

Lecture 07:
The “Benchmark Model” and
“measurable” distances

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1 The Benchmark Model for the real Universe

- Up to this point, we’ve only considered “simplistic” universes which are either empty, or which just consist of a single component (e.g., matter *or* radiation, etc.).
- However, we know that the real Universe contains (at least) three main components: radiation, matter and Dark Energy (I say “at least”, since there may be other components that are too insignificant to currently measure).
- Thankfully, things are made easier by different components adding linearly in the Friedmann Equation, i.e.,:

$$\varepsilon_{\text{tot}} = \varepsilon_{\text{R}} + \varepsilon_{\text{M}} + \varepsilon_{\text{D}} \quad (1)$$

- Which means the F.E. for the real Universe becomes:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \sum_i \varepsilon_i - \frac{\kappa c^2}{R_0^2 a(t)^2} \quad (2)$$

- From this, and by using the relationships derived in previous lectures:

$$H = \left(\frac{\dot{a}}{a}\right) \quad (3)$$

$$\Omega_{i,0} = \frac{\varepsilon_{i,0}}{\varepsilon_{c,0}} \quad (4)$$

$$\varepsilon_{0,c} = \frac{3H_0^2 c^2}{8\pi G} \quad (5)$$

$$H_0^2(1 - \Omega_0) = \frac{-\kappa c^2}{R_0^2} \quad (6)$$

$$\varepsilon(a)_i = \varepsilon_{i,0} a^{-3(1+\omega)} \quad (7)$$

we get:

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{\text{p},0}}{a^4} + \frac{\Omega_{\text{m},0}}{a^3} + \Omega_{\text{d},0} + \frac{1 - \Omega_0}{a^2} \quad (8)$$

- meaning:

$$\frac{da}{dt} = H_0 \left(\frac{\Omega_{p,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{d,0}a^2 + 1 - \Omega_0 \right)^{1/2} \quad (9)$$

- So, to obtain $a(t)$, we need to solve the following:

$$\int_0^a \frac{da}{\left(\frac{\Omega_{p,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{d,0}a^2 + 1 - \Omega_0 \right)^{1/2}} = H_0 t \quad (10)$$

- For the real Universe:

$$\Omega_{p,0} = 9 \times 10^{-5} \quad (11)$$

$$\Omega_{m,0} = 0.31 \quad (12)$$

$$\Omega_{d,0} = 0.69 \quad (13)$$

$$\Omega_0 = 1.00 \quad (14)$$

$$H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc} \quad (15)$$

- Unfortunately, this cannot be solved analytically, so instead we solve it numerically using computers to integrate “under the curve”.
- See notes on lecture slides...