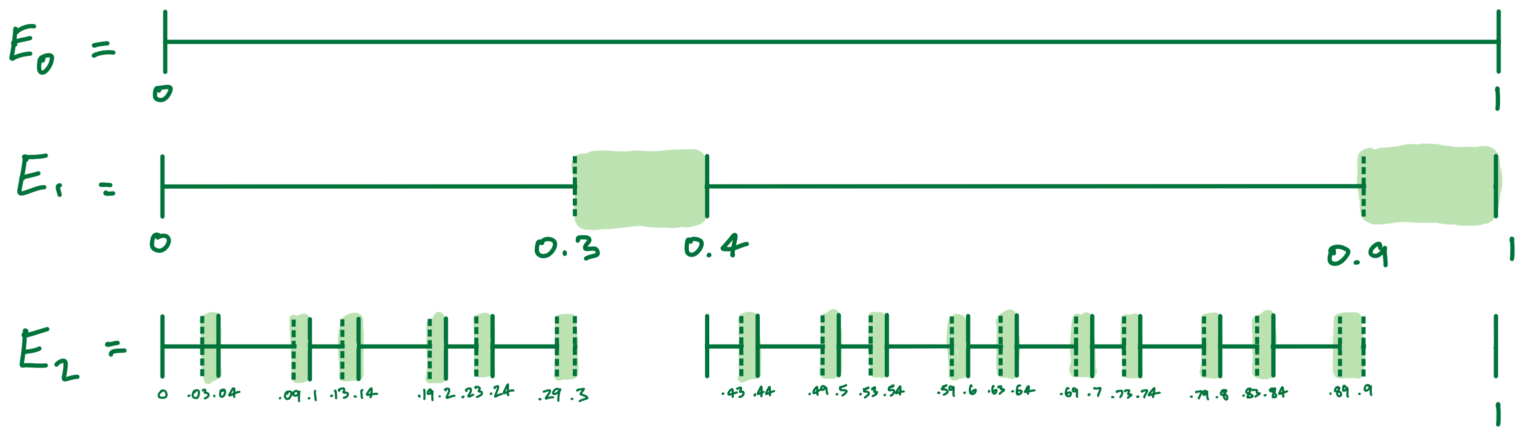
20. **(Review MT 1 - Chifan)** Let E be the subset of all elements in $[0,1]$ which do not contain the digits 3 and 9 in their decimal expansion. Is E Lebesgue measurable? If yes, find its measure.

Let $F_n = \left\{ x \mid \begin{array}{l} n^{th} \text{ digit of decimal expansion} \\ \text{of } x \text{ is not } 3 \text{ or } 9 \end{array} \right\}$ and $F_0 = [0,1]$.

Then $E = \bigcap_{n=0}^{\infty} F_n$ is measurable as it is a countable intersection of half open intervals. Let $E_n = \bigcap_{k=0}^{n} F_k$.



$(E_k \setminus E_{k+1}$ highlighted$)$

Note E_{k+1} is constructed by removing $8^{k-1} \cdot 2$ intervals of length $\frac{1}{10^k}$ from E_k, so

$$m(E) = \lim_{n \to \infty} m(E_n)$$

$$= m(E_0) - \sum_{k=0}^{\infty} 8^k \cdot \frac{2}{10^{k+1}}$$

$$= 1 - \sum_{k=0}^{\infty} \left(\frac{2}{10}\right)\left(\frac{8}{10}\right)^k$$

$$= 1 - \frac{2/10}{1 - 8/10}$$

$$= 0 \; //$$