Lecture 09: The radiation content of the Universe: The Cosmic Microwave Background Part 1

Dr. James Mullaney

March 6, 2018

1 The radiation content of the Universe

- Galaxy surveys tell us that the density of stars in the Universe corresponds to about 1.7 L $\odot$ Mpc^{-3}.
- If, as an upper limit, we assume that these stars have emitted photons since the beginning of the Universe, 4.5×10^{17} s ago, it corresponds to an energy density arising from stellar photons of: 10^{15} Jm^{-3}.
- The other main source of photons in the Universe is the CMB.
- The spectrum of the CMB is a black body of $T = 2.7\ K$, so we can calculate its energy density using:

$$E = \alpha T^4 \tag{1}$$

where

$$\alpha = \frac{\pi^2}{15} \frac{k^4}{\hbar^3 c^3} = 7.566 \times 10^{-16}\ \mathrm{J\ m^{-3}\ K^{-4}} = 0.2606\ \mathrm{MeV\ m^{-3}} \tag{2}$$

and k is the Boltzmann constant.

- So the energy density of the CMB is *at least* a factor of 40 times that of stellar photons.
- Despite this, the energy density of the CMB still represents only a small fraction of the critical density of our Universe (i.e., $\Omega_{\mathrm{p}} = 5.35 \times 10^{-5}$).
- However, because the energy of each CMB photon is very low, $hf_{\mathrm{mean}} = 6.34 \times 10^{-4}$ eV, the number density of CMB photons (i.e., the number of photons per unit volume) is very high:

$$n_{\mathrm{p}} = \frac{0.2606 \times 10^6}{6.34 \times 10^{-4}} = 4.107 \times 10^8\ \mathrm{m^{-3}} \tag{3}$$

- Indeed, compared to the number density of Baryons:

$$n_{\mathrm{b}} = \frac{\Omega_{\mathrm{b},0}\varepsilon_{c,0}}{E_{\mathrm{b}}} = \frac{0.048 \times 4890\ \mathrm{MeV\ m^{-3}}}{939\ \mathrm{MeV}} = 0.25\ \mathrm{m^{-3}} \tag{4}$$

2 Recombination

- In an expanding universe, such as our own, as we go further back in time the scale factor, a, gets smaller and smaller.
- This, in turn, means that the wavelength of CMB photons get shorter and shorter, meaning their energies get higher and higher.
- Since the mean energy, E, of a CMB photon is related to temperature, T, via $E = 2.7kT$ (where k is the Boltzman constant), we can calculate the temperature of the Universe as a function of a:

$$hf_{\text{mean}} = \frac{hc}{\lambda_{\text{mean}}} = \frac{hc}{a\lambda_0} = 2.7kT(a) \tag{5}$$

  or, equivalently, $T(a) = T_0/a$, where T_0 is the temperature of the CMB today (i.e., 2.755 K).
- If we go far enough back in time, then the energies of a large proportion of CMB photons exceed that of the ionisation energy of Hydrogen.
- And since there are 1.6 billion CMB photons for every Baryon, the instant an electron binds to a proton to form a Hydrogen atom, it gets blasted apart by an ionising photon.
- However, as the Universe expands, the photons and electrons move further away from each other, so they interact less.
- If nothing were to change, the distance between the photons and electrons would become so large that they would eventually stop interacting.
- Before that happens, however, we get recombination.
- As the Universe expands, the energy of the CMB photons drops as their wavelengths increase.
- Recombination occurs when there are insufficient numbers of photons with energy greater than the ionisation energy of Hydrogen to keep it ionised.
- When does this occur?
- One guess would be when the mean energy per photon drops below the ionisation energy of Hydrogen. With the mean photon energy given by $2.7kT$, this corresponds to:

$$T_{\text{rec}} = \frac{13.6}{2.7k} = \frac{13.6}{2.7 \times 8.6 \times 10^{-5}} \sim 60,000 \text{ K} \tag{6}$$

  which occured when $a = 2.7/60000 = 4.5 \times 10^{-5}$, or $z \sim 22,000$.
- However, it turns out this is a poor guess.
- The reason being that, with 1.6 billion photons per baryon, the Universe doesn't need roughly *half* the CMB to be able to ionise H, it needs a much smaller fraction.