Problem R-1A. Determine if the following statement is true or false. If true, provide a proof; if false, provide a counterexample or show in some fashion why the statement is false. In either case, feel free to use results from the Prep Sheet.

(True or False?) Let $f : [0,1] \to \mathbb{R}$ be a function such that $f \in AC([\epsilon, 1])$ for all $\epsilon > 0$. Assume that f is continuous at 0, and $f \in BV([0,1])$. Then $f \in AC([0,1])$.

We know that a function is AC iff it is continuous, of bounded variation, & maps sets of measure zero to sets of measure zero.

We are given the first two on $[0,1]$, so we want to show the third.

Let $N \subsetneq [0,1]$ be a set of measure zero. Then

$m(N \cap [1/n, 1]) = 0$, so

$$m(f(N)) = \lim_{n\to\infty} m(f(N \cap [1/n, 1]))$$

$$= 0$$