Lecture 3:
The Friedmann Equation

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1 The Robertson Walker metric

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- Saw in lecture 2 that an isotropic, homogeneous Universe can be fully described by just three numbers: κ, R_0, $a(t)$.
- $a(t)$ is paricularly important: it tells us how the Universe expands and contracts over time.
- $a(t)$ also enables us to relate redshifts (which are easily measured) to distances (which are much more difficult to measure).
- However, let's first focus on curvature, described by κ and R_0.

1.1 What is curvature in the context of a universe?

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- Curvature in a universe would manifest itself in terms of the perceived sizes of distant objects.
- The *perceived* size of an object is the angular size it subtends.
- As such, it is convenient to think in terms of a triangle, with the object at one end, and the angular size at the other.
- In a flat universe, the angles of this triangle all add up to 180 degrees, and the objects subtends the angle that we have come to expect for flat geometry.
- In a negatively curved universe, the interior angles of the triangle add up to less than 180 degrees, and the object subtends a *smaller* angle that we would expect. This would be witnessed as distant galaxies appearing disproportionately smaller than nearby ones.
- By contrast, in a negatively curved universe, the interior angles of the triangle add up to more than 180 degrees, and the object subtends a *larger* angle that we would expect. This would be witnessed as distant galaxies appearing disproportionately larger than nearby ones.
- Such disproportionately large or small distant galaxies are not seen when we look to higher and higher distances (i.e., redshifts), so we can conclude that *if* the Universe is curved, then its radius of curvature, R_0 is much larger than the size of the observable Universe.

2 Relating curvature to content

- General relativity tells us that a universe's curvature is dictated by its content (whether mass or energy, since they are one and the same thing in relativity).
- The *Field Equation* links the two - it tells spacetime ($G_{\mu,\nu}$) how to curve in the presence of stress-energy ($T_{\mu,\nu}$).
- Unfortunately, both $G_{\mu,\nu}$ and $T_{\mu,\nu}$ are 4×4 tensors (i.e., matrices), and the equation as a whole represents ten non-linear second-order differential equations!
- Thus, in general, it can be extremely difficult to solve for $G_{\mu,\nu}$. **Slide 5**
- However, on large scales, we can make some sweeping simplifications.
- On very large scales, we can ignore the "clumpy" nature of the Universe and instead describe it as being filled with a uniform (i.e., homogeneous), isotropic gas or pressure $P(t)$ and (energy) density $\varepsilon(t)$.
- In such a case, then $T_{\mu,\nu}$ only depents on $P(t)$ and $\varepsilon(t)$, and the metric is given by the Robertson Walker metric.
- Our goal, therefore, is simply to relate $a(t)$, κ, and R_0 to $P(t)$ and $\varepsilon(t)$.

3 The Friedmann Equation