Derive the n^{th} order Taylor polynomial for e^x. What value of n will guarantee an error or no more than 10^{-10} for any x with $|x| < 1.2$?

The n^{th} order Taylor polynomial for $f(x) = e^x$ (centered at 0) is

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^n = \sum_{k=0}^{n} \frac{e^0}{k!} x^k$$

$$= \sum_{k=0}^{n} \frac{x^k}{k!}$$

with error term

$$r_n(x) = \frac{f^{(n+1)}(\alpha)}{(n+1)!} x^{n+1} \qquad 0 \le \alpha \le x$$

$$= \frac{e^\alpha}{(n+1)!} x^{n+1}$$

$$\le \frac{e^{1.2}}{(n+1)!} (1.2)^{n+1} \qquad \text{(as } e^x \text{ is increasing)}$$

So we want to solve

$$\frac{e^{1.2}}{(n+1)!} (1.2)^{n+1} \le 10^{-10}$$

If you had a calculator you could see $n \ge 14$.