Problems Class I

Dr. James Mullaney

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1. Show that the observed surface brightness of a galaxy – the flux per unit solid angle – falls with distance as:
$$S = \frac{L}{4\pi d_p(t_0)^2}\frac{1}{(1+z)^4}$$
where L is luminosity of all the stars contained within a square kiloparcsec of the galaxy.

2. If my telescope can measure flux to within an accuracy of 10%, at what redshifts do I have to worry about which cosmological distance I use when calculating the luminosity of a galaxy?

3. Derive the following form of the Friedmann Equation:
$$\dot{a}^2 = H_0^2\left(\frac{\Omega_{\mathrm{r},0}}{a^2} + \frac{\Omega_{\mathrm{m},0}}{a} + \Omega_{\kappa,0} + \Omega_{\mathrm{D},0}a^2\right) \tag{1}$$
   (a) From Eq. 1, show that a universe in which $\kappa = +1$ and $\Omega_{\mathrm{D},0} = 0$ will reach a maximum value of a before recollapsing.
   (b) What would the recollapse mean in terms of motions of bodies on sub-cosmological scales (i.e., galaxy groups, galaxies, planetary systems, you, me). Justify your answer.
   (c) By differentiating Eq. 1 in the case of where the universe is flat and the radiation density is negligible (i.e., a reasonable description of today's Universe), find the condition under which the expansion of the universe will currently be accelerating.

4. The metal-poor star HE 1523-0901 has an age of 13.4 ± 2.2 Gyr based on uranium radiochemical dating (Frebel et al., 2007, *ApJ*, 660, L117). Assuming a matter-only model, calculate the value of H_0 that this would imply, if the first stars formed 100 Myr after the Big Bang.

5. In the 1950s it was generally-accepted that $H_0 \sim 500$ km s^{-1}, with a random error estimated to be around 10-20%.
   (a) Why do you think this early measurement differed so much from today's generally-accepted value of the Hubble constant, $H_0 \approx 70$ km s^{-1}?
   (b) Why did the generally-accepted value of H_0 in 1950 lead most astronomers to believe in the steady-state model of the Universe?