5. **(Review Final - Chifan)** Find all entire functions f satisfying $|f(z)| \geq e^{2Im(z)}$ for every $z \in \mathbb{C}$.

Let f be such a function. As f is nonzero, $\frac{1}{f}$ is entire and $\left|\frac{1}{f(z)}\right| \leq e^{-2Im(z)} = |e^{2iz}|$. Then $\left|\frac{1}{f(z)} e^{-2iz}\right| \leq 1$, so by Liouville's, $\frac{1}{f(z)} e^{-2iz} = w$ for some $w \in \mathbb{C} \setminus \{0\}$.

Therefore $f(z) = \zeta e^{2iz}$, where $\zeta = \frac{1}{w}$.