



# Optimal-observable analysis of possible non-standard top-quark couplings in $pp \rightarrow t\bar{t}X \rightarrow \ell^+ X'$

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## ABSTRACT

Possible non-standard top-quark interactions with the gluon and the  $W$  boson induced by  $SU(3) \times SU(2) \times U(1)$  gauge-invariant dimension-6 effective operators are studied for hadron-collider experiments. Current limits on top-gluon couplings are presented by using the latest experimental data of  $t\bar{t}$  productions at the Tevatron and the Large Hadron Collider (LHC). The optimal-observable procedure is applied to the charged-lepton distributions in  $pp \rightarrow t\bar{t}X \rightarrow \ell^+ X'$  ( $\ell = e$  or  $\mu$ ) at the LHC in order to estimate the expected statistical uncertainties in measurements of those non-standard top-gluon and top- $W$  couplings that contribute to this process in the leading order.

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## 1. Introduction

The Large Hadron Collider (LHC) is currently one of the most powerful facilities for probing possible new physics beyond the standard model (BSM) [1]. In collider experiments such as the LHC, there could be two kinds of approaches to new-physics studies: One is a direct search for new particles which do not belong to the standard model (SM). The other is an indirect search, which tries to find certain deviations from the SM predictions in processes having only well-known particles in their initial and final states.

In the former case, any non-standard particles once being discovered, we would get a lot of information about new models which contain those particles. However, it is not that easy to enable such high-energy collisions which are enough energetic to create new heavy particles. On the other hand, in the latter case, observable deviations might appear even at an energy far below a typical new-physics scale since they could be induced by quantum effects of non-SM heavy particles. Of course a large number of events are needed in order to establish a new-physics evidence at a statistically-significant level, because such signals would generally be quite small, considering the remarkable success of the SM in various phenomena below  $O(10^2)$ -GeV scale.

Standard particles have been satisfactorily “re”discovered today except for the Higgs boson at the LHC, including top quarks. This is the first time for the top quark to be measured at a facility other than the Tevatron although almost twenty years have passed since its discovery there [2,3]. This situation, in which more precise tests of top-quark properties can be continued, is quite meaningful for new-physics studies after the Tevatron is shutdown: The top quark is the heaviest particle of all standard particles, indicating that precise measurements of this quark might be able to open windows for the BSM [4].

In various direct searches of new particles up to today, any plausible candidates have not been discovered yet, which definitely requires indirect BSM searches. Considering this circumstance, we pay attention in this work to  $t\bar{t}$  productions followed by  $t \rightarrow \ell^+ X$  ( $\ell = e$  or  $\mu$ ) at hadron colliders, and investigate relevant non-standard top-quark couplings  $t\bar{t}g$ ,  $t\bar{t}g\gamma$  and  $t\bar{t}W$  based on the effective Lagrangian which is constructed with SM particles alone. So far, a number of authors have studied this issue in similar frameworks to extract BSM signals at hadron colliders [5–25]. Since many of them are however focusing mainly on CP-violating observables, we study top-quark couplings which induce CP-conserving as well as CP-violating interactions. We first give updated constraints on the top-gluon couplings coming from the latest data on the  $t\bar{t}$  total cross sections at the Tevatron and the LHC. Then we perform an optimal-observable analysis (OOA) to estimate the expected statistical uncertainties in measuring the top-gluon and top- $W$  couplings that contribute to  $pp \rightarrow t\bar{t}X \rightarrow \ell^+ X'$

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under a linear approximation for current and future LHC experiments, which is our final goal here.

## 2. Framework

The effective-Lagrangian approach, in which its low-energy form reproduces the SM, is one of the most promising methods to describe new-physics phenomena when the energy of our experimental facility is not high enough to produce new particles. Assuming any non-SM particles too heavy to appear as real ones, the effective Lagrangian is given as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i (C_i \mathcal{O}_i + \text{h.c.}), \quad (1)$$

where  $\mathcal{L}_{\text{SM}}$  is the SM Lagrangian,  $\mathcal{O}_i$  mean  $SU(3) \times SU(2) \times U(1)$  gauge-invariant operators of mass-dimension 6 involving only the SM fields [26,27] and their coefficients  $C_i$  parameterize virtual effects of new particles at an energy less than the assumed new-physics scale  $\Lambda$ .<sup>1</sup> In this framework, all the form factors related to  $C_i$  are dealt with as constant parameters, without supposing any specific new-physics models.

All the dimension-6 operators were initially listed by Buchmüller and Wyler [26]. However, it was pointed out that some operators there are related with others through equations of motion, which means that they are not independent of each other [28]. Then, the whole related operators were rearranged to get rid of this redundancy in Refs. [29,30].

Following the notation of Ref. [29], the relevant effective Lagrangian which describes non-standard interactions of the third generation for the parton-level process  $q\bar{q}/g\bar{g} \rightarrow t\bar{t} \rightarrow \ell^+ X$  is given in [24] as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{t\bar{t}g,gg} + \mathcal{L}_{tbW}: \quad (2)$$

$$\mathcal{L}_{t\bar{t}g,gg} = -\frac{1}{2} g_s \sum_a \left[ \bar{\psi}_t(x) \lambda^a \gamma^\mu \psi_t(x) G_\mu^a(x) - \bar{\psi}_t(x) \lambda^a \frac{\sigma^{\mu\nu}}{m_t} (d_V + i d_A \gamma_5) \psi_t(x) G_{\mu\nu}^a(x) \right], \quad (3)$$

$$\mathcal{L}_{tbW} = -\frac{1}{\sqrt{2}} g \left[ \bar{\psi}_b(x) \gamma^\mu (f_1^L P_L + f_1^R P_R) \psi_t(x) W_\mu^-(x) + \bar{\psi}_b(x) \frac{\sigma^{\mu\nu}}{M_W} (f_2^L P_L + f_2^R P_R) \psi_t(x) \partial_\mu W_\nu^-(x) \right], \quad (4)$$

where  $g_s$  and  $g$  are the  $SU(3)$  and  $SU(2)$  coupling constants,  $P_{L/R} \equiv (1 \mp \gamma_5)/2$ ,  $d_V, d_A$  and  $f_{1,2}^{L,R}$  are form factors defined as

$$\begin{aligned} d_V &\equiv \frac{\sqrt{2} v m_t}{g_s \Lambda^2} \text{Re}(C_{uG\phi}^{33}), & d_A &\equiv \frac{\sqrt{2} v m_t}{g_s \Lambda^2} \text{Im}(C_{uG\phi}^{33}), \\ f_1^L &\equiv V_{tb} + C_{\phi q}^{(3,33)*} \frac{v^2}{\Lambda^2}, & f_1^R &\equiv C_{\phi\phi}^{33*} \frac{v^2}{2\Lambda^2}, \\ f_2^L &\equiv -\sqrt{2} C_{dW}^{33*} \frac{v^2}{\Lambda^2}, & f_2^R &\equiv -\sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}, \end{aligned} \quad (5)$$

with  $v$  being the Higgs vacuum expectation value and  $V_{tb}$  being  $(tb)$  element of Kobayashi–Maskawa matrix. In particular,  $d_V$  and  $d_A$  are the so-called chromomagnetic- and chromoelectric-dipole moments, respectively.

In the following calculations, we use the above effective Lagrangian for top-quark interactions and the usual SM Lagrangian for the other interactions which are not affected by top quarks hereafter.

## 3. Current bounds on $d_V$ and $d_A$

Experimental constraints on  $d_V$  and  $d_A$  were derived in our preceding works [20,24]. Since LHC data of top-pair productions have been updated, however, we also re-derive new limits on  $d_V$  and  $d_A$  combining the recent data from the Tevatron for  $\sqrt{s} = 1.96$  TeV and the LHC for  $\sqrt{s} = 7$  TeV assuming  $m_t = 172.5$  GeV:

$$\begin{aligned} \sigma_{\text{exp}} &= 7.50 \pm 0.48 \text{ [pb]} \quad (\text{CDF [31]}), \\ &= 7.56_{-0.56}^{+0.63} \text{ [pb]} \quad (\text{D0 [32]}), \\ &= 187_{-21}^{+22} \text{ [pb]} \quad (\text{ATLAS [33]}), \\ &= 165.8 \pm 13.3 \text{ [pb]} \quad (\text{CMS [34]}). \end{aligned}$$

The corresponding theoretical predictions in the SM taking into account QCD higher-order corrections have been given by several authors [35–39]. Instead of averaging them, we take here the following values:

$$\sigma_{\text{SM}} = 7.29_{-0.85}^{+0.79} \text{ [pb]} \quad \text{for Tevatron}, \quad (6)$$

$$= 165_{-16}^{+11} \text{ [pb]} \quad \text{for LHC}, \quad (7)$$

which were used in [31,33]. They are consistent with the others, and therefore this choice does not cause any serious problem to our results shown below except for some tiny difference.

Adding those theoretical uncertainties to the above experimental errors quadratically, we compare thus-obtained “effective” experimental data

$$\begin{aligned} \sigma_{\text{exp}}^{\text{eff}} &= 7.50_{-0.98}^{+0.92} \text{ [pb]} \quad \text{for CDF}, \\ &= 7.56_{-1.02}^{+1.01} \text{ [pb]} \quad \text{for D0}, \\ &= 187_{-26}^{+25} \text{ [pb]} \quad \text{for ATLAS}, \\ &= 166_{-21}^{+17} \text{ [pb]} \quad \text{for CMS}, \end{aligned}$$

as our inputs with

$$\sigma = \sigma_{\text{SM}} + \Delta\sigma(d_V, d_A), \quad (8)$$

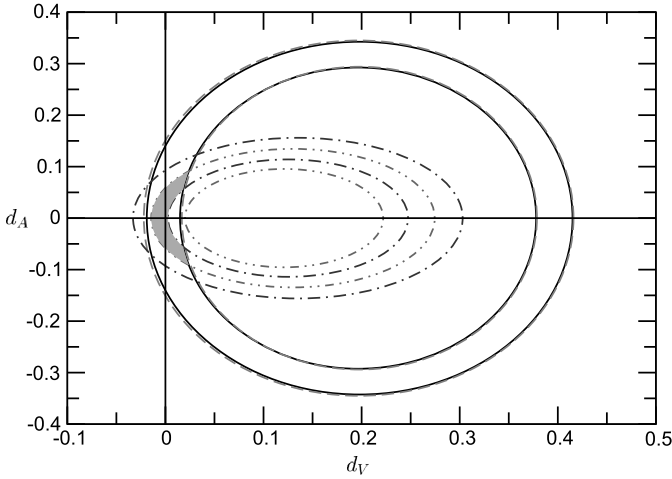
where  $\sigma_{\text{SM}}$  represents the central values in the above Eqs. (6), (7),  $\Delta\sigma(d_V, d_A)$  is the non-SM contribution calculated based on the effective Lagrangian, and the explicit forms of the corresponding parton-level quantities, i.e.,  $\Delta\sigma_{q\bar{q}}(d_V, d_A)$  and  $\Delta\sigma_{g\bar{g}}(d_V, d_A)$  are found in the first article of Ref. [20].

Numerical analyses are performed the same manner (including the parton-distribution functions) as in [20] and the whole result is shown in Fig. 1. The shaded part there expresses the region of  $d_V$  and  $d_A$  allowed by the above Tevatron and LHC data altogether. This shows clearly that both  $d_V$  and  $d_A$  are well restricted though there remains some small room for non-SM contribution.

## 4. Optimal-observable analysis

The optimal-observable analysis (OOA) is a method that could systematically estimate the expected statistical uncertainties of measurable parameters. Here we do not go into its detailed description and just show how to know the uncertainties thereby, as more theoretical aspects of this method are presented in other papers [40–43].

<sup>1</sup> Since dimension-5 operators violate the lepton-number conservation, they are not treated hereafter. Therefore, we deal with dimension-6 operators, which give the largest contributions in relevant processes.



**Fig. 1.** The  $d_{V,A}$  region allowed by the Tevatron and LHC data altogether (the shaded part). The solid curves, the dashed curves, the dash-dotted curves and the dash-dot-dotted curves are respectively from CDF, D0, ATLAS and CMS data.

First, in order to apply the OOA to the process  $pp \rightarrow t\bar{t}X \rightarrow \ell^+ X'$ , we express its differential cross section (the angular and energy distribution of the charged-lepton  $\ell^+$ ) which we derived in our previous paper [24] as follows:

$$\begin{aligned} \frac{d^2\sigma_\ell}{dE_\ell d\cos\theta_\ell} = & f_{\text{SM}}(E_\ell, \cos\theta_\ell) \\ & + d_V f_{d_V}(E_\ell, \cos\theta_\ell) + d_R f_{d_R}(E_\ell, \cos\theta_\ell) \\ & + d_V^2 f_{d_V^2}(E_\ell, \cos\theta_\ell) + d_A^2 f_{d_A^2}(E_\ell, \cos\theta_\ell) \\ & + \dots, \end{aligned} \quad (9)$$

where  $f_{\text{SM}}(E_\ell, \cos\theta_\ell)$  denotes the SM contribution, all the other  $f_I(E_\ell, \cos\theta_\ell)$  describe the non-SM terms corresponding to their coefficients, and  $d_R$  is defined as

$$d_R \equiv \text{Re}(f_2^R) M_W / m_t. \quad (10)$$

The explicit forms of  $f_I(E_\ell, \cos\theta_\ell)$  at the parton level are easily found in the relevant formulas in [24].

Since the magnitude of  $d_V$  and  $d_A$  has been shown small as in Fig. 1, we neglect any contribution from terms quadratic (or higher) in those non-SM parameters hereafter.<sup>2</sup> Note that all  $d_A$  contributions disappear under this linear approximation, because there is no term proportional to  $d_A$ . Thus, the angular and energy distributions of a decay lepton are written as

$$\frac{d\sigma_\ell}{d\cos\theta_\ell} = g_1(\cos\theta_\ell) + d_V g_2(\cos\theta_\ell), \quad (11)$$

$$\frac{d\sigma_\ell}{dE_\ell} = h_1(E_\ell) + d_V h_2(E_\ell) + d_R h_3(E_\ell), \quad (12)$$

where  $g_i(\cos\theta_\ell)$  and  $h_i(E_\ell)$  are given by

$$g_i(\cos\theta_\ell) = \int dE_\ell f_i(E_\ell, \cos\theta_\ell),$$

$$h_i(E_\ell) = \int d\cos\theta_\ell f_i(E_\ell, \cos\theta_\ell)$$

<sup>2</sup> The quadratic terms of  $f_{1,2}^{L,R}$  have already been neglected in our previous paper [24]. Indeed, studies at the Tevatron suggest that those contributions are tiny: See [44] for the latest data.

with  $i = 1, 2$  and  $3$  corresponding to  $I = \text{SM}, d_V$  and  $d_R$ , respectively. Here should be one comment about the angular distribution: We can thereby probe exclusively the  $d_V$  term, since any contribution from  $d_R$  disappears within our approximation as a result of the decoupling theorem found in [45–47].

Secondly, we calculate the following matrices:

$$M_{ij}^c \equiv \int d\cos\theta_\ell \frac{g_i(\cos\theta_\ell) g_j(\cos\theta_\ell)}{g_1(\cos\theta_\ell)} \quad (i, j = 1, 2), \quad (13)$$

$$M_{ij}^E \equiv \int dE_\ell \frac{h_i(E_\ell) h_j(E_\ell)}{h_1(E_\ell)} \quad (i, j = 1, 2, 3) \quad (14)$$

and their inverse matrices  $X_{ij}^{c,E}$ , all of which are apparently symmetric. Then the statistical uncertainties for the measurements of couplings  $d_V$  and  $d_R$  could be estimated by

$$|\delta d_V| = \sqrt{X_{22}^c \sigma_\ell / N_\ell} = \sqrt{X_{22}^c / L} \quad (15)$$

through the angular distribution, and

$$|\delta d_V| = \sqrt{X_{22}^E \sigma_\ell / N_\ell} = \sqrt{X_{22}^E / L}, \quad (16)$$

$$|\delta d_R| = \sqrt{X_{33}^E \sigma_\ell / N_\ell} = \sqrt{X_{33}^E / L} \quad (17)$$

via the energy distribution, where  $\sigma_\ell$ ,  $N_\ell$  and  $L$  denote the total cross section, the number of events and the integrated luminosity for the process, respectively.

We are now ready to carry out necessary numerical calculations. Below we show the elements of  $M^{c,E}$  computed for  $\sqrt{s} = 7, 8, 10$ , and  $14$  TeV, assuming  $m_t = 173$  GeV.<sup>3</sup>

#### (1) The angular distribution

##### (1-1) $\sqrt{s} = 7$ TeV

$$\begin{aligned} M_{11}^c &= 23.102, \\ M_{12}^c &= -245.412, \\ M_{22}^c &= 2607.340. \end{aligned}$$

##### (1-2) $\sqrt{s} = 8$ TeV

$$\begin{aligned} M_{11}^c &= 33.234, \\ M_{12}^c &= -353.598, \\ M_{22}^c &= 3762.753. \end{aligned}$$

##### (1-3) $\sqrt{s} = 10$ TeV

$$\begin{aligned} M_{11}^c &= 59.333, \\ M_{12}^c &= -632.179, \\ M_{22}^c &= 6736.735. \end{aligned}$$

##### (1-4) $\sqrt{s} = 14$ TeV

$$\begin{aligned} M_{11}^c &= 134.052, \\ M_{12}^c &= -1428.300, \\ M_{22}^c &= 15220.286. \end{aligned}$$

<sup>3</sup> We performed our analysis of the  $t\bar{t}$  total cross section for  $m_t = 172.5$  in Section 3 since this value was assumed in the Tevatron and LHC data. Hereafter, however, we would like to use  $m_t = 173$  GeV considering the latest world average [48].

## (2) The energy distribution

(2-1)  $\sqrt{s} = 7$  TeV

$$\begin{aligned}
M_{11}^E &= 23.102, \\
M_{12}^E &= -245.412, \\
M_{13}^E &= 0.000, \\
M_{22}^E &= 2607.658, \\
M_{23}^E &= -1.974, \\
M_{33}^E &= 15.323.
\end{aligned}$$

(2-2)  $\sqrt{s} = 8$  TeV

$$\begin{aligned}
M_{11}^E &= 33.234, \\
M_{12}^E &= -353.598, \\
M_{13}^E &= 0.000, \\
M_{22}^E &= 3763.252, \\
M_{23}^E &= -2.880, \\
M_{33}^E &= 21.124.
\end{aligned}$$

(2-3)  $\sqrt{s} = 10$  TeV

$$\begin{aligned}
M_{11}^E &= 59.333, \\
M_{12}^E &= -632.179, \\
M_{13}^E &= 0.000, \\
M_{22}^E &= 6737.696, \\
M_{23}^E &= -5.120, \\
M_{33}^E &= 35.233.
\end{aligned}$$

(2-4)  $\sqrt{s} = 14$  TeV

$$\begin{aligned}
M_{11}^E &= 134.052, \\
M_{12}^E &= -1428.300, \\
M_{13}^E &= 0.000, \\
M_{22}^E &= 15222.443, \\
M_{23}^E &= -10.953, \\
M_{33}^E &= 72.264.
\end{aligned}$$

Here, all  $M_{13}^E$  became zero for the same reason as vanishing  $d_R$  terms in the angular distribution, i.e., the decoupling theorem [45–47]. Using the inverse matrices calculated from the above elements and Eqs. (15)–(17), we can estimate the statistical uncertainties of the relevant couplings.

Before giving the results, however, we should comment about a stability problem in inverse-matrix computations: We noticed that the results fluctuate to a certain extent (beyond our expectation) depending on to which decimal places of  $M^{c,E}$  we take into account as our input data. Therefore we calculate  $X^{c,E}$  for the above  $M^{c,E}$  and also for what are obtained by rounding those  $M^{c,E}$  off to two decimal places. We then use their mean values to get  $\delta d_V$  and  $\delta d_R$  with “errors”, which are the differences between the mean values and the maximum/minimum. Let us explain these treatments more specifically by taking (1-1) as an example: For those different inputs, we get

$$\begin{aligned}
\sqrt{X_{22}^c} &= 1.73 \quad \text{for } M_{11}^c = 23.102, \dots, \\
&= 2.57 \quad \text{for } M_{11}^c = 23.10, \dots,
\end{aligned}$$

**Table 1**

Estimated statistical uncertainties of  $d_V$  (multiplied by  $\sqrt{L}$ ) from the angular distribution of a decay lepton.

$\sqrt{s}$ [TeV]	$ \delta d_V  (\times \sqrt{L})$
7	$2.15 \pm 0.42$
8	$2.25 \pm 0.96$
10	$1.12 \pm 0.12$
14	$0.73 \pm 0.02$

**Table 2**

Estimated statistical uncertainties of  $d_V$  and  $d_R$  (multiplied by  $\sqrt{L}$ ) from the energy distribution of a decay lepton.

$\sqrt{s}$ [TeV]	$ \delta d_V  (\times \sqrt{L})$	$ \delta d_R  (\times \sqrt{L})$
7	$1.86 \pm 0.28$	$0.35 \pm 0.02$
8	$1.70 \pm 0.51$	$0.32 \pm 0.05$
10	$0.98 \pm 0.08$	$0.22 \pm 0.01$
14	$0.65 \pm 0.02$	0.15

which lead to the mean value 2.15, the maximum 2.57, and the minimum 1.73, and resultant errors  $\pm 0.42$ , which are derived by  $2.57 - 2.15$  and  $1.73 - 2.15$ . These uncertainties are of course independent of the statistical ones which the OOA gives, i.e.,  $\delta d_{V,R}$  themselves, so we should show both uncertainties separately as different errors.

This instability is due to the large contribution from the  $d_V$  coupling, which is dependent of gluon momenta and could therefore dominate. That is, we are consequently forced to treat large and small figures simultaneously in one matrix. In fact, we confirmed that any similar problem does not appear if we carry out computations for  $M^E$  and  $X^E$  without  $d_V$  terms, in which there is no big difference between the SM- and  $d_R$ -term contributions.

We also have to explain how we can take into account QCD higher-order corrections: All the numerical computations of  $M_{ij}^{c,E}$  in this section were done with the tree-level formulas. In order to include QCD corrections there, we multiply the tree cross sections by the  $K$ -factor. This factor disappears in the combination  $X_{ii}^{c,E} \sigma_\ell$  and remains only in  $N_\ell (= L \sigma_\ell)$  when we estimate  $\delta d_{V,R}$ . Therefore the luminosity  $L$  discussed in the following should be understood as an effective one including  $K$  (and also the lepton detection efficiency  $\epsilon_\ell$ ).

We now show the whole results in Tables 1 and 2 for the angular and energy distributions, respectively. The uncertainties we encountered here are hard to eliminate, but still the results will tell us the necessary luminosity for reaching the precision which we aim to realize. For example, we need at least  $L \simeq 1500 \text{ pb}^{-1}$  ( $= 1.5 \text{ fb}^{-1}$ ) in order to achieve  $|\delta d_V| \simeq O(10^{-2})$  in case of measuring the angular distribution at the LHC whose colliding energy is 8 TeV. As mentioned, those  $L$  should be divided by  $K \simeq 1.5$  and  $\epsilon_\ell$ . If we assume  $\epsilon_\ell = 0.5$ , the resultant  $L$  increases slightly, which however hardly affects our conclusion.

Here we performed all the angular and energy integrations to compute  $M^{c,E}$  over the full kinematically-allowed ranges for simplicity, but it will be useful to see how our results are affected by specific experimental conditions. As a typical example, we take the transverse-momentum cut for  $\ell^+$  in the energy distribution:  $p_{\ell T} \geq p_{\ell T}^{\min}$ , which leads to

$$-\sqrt{1 - (p_{\ell T}^{\min}/E_\ell)^2} \leq \cos \theta_\ell \leq +\sqrt{1 - (p_{\ell T}^{\min}/E_\ell)^2}$$

with  $E_\ell \geq p_{\ell T}^{\min}$ . The results for  $p_{\ell T}^{\min} = 20 \text{ GeV}$  (see, e.g., [33,49]) are

$$|\delta d_V| (\times \sqrt{L}) = 2.30 \pm 0.51,$$

$$|\delta d_R|(\times\sqrt{L}) = 0.69 \pm 0.12 \quad (18)$$

for  $\sqrt{s} = 8$  TeV, where our results with no experimental cuts were most unstable. We find there is no change in the instability of  $\delta d_V$  while that of  $\delta d_R$  has increased. However, the relative size of it compared to  $\delta d_R$  itself ( $= 0.69$ ) is almost the same as that in [Table 2](#):  $16\% \rightarrow 17\%$  ( $30\% \rightarrow 22\%$  in  $\delta d_V$ ). Therefore, they tell us that introducing a lepton- $p_T$  cut will not seriously affect the stability of  $X^{c,E}$ , although the expected uncertainties themselves become a bit larger, which we can take into account through the detection efficiency as will be discussed in the last section.

## 5. Summary

Possible non-standard top-gluon and top- $W$  couplings were studied for hadron-collider experiments in a model-independent way. Those couplings are derived as parameters which characterize the effects of dimension-6 effective operators based on a scenario of Buchmüller and Wyler [\[26\]](#). More specifically, we studied  $d_{V,A}$  couplings which contribute to top-pair productions and  $d_R$  coupling which affects leptonic decays of a top quark as non-standard interactions.

First, we updated the bounds of  $d_V$  and  $d_A$  from the current Tevatron and LHC data of top-pair productions and showed that the allowed area of  $d_{V,A}$  became narrower around  $(d_V, d_A) = (0, 0)$ . Based on this observation, we adopted a linear approximation for the non-standard parameters in the subsequent optimal-observable analysis (OOA) of  $pp \rightarrow t\bar{t}X \rightarrow \ell^+ X'$ . In this approximation, all the contributions from  $d_A$  were neglected because the leading order of  $d_A$  terms is quadratic. Therefore, the OOA was performed for  $d_V$  and  $d_R$  to estimate their expected statistical uncertainties using the angular and energy distributions of the final lepton.

Measuring the energy distribution could study both  $d_V$  and  $d_R$  at the same time with a higher precision than the case of the angular distribution. Meanwhile, since the angular distribution is affected by only  $d_V$  contributions within our approximation, this one is suitable for exploring the mechanism of top-pair productions exclusively though the precision of  $d_V$  is slightly lower than that from the energy distribution.

We found that there were some ambiguities in inverse-matrix computations depending on where to round the input data (i.e.,  $M_{ij}^{c,E}$ -elements) off, and consequently estimated statistical errors were affected thereby to a certain extent. However, our conclusion about the necessary luminosity and new-physics scale  $\Lambda$  hardly receive that serious influence, because those ambiguities do not change the order of the results.

As for future prospects, if the LHC is going to be steadily upgraded and  $L = 500 \text{ fb}^{-1}$  is achieved at  $\sqrt{s} = 14$  TeV, both  $d_V$  and  $d_R$  could be determined with  $|\delta d_V| \sim O(10^{-3})$  and  $|\delta d_R| \sim O(10^{-4})$  from the energy distribution: Here,  $|\delta d_V| \sim 0.001$  ( $|\delta d_R| \sim 0.0001$ ) means that contribution from effective operator  $\mathcal{O}_{uG\phi}^{33} (\mathcal{O}_{uW}^{33})$  is suppressed by  $\Lambda \gtrsim 7$  (20) TeV if the center value of measured  $d_V$  ( $d_R$ ) is close to zero and  $C_{uG\phi}^{33} (C_{uW}^{33}) \sim 1$ , which we estimated from Eqs. (5) and (10).

Finally, we did not discuss here any realistic experimental conditions like effects of event-selection cuts and/or backgrounds except for the discussion in the last paragraph of the preceding section. This was partly because it was not our purpose to go into such details in this Letter, although it is important since they decrease actual event number, but also because the OOA can work under any experimental conditions. All we have to do is to take account of them properly and accept the estimated luminosity as an effective one including all those effects. This is closely related to the detection efficiency  $\epsilon_\ell$ , which we discussed at the end of the

previous section. Indeed we are ready to re-calculate every quantity obtained in this Letter, correspondingly to any lepton angle- and/or energy-cut conditions since we gave all the formulas used here analytically in Ref. [\[24\]](#).

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