Exercise 46

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Introduction to Electrodynamics

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Solution

Verified

Step 1

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Just apply the divergence operator to the given electric field to get the answer:

$$
\begin{aligned}
\nabla \cdot \vec{E} =& \frac{1}{r^2}\frac{\partial}{\partial r}(3kr) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{k}{r}2\sin^2\theta\cos\theta\sin\phi\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{k}{r}\sin\theta\cos\phi\right) \\
=& \frac{3k}{r^2} + \frac{2k\sin\phi}{r^2\sin\theta}(2\sin\theta\cos^2\theta - \sin^3\theta) - \frac{k}{r^2}\sin\phi \\
=& \frac{3k}{r^2} + \frac{k}{r^2}(4\cos^2\theta - 2\sin^2\theta - 1)\sin\phi \\
=& \frac{3k}{r^2} + \frac{3k}{r^2}(\cos^2 - \sin^2\theta)\sin\phi \\
=& \frac{3k}{r^2}(1 + \cos(2\theta)\sin\phi)
\end{aligned}
$$

The charge density is then:

$$\rho = \nabla \cdot \vec{E}\epsilon_0 = \boxed{\frac{3k\epsilon_0}{r^2}(1 + \cos(2\theta)\sin\phi)}$$

Result

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$$\boxed{\rho = \frac{3k\epsilon_0}{r^2}(1 + \cos(2\theta)\sin\phi)}$$

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