1. **Spring2019.** Let G be a simple group with 360 elements, and let H be a proper subgroup. Let $\mathcal{C}_l = \{gH | g \in G\}$ be the set of left cosets of H in G, and let $\mathcal{C}_r = \{Hg | g \in G\}$ be the set of right cosets of H in G. Suppose $C_l = C_r$ as subsets of $\mathcal{P}(G)$. Find the order of H.

$\Downarrow$

for $g_1H \in \mathcal{C}_l$, $\exists!\ Hg_2 \in \mathcal{C}_r$ st $g_1H = Hg_2$

$\Rightarrow g_1 \in Hg_2$

$\Rightarrow \exists\ h \in H$ st $g_1 = hg_2$

$\Rightarrow g_1 g_2^{-1} \in H$ so $(g_1 g_2^{-1})^{-1} = g_2 g_1^{-1} \in H$

so $g_1H = Hg_2$

$= Hg_2(g_1^{-1}g_1)$

$= H(g_2 g_1^{-1})g_1$

$= Hg_1$ by

$\therefore H \trianglelefteq G \Rightarrow H = \{id_G\}$ as $H \subsetneq G$ & G is simple

$\Rightarrow |H| = 1$ //