6) **[Liz]**

What is the Poisson Kernel in the unit circle? What is the solution of the Dirichlet boundary condition on the boundary condition with continuous boundary data. Prove the solution is unique.

Let B_R^n denote the ball of radius R in $\mathbb{R}^n$. Then the Poisson kernel is given by

$$P(x,\sigma) = \frac{R^2 - x^2}{R\,\omega_{n-1}\,|x-\sigma|^n}$$

where $\sigma \in \partial B_R^n$ and ω_{n-1} is the surface area of S^{n-1}. The Laplace Dirichlet problem on B_R^n

$$\begin{cases} \Delta u = 0 & \text{in } B_R^n \\ u = g & \text{on } \partial B_R^n \end{cases}$$

is solved by

$$u(x) = \int_{\partial B_R^n} P(x,\sigma)\, g(\sigma)\, d\sigma$$

In polar coordinates (for $\mathbb{R}^2$), we have

$$u(r,\theta) = \frac{R^2 - r^2}{2\pi} \int_0^{2\pi} \frac{g(\phi)}{R^2 + r^2 - 2Rr\cos(\theta-\phi)}\, d\phi$$

Uniqueness: let u, v be two solutions to the Laplace Dirichlet problem.

Then $w = u - v$ solves

$$\begin{cases} \Delta w = 0 & \text{in } B_R^n \\ w = 0 & \text{on } \partial B_R^n \end{cases}$$

By the maximum principle, $w \equiv 0$. Thus $u = v$. //