32. (***Javier and Zhihua***) Determine and show the degree of precision of Simpson's rule on the reference interval [-1,1], $\int_{-1}^{1} f(x)dx \approx \frac{1}{3}f(-1) + \frac{4}{3}f(0) + \frac{1}{3}f(1)$.

Let $f_m(x) = x^m$. Then the degree of precision of the method is the maximal value of m for which the method is still exact; we show this is $m=3$.

$m=0$

LHS: $\int_{-1}^{1} f_0(x) = 2$ RHS: $\frac{1}{3}f_0(-1) + \frac{4}{3}f_0(0) + \frac{1}{3}f_0(1)$

$= \frac{6}{3} = 2$ ✓

$m=1$

LHS: $\int_{-1}^{1} f_1(x) = 0$ RHS: $\frac{1}{3}f_1(-1) + \frac{4}{3}f_1(0) + \frac{1}{3}f_1(1)$

$= \frac{1}{3} - \frac{1}{3} = 0$ ✓

$m=2$

LHS: $\int_{-1}^{1} f_2(x) = \frac{2}{3}$ RHS: $\frac{1}{3}f_2(-1) + \frac{4}{3}f_2(0) + \frac{1}{3}f_2(1)$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ ✓

$m=3$

LHS: $\int_{-1}^{1} f_3(x) = 0$ RHS: $\frac{1}{3}f_3(-1) + \frac{4}{3}f_3(0) + \frac{1}{3}f_3(1)$

$= \frac{1}{3} - \frac{1}{3} = 0$ ✓

$m=4$

LHS: $\int_{-1}^{1} f_4(x) = \frac{2}{5}$ RHS: $\frac{1}{3}f_4(-1) + \frac{4}{3}f_4(0) + \frac{1}{3}f_4(1)$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ ☹

Since the formula fails for $m=4$, the degree of precision is 3.