Problem 5.1 (HW09-2019)**.** Show that the following system does not have any closed orbits in the region $x, y > 0$:

$$\dot{x} = x(2 - x - y)$$
$$\dot{y} = y(4x - x^2 - 3)$$

Since $\{x, y > 0\}$ is simply connected, we can use Bendixon's criterion to show the nonexistence of closed orbits.

Let $f(x,y) = x(2 - x - y)$ and $g(x,y) = y(4x - x^2 - 3)$. We want to show $\operatorname{div}([f\ g]^T)$ is nonzero & does not change sign. So:

$$\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} = 2 - 2x - y + 4x - x^2 - 3$$
$$= -x^2 + 2x - y - 1$$
$$= -(x-1)^2 - y$$
$$< 0 \quad \forall\ x, y > 0$$

By the Bendixon criterion we have no periodic orbits.