

25. (Victoria and Fatemeh) Determine the symbol/stability of Heun's method.

Heun's method: $x_{k+1} = x_k + \frac{h}{2} (f(t_k, x_k) + f(t_{k+1}, z_{k+1}))$

$$z_{k+1} = x_k + hf(t_k, x_k)$$

Let $f(t_k, x_k) = \lambda x_k$. Then

$$x_{k+1} = x_k + \frac{h}{2} (\lambda x_k + \lambda (x_k + \lambda h x_k))$$

$$= x_k + \frac{h}{2} (\lambda^2 h x_k + 2\lambda x_k)$$

$$= \frac{\lambda^2 h^2}{2} x_k + \lambda h x_k + x_k$$

$$\Rightarrow \text{symbol of our method is } \frac{z^2}{2} + z + 1.$$

Let $f(t_k, x_k) = -\lambda x_k$. Then

$$x_{k+1} = x_k + \frac{h}{2} (-\lambda x_k - \lambda (x_k - \lambda h x_k))$$

$$= x_k + \frac{h}{2} (\lambda^2 h x_k - 2\lambda x_k)$$

$$= \frac{\lambda^2 h^2}{2} x_k - \lambda h x_k + x_k$$

$$\Rightarrow \text{symbol of our method is } \frac{z^2}{2} - z + 1.$$