1. **Fall2005.** Show that the $\underset{R}{\underline{\text{ring of 3-by-3 matrices}}}$ over a field is simple.

(simple: only two-sided ideals are 0 and R)

Let $\mathcal{I}$ be a nonzero ideal in R. Let $M \neq 0 \in \mathcal{I}$. Assume WLOG that $\alpha = M_{ij}$ (for fixed i, j) is nonzero. Let B_{ij} (we briefly unfix i, j) be the matrix with the (i,j) entry equal to 1 and the rest zero.

Then $B_{ii} A B_{jj} = \alpha B_{ij}$ (please refix), and in turn $\alpha^{-1}(\alpha B_{ij}) = B_{ij}$.

$$(b^{-1}) \overset{B_{11}}{\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}} \overset{A}{\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}} \overset{B_{22}}{\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}} = (b^{-1}) \overset{a_{12}B_{12}}{\begin{bmatrix}0&b&0\\0&0&0\\0&0&0\end{bmatrix}}$$

$$B_{11}A = \begin{bmatrix}a&b&c\\0&0&0\\0&0&0\end{bmatrix} \qquad B_{22}A = \begin{bmatrix}0&b&0\\0&e&0\\0&h&0\end{bmatrix}$$

$$= \overset{B_{12}}{\begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}}$$

We can make any B_{ij} (really unfixing now) in this fashion. All of them additively generate $\mathcal{I}$, so $\mathcal{I} = R$ and we are finished.