Problem C-4B. Let $f(z) = e^{2\pi i z}$. We know that f maps the upper half plane onto the inside of the punctured disc $\mathbb{D} \setminus \{0\}$. Given $B > 0$, let $H(B)$ be that part of the upper half plane consisting of those complex number $x + iy$ with $y \geq B$. What is the image of $H(B)$ under f ? Give a concrete and detailed description.

We have:

$$e^{2\pi i z} = e^{2\pi i (x+iy)}$$

$$= e^{2\pi i x} e^{-2\pi y}$$

$$= e^{-2\pi y}\left(\cos(2\pi x) + i\sin(2\pi x)\right)$$

Since $y \geq B$, $0 < |f(w)| \leq e^{-2\pi B}$ for $w \in H(B)$. Also since x is an arbitrary real number, we see that the argument of $f(z)$ can be any value in $[0, 2\pi]$.

We have thus described a punctured disk at the origin of radius $e^{-2\pi B}$; in particular any strip of width 1 will be mapped to this disk. What f does to $H(B)$ is map <u>all</u> these strips to the disk on top of each other.