Exercise 28

Chapter 2, Page 88

Introduction to Electrodynamics
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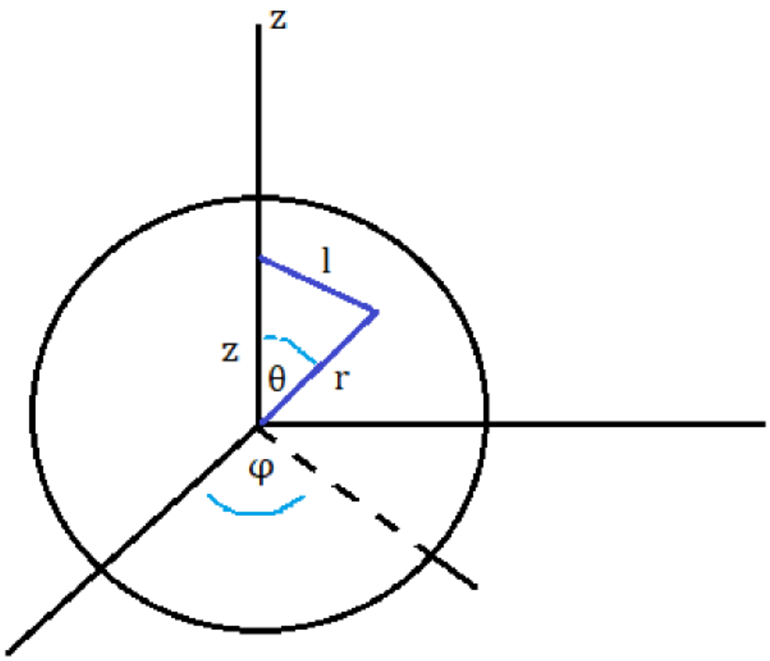
Solution

Verified

Step 1

We will choose to orient the axes so that the point P is on the z-axis, as on the picture.



Step 2

Because of the limitations of this site I will be using letter l as the distance from the source to point of interest.

Now just integrate the charge distribution using the Eq. 2.29):

$$\begin{aligned} V =& \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho}{l} dV \\ =& \frac{\rho}{4\pi\epsilon_0}\int_0^R\int_0^{2\pi}\int_0^{\pi} \frac{r^2 dr \sin\theta d\theta d\phi}{\sqrt{r^2+z^2-2rz\cos\theta}} \\ =& \frac{\rho}{2\epsilon_0}\int_0^R\int_0^{\pi} \frac{r^2 dr \sin\theta d\theta}{\sqrt{r^2+z^2-2rz\cos\theta}} \end{aligned}$$

The angular integral is:

$$\begin{aligned} \int_0^{\pi} \frac{r^2 \sin\theta d\theta}{\sqrt{r^2+z^2-2rz\cos\theta}} =& \frac{1}{2rz}\int_0^{\pi} \frac{d(r^2+z^2-2rz\cos\theta)}{\sqrt{r^2+z^2-2rz\cos\theta}} \\ =& \frac{1}{2rz} 2\sqrt{r^2+z^2-2rz\cos\theta}\Big|_0^{\pi} = \frac{|r+z|}{rz} - \frac{|r-z|}{rz} \\ =& \begin{cases} 2/z & , r < z \\ 2/r & , r > z \end{cases} \end{aligned}$$

Then, the radial integral has two parts and the final potential is:

$$\begin{aligned} V =& \frac{\rho}{2\epsilon_0}\left(\int_0^z \frac{2}{z} r^2 dr + \int_z^R \frac{2}{r} r^2 dr\right) = \frac{\rho}{2\epsilon_0}\left(\frac{2}{3}z^2 + R^2 - z^2\right) \\ =& \frac{\rho}{2\epsilon_0}\left(R^2 - \frac{z^2}{3}\right) \end{aligned}$$

$$Q = \rho \frac{4}{3}\pi R^3$$

$$= \boxed{\frac{Q}{8\pi\epsilon_0}\left(\frac{3}{R} - \frac{z^2}{R^3}\right)}$$

Result

$$\boxed{V = \frac{Q}{8\pi\epsilon_0}\left(\frac{3}{R} - \frac{z^2}{R^3}\right)}$$

, agreeing with result of Problem 2.21).

Rate this solution

Exercise 27

Exercise 29

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