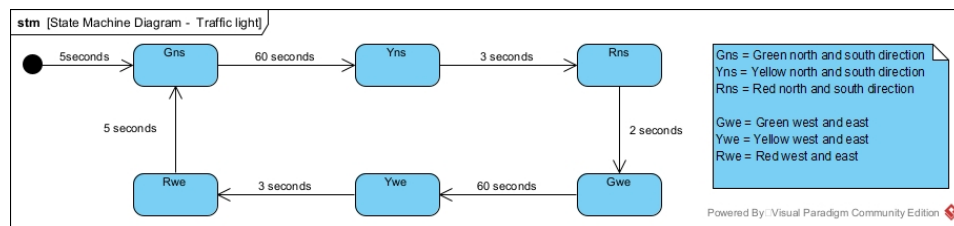


Fig. 7: State machine diagram.

The system in Figure 8. shows the flow of the system in respect to the specified time intervals listed in section above. There are six transitions (t0, t1, t2, t3, t4, and t5) with deterministic firing times; red is the signal time in direction ns (we), green is the green signal time, and yellow is yellow for the signal time in direction ns (we). A proper analysis and test of the system was done using Tapaal, the system simulates repeatedly as required with no deadlocks, also structurally bounded and live.

The traffic light system's TPNs model can be built in accordance with the specifications as seen in Figure 7. The starting state of the traffic control system model is depicted in Figure 7 as the red states.

From Figure 8, when t4 is fired, a token is moved into Green-ns and P2, respectively, after 5 seconds. The green light is now on, allowing northbound and southbound traffic to proceed through the crossing. The green light should then be turned off after the length of t1. The green light has apparently been on for 60 seconds. And for the next transition, as t0 lasts for 3 seconds, the yellow light stays on for 3 seconds. The token is then transferred back into Red-ns. In 2 seconds (following firing of t5), a token in Red-we moves into Green-we. The signal light T3 can fire when Green-we becomes green, the Green-we stays on for 60 seconds and then the T3 fires in, Due to the duration of t2, the yellow light is on for 3 seconds. The token is finally placed back into Red-we.

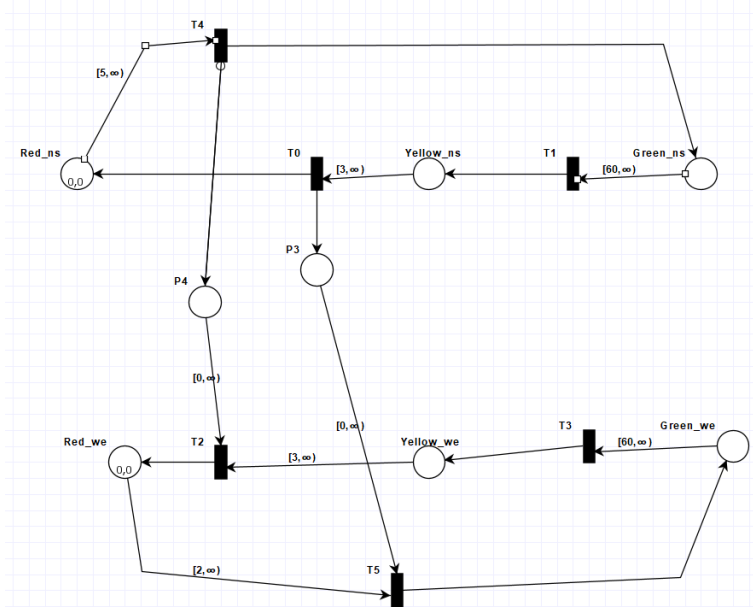


Fig. 8: Traffic light timed petri net model.

#### 4 Verification and validation of the systems Timed Petri Net model

As a first step, we define a marking function, which assigns a finite multiset of nonnegative real numbers to each place. Real numbers are used to represent the ages of tokens at a given location. The ages of tokens should also respect the invariant of the place where the token is located. A marked TPN consists of a pair  $(N, M_0)$  where  $N$  is a TPN and  $M_0$  is an initial marking on  $N$  where all tokens have the same age.

**Definition 1 (Enabledness).** Let  $N = (P, T, IA, OA, Transport, Inhib, Inv)$  be a TAPN. We say that a transition  $t \in T$  is enabled in a marking  $M$  by tokens if

- for all input arcs except the inhibitor arcs there is a token in the input place with an age satisfying the age guard of the arc, i.e. [Ja11]
- for all inhibitor arcs there is no token in the input place of the arc with an age satisfying the age guard of the arcs respectively, i.e. [Ja11]
- for all input arcs and output arcs which constitute a transport arc the age of the input token must be equal to the age of the output token and satisfy the invariant of the output place, i.e. [Ja11]
- for all output arcs that are not part of a transport arc the age of the output token is 0, i.e. [Ja11]

**Definition 2 (Firing Rule).** *Let  $N = (P, T, IA, OA, Transport, Inhib, Inv)$  be a TAPN,  $M$  a marking on  $N$  and  $t \in T$  a transition. If  $t$  is enabled in the marking  $M$  by tokens  $In$  and  $Out$  then it can fire and produce a marking  $M'$  [Ja11]*

**Definition 3 (Time Delay).** *Let  $N = (P, T, IA, OA, Transport, Inhib, Inv)$  be a TAPN and  $M$  a marking on  $N$ . A time delay  $d \geq 0$  is allowed in  $M$  if  $(x + d) \leq Inv(p)$  for all  $p \in P$  and all  $x = M(p)$ , i.e. by delaying  $d$  time units no token violates any of the age invariants. By delaying  $d$  time units in  $M$  we reach a marking  $M_0$  [Ja11]*

In TPN there is a timed transition system where states are markings of  $N$ , so if there are two markings  $M$  and  $M_0$  we can reach the marking  $M_0$  by firing some transition in  $M$  and by delaying the time units in  $M$  we can reach the marking  $M_0$ . That is to say that a marking  $M_0$  is reachable from a marking  $M$  if  $M_0$  is reachable from  $M$ .

To properly verify a TPNs model and make sure all the above statements are met, the questions of the Petri net problems like reachability and boundedness in the TPN context must be checked. The reachability of the basic timed-arc Petri net model is undecidable (with only ordinary arcs and no age invariants). However, coverability, boundedness, and other issues remain decidable for the basic TPN model [Ja11].

For the above Model (Figure 8), I checked if every reachable marking in the net satisfies the given property and if there is a trace on which all markings satisfy the given property. The results were true and successful, the system was deadlock free and live.

## 5 Conclusion

In this study, TPNs models for a traffic light level crossing systems are proposed. It is important to note that the presented models make use of the idea of hybrid systems. They can be used for timing analysis. In particular, the transition used is timed. Which is therefore used to model the traffic light behaviour. It is important to note that the emerging scenarios can be accurately identified using the proposed models. Upon investigation, the proposed system model is free of deadlocks, exhibits repetitive behavior, and is structurally bounded and live. The analysis of the models proves so. Based on the validation provided by the application example, Timed Petri nets can be considered a promising formalism for modelling CPSS-type systems.

## 6 Declaration of Originality

I, Izuchukwu George Enekwa, herewith declare that I have composed the present paper and work by myself and without the use of any other than the cited sources and aids. Sentences or parts of sentences quoted literally are marked as such; other references with regard to the statement and scope are indicated by full details of the publications concerned. The paper

and work in the same or similar form have not been submitted to any examination body and have not been published. This paper was not yet, even in part, used in another examination or as a course performance. I agree that my work may be checked by a plagiarism checker.

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