ODE Midterm II
Fall 2022
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1. (20 points) Consider

$$\dot{x} = 2x + xy$$
$$\dot{y} = -y + x^2$$

Find the quadratic approximation for S and U, the stable and unstable manifolds of the equilibrium (0,0). Draw the phase portrait and sketch S, U, and the corresponding eigenspaces (E^s, E^u).

2. (20 points) Consider

$$\dot{x} = x^2 - e^t x + e^t$$

(a) Show the function $x(t) = e^t$ is a solution to the above differential equation with initial condition $x(0) = 1$.

(b) Write down the variational equation associated with the above intial value problem and compute its principal fundamental matrix solution $\Phi(t)$.

3. (20 points) Consider the system

$$\dot{x} = -x^3 - x^2 y$$
$$\dot{y} = -y + x^3$$

Determine the stability of the origin using the Lyapunov function method. What is the basin of attraction?