3. Let U be an open subset of $\mathbb{R}^n$. Show that the exterior derivative $d : \Omega^*(U) \to \Omega^*(U)$ satisfies the *cocycle condition*; that is, $d^2 = 0$.

Let $\omega = f dx^I \in \Omega^*(U)$. Then

$$d^2\omega = d(d\omega)$$

$$= d\left(\sum \frac{\partial f}{\partial x^i} dx^i \wedge dx^I\right)$$

$$= \sum \frac{\partial^2 f}{\partial x^j x^i} dx^j \wedge dx^i \wedge dx^I$$

In this sum, if $i=j$, then $dx^i \wedge dx^i = 0$ so the term is 0. If not, there is a corresponding $dx^i \wedge dx^j$ to each $dx^j \wedge dx^i$, & the alternating property of the wedge product causes these terms to annihilate. Thus the entire sum is 0, as desired.