Exercise 1

Chapter 8, Page 360

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Introduction to Electrodynamics

ISBN: 9780321856562

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Solution ✔ Verified

Step 1

a)

The magnetic field is nonzero only in between of the two conductors, and so is electric field. The magnetic field is:

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

, where ϕ curls in the direction of the inside flowing current. Now, the electric field. In between the conductors the potential obeys the Laplace's equation. Say $V(a) = V$ and $V(b) = 0$, then:

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) = 0 \implies s\frac{\partial V}{\partial s} = C$$

$$\frac{\partial V}{\partial s} = \frac{C}{s} \implies V(s) = C\ln s + D$$

$$V(b) = 0 \implies D = -C\ln b \qquad V(a) = V \implies C\ln\frac{a}{b} = V \implies C = \frac{V}{\ln(a/b)}$$

$$V(s) = V\frac{\ln s}{\ln(a/b)}$$

The electric field is then:

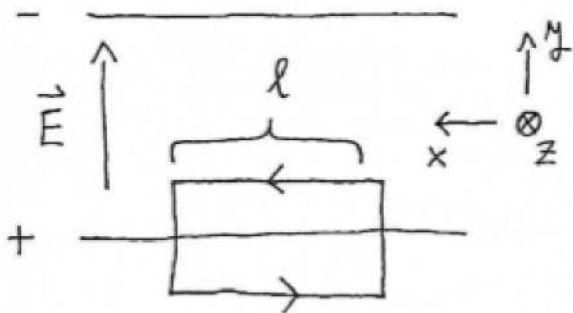$$\vec{E} = -\frac{\partial V}{\partial s}\hat{s} = -\frac{V}{s\ln(a/b)}\hat{z}$$

The Poynting vector is then:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = -\frac{VI}{2\pi\ln(a/b)s^2}\hat{s}\times\hat{\phi}$$
$$= -\frac{VI}{2\pi\ln(a/b)s^2}\hat{z}$$

The total power passing the annular section between the radii a and b in the z-direction is then:

$$P = \int_a^b\int_0^{2\pi}\vec{S}\cdot d\vec{A} = \int_a^b\int_0^{2\pi}S\,dr\,r\,d\phi$$
$$= -\frac{VI}{2\pi\ln(a/b)}\int_a^b\frac{ds}{s}\int_0^{2\pi}d\phi = -\frac{VI}{\ln(a/b)}\ln s\Big|_a^b$$
$$= -\frac{VI}{\ln(a/b)}\ln\frac{b}{a}$$
$$= \boxed{VI}$$

8.1)

b)

Refer to the picture. On the lower plate the current is flowing out of the page, and on the upper it's the opposite. By taking an Amperian loop, the field is in the x-direction and:

$$lB = \mu_0 lK \implies \vec{B} = \mu_0 K\hat{x}$$

, where the magnetic field is zero below the lower and above upper plate. The poynting vector is:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \frac{1}{\mu_0}\frac{V}{h}\hat{y} \times K\mu_0\hat{x} = -\frac{VK}{h}\hat{z}$$

By choosing the surface over which we'll integrate the Poynting vector to be positive in the negative z-direction:

$$P = AS = \frac{VK}{h}hl = V(Kl) = \boxed{VI}$$

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$$\boxed{P = VI}$$, in both cases.

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