

ABSTRACT ALGEBRA EXAM 1B  
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1. (30 points) Determine the smallest positive integer  $n$  such that  $S_n$  has an element of order 210.
2. (30 points) Consider the subgroup  $H = \langle r^{15}, s \rangle$  of  $D_{80}$ , the group of symmetries of a regular 40-gon. Determine the order of  $H$ .
3. (40 points) Let  $G$  be an abelian group. Prove the function  $\phi : G \rightarrow G$  given by  $\phi(x) = x^{-1}$  is an automorphism.
4. (40 points) Let  $G$  be a group with a normal subgroup  $N$  of order 5 with  $G/N \simeq S_3$ . Prove:
  - (a)  $|G| = 30$ , and
  - (b)  $G$  has a normal subgroup of order 15.
  - (c)  $G$  has exactly three subgroups of order 10, and none are normal.