

Exercise 58

Chapter 7, Page 350

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Introduction to Electrodynamics

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a)

The same flux passes through all the turns of the primary and the secondary, so:

$$\Phi_1 = L_1 I_1 + M I_2 \quad \Phi_2 = L_2 I_2 + M I_1$$

$$\Phi_1 = \Phi_2$$

$$L_1 I_1 + M I_2 = L_2 I_2 + M I_1$$

This is true for any currents I_1 and I_2 , so it has to be true for $I_1 = 0$ and $I_2 = 0$ cases as well:

$$M I_2 = L_2 I_2 \quad (1)$$

$$L_1 I_1 = M I_1 \quad (2)$$

$$(1)/(2) \implies \frac{M}{L_1} = \frac{L_2}{M} \implies L_1 L_2 = M^2$$

, hence $M = \sqrt{L_1 L_2}$.

b)

Going around the primary's circuit, and using the fact that the sum of potentials is zero when going around the circle:

$$\mathcal{E}_1 - L_1 \dot{I}_1 - M \dot{I}_2 \implies \boxed{L_1 \dot{I}_1 + M \dot{I}_2 = V_1 \cos(\omega t)} \quad (1)$$

While, for the secondary's circuit:

$$\mathcal{E}_2 = I_2 R = -\dot{\Phi}_2 = -L_2 \dot{I}_2 - M \dot{I}_1 \implies \boxed{L_2 \dot{I}_2 + M \dot{I}_1 = -I_2 R} \quad (2)$$

c)

From (1) express \dot{I}_1 , and plug into (2), using the result from a) part of the problem:

$$\begin{aligned}\dot{I}_1 &= \frac{V_1}{L_1} \cos(\omega t) - \sqrt{\frac{L_2}{L_1}} \dot{I}_2 \\ L_2 \dot{I}_2 + \sqrt{L_1 L_2} \frac{V_1}{L_1} \cos(\omega t) - \sqrt{L_1 L_2} \sqrt{\frac{L_2}{L_1}} \dot{I}_2 &= -I_2 R\end{aligned}$$

$$\boxed{I_2 = -\sqrt{\frac{L_2}{L_1}} \frac{V_1}{R} \cos(\omega t)}$$

Now put back the I_2 into the equation (1) to get the current I_1 :

$$\begin{aligned}L_1 \dot{I}_1 + \sqrt{L_1 L_2} \omega \sqrt{\frac{L_2}{L_1}} \frac{V_1}{R} \sin(\omega t) &= V_1 \cos(\omega t) \\ \dot{I}_1 &= \frac{V_1}{L_1} \left(\cos(\omega t) - \frac{L_2 \omega}{R} \sin(\omega t) \right)\end{aligned}$$

Integrating this, and eliminating the constant term (since I_1 has no DC component) we get:

$$\boxed{I_1 = \frac{V_1}{L_1} \left(\frac{1}{\omega} \sin(\omega t) + \frac{L_2}{R} \cos(\omega t) \right)}$$

d)

For an long solenoid of length l and cross-section S the inductivity is:

$$L = \frac{\Phi}{I} = \frac{BSN}{I} = \frac{\mu_0 n I S N}{I} = \mu_0 S \frac{N^2}{l}$$

Hence, if two inductors have same cross sections and lengths (or same ratios of length/cross section):

$$\sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2}$$

Now, in our case the ratios of the voltages on the primary and secondary are:

$$\frac{V_1}{V_2} = \frac{V_1 \cos(\omega t)}{I_2 R} = -\sqrt{\frac{L_1}{L_2}} \Rightarrow \boxed{\frac{V_1}{V_2} = -\frac{N_1}{N_2}}$$

e)

Power on the secondary is:

$$P_2 = V_2 I_2 = I_2 R I_2 = R I_2^2 = \frac{L_2}{L_1} \frac{V_1^2}{R} \cos^2(\omega t)$$

$$\langle P_2 \rangle = \frac{1}{2} \frac{L_2}{L_1} \frac{V_1^2}{R}$$

, since the average of the cosine squared is 1/2. Now, the average power on the primary is:

$$P_1 = V_1 I_1 = \frac{V_1^2}{L_1} \left(\frac{1}{\omega} \cos(\omega t) \sin(\omega t) + \frac{L_2}{R} \cos^2(\omega t) \right)$$

$$\langle P_1 \rangle = \frac{1}{2} \frac{L_2}{L_1} \frac{V_1^2}{R}$$

, since the mixed sine-cosine term will disappear by averaging. Hence, the average powers are equal.

Result

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a) $M = \sqrt{L_1 L_2}$

b) $L_1 \dot{I}_1 + M \dot{I}_2 = V_1 \cos(\omega t) \quad L_2 \dot{I}_2 + M \dot{I}_1 = -I_2 R$

c) $I_2 = -\sqrt{\frac{L_2}{L_1}} \frac{V_1}{R} \cos(\omega t) \quad I_1 = \frac{V_1}{L_1} \left(\frac{1}{\omega} \sin(\omega t) + \frac{L_2}{R} \cos(\omega t) \right)$

d) $\frac{V_1}{V_2} = -\frac{N_1}{N_2}$

e) $\langle P_1 \rangle = \langle P_2 \rangle = \frac{1}{2} \frac{L_2}{L_1} \frac{V_1^2}{R}$

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