Abstract Algebra Exam 1A

Fall 2021

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1. (20 points) Let G be a group and $N \leq G$ a subgroup. Give any four conditions which are equivalent to N being a normal subgroup of G.

2. (30 points) Draw the subgroup lattice for the cyclic group $\langle x \rangle$, where x has order 36.

3. (30 points) Let $N \leq H \leq G$ and assume that both N and H are normal subgroups of G. You may further assume that H/N is a subgroup of G/N. Prove that H/N is normal in G/N.

4. (30 points) Let H, K be normal subgroups of a group G, and assume $H \cap K = \{e\}$. Prove that $xy = yx$ for all $x \in H,\ y \in K$.