Lecture 07: The “Benchmark Model” and “measurable” distances

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1 The Benchmark Model for the real Universe

- Up to this point, we’ve only considered “simplistic” universes which are either empty, or which just consist of a single component (e.g., matter *or* radiation, etc.).
- However, we know that the real Universe contains (at least) three main components: radiation, matter and Dark Energy (I say “at least”, since there may be other components that are too insignificant to currently measure).
- Thankfully, things are made easier by different components adding linearly in the Friedmann Equation, i.e.,:

$$\varepsilon_{\rm tot} = \varepsilon_{\rm R} + \varepsilon_{\rm M} + \varepsilon_{\rm D} \tag{1}$$

- Which means the F.E. for the real Universe becomes:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\sum_i \varepsilon_i - \frac{\kappa c^2}{R_0^2 a(t)^2} \tag{2}$$

- From this, and by using the relationships derived in previous lectures:

$$H = \left(\frac{\dot{a}}{a}\right) \tag{3}$$

$$\Omega_{i,0} = \frac{\varepsilon_{i,0}}{\varepsilon_{c,0}} \tag{4}$$

$$\varepsilon_{0,c} = \frac{3H_0^2 c^2}{8\pi G} \tag{5}$$

$$H_0^2(1-\Omega_0) = \frac{-\kappa c^2}{R_0^2} \tag{6}$$

$$\varepsilon(a)_i = \varepsilon_{i,0} a^{-3(1+\omega)} \tag{7}$$

we get:

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{\rm p,0}}{a^4} + \frac{\Omega_{\rm m,0}}{a^3} + \Omega_{\rm d,0} + \frac{1-\Omega_0}{a^2} \tag{8}$$

- meaning:

$$\frac{da}{dt} = H_0 \left(\frac{\Omega_{\mathrm{p},0}}{a^2} + \frac{\Omega_{\mathrm{m},0}}{a} + \Omega_{\mathrm{d},0} a^2 + 1 - \Omega_0 \right)^{1/2} \tag{9}$$

- So, to obtain $a(t)$, we need to solve the following:

$$\int_0^a \frac{da}{\left(\frac{\Omega_{\mathrm{p},0}}{a^2} + \frac{\Omega_{\mathrm{m},0}}{a} + \Omega_{\mathrm{d},0} a^2 + 1 - \Omega_0 \right)^{1/2}} = H_0 t \tag{10}$$

- For the real Universe:

$$\Omega_{p,0} = 9 \times 10^{-5} \tag{11}$$
$$\Omega_{m,0} = 0.31 \tag{12}$$
$$\Omega_{d,0} = 0.69 \tag{13}$$
$$\Omega_0 = 1.00 \tag{14}$$
$$H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc} \tag{15}$$

- Unfortunately, this cannot be solved analytically, so instead we solve it numerically using computers to integrate "under the curve".

- See notes on lecture slides...

2 Luminosity Distance

$$d_L = \left(\frac{L}{4\pi F} \right)^{1/2} \tag{16}$$

$$d_A = d_p(t_{\mathrm{em}}) = \frac{d_p(t_{\mathrm{ob}})}{1+z} \tag{17}$$