

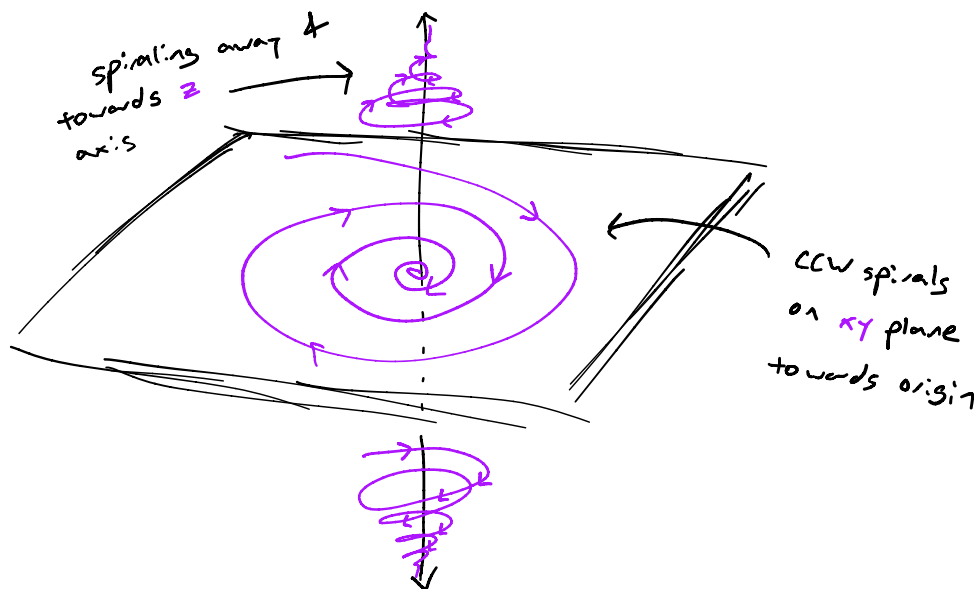
Problem 1.1 (Exam 1, Fall 2019). Find the general solution for $\dot{X} = AX$ where

$$A = \begin{bmatrix} -2 & 1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Sketch the phase portrait and determine the stable and unstable subspaces.

We see upon inspection that A is in canonical form, & we can read off its eigenvalues: $\lambda = -2 \pm i, 3$. Thus our general solution is

$$X(t) = c_1 e^{-2t} \begin{bmatrix} \cos t \\ -\sin t \\ 0 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} \sin t \\ \cos t \\ 0 \end{bmatrix} + c_3 e^{3t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



stable subspace: O

unstable subspace: z -axis