Problem R-5B. Let $\{p_n\}_{n=1}^{\infty}$ be a sequence of polynomials that converges uniformly to some $f \in C([a,b])$, which is **not** a polynomial. Prove that

$$\sup_n \text{degree}\,(p_n) = +\infty.$$

Let W_k be the space of polynomials with degree $\leq k$. Then W_k is a finite dimensional normed vector space. Since any two such spaces of the same dimension are isomorphic, W_k is isomorphic to $\mathbb{R}^k$, which is a Banach space. Thus W_k is Banach, & any convergent sequence here converges to another polynomial in W_k. We conclude that if $p_n \to f$ not a polynomial, then we must escape W_k & so $\sup_n \deg p_n = \infty$.