(10 points) **(C-1)** Consider the meromorphic function

$$f(z) := \frac{\log(1+z^2)}{z^4} \qquad (z \neq 0).$$

Find the residue of $f$ at $z = 0$.

We'll find the Laurent series of $f$ at $0$, and then inspect the coefficient on $z^{-1}$ to get the residue. We have:

$$\log(1+z) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^k}{k}$$

$$\Rightarrow \quad \log(1+z^2) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^{2k}}{k}$$

$$\Rightarrow \quad \frac{\log(1+z^2)}{z^4} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^{2k-4}}{k}$$

There is no $z^{-1}$ term in this series, so our residue is $0$.