

26. (*Ashwin and Hemanth*) Determine the symbol/stability of the Trapezoidal rule.

$$\text{trapezoid: } x_{k+1} = x_k + \frac{h}{2} (f(t_k, x_k) + f(t_{k+1}, x_{k+1}))$$

Let $f(t_k, x_k) = \lambda x_k$. Then

$$\begin{aligned} x_{k+1} &= x_k + \frac{h}{2} (\lambda x_k + \lambda x_{k+1}) \\ &= x_k + \frac{\lambda h}{2} x_k + \frac{\lambda h}{2} x_{k+1} \end{aligned}$$

$$\Rightarrow x_{k+1} - \frac{\lambda h}{2} x_{k+1} = x_k + \frac{\lambda h}{2} x_k$$

$$\Rightarrow x_{k+1} \left(1 - \frac{\lambda h}{2} \right) = x_k \left(1 + \frac{\lambda h}{2} \right)$$

$$\Rightarrow x_{k+1} = x_k \left(\frac{1 + \frac{\lambda h}{2}}{1 - \frac{\lambda h}{2}} \right)$$

$$\Rightarrow \text{symbol is } \frac{1 + \frac{z}{2}}{1 - \frac{z}{2}}$$