# Exercise 36

Chapter 2, Page 97

Introduction to Electrodynamics

ISBN: 9780321856562

Table of contents

## Solution

Verified

### Step 1



**(a)** We have two concentric spherical shells, of radii $a$ and $b$. The inner one carries a charge $q$, and the outer one a charge $-q$. We need to calculate the energy of this configuration, first using equation 2.45, that is,

$$W = \frac{\epsilon_0}{2} \int E^2 d\tau$$

where $E$ is the electric field. The electric is,

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} (a < r < b)$$

and zero elsewhere. Substitute into the above equation, we get,

$$\begin{aligned} W &= \frac{\epsilon_0}{2} \left( \frac{q}{4\pi\epsilon_0} \right)^2 \int_a^b \left( \frac{1}{r^2} \right)^2 4\pi r^2 dr \\ &= \frac{q^2}{8\pi\epsilon_0} \int_a^b \frac{1}{r^2} \\ &= \frac{q^2}{8\pi\epsilon_0} \left( \frac{1}{a} - \frac{1}{b} \right) \end{aligned}$$

$$\boxed{W = \frac{q^2}{8\pi\epsilon_0} \left( \frac{1}{a} - \frac{1}{b} \right)}$$

**(b)** Now we need to find the total work using the the superposition principle (equation 2.47) and the results of example 2.8, that are,

**Step 2**



hence,

$$\begin{aligned}
\int \mathbf{E}_1 \cdot \mathbf{E}_2 d\tau = \int \mathbf{E}_1 \cdot \mathbf{E}_2 d\tau \\
&= -\left(\frac{1}{4\pi\epsilon_0}\right)^2 q^2 \int_b^{\infty} \frac{1}{r^4} 4\pi r^2 dr \\
&= \left(\frac{4\pi}{4\pi\epsilon_0}\right)^2 q^2 \left[\frac{1}{r}\right]_b^{\infty} \\
&= -\frac{q^2}{4\pi\epsilon_0 b}
\end{aligned}$$

substitute into the first equation in part (b), "the total work", with this result and the works $W_1$ and $W_2$, so we get,

$$\begin{aligned}
W &= \frac{1}{8\pi\epsilon_0} q^2 \left(\frac{1}{a} + \frac{1}{b} - \frac{2}{b}\right) \\
&= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)
\end{aligned}$$

$$\boxed{W = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

**Result**



$$\textbf{(a)}\ W = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\textbf{(b)}\ W = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Rate this solution