36. **(Qual Spring 2007 #5)** Let $F_k \subset [0,1]$ for all $k \in \mathbb{N}$ be measurable sets, such that there exists $\delta > 0$ with $m(F_k) \geq \delta$ for all k. Assume the sequence $a_k \geq 0$ satisfies

$$\sum_{k=1}^{\infty} a_k \chi_{F_k}(x) < \infty$$

for almost every $x \in [0,1]$. Show that $\sum_{k=1}^{\infty} a_k < \infty$.

We have:

$$\sum_{k=1}^{\infty} a_k \chi_{F_k} = \lim_{n\to\infty} \sum_{k=0}^{n} a_k \chi_{F_k}$$

$$\Rightarrow \underbrace{\int_0^1 \sum_{k=1}^{\infty} a_k \chi_{F_k}(x)}_{<\infty} = \int_0^1 \lim_{n\to\infty} \sum_{k=1}^{n} a_k \chi_{F_k}(x)$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} a_k\, m(F_k)$$

$$\geq \lim_{n\to\infty} \sum_{k=1}^{n} a_k \delta$$

$$= \sum_{k=1}^{\infty} a_k \delta$$

and the LHS is finite, so the RHS is also finite. Thus

$$\frac{1}{\delta} \sum_{k=1}^{\infty} a_k \delta = \sum_{k=1}^{\infty} a_k < \infty$$