5. Solve(formula as an integral is enough) the problem $u_t = u_{xx}$ in R^2_+ with $u(x,0) = (e^x - 1)^+$ and prove the solution is unique by showing that the solution with required estimate so the uniqueness theorem can be applied.

Convolute the heat kernel with the initial condition to obtain the solution:

$$u(x,t) = \int_{\mathbb{R}^+} \Gamma(x-\xi)\, g(t)\, d\xi$$

$$= \int_{\mathbb{R}^+} \frac{1}{\sqrt{4\pi t}} \exp\left(-(x-\xi)^2/4t\right)(e^\xi - 1)\, d\xi$$

This solution obeys the inequality $u(x,0) \leq Ce^{k|x^2}$ & so is unique.