Problems Class I

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March 18, 2018

Equations and constants

The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{M,0} = 0.31,\ \Omega_{D,0} = 0.69,\ \Omega_{R,0} = 9 \times 10^{-5},\ H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Parsec in SI units: 1 pc $= 3.09 \times 10^{16}$ m

Questions

1. Show that the observed surface brightness of a galaxy – the flux per unit solid angle – falls with distance as:

   $$S = \frac{L}{4\pi}\frac{1}{(1+z)^4}$$

   where L is the luminosity per unit physical area (e.g., per square kiloparsec) of the galaxy.

2. If my telescope can measure flux to within an accuracy of 10%, at what redshifts do I have to worry about which cosmological distance I use when calculating the luminosity of a galaxy?

   The Coma supercluster of galaxies is 99 Mpc away. Do I need to worry about which cosmological distance I should use when calculating the luminosity of galaxies in the Coma supercluster?

3. Derive the following form of the Friedmann Equation:

$$\dot{a}^2 = H_0^2 \left(\frac{\Omega_{\rm r,0}}{a^2} + \frac{\Omega_{\rm m,0}}{a} + \Omega_{\kappa,0} + \Omega_{\rm D,0} a^2 \right) \tag{1}$$

   (a) From Eq. 1, show that a universe in which $\kappa = +1$ and $\Omega_{\rm D,0} = 0$ will reach a maximum value of a before recollapsing.

   (b) What would the recollapse mean in terms of motions of bodies on sub-cosmological scales (i.e., galaxy groups, galaxies, planetary systems, you, me). Justify your answer.

   (c) By differentiating Eq. 1 in the case of where the universe is flat and the radiation density is negligible (i.e., a reasonable description of today's Universe), find the condition under which the expansion of the universe will currently be accelerating.

4. The metal-poor star HE 1523-0901 has an age of 13.4 ± 2.2 Gyr based on uranium radiochemical dating (Frebel et al., 2007, *ApJ*, 660, L117). Assuming a matter-only model, calculate the value of H_0 that this would imply, if the first stars formed 100 Myr after the Big Bang.

5. In the 1950s it was generally-accepted that $H_0 \sim 500$ km s^{-1}, with a random error estimated to be around 10-20%.

   (a) Why do you think this early measurement differed so much from today's generally-accepted value of the Hubble constant, $H_0 \approx 70$ km s^{-1}?

   (b) Why did the generally-accepted value of H_0 in 1950 lead most astronomers to believe in the steady-state model of the Universe?