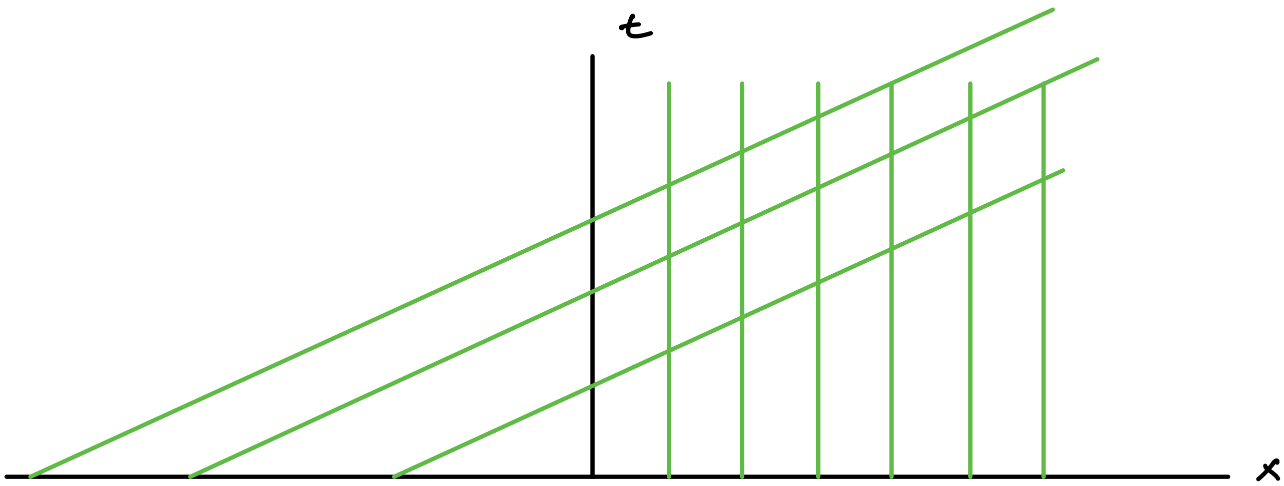
1) **[George]**

What is the Rankine-Hugoniot condition for $u_t + (u^2)_x = 0$. Solve its Riemann problem.

Adding some initial conditions, we consider the system

$$\begin{cases} u_t + (u^2)_x = 0 \\ u(x,0) = \chi_{(-\infty,0)} \end{cases}$$

Letting $f(u) = u^2$, we can find the slopes of our characteristics by calculating $f'(u) = \frac{dx}{dt} = 2u$, or $\frac{dt}{dx} = \frac{1}{2u}$:

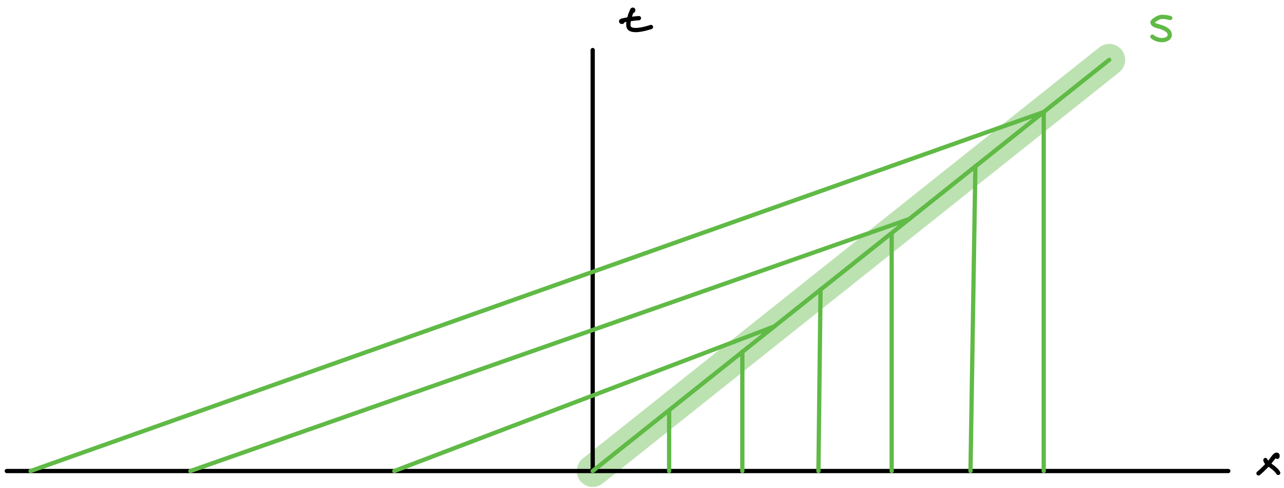


We can see intersecting characteristics, meaning we need to find a shock curve to split the region of intersection & find a piecewise solution. We can do this with the Rankine-Hugoniot condition, which will give us the slope of the shock curve $x = s(t)$:

$$\dot{s} = \frac{f(u|_{x>0}) - f(u|_{x<0})}{u|_{x>0} - u|_{x<0}} = \frac{0-1}{0-1} = 1$$

Note the entropy condition holds; i.e., $f'(u|_{x>0}) < \dot{s} < f'(u|_{x<0})$.

Our shock needs to begin at $(0,0)$, meaning s is explicitly given by $x = s(t) = t$:



Our piecewise solution follows, splitting ahead of & behind the shock:

$$u(x,t) = \begin{cases} 1 & x < t \\ 0 & x > t \end{cases}$$