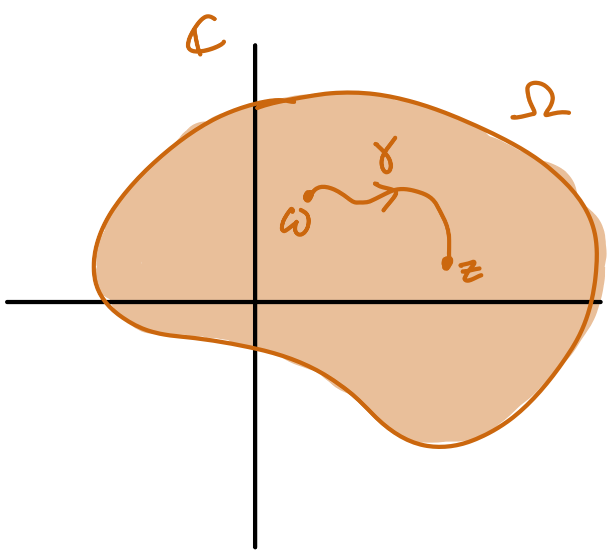
13. **(Stein Chapter 3 Thm 6.2)** If f is a nowhere vanishing holomorphic function in a simply connected region Ω, then there exists a holomorphic function g on Ω such that $f(z) = e^{g(z)}$.

Let $\omega \in \Omega$, and write $f(\omega) = e^{w}$ for some $w \in \mathbb{C}$ (which we can do because f does not vanish in Ω). For any $z \in \Omega$, let γ be a path in Ω connecting ω to z. Define $g(z) = \int_\gamma \frac{f'(\zeta)}{f(\zeta)} d\zeta + w$. Note the choice of γ doesn't matter because Ω is simply connected. Then $g'(z) = \frac{f'(z)}{f(z)}$ and we have:

$$\frac{d}{dz}\left(\frac{f(z)}{e^{g(z)}}\right) = \frac{f'(z)e^{g(z)} - f(z)g'(z)e^{g(z)}}{e^{2g(z)}}$$

$$= \frac{f'(z)e^{g(z)} - f(z)\left(\frac{f'(z)}{f(z)}\right)e^{g(z)}}{e^{2g(z)}}$$



$$= \frac{f'(z)e^{g(z)} - f'(z)e^{g(z)}}{e^{2g(z)}}$$

$$= 0$$

$$\Longrightarrow \frac{f(z)}{e^{g(z)}} = \lambda \in \mathbb{C}$$

And since $g(\omega) = w$ (γ becomes trivial), $\frac{f(\omega)}{e^{g(\omega)}} = \frac{e^w}{e^w} = 1$, so $f(z) = e^{g(z)}$ for $z \in \Omega$, as desired.