1. **(Qual Fall 2007 #3)** Let A be a subset of $\mathbb{R}$ with the property that for each $\epsilon > 0$ there are Lebesgue measurable sets B and C such that $B \subset A \subset C$ and $m(C \cap B^c) < \epsilon$. Show that A is measurable.

C measurable $\Rightarrow$ $\exists$ open set $\mathcal{O}$ containing C such that $m^*(\mathcal{O} \setminus C) < \varepsilon$.

Note $\mathcal{O}$ also contains A. Then

$$\begin{aligned} m^*(\mathcal{O} \setminus A) &\leq m^*(\mathcal{O} \setminus B) \quad (\text{as } B \subseteq A) \\ &= m^*(\mathcal{O} \setminus C) + m^*(C \setminus B) \\ &< 2\varepsilon \end{aligned}$$

So $\mathcal{O}$ is an open set containing A such that $m^*(\mathcal{O} \setminus A) < \varepsilon$ (after some ε shuffling). Thus A is measurable.