Fundamental Groups and Covering Spaces Midterm
Fall 2021
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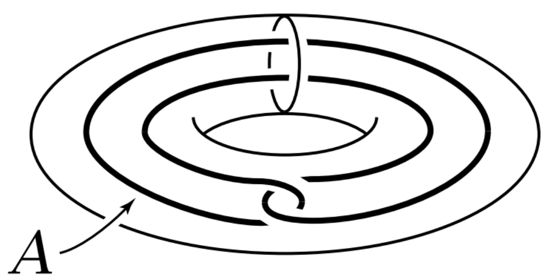
1. Let $X = \mathbb{R}^3 \setminus Z$ where $Z \subset \mathbb{R}^3$ is the z-axis (so Z is homeomorphic to $\mathbb{R}$.) Show that X is not simply connected.

2. Identify a torus $S^1 \times S^1$ with a square S whose vertical sides are glued and horizontal sides are glued.
   (a) Sketch a loop (in the square S) that represents the element $(2, 3) \in \mathbb{Z} \times \mathbb{Z} \cong \pi_1(S^1 \times S^1)$.
   (b) Do the same for $(6, 4)$.

3. Find a map $f : S^1 \to S^1$ that yields the mapping cylinder structure of a Möbius band.

4. Let $f : I \to \mathbb{R}^3$ be a loop whose image is the unit circle and $g : I \to \mathbb{R}^3$ be a trefoil knot. Are f and g homotopic?

5. Show that there are no retractions $r : X \to A$ in the following cases:
   (a) $X = S^1 \times D^2$ and A is the circle shown below.



   (b) X is the Möbius band and A is the boundary of X.

6. (a) Find a cell-complex structure of a genus 2 orientable surface.
   (b) Do the same for general genus g.