Science / Physics / Introduction to Electrodynamics (4th Edition)

Exercise 15

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Introduction to Electrodynamics

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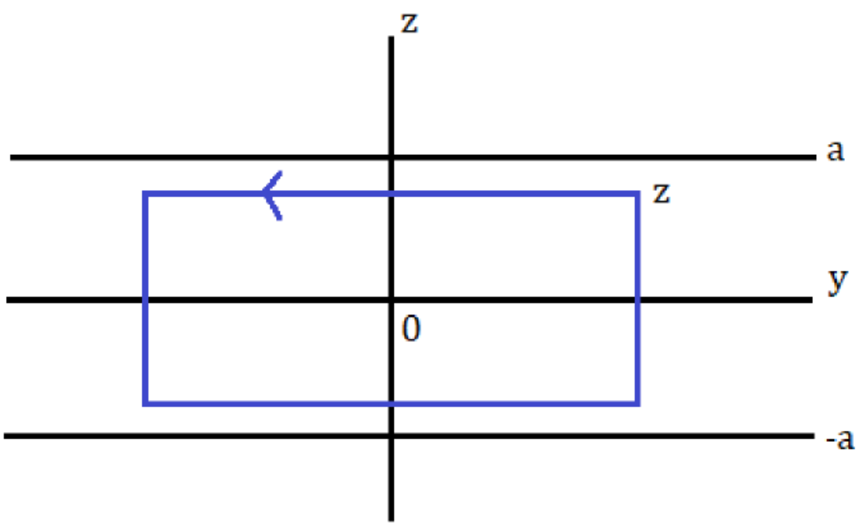
Solution

Verified

Step 1

Refer to the Fig. 5.41). For $z > 0$ there is more current below this point than above, so the field will point in the $-y$ direction, and for $z < 0$ it will be the opposite. By symmetry, $B = 0$ at $z = 0$.

Take a rectangular Amperian loop like on the picture:



Step 2

Using the Ampere law, for $z < a$:

$$2LB = \mu_0(2z)LJ \implies B = \mu_0 Jz \implies \vec{B} = \boxed{\mu_0 Jz\hat{\mathbf{y}}}$$

For $z > a$ the Ampere law gives the magnetic field of constant magnitude, no matter how far away from the plane we are:

$$2LB = \mu_0(2a)LJ \implies \vec{B} = \boxed{\pm\mu_0 aJ\hat{\mathbf{y}}}$$

Result

$$\boxed{\vec{B}_{in} = \mu_0 zJ\hat{\mathbf{y}} \qquad \vec{B}_{out} = \pm\mu_0 aJ\hat{\mathbf{y}}}$$

, positive for $z < 0$ and negative for $z > 0$.