51. **(Homework 5 - Chifan)** Suppose $f \geq 0$ and f is integrable. If $\alpha > 0$ and $E_\alpha = \{x : f(x) > \alpha\}$ prove that $m(E_\alpha) \leq \frac{1}{\alpha}\int_{\mathbb{R}} f(x)dx$.

Since f is integrable,

$$\left.\begin{aligned} \infty > \int_{\mathbb{R}} f(x)\,dx &\geq \int_{E_\alpha} f(x)\,dx \\ &\geq \alpha \int_{\mathbb{R}} \chi_{E_\alpha}(x)\,dx \end{aligned}\right\} \text{this is legal because } f \text{ is nonnegative}$$

$$= \alpha\, m(E_\alpha)$$

$$\Rightarrow m(E_\alpha) \leq \frac{1}{\alpha}\int_{\mathbb{R}} f(x)\,dx \quad //$$