2. (**Dr. Ayati: S13 Midterm** – ***Ashwin and Hemanth***) Prove that if $A = MM^T$ for $M \in \mathbb{R}^{n\times n}$ is nonsingular, then A is symmetric positive definite.

symmetric

$$A^T = (MM^T)^T$$
$$= MM^T$$
$$= A$$

positive definite

Since A is nonsingular, $\det A \neq 0$. Then as $\det A = \det M \det M^T$, we also have $\det M, \det M^T \neq 0$, i.e., M and M^T are nonsingular.

Let x be a nonzero vector. Then

$$x^T A x = x^T M M^T x$$
$$= (M^T x)^T M^T x$$
$$= \| M^T x \|_2$$
$$> 0 \quad \text{as } M^T \text{ is nonsingular}$$