Imperial College London
Department of Mathematics
MSc in Mathematics and Finance
Academic year 2022–2023, Autumn term

MATH70110 Quantitative Risk Management

Assessed Coursework 2

Due date: 9 December 2022, 11:59pm

Instructions

- This assignment accounts for 10% of the total module score.
- Work in the same group as you did in the first assignment. Each member in the group will receive the same score.
- There is no formal word limit but ideally your work should not exceed 10 pages under a reasonable document layout (excluding tables, figures and other illustrations).
- You can use any programming languages you like but Python is recommended.
- You will not be penalised for "bad programming styles" provided that correct answers are generated. There is no need to provide comments to your codes or Python notebook.
- One written pdf document and relevant program file(s) to be sent to `a.tse@imperial.ac.uk` by the deadline. Indicate all members' names and student ids on the report clearly and Cc everyone in the submission email.

Risk Forecasting with Extreme Value Theory

Pietro[1] has changed his mind. Instead of shorting US equities via buying the DOG ETF, he wants to make a long equity investment and has become bullish about Tesla (TSLA)[2] after following Elon Musk[3] on Twitter. Help Pietro forecast the risk of this

[1] Still a fictitious character.
[2] This does not constitute financial advice. Please invest responsibly.....
[3] Perhaps a real character.

investment by considering the daily (relative) linearised losses

$$\tilde{L}_{t+1}^{\Delta} = -r_{t+1}$$

where r_{t+1} is the daily log-return (in percentage) on TSLA between date t and $t+1$.

The daily closing prices[4] of TSLA from 26 November 2012 to 25 November 2022 are provided in the file `QRM-2022-cw2-data.csv`.

To reduce the computational effort, we will not use the rolling-window estimation procedure this time but instead we will just estimate each relevant model only once. All the log-return values prior to 26 November 2021 are considered as "training data" which will be used to estimate the model parameters. All the log-return values on or after 26 November 2021 are considered as "testing data" which will be used for backtesting.

(i) Fit a standard GARCH(1,1) model, with constant conditional mean under the assumption of standard normal errors. Perform the estimation using the training data set only. Report the model fitting results and analyse the standardised residuals $\hat{Z}_1, ..., \hat{Z}_n$. Do the residuals appear to be iid?

(ii) Fit a normalised Student t distribution to the standardised residuals from part (i). Report the estimated degree of freedom and assess the quality of fit briefly.

(iii) In parallel, fit a generalised Pareto distribution (GPD) to the standardised residuals exceeding a threshold $u > 0$. Make your choice of u, which should be based on the sample mean excess function plot of the standardised residuals. Report the fitted model and assess the quality of fit briefly.

(iv) Compute several sets of VaR and ES forecast on each date in the testing data set at 95% and 99% confidence levels, based on the following different specifications of the strict white noise component Z in the GARCH(1,1) model:

- Z follows a standard normal distribution;
- Z follows a normalised Student t distribution;
- Z is such that its excess distribution (over u) is described by GPD.

Describe clearly how you compute the VaR and ES under each specification. Display the risk forecasts graphically. Backtest the VaR forecasts by considering unconditional coverage tests only and compare the results under different specifications. You are not required to look at the (joint coverage-)independence tests for VaR nor to backtest the ES.

[4] Adjusted for TSLA stock split already which happened in August 2022.

Other tips, recommendations and best practices

- You don't need to address this in the assignment, but it is beneficial for you to reflect on how the suggested procedures above subtly differ from what you have done in Assignment 1.
- Again, you can use any available packages provided that they are mentioned in the report. Useful ones include the arch[5] and perhaps the pyextremes[6] packages for Python, and the rugarch/fGarch[7] and QRM[8] packages for R.
- Refer to the guidelines in Assignment 1 again for the best practices related to tables, figures, references, etc.

[5] See https://pypi.org/project/arch/.
[6] See https://georgebv.github.io/pyextremes/.
[7] See https://cran.r-project.org/web/packages/rugarch/ and https://cran.r-project.org/web/packages/fGarch/index.html.
[8] See https://cran.r-project.org/web/packages/QRM/QRM.pdf.