

# 16-720B Homework 4 Write-up

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## Q1.1

Consider the point  $\mathbf{w}$  where the principle axes of the two cameras intersect, and we can see that  $\tilde{\mathbf{x}}_1 = [0, 0, 1]^T$  and  $\tilde{\mathbf{x}}_2 = [0, 0, 1]^T$  corresponding one point in 3D. Therefore

$$\tilde{\mathbf{x}}_2^T \mathbf{E} \tilde{\mathbf{x}}_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} & \mathbf{E}_{13} \\ \mathbf{E}_{21} & \mathbf{E}_{22} & \mathbf{E}_{23} \\ \mathbf{E}_{31} & \mathbf{E}_{32} & \mathbf{E}_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \mathbf{E}_{33} = 0 \quad (1)$$

Since two cameras are normalized, the intrinsic matrices for them are identity:  $\mathbf{K}_1 = \mathbf{K}_2 = \mathbf{I}$ . Then  $\mathbf{E} = \mathbf{K}_1^T \mathbf{F} \mathbf{K}_2 = \mathbf{E}$ . Therefore,  $\mathbf{E}_{33} = \mathbf{F}_{33} = 0$ .

## Q1.2

Suppose the cameras are normalized in the sense that their intrinsic matrices are both identity. Now that the translation and rotation from camera 1 to camera 2 are

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{t} = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix} \quad (2)$$

According to the definition of essential matrix

$$\tilde{\mathbf{x}}_2^T \mathbf{t} \times \mathbf{R} \tilde{\mathbf{x}}_1 = \tilde{\mathbf{x}}_2^T \mathbf{t}_\times \mathbf{R} \tilde{\mathbf{x}}_1 = \tilde{\mathbf{x}}_2^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tilde{\mathbf{x}}_1 \quad (3)$$

$$= \tilde{\mathbf{x}}_2^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \tilde{\mathbf{x}}_1 = \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -t_x \\ t_x y_1 \end{bmatrix} \quad (4)$$

$$= -t_x y_2 + t_x y_1 = 0 \quad (5)$$

$$\Rightarrow y_1 = y_2 \quad (6)$$

For a certain epipolar line in camera 2, it has a fixed epipole with fixed  $y_1$  in camera 1, and thus every 3D point on the corresponding incident ray to camera 1 has a projection with a fixed  $y_2 = y_1$  in camera 2, which means the epipolar line has a fixed  $y$  coordinate, and thus parallel to x-axis.

The above deduction also holds if camera 1 and 2 are exchanged.

### Q1.3

Assume  $(\mathbf{R}_i, \mathbf{t}_i)$  and  $(\mathbf{R}_j, \mathbf{t}_j)$  are the rotation and translation at time  $i$  and time  $j$ . Then for a point  $\mathbf{w}$  in the 3D world

$$\begin{aligned}\lambda_i \tilde{\mathbf{x}}_i &= \mathbf{R}_i \mathbf{w} + \mathbf{t}_i, & \lambda_j \tilde{\mathbf{x}}_j &= \mathbf{R}_j \mathbf{w} + \mathbf{t}_j \\ \Rightarrow \mathbf{w} &= \mathbf{R}_i^T (\lambda_i \tilde{\mathbf{x}}_i - \mathbf{t}_i) \\ \Rightarrow \lambda_j \tilde{\mathbf{x}}_j &= \mathbf{R}_j \mathbf{R}_i^T (\lambda_i \tilde{\mathbf{x}}_i - \mathbf{t}_i) + \mathbf{t}_j \\ \Rightarrow \lambda_j \tilde{\mathbf{x}}_j &= \mathbf{R}_j \mathbf{R}_i^T \lambda_i \tilde{\mathbf{x}}_i - \mathbf{R}_j \mathbf{R}_i^T \mathbf{t}_i + \mathbf{t}_j \\ \Rightarrow \lambda_j \tilde{\mathbf{x}}_j &= \lambda_i \mathbf{R}_{rel} \tilde{\mathbf{x}}_i + \mathbf{t}_{rel}\end{aligned}\tag{7}$$

Therefore

$$\mathbf{R}_{rel} = \mathbf{R}_j \mathbf{R}_i^T, \quad \mathbf{t}_{rel} = \mathbf{t}_j - \mathbf{R}_j \mathbf{R}_i^T \mathbf{t}_i\tag{8}$$

Q1.4