Exercise 14

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Introduction to Electrodynamics

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Solution

Verified

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Step 1

a)

Using an Amperian loop of radius $s < a$ we find that the inside magnetic field is zero. Outside the loop encloses the current I so the field is that of the wire carrying current I:

$$\vec{B} = \boxed{\begin{cases} 0 & , s < a \\ \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & , s > a \end{cases}}$$

b)

First we need to find J:

$$I=\int_S J dS=2\pi k\int_0^a s^2 ds=\frac{2\pi}{3}a^3 k \implies k=\frac{3I}{2\pi a^3} \implies J=\frac{3Is}{2\pi a^3}$$

Now, using an Amperian loop with $s<a$:

$$2\pi Bs=\mu_0\frac{3I}{2\pi a^3}2\pi\int_0^s s^2 ds=\mu_0 I\frac{s^3}{a^3} \implies \vec{B}=\frac{\mu_0 I s^2}{2\pi a^3}\hat{\phi}$$

Outside all the current is within the loop so the field is the same as in the **a)** case for $s>a$:

$$\vec{B}=\boxed{\begin{cases}\dfrac{\mu_0 I s^2}{2\pi a^3}\hat{\phi} & , s<a \\ \\ \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & , s>a\end{cases}}$$

Result

a)

$$\boxed{\vec{B} = \begin{cases} 0 & , s < a \\ \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & , s > a \end{cases}}$$

b)

$$\boxed{\vec{B} = \begin{cases} \dfrac{\mu_0 I s^2}{2\pi a^3}\hat{\phi} & , s < a \\ \\ \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & , s > a \end{cases}}$$