Top quark differential cross sections: how to simultaneously unfold 2016-18 data

TOP PAG meeting, May 14, 2019

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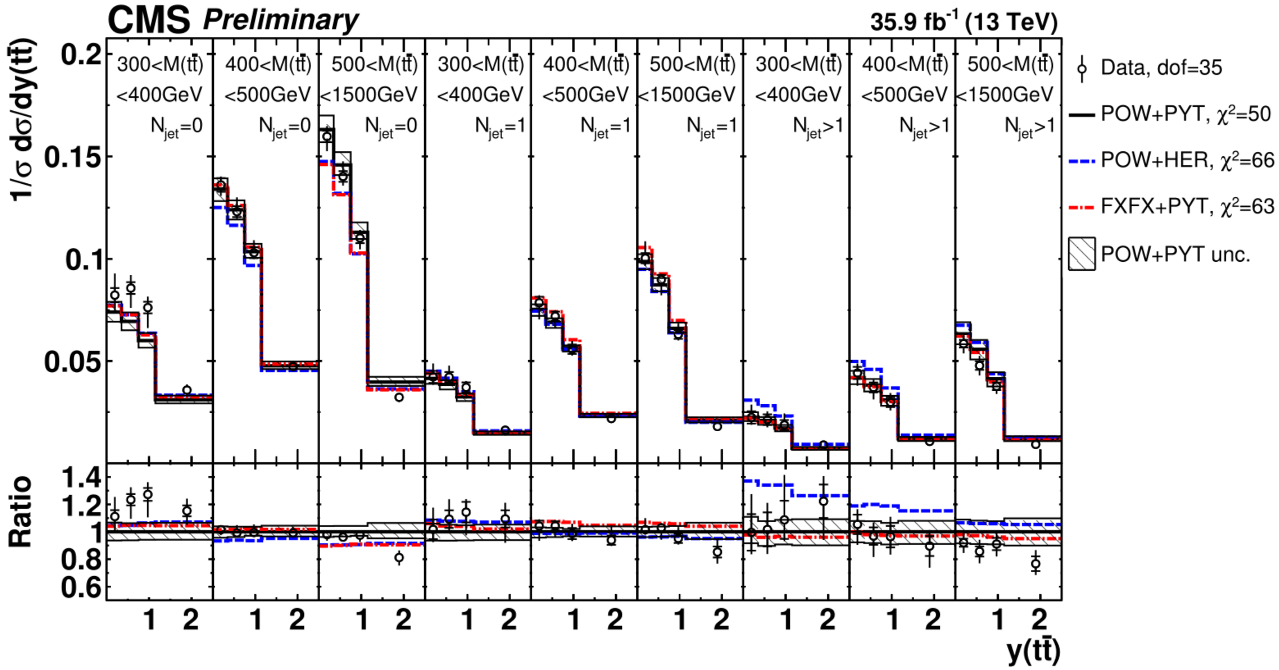
Introduction

- Measured already up to 3D tt cross sections with 2016 data alone:



TOP-18-004, 1904.05237

using TUnfold

- Discuss today three suitable variants for combined 2016-18 unfolding:

Analysis type

  - TUnfold after background subtraction

High stat. low background:
tt, TOP-18-004, TOP-17-002,..

  - TUnfold after template fit of signal yields

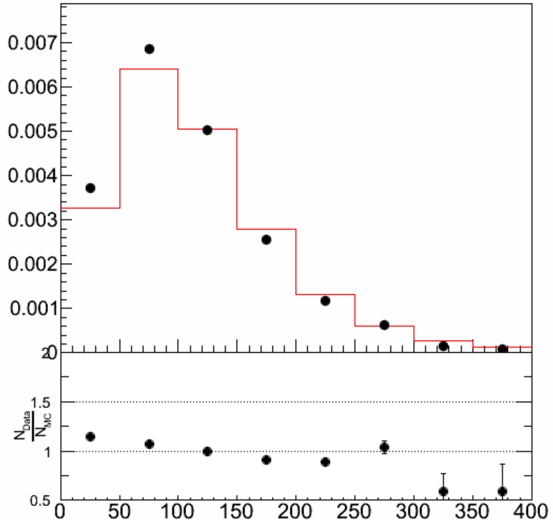
High background
Single t, TOP-17-023

  - Combine tool: direct Max. Lh. fit of dσ/dx

#unfolded bins not too high

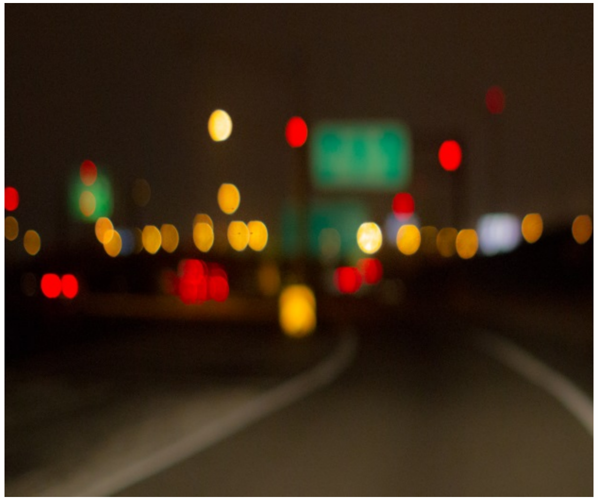
Unfolding of λ

True level

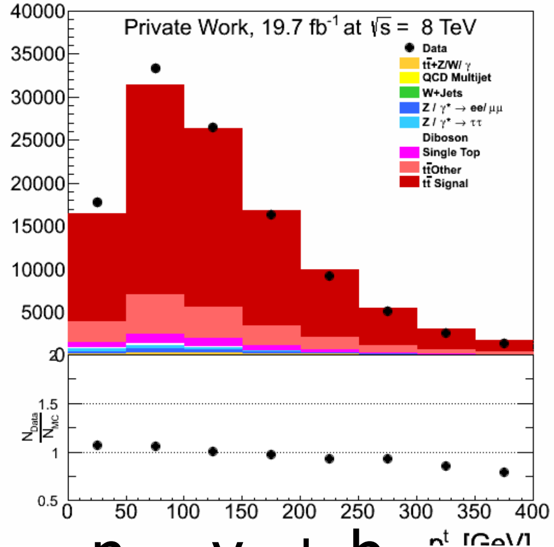


λ

Detector effects + background



Smeared space



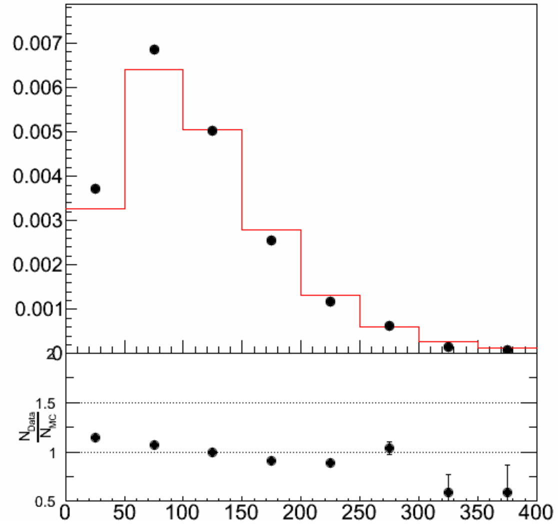
$n = y + b$

Unfolding of λ

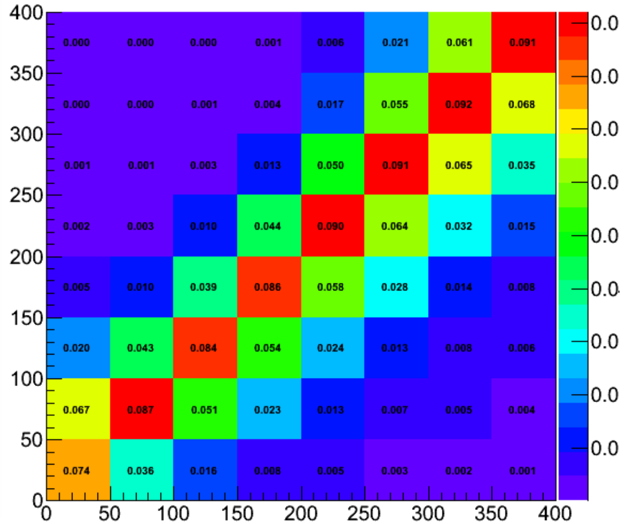
Detector effects + background

Smeared space

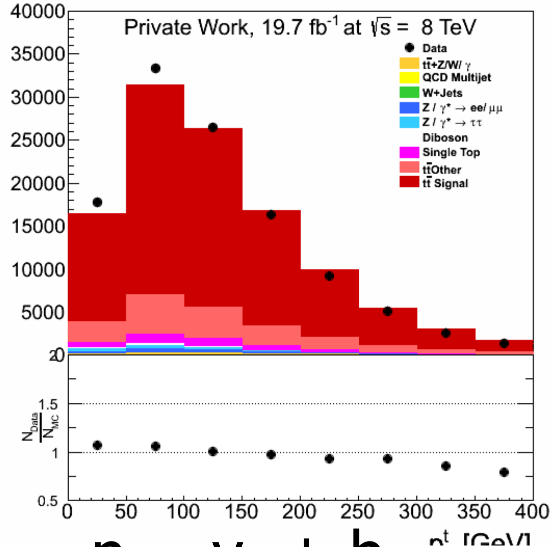
True level



λ



$K\lambda + b$



$n = y + b$

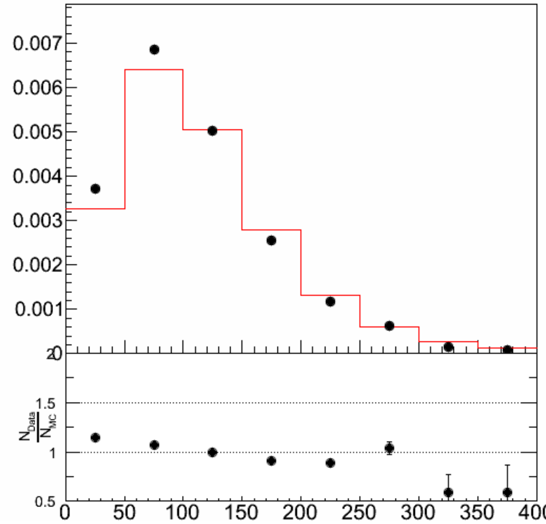
- Unfolding means estimating λ from n, → $\hat{\lambda}$

Unfolding of λ

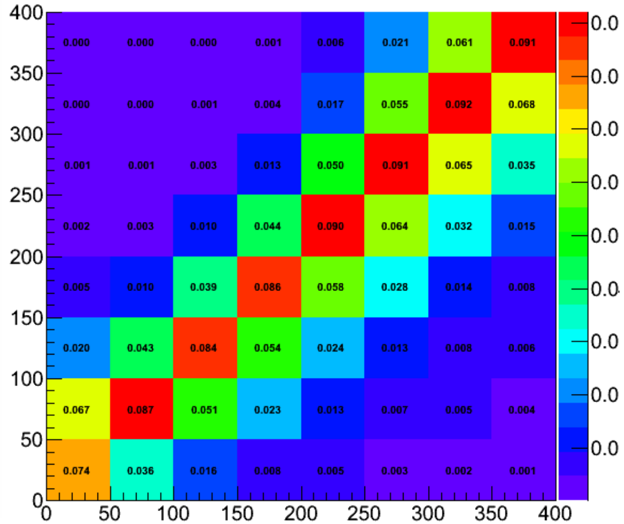
Detector effects + background

Smeared space

True level



λ



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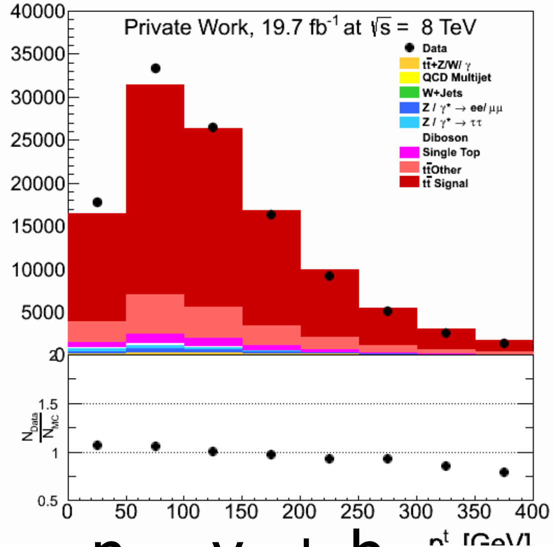


$n = y + b$

- Unfolding means estimating λ from n, → $\hat{\lambda}$

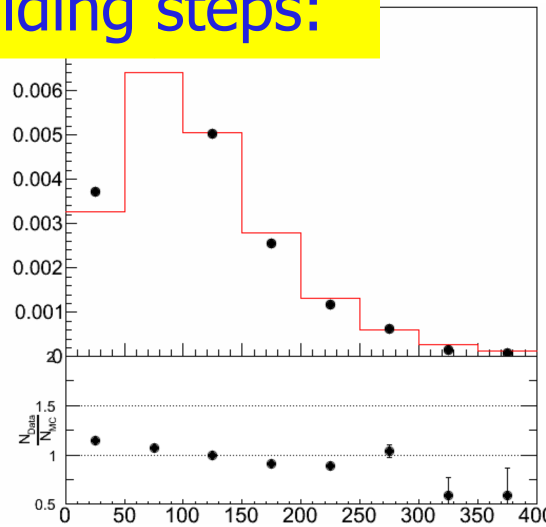
Use '16' for '2016', etc.

- Note: Data combination can be always done
  1. at the very beginning, unfolding $n = n_{16}+n_{17}+n_{18}$
     + increase directly statistics, stabilise results
     – not optimal if data quality varies a lot
  2. after unfolding, combining $\hat{\lambda}_{16}$, $\hat{\lambda}_{17}$, $\hat{\lambda}_{18}$
     – if BLUE χ^2 is used → gaussian uncertainty approximations

Unfolding of λ

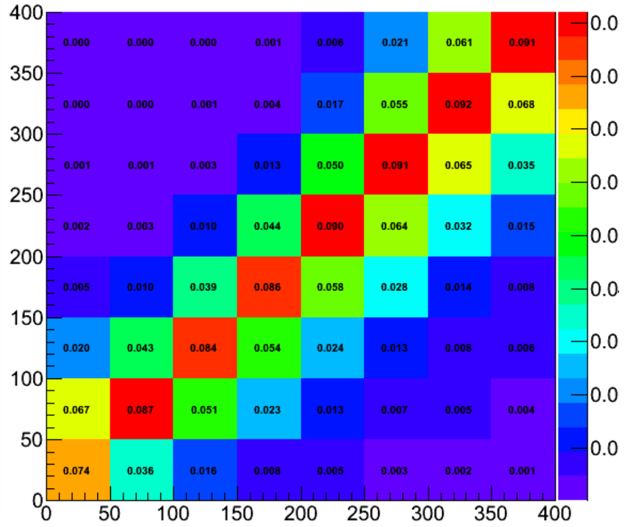
Unfolding steps:

True level



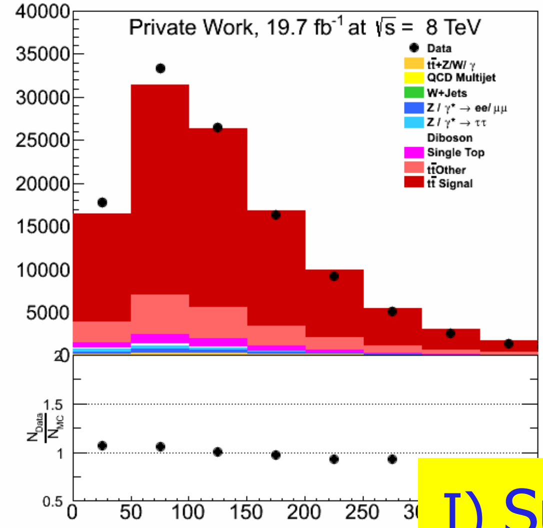
λ

Detector effects + background



$K\lambda + b$

Smeared space



$n = y + b$

I) Subtract: $y = N - b$

II) Max. Likelihood:
gaussian approximation

Same # of λ and y bins: $\lambda = K^{-1} y$

More y bins: minimize $\chi^2 = [\mathbf{y} - \mathbf{K}\lambda]^t \, \mathbf{V_y}^{-1} \, [\mathbf{y} - \mathbf{K}\lambda]$

Often instable high frequency components of unfolded λ → regularisation

TUnfold:

$$\chi^2 = [\mathbf{y} - \mathbf{K}\lambda]^t \, \mathbf{V_y}^{-1} \, [\mathbf{y} - \mathbf{K}\lambda] \; + \boldsymbol{\tau}^2 ||\mathbf{L}(\lambda - \lambda_\mathbf{0})||^2$$

Bias vector

Regularisation strength

Matrix: unity, 1st or 2nd derivatives

TUnfold programme

www.desy.de/~sschmitt

$$\chi^2 = [\mathbf{y} - \mathbf{K}\lambda]^t \, \mathbf{V_y}^{-1} \, [\mathbf{y} - \mathbf{K}\lambda] \; + \boldsymbol{\tau}^2 ||\mathbf{L}(\lambda - \lambda_\mathbf{0})||^2$$

- Regularisation schemes: → choice of τ value
  - L-curve (balance of the two χ^2 terms)
  - Minimum global correlation in covariance matrix of λ
- For multi-differential cross sections: automatic internal mapping of multi-D variables to 1D vectors → TUnfold takes care of regularisation in the multi-D phasespace.

- Systematic uncertainties: Need to repeat TUnfolding with varied response matrices K

Simultaneous TUnfolding of 2016-18 data

- Use background subtracted yield vectors y_{16}, y_{17} and y_{18}, define

$$\boldsymbol{y} = \begin{pmatrix} y_{16} \\ y_{17} \\ y_{18} \end{pmatrix};\ \mathbf{V_y} = \begin{pmatrix} V_{16,16} & V_{16,17} & V_{16,18} \\ V_{16,17} & V_{17,17} & V_{17,18} \\ V_{16,18} & V_{17,18} & V_{18,18} \end{pmatrix};\quad \mathbf{K} = \begin{pmatrix} K_{16} \\ K_{17} \\ K_{18} \end{pmatrix}$$

- In most simple variant V is just diagonal matrix $\propto$ event counts
- use Tunfold as usual → will min. following χ^2 to find estimator $\hat{\lambda}$

$$\chi^2 = [\mathbf{y} - \mathbf{K}\lambda]^t\, \mathbf{V_y}^{-1}\, [\mathbf{y} - \mathbf{K}\lambda]\ + \boldsymbol{\tau}^2 ||\mathbf{L}(\lambda - \lambda_0)||^2$$

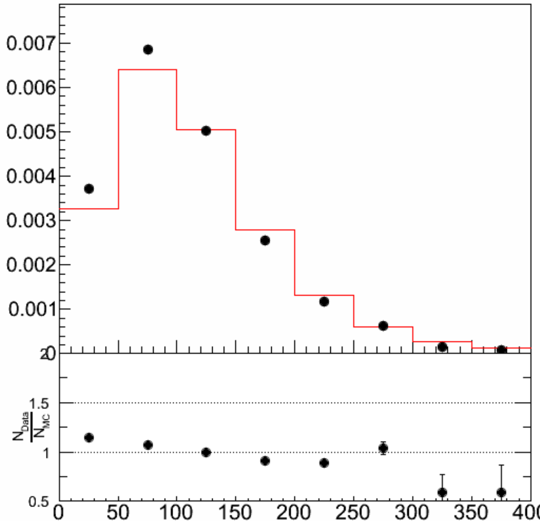
⚠ If we would first unfold separately 2016, 2017 and 2018 and then combine, we would count the regularisation term 3 times!

Unfolding of λ

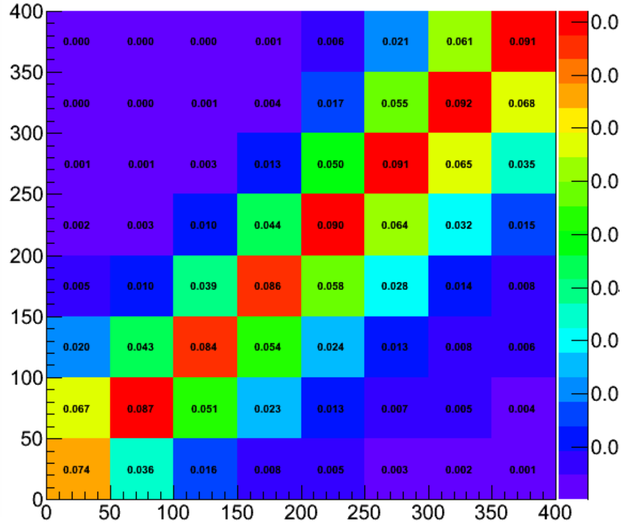
Detector effects + background

Smeared space

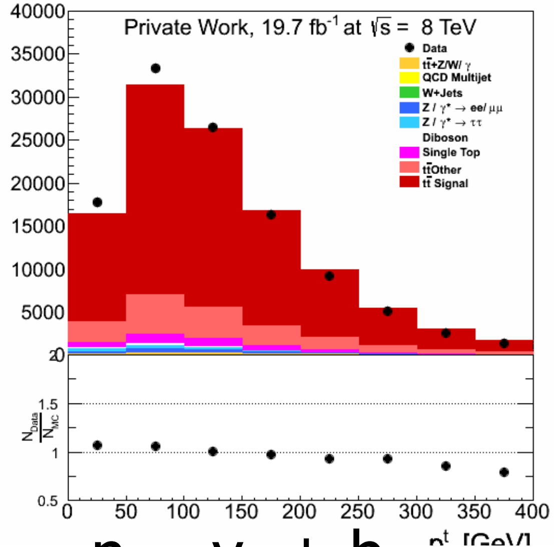
True level



λ



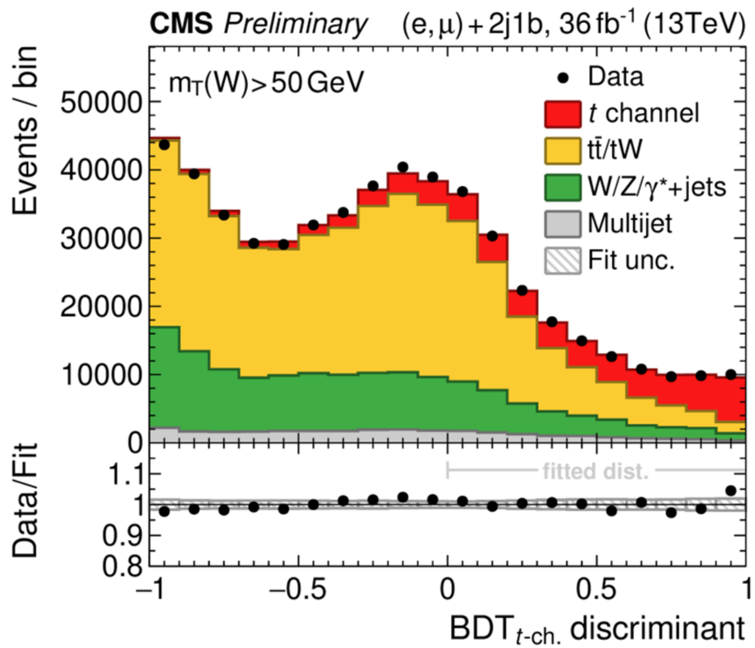
$K\lambda + b$



$n = y + b$

Alternative variant

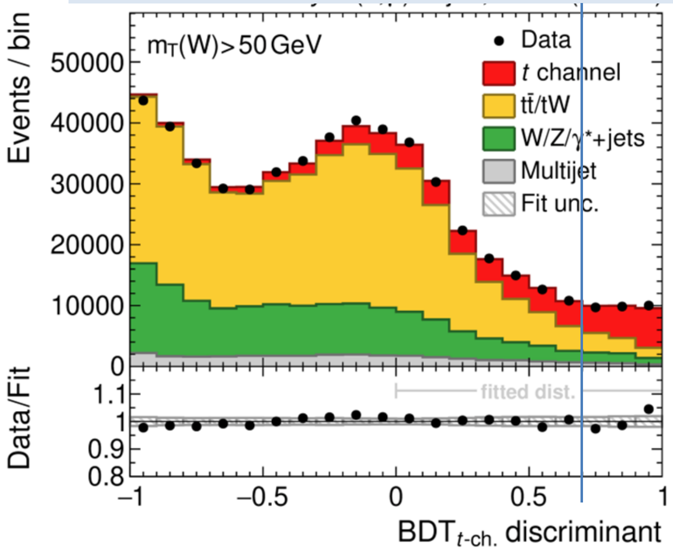
- For background dominated analyses:
  → first estimate bin-wise signal yields y_j in template fit to discriminator distribution, then feed to TUnfold
  → strategy developed for t-channel single top production TOP-17-023



Pre-unfolding template fit in TOP-17-023

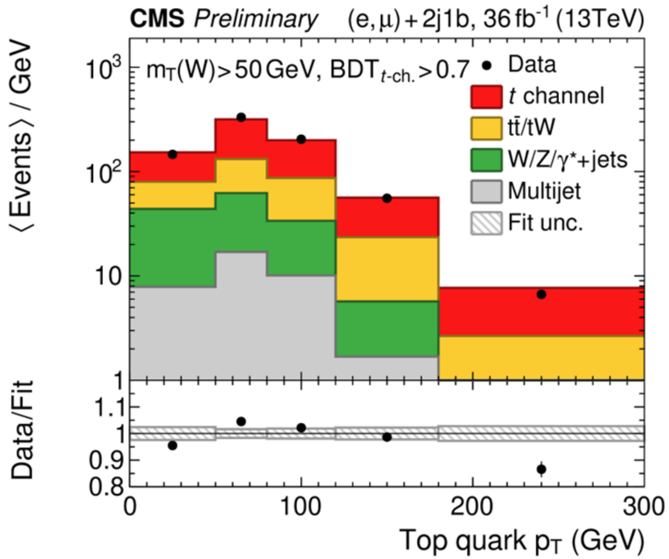
M. Komm, using theta tool

Discriminator i-bins



For each bin j of spectrum to unfold, fit signal yield y_j to the counts in the i-bins of the discriminator

$p_T(t)$ j-bins – control plot



$$L \sim \prod_{j-bins} \prod_{i-bins} exp[-(y_j s_{ij} + b_{ij})] \cdot (y_j s_{ij} + b_{ij})^{n_{ij}} \cdot \prod \text{Constraints}$$

s_{ij}: fraction of signal y_j expected in bin i

b_{ij}: total expected background in bin ij

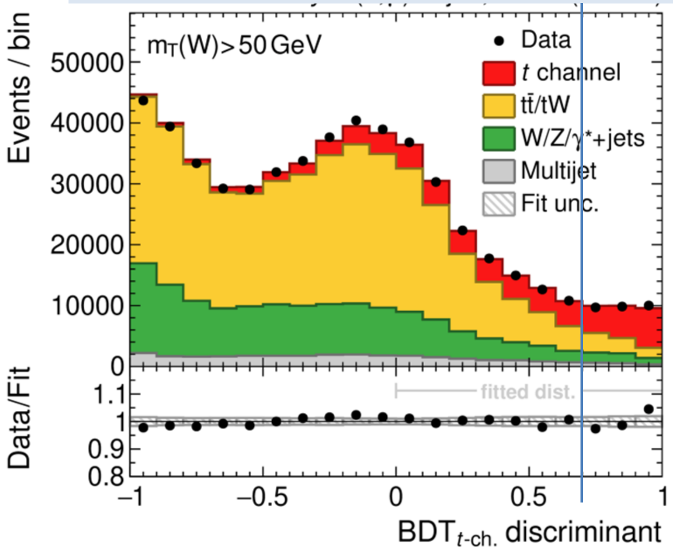
n_{ij}: observed event count

- b_{ij} depend on tt and other bgr. normalisations, fitted for each bin j
- s_{ij} and b_{ij} depend on other nuisance pars that are also fitted
- All y_j fitted simultaneously, → then feed to TUnfold

Pre-unfolding template fit in TOP-17-023

M. Komm, using theta tool

Discriminator i-bins



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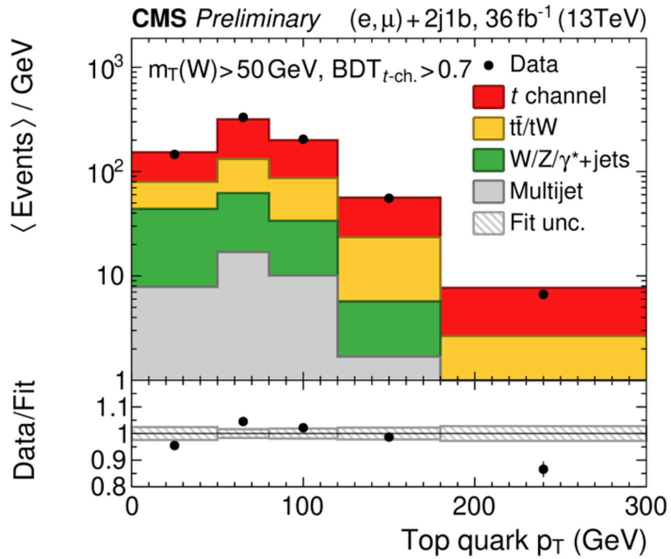
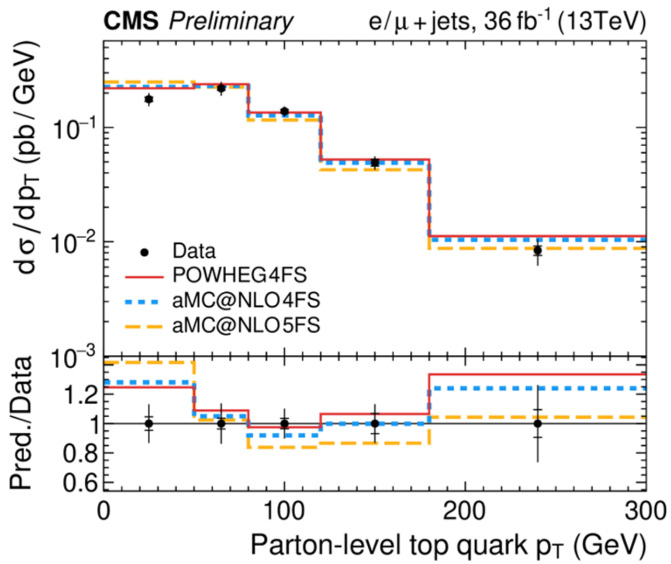


$$L \sim \prod_{j-bins} \prod_{i-bins} exp[-(y_j s_{ij} + b_{ij})] \cdot (y_j s_{ij} + b_{ij})^{n_{ij}} \cdot \prod \text{Constraints}$$

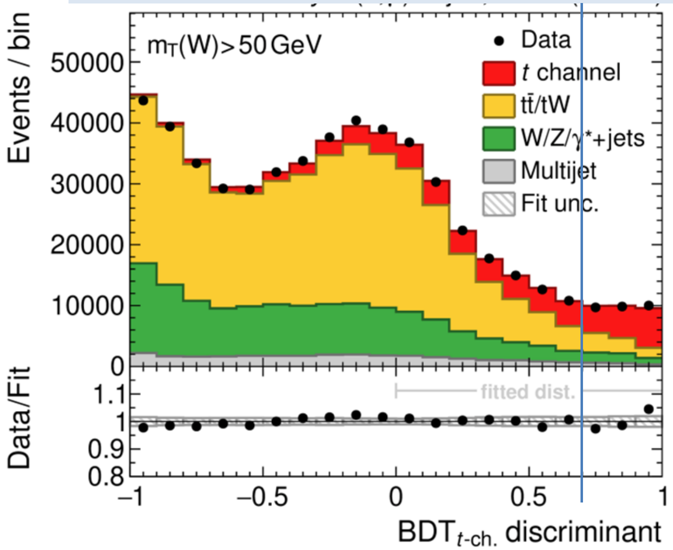
Final step: TUnfold total signals $y_j \rightarrow \hat{\lambda}$



Pre-unfolding template fit in TOP-17-023

M. Komm, using theta tool

Discriminator i-bins



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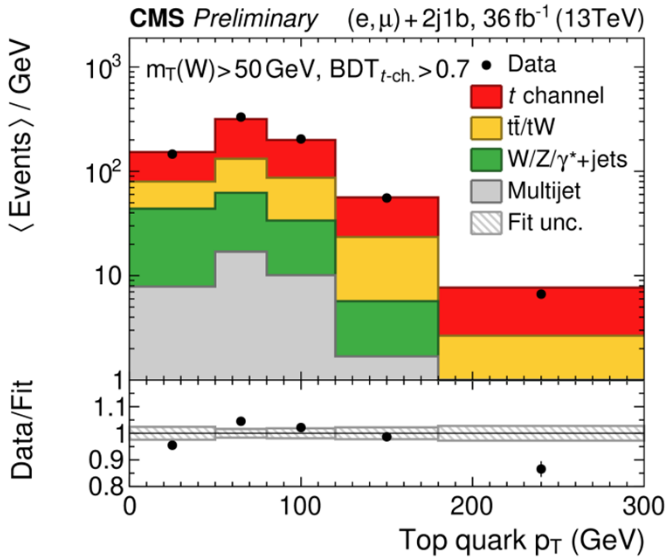


For each bin j of spectrum to unfold, fit signal yield y_j to the counts in the i-bins of the discriminator

$$L \sim \prod_{j-bins} \prod_{i-bins} exp[-(y_j s_{ij} + b_{ij})] \cdot (y_j s_{ij} + b_{ij})^{n_{ij}} \cdot \prod \text{Constraints}$$

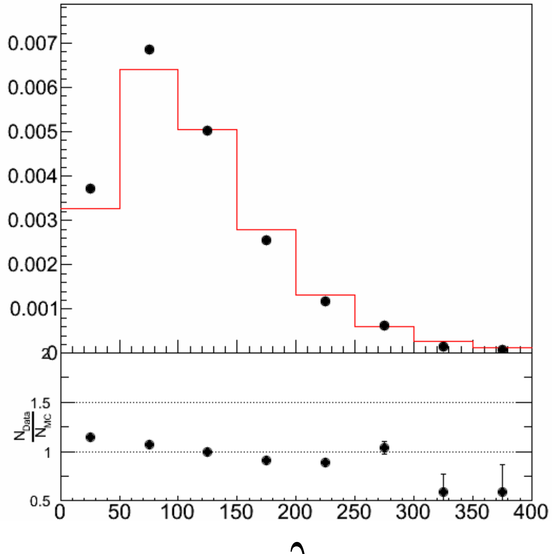
- Simultaneous 2016-18 analysis:
  $L = L_{16} \cdot L_{17} \cdot L_{18}$ → fit y_{16}, y_{17} and y_{18}, feed to TUnfold, as on p.8

Unfolding of λ

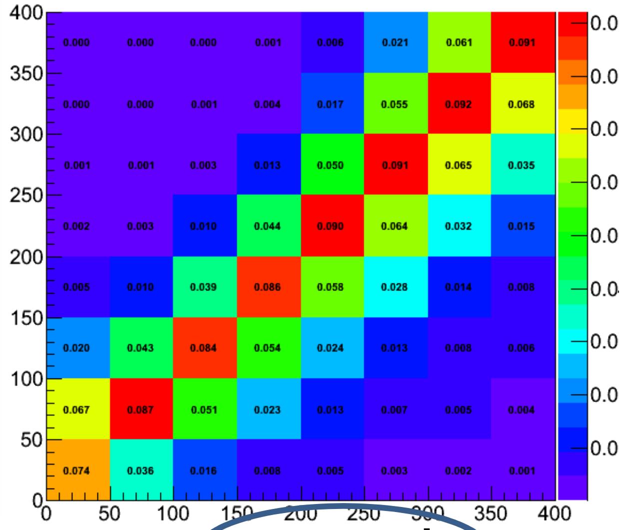
Detector effects + background

Smeared space

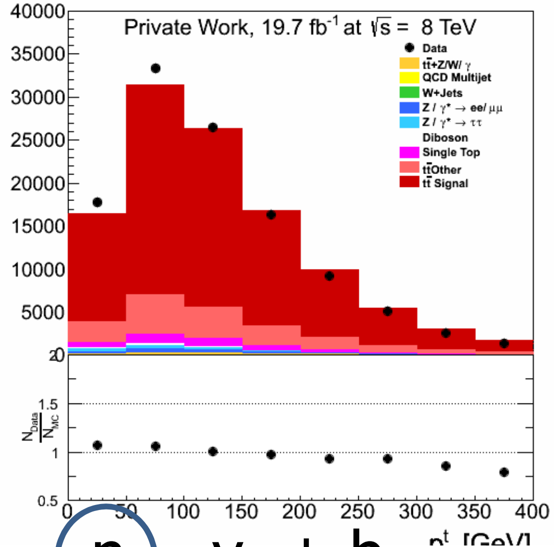
True level



λ



$K\lambda + b$



$n = y + b$

Unfolding with combine tool

https://cms-analysis.github.io/HiggsAnalysis-CombinedLimit/part3/regularisation

$$L \sim \prod_{j-bins} exp\left[-\left(\sum_{m-bins} K_{jm}\lambda_m + b_j\right)\right] \cdot \left(\sum_{m-bins} K_{jm}\lambda_m + b_j\right)^{n_j} \cdot \prod \text{Constraints}$$

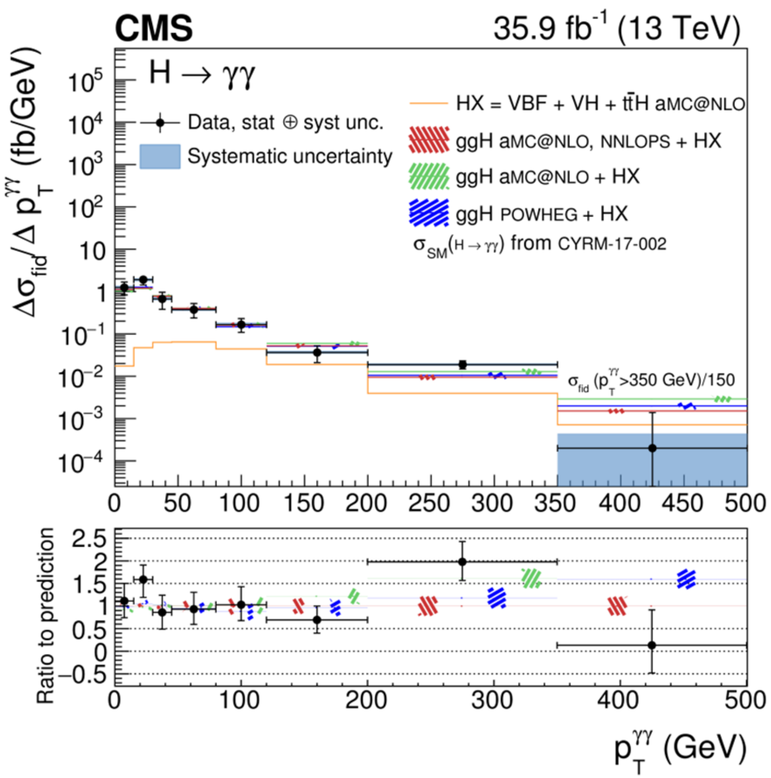
- Rigorous Max. Likelihood fit of λ; 2016-18 analysis: $L = L_{16} \cdot L_{17} \cdot L_{18}$
- K_{jm} and b_j can depend on nuisance parameters → use all related tools in combine (template morphing, log-normal constraints, …)
- Extendable to template fit vs discriminator bins i

Example: HIG-17-025

All CMS Higgs dσ/dx measurements based on combine tool approach

New: now also available with Tikhonov regularisation, docu see

https://cms-analysis.github.io/HiggsAnalysis-CombinedLimit/part3/regularisation



Unfolding with combine tool

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- Rigorous Max. Likelihood fit of λ; 2016-18 analysis: $L = L_{16} \cdot L_{17} \cdot L_{18}$
- K_{jm} and b_j can depend on nuisance parameters → use all related tools in combine (template morphing, log-normal constraints, …)
- Extendable to template fit vs discriminator bins i

Summary

- Presented 3 ways to simultaneously unfold separate 2016, 2017 and 2018 detector level data, to obtain best $\hat{\lambda} = d\sigma/dx$ from RUN II.

- Adapt strategies to your problem:

High stat. low background: Examples: tt, TOP-18-004, TOP-17-002	→ TUnfold after backgr. subtraction
High background: Examples: t, TOP-17-023	→ TUnfold after template fit of detector level signal yields y_j
#unfolded bins not too high	→ Combine tool direct Maximum Likelihood Fit → $\hat{\lambda}$

- 2016-18 combination based on building total χ^2 or likelihood
- Note: assessment of correlated uncertainties between the periods is one of the major tasks/challenges! (beyond scope of talk)

Backup slides