

2) [Yutian, Victoria]

Solve  $u = u(x, t)$

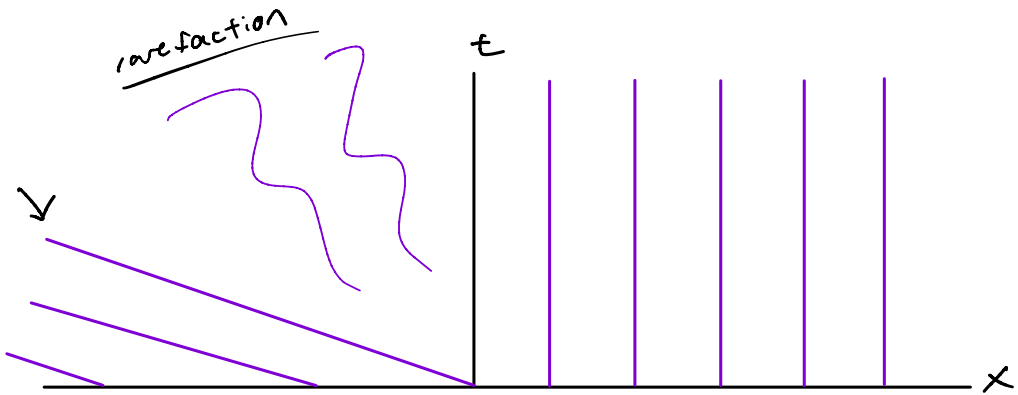
$$\begin{cases} u_t - 2(u^2)_x = 0 & x \in \mathbb{R}^1, t \geq 0 \\ u(x, 0) = 1 & x \leq 0 \\ u(x, 0) = 0 & x > 0 \end{cases}$$

flux equation:  $f(u) = -2u^2$

$$\Rightarrow \frac{1}{f'(u)} = -\frac{1}{4u}$$

slopes of characteristics:  $x \leq 0$ :  $\frac{dt}{dx} = -\frac{1}{4}$

$x > 0$ :  $\frac{dt}{dx} = -\frac{1}{0}$  so we will say  $\infty$



To get a piecewise solution in the rarefaction zone, we calculate

$$(f')^{-1}\left(\frac{x}{t}\right) = -\frac{x}{4t}. \quad \text{Thus}$$

$$u(x, t) = \begin{cases} 1 & x \leq -4t \\ -\frac{x}{4t} & -4t < x \leq 0 \\ 0 & x > 0 \end{cases}$$