

11. (Dr. Ayati: F22 Midterm – Abdul and Bakhtiar) Apply two steps of Newton's method to find a root of $f(x) = \cos(x)$ with initial guess $x_0 = \pi/4$.

Newton's method:
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$f(x) = \cos x$$
$$f'(x) = -\sin x$$
$$x_0 = \pi/4$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= \pi/4 - \frac{\cos(\pi/4)}{-\sin(\pi/4)}$$

$$= \pi/4 + 1$$

$$= \frac{\pi + 4}{4}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \frac{\pi + 4}{4} - \frac{\cos\left(\frac{\pi + 4}{4}\right)}{-\sin\left(\frac{\pi + 4}{4}\right)}$$