Interplanetary Transfer Project

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AERO 423 - Fall 2021
10 December 2021

Introduction:

The purpose of this project is to obtain the maximum 'kick' or impulse from an interplanetary fly-by starting at planet Aggie and performing a fly-by of planet Longhorn. The task is to iterate through both the true anomaly and the periapse altitude in order to find which combination will maximize the delta V. The range of these values as well as the approach is determined by the last three digits of my UIN which are 116. The last digit being even means I will use a sun-side approach (trailing side fly-by), the second to last digit means my range for true anomaly is between -30 and -39 degrees, and the third to last digit corresponds to a periapse altitude range of 300 to 400 km.

Computation:

In order to find delta V for each combination I ran a for loop through the true anomaly(θ) and a nested loop through each altitude in order to get every combination. Below are the steps and equations used for just one combination.

First we will go over the given constants for this problem:

$$\mu_s = 1.3 * 10^{11}\ km^3/s^2$$

$$\mu_L = 355000\ km^3/s^2$$

$$R_A = 1\ AU$$

$$R_L = 0.75\ AU$$

$$r_L = 5500\ km$$

$$\theta = [-39,\ -30]$$

$$r\ (alt.) = [300,\ 400]$$

We start by finding the eccentricity and angular momentum of the pre fly-by ellipse:

$$e_1 = (R_A - R_L)/(R_A + R_L cos(\theta))$$

$$h_1 = \sqrt{R_A \mu_s (1 - e_1)}$$

Next, calculate the radial and tangential velocities at the inbound crossing of planet Longhorns sphere of influence:

$$V_{r_1} = (\mu_s/h_1) e_1 sin(\theta)$$

$$V_{\perp_1} = h_1/R_L$$

These values are the components of the $V_1^{(v)}$ vector that is needed to find the delta V and the hyperbolic excess velocity:

$$V_1^{(v)} = V_{\perp_1}\hat{u}_V - V_{r_1}\hat{u}_s$$

$$|V_1^{(v)}| = \sqrt{V_{\perp_1}^{\ 2} + V_{r_1}^{\ 2}}$$

Next, we need to look at calculations for the fly-by hyperbola. To start, find the velocity of planet Longhorn in its circular orbit around the star:

$$V = \sqrt{\mu_s/R_L}\ \hat{u}_V$$

Then we can use this vector and the $V_1^{(v)}$ vector to find the hyperbolic excess velocity v_∞:

$$v_{\infty_1} = V_1^{(v)} - V$$

Since this results in a vector, we can easily find the magnitude. Now we will look at the periapse altitude:

$$r_p = r_L + r(alt)$$

We will use these values to find the eccentricity and angular momentum for the hyperbolic orbit:

$$e = 1 + (r_p v_\infty^{\ 2})/\mu_L$$

$$h = r_p\sqrt{v_\infty^{\ 2} + (2\mu_L)/r_p}$$

Then we find the turn angle and true anomaly of the asymptote:

$$\delta = 2sin^{-1}(1/e)$$

$$\theta_\infty = cos^{-1}(-1/e)$$

Finally, from the v_{∞_1} vector, we can use the components to find the angle between v_{∞_1} and V at inbound crossing:

$$\phi_1 = tan^{-1}(\hat{u}_V/\hat{u}_s)$$

Where $\hat{u}_V$ and $\hat{u}_s$ are the corresponding components of the v_{∞_1} vector.

Finally we can begin analyzing the last steps of the process, the sun side approach. To start, find the angle between v_{∞_1} and V at outbound crossing:

$$\phi_2 = \phi_1 - \delta$$

Then we can find the new v_{∞_2}:

$$v_{\infty_2} = v_\infty[cos(\phi_2)\hat{u}_V + sin(\phi_2)\hat{u}_s]$$

Now, find the value of $V_2^{(v)}$:

$$V_2^{(v)} = V - v_{\infty_2}$$

Again, this will give us a vector that we can find the magnitude of and finally compute our total delta V.

$$\Delta v = |V_2^{(v)} - V_1^{(v)}|$$

Below are the intermediate values for the above calculations using $\theta = -30$ deg and r(alt) = 400 km:

Intermediate Calculations	
e_1	0.1516
h_1	4.062e09 km^2/s
V_{r_1}	-2.4252 km/s
$V_{\perp_1}$	36.2042 km/s
$V_1^{(v)}$	[36.2042 2.4252] km/s
$\lvert V_1^{(v)}\rvert$	36.2853 km/s
V	[34.0391 0] km/s
v_{∞_1}	[2.1650 -2.4252] km/s
v_∞	3.2510 km/s
r_p	5900 km
e	1.1757
h	6.751e04 km^2/s
δ	2.0342 rad
θ_∞	2.5879 rad

ϕ_1	0.8420 rad
ϕ_2	-1.1922 rad
v_{∞_2}	[1.2016 -3.0208] km/s
$V_2^{(v)}$	[35.2408 -3.0208] km/s
$\lvert V_2^{(v)}\rvert$	35.3700 km/s
Δv	0.9153 km/s

Table 1: Intermediate Calculation Values for finding delta V

Results:

After iterating through each possible combination of true anomaly and periapsis altitude and plotting them, we get the following contour plot:

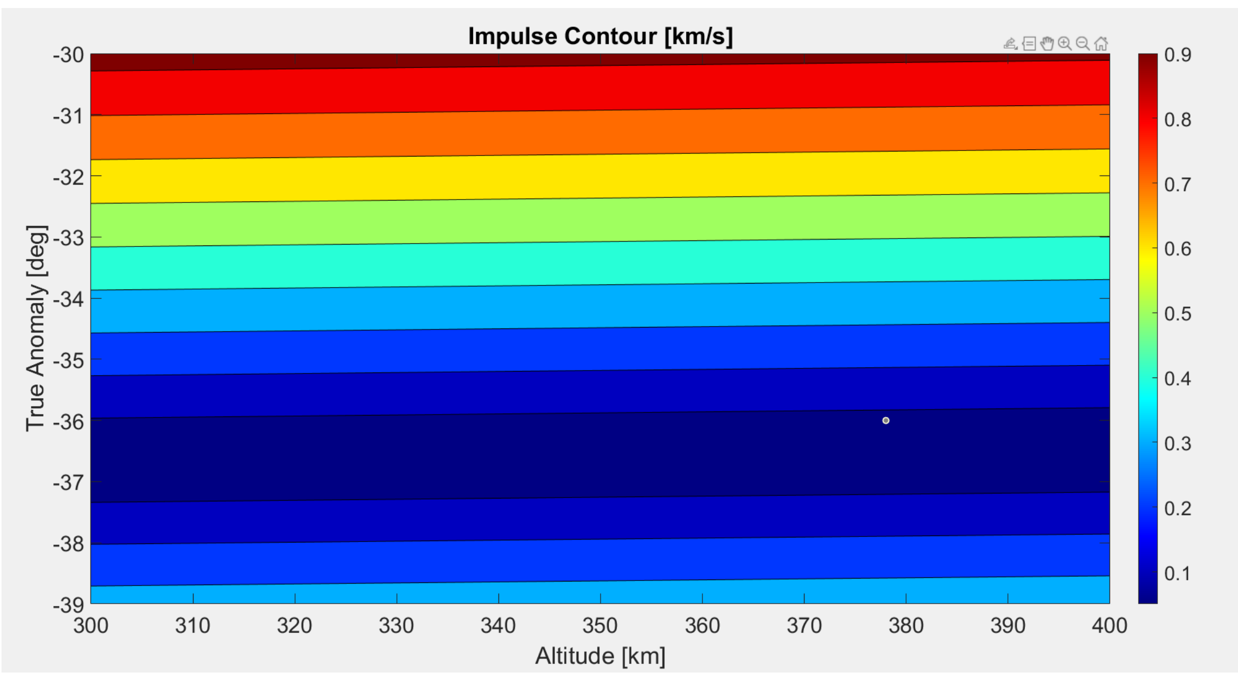

Figure 1: Contour Plot of delta V as a Function of True Anomaly and Altitude

From the contour plot, it is obvious that the highest value of delta V lies somewhere near the top of the plot at the $\theta = -30$ deg area. After further inspection, we find that the maximum delta V value occurs at the minimum altitude and the maximum true anomaly:

X 300
Y -30
Level 0.939299

This was to be expected as mentioned in the project outline. The maximum delta V from the fly-by occurs at 300 km altitude and a true anomaly of -30 deg and gives a value of 0.939299 km/s.

Conclusions:

As expected, the maximum delta V found lies at the extremes of the given parameters for true anomaly and altitude. Some other things to consider are the fact that an altitude of 300 km is very close to the surface of the planet. Flying too close to the surface can cause problems and be dangerous for the team of astrodynamicist on board the spacecraft. There may be too much gravity or other effects that have not been taken into account that may cause the spacecraft to crash or be pulled into the orbit of planet Longhorn. The true anomaly can have similar effects as it changes the alignment of the target fly-by planet with respect to the incoming spacecraft. This will change the eccentricity and radial velocity of the initial transfer ellipse and is something that needs to be considered when planning a journey such as this. Just because these specific values result in the maximum delta V does not mean they will always be the most practical methods for achieving a maximum value, especially when the safety of the astronauts is paramount.

Appendix

MATLAB Code:

```
% given
mu_s = 130000000000; %km^3/s^2
mu_L = 355000; % km^3/s^2
AU = 149597870.691; %km
R_A = 1*AU; %km
R_L = 0.75*AU; %km
r_L = 5500; %km
theta_vec = linspace(-39, -30, 10); % deg
Z_p_vec = linspace(300, 400, 51); % km
dV_matrix = zeros(length(theta_vec));
iterations = 0;

for i = 1:length(theta_vec)
   % Find V1v pre fly-by ellipse
   theta = theta_vec(i);
   e_1 = (R_A - R_L)/(R_A + R_L*cos(theta*(pi/180)));
   h_1 = sqrt(R_A*mu_s*(1-e_1));
   v_tan = h_1/R_L; %km/s
   v_rad = (mu_s/h_1)*e_1*sin(theta*(pi/180)); %km/s
   V1v_vector = [v_tan -v_rad]; % putting rad and tan components together
   V1v = sqrt(v_tan^2 + v_rad^2); %km/s magnitude of V1v vector
   V_cL = sqrt(mu_s/R_L); %circular orbit velocity of planet longhorn
   V_cL_vector = [V_cL 0]; % circular orbit in vector form
   v_inf1_vector = V1v_vector - V_cL_vector;
   v_inf = sqrt(dot(v_inf1_vector, v_inf1_vector)); %finding v_inf
   % using these values for theta, iterate through every altitude
   for j = 1:length(Z_p_vec)
      Z_p = Z_p_vec(j); % altitude
      r_p = r_L + Z_p; %radius
      h = r_p*sqrt(v_inf^2 + ((2*mu_L)/r_p));
      e = 1 + ((r_p*v_inf^2)/mu_L); %hyperbolic eccentricity
      turn = 2*asin(1/e); % turn angle
      theta_inf = acos(-(1/e)); %hyperbolic true anomaly
      phi_1 = atan(v_inf1_vector(2) / v_inf1_vector(1));
      phi_2 = phi_1 - turn;
      v_inf2__vector = v_inf*[cos(phi_2) sin(phi_2)];
      V2v_vector = V_cL_vector + v_inf2__vector;
      V2v = sqrt((V2v_vector(1))^2 + (V2v_vector(2))^2); % second impulse
```

```
        % calculate delta V at specific theta and altitude
        delta_V = abs(V2v - V1v);
        % add delta v to correct position in matrix
        dV_matrix(i, j) = delta_V;
        % count total iterations
        iterations = iterations + 1;
    end
end

% plot contour map
figure
contourf(Z_p_vec, theta_vec, dV_matrix)
colormap jet
colorbar('vertical');
ylabel('True Anomaly [deg]');xlabel('Altitude [km]')
title('Impulse Contour [km/s]');set(gca, 'fontsize', 18)
```