96. **(TNA Homework 1 - Dr. Stewart)** Give two norms on infinite sequences $(x_1, x_2, ...)$ that are not equivalent.

Consider the ℓ_1 and ℓ_∞ norms, given by (for $x \in \mathbb{R}^\infty$)

$$\|x\|_{\ell_1} = \sum_i |x_i| \quad \text{and}$$

$$\|x\|_{\ell_\infty} = \sup_i |x_i|$$

We claim these are not equivalent norms in $\mathbb{R}^\infty$, that is, $\nexists\ C, D > 0$ such that

$$C\|x\|_{\ell_1} \leq \|x\|_{\ell_\infty} \leq D\|x\|_{\ell_1}$$

Assume to the contrary that such C and D exist. Define a sequence Y by $Y_i \equiv 1$. Then $\|Y\|_{\ell_1} = \infty$ and $\|Y\|_{\ell_\infty} = 1$, and it is obvious our inequality cannot hold. //