25. (***Victoria and Fatemeh***) Determine the symbol/stability of Heun's method.

Heun's method: $x_{k+1} = x_k + \frac{h}{2}\left(f(t_k, x_k) + f(t_{k+1}, z_{k+1})\right)$

$$z_{k+1} = x_k + hf(t_k, x_k)$$

Let $f(t_k, x_k) = \lambda x_k$. Then

$$x_{k+1} = x_k + \frac{h}{2}\left(\lambda x_k + \lambda(x_k + \lambda h x_k)\right)$$

$$= x_k + \frac{h}{2}\left(\lambda^2 h x_k + 2\lambda x_k\right)$$

$$= \frac{\lambda^2 h^2}{2} x_k + \lambda h x_k + x_k$$

$\Longrightarrow$ symbol of our method is $\frac{z^2}{2} + z + 1$.

Let $f(t_k, x_k) = -\lambda x_k$. Then

$$x_{k+1} = x_k + \frac{h}{2}\left(-\lambda x_k - \lambda(x_k - \lambda h x_k)\right)$$

$$= x_k + \frac{h}{2}\left(\lambda^2 h x_k - 2\lambda x_k\right)$$

$$= \frac{\lambda^2 h^2}{2} x_k - \lambda h x_k + x_k$$

$\Longrightarrow$ symbol of our method is $\frac{z^2}{2} - z + 1$.