Exercise 25

Chapter 5, Page 248

Introduction to Electrodynamics

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Solution

Verified

Step 1

Take the divergence, keeping in mind that the magnetic field is uniform in strenght and direction (that is, a constant vector):

$$\nabla \cdot \vec{A} = -\frac{1}{2}\nabla \cdot (\vec{r} \times \vec{B}) = -\frac{1}{2}[\vec{B} \cdot (\nabla \times \vec{r}) - \vec{r} \cdot (\nabla \times \vec{B})] = 0$$

, since the position vector is irrotational. Now take the curl:

$$\begin{aligned} \nabla \times \vec{A} &= -\frac{1}{2}\nabla \times (\vec{r} \times \vec{B}) \\ &= -\frac{1}{2}[(\vec{B} \cdot \nabla)\vec{r} - (\vec{r} \cdot \nabla)\vec{B} + \vec{r}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{r})] \end{aligned}$$

$$\nabla \cdot \vec{r} = 3 \qquad (\vec{B} \cdot \nabla)\vec{r} = \vec{B}$$

$$= -\frac{1}{2}[\vec{B} - 0 + 0 - 3\vec{B}] = \boxed{\vec{B}}$$

, proving that this vector potential does indeed produce an uniform magnetic field.

This is not an unique solution. You can replace $\vec{r}$ with $\vec{r} + \vec{d}$, for any constant vector $\vec{d}$ and the same result would be reached.

Result

The vector potential does produce the magnetic field. This is not an unique result, as replacing $\vec{r} \to \vec{r} + \vec{d}$, $\vec{d}$ a constant vector, produces the same result.

Rate this solution

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