16-720B Homework 3 Write-up

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Q1.1

- $\frac{\partial \mathcal{W}(\mathbf{x};\mathbf{p})}{\partial \mathbf{p}^T}$ is the graident of the warped coordinates over the warping parameter $\mathbf{p}$, which is:

$$\frac{\partial \mathcal{W}(\mathbf{x};\mathbf{p})}{\partial \mathbf{p}^T} = \frac{\partial \mathbf{x} + \mathbf{p}}{\partial \mathbf{p}^T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{1}$$

- For the iterative process, replace $\mathbf{p}$ with $\mathbf{p} + \Delta\mathbf{p}$ in Eq. (2) of the handout, and then

$$\begin{aligned} \mathcal{I}_{t+1}(\mathbf{x} + \mathbf{p} + \Delta\mathbf{p}) - \mathcal{I}_t(\mathbf{x}) &= \mathcal{I}_{t+1}(\mathbf{x} + \mathbf{p}) + \frac{\partial \mathcal{I}_{t+1}(\mathbf{x} + \mathbf{p})}{\partial (\mathbf{x} + \mathbf{p})^T} \Delta\mathbf{p} - \mathcal{I}_t(\mathbf{x}) & (2) \\ &= \nabla \mathcal{I}_{t+1}(\mathbf{x} + \mathbf{p}) \Delta\mathbf{p} - (\mathcal{I}_t(\mathbf{x}) - \mathcal{I}_{t+1}(\mathbf{x} + \mathbf{p})). & (3) \end{aligned}$$

  Therefore the Eq. 2 of the handout in vector form is (Note that each $\nabla \mathcal{I}_{t+1}(\mathbf{x} + \mathbf{p})$ are of shape 1×2.)

$$\arg\min_{\Delta\mathbf{p}} \left\| \begin{bmatrix} \nabla \mathcal{I}_{t+1}(\mathbf{x}_1 + \mathbf{p}) \\ \nabla \mathcal{I}_{t+1}(\mathbf{x}_2 + \mathbf{p}) \\ \cdots \\ \nabla \mathcal{I}_{t+1}(\mathbf{x}_D + \mathbf{p}) \end{bmatrix} \Delta\mathbf{p} - \begin{bmatrix} \mathcal{I}_t(\mathbf{x}_1) - \mathcal{I}_{t+1}(\mathbf{x}_1 + \mathbf{p}) \\ \mathcal{I}_t(\mathbf{x}_2) - \mathcal{I}_{t+1}(\mathbf{x}_2 + \mathbf{p}) \\ \cdots \\ \mathcal{I}_t(\mathbf{x}_D) - \mathcal{I}_{t+1}(\mathbf{x}_D + \mathbf{p}) \end{bmatrix} \right\| = \arg\min_{\Delta\mathbf{p}} \|\mathbf{A}\Delta\mathbf{p} - \mathbf{b}\| \tag{4}$$

  The big matrix and the big vector on the L.H.S. of the above equation are the $\mathbf{A}$ and $\mathbf{b}$.

- To solve for the least square solution of Eq. 4, we need to compute $(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$. Therefore, we must have $\mathbf{A}^T\mathbf{A}$ to be invertible.