1. Solve the problem $u_t = u_{xx}$ in R^2_+ with $u(x,0) = e^x$ and state that n what sense the solution is unique.

The solution is given by

$$u(x,t) = \int_{\mathbb{R}} \Gamma(x-\xi, t) e^{\xi} \, d\xi$$

$$= \frac{1}{\sqrt{4\pi t}} \int_{\mathbb{R}} e^{\left(|x-\xi|^2/4t\right) + \xi} \, d\xi$$

Or $u(x,t) = e^{x+t}$ upon inspection. This solution is unique in the sense that it is unique among the functions with growth rate bounded by $Ce^{\alpha|x|^2}$.