6. **(Qual Fall 2007 #4)** If $f \in L^1(\mathbb{R})$ is it true that $\lim_{|x|\to\infty} f(x) = 0$?

No. Consider $\chi_{\mathbb{Q}}$, the indicator function of the rationals. Then

$\int_{\mathbb{R}} \chi_{\mathbb{Q}} \, d\mu = 0$ as $m(\mathbb{Q}) = 0$ but $\lim_{|x| \to \infty} \chi_{\mathbb{Q}}(x)$ does not exist.