1. **Fall2005.** Show that the ring of 3-by-3 matrices over a field is simple.

Let our ring be R, and our field be F.

WTS: If $\mathcal{I} \subseteq R$ is a nonzero ideal, then $\mathcal{I} = R$.

Let $M \neq 0 \in \mathcal{I}$. Assume the (i,j) entry of M is nonzero WLOG.

Let A be a matrix that has a 1 at the (i,i) position, and let B be a matrix that has a 1 at the (j,j) position.

Then the matrix AMB has a single nonzero entry, say $a \in F$, at the (i,j) position. In particular $AMB \in \mathcal{I}$. We can shift the position of a just by conjugating by appropriate permutation matrices, and we can change the value of a to any $\lambda \in F$ by multiplying by $a^{-1}\lambda I$.

So all the matrices with a single nonzero entry are in $\mathcal{I}$, and so are all their sums; this means any matrix in R can be constructed via sums of matrices in $\mathcal{I}$. Thus $\mathcal{I} = R$ and we are finished.