

Assume f and g are continuous functions. Assuming compatible domains & ranges, show that $f \circ g$ is continuous.

Fix $\epsilon > 0$. WTS: $\exists \delta$ s.t.

$$|x - y| < \delta \Rightarrow |f(g(x)) - f(g(y))| < \epsilon$$

Since f is continuous, we know $\exists \delta_1$ such that

$$|x - y| < \delta_1 \Rightarrow |f(x) - f(y)| < \epsilon$$

Since g is continuous, we know $\exists \delta_2$ such that

$$|x - y| < \delta_2 \Rightarrow |g(x) - g(y)| < \delta_1$$

Let $\delta = \delta_2$. Then

$$\begin{aligned} |x - y| < \delta &\Rightarrow |g(x) - g(y)| < \delta_1 \\ &\Rightarrow |f(g(x)) - f(g(y))| < \epsilon \end{aligned}$$

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