27. (***Liz and James***) Determine the symbol/stability of the Midpoint rule.

midpoint rule: $x_{k+1} = x_k + hf\left(\frac{1}{2}(t_k + t_{k+1}), \frac{1}{2}(x_k + x_{k+1})\right)$

Let $f(t,x) = \lambda x$. Then

$$x_{k+1} = x_k + h\left(\frac{\lambda}{2}(x_k + x_{k+1})\right)$$

$$= x_k + \frac{\lambda h}{2} x_k + \frac{\lambda h}{2} x_{k+1}$$

$$\Rightarrow x_{k+1}\left(1 - \frac{\lambda h}{2}\right) = x_k\left(1 + \frac{\lambda h}{2}\right)$$

$$\Rightarrow x_{k+1} = \frac{1 + \frac{\lambda h}{2}}{1 - \frac{\lambda h}{2}}$$

$\Rightarrow$ symbol is $\frac{1+z}{1-z}$

Let $f(t,x) = -\lambda x$. Then

$$x_{k+1} = x_k - h\left(\frac{\lambda}{2}(x_k + x_{k+1})\right)$$

$$= x_k - \frac{\lambda h}{2} x_k - \frac{\lambda h}{2} x_{k+1}$$

$$\Rightarrow x_{k+1}\left(1 + \frac{\lambda h}{2}\right) = x_k\left(1 - \frac{\lambda h}{2}\right)$$

$$\Rightarrow x_{k+1} = \frac{1 - \frac{\lambda h}{2}}{1 + \frac{\lambda h}{2}}$$

$\Rightarrow$ symbol is $\frac{1-z}{1+z}$