Exercise 1

Chapter 7, Page 301

Introduction to Electrodynamics

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Solution

Verified

4.5 (8 ratings)

Step 1

a)

Integrating the Ohm's law we get:

$$I = \oint_S \vec{J} \cdot d\vec{S} = \sigma \oint_S \vec{E} \cdot d\vec{S} = \sigma \frac{Q}{\epsilon_0} \tag{1}$$

, where Q is the charge on the inner sphere. The potential difference between the shells is:

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{l} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \tag{2}$$

Using (1) and (2) the current is:

$$I = \frac{\sigma}{\epsilon_0} \Delta V \frac{4\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \boxed{4\pi\sigma \frac{\Delta V ab}{b-a}}$$

Step 2

b)

Uisng the Ohm's law the resistance is $R = \Delta V / I$, so:

$$R = \boxed{\frac{1}{4\pi\sigma}\frac{b-a}{ab}}$$

c)

If $b >> a$ then the resistance of the setup is just:

$$R = \frac{1}{4\pi\sigma a}$$

If we let $b \to \infty$, then we can say that this would be the resistance of one sphere surrounded by the weakly conducing material. The resistance of two faraway spheres is twice that, so the current flowing between them is:

$$I = \frac{\Delta V}{R} = \Delta V \frac{4\pi\sigma a}{2} = \boxed{2\pi\sigma a \Delta V}$$

Result

$$\textbf{a)}\ \boxed{I = 4\pi\sigma\frac{\Delta V ab}{b-a}}$$

$$\textbf{b)}\ \boxed{R = \frac{1}{4\pi\sigma}\frac{b-a}{ab}}$$

$$\textbf{a)}\ \boxed{I = 2\pi\sigma a \Delta V}$$