4. (***Javier and Zhihua***) Show that the LDL^T can be numerically unstable even when it succeeds, by considering the matrix

$$\begin{bmatrix} \epsilon & 1 \\ 1 & 1 \end{bmatrix}$$

with $0 \neq \epsilon \approx 0$.

The LDL^T decomposition can be found symbolically:

$$\begin{bmatrix} \varepsilon & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l & 1 \end{bmatrix} \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_1 & 0 \\ d_1 l & d_2 \end{bmatrix} \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_1 & d_1 l \\ d_1 l & d_1 l^2 + d_2 \end{bmatrix}$$

$$\Rightarrow d_1 = \varepsilon$$

$$\Rightarrow l = 1/\varepsilon$$

$$\Rightarrow d_2 = 1 - 1/\varepsilon$$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 \\ 1/\varepsilon & 1 \end{bmatrix} \quad \& \quad D = \begin{bmatrix} \varepsilon & \\ & 1 - 1/\varepsilon \end{bmatrix}$$

But $\lim_{\varepsilon \to 0} \left| 1/\varepsilon \right| = \lim_{\varepsilon \to 0} \left| 1 - 1/\varepsilon \right| = \infty$, so we have blowup occurring as $|\varepsilon| \to 0$.