

Exercise 23

Chapter 7, Page 327

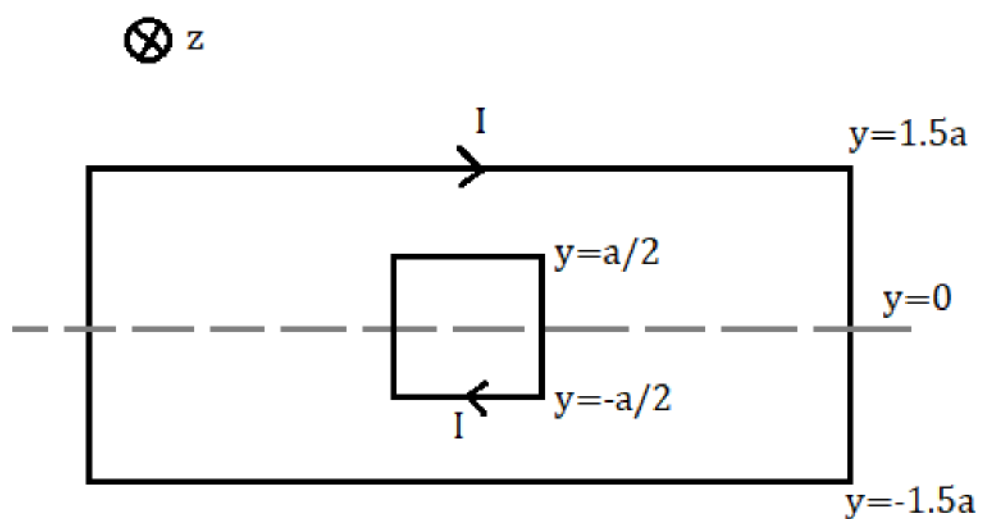


Introduction to Electrodynamics

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Instead of finding the flux through the big loop due to the current in the small one, find the flux through the small one due to the large one, same current I flowing through it. The magnetic field of the big loop is just the field of two infinite wires, one at $y = -(3/2)a$ and one at $y = (3/2)a$. Both contributions point downwards.

$$\begin{aligned}
 \Phi &= \int_{S_{\text{small}}} B_{\text{big}} dS = \frac{\mu_0 I}{2\pi} a \int_{-a/2}^{a/2} \left[\frac{1}{y + \frac{3}{2}a} - \frac{1}{y - \frac{3}{2}a} \right] dy \\
 &= \frac{\mu_0 I}{2\pi} a \left[\ln \left(y + \frac{3}{2}a \right) \Big|_{-a/2}^{a/2} - \ln \left(y - \frac{3}{2}a \right) \Big|_{-a/2}^{a/2} \right] \\
 &= \frac{\mu_0 I}{2\pi} a \left[\ln \frac{2a}{a} - \ln \frac{a}{2a} \right] \\
 &= \frac{\mu_0 I}{2\pi} a 2 \ln 2 = \frac{\mu_0 I}{\pi} a \ln 2
 \end{aligned}$$

The current is $I = kt$, so the EMF induced in the small loop, which is by reciprocity equal to the EMF that would be induced in the big loop had the current been flowing through the small loop is:

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0}{\pi} ka \ln 2$$

Step 3

By right-hand rule the flux through the surface of the small loop is into the page (z -direction). This means that all of the "down" flux passes through the surface spanned by the big loop, but not all of the "up" flux (some is outside of the big loop). Thus the net flux is down and increasing, and the big loop will counter this with induced up flux, which by right hand rule means the current will be **counterclockwise**.

Result

$$\mathcal{E} = \frac{\mu_0}{\pi} ka \ln 2$$

, and the current in the big loop will flow **counterclockwise**.

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