16-720B Homework 4 Write-up

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November 5, 2019

Q1.1

Consider the point $\mathbf{w}$ where the principle axes of the two cameras intersect, and we can see that $\tilde{\mathbf{x}}_1 = [0,0,1]^T$ and $\tilde{\mathbf{x}}_2 = [0,0,1]^T$ corresponding one point in 3D. Therefore

$$\tilde{\mathbf{x}}_2^T \mathbf{E} \tilde{\mathbf{x}}_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} & \mathbf{E}_{13} \\ \mathbf{E}_{21} & \mathbf{E}_{22} & \mathbf{E}_{23} \\ \mathbf{E}_{31} & \mathbf{E}_{32} & \mathbf{E}_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \mathbf{E}_{33} = 0 \tag{1}$$

Since two cameras are normalized, the intrinsic matrices for them are identity: $\mathbf{K}_1 = \mathbf{K}_2 = \mathbf{I}$. Then $\mathbf{E} = \mathbf{K}_1^T \mathbf{F} \mathbf{K}_2 = \mathbf{E}$. Therefore, $\mathbf{E}_{33} = \mathbf{F}_{33} = 0$.

Q1.2

Suppose the cameras are normalized in the sense that their intrinsic matrices are both identity: $\mathbf{K}_1 = \mathbf{K}_2 = \mathbf{I}$.

Now that the translation and rotation from camera 1 to camera 2 are

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{t} = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

And thus the essential matrix are

$$\mathbf{E} = \mathbf{t}_\times \mathbf{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \tag{3}$$

Therefore for an epipolar line in camera 1 $\mathbf{l}_1^T \tilde{\mathbf{x}}_1 = 0$ and $\tilde{\mathbf{x}}_2^T \mathbf{E} \tilde{\mathbf{x}}_1 = 0$, where $\tilde{\mathbf{x}}_2$ is a fixed point on the image plane of camera 2 resulting from the ray corresponding to the epipolar line, then we can see that

$$\mathbf{l}_1^T = \tilde{\mathbf{x}}_2^T \mathbf{E} = \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & t_x & -t_x y_2 \end{bmatrix} \tag{4}$$

Similarly we can see that any epipolar line in camera 1 has $\mathbf{l}_2^T = [0 \; -t_x \; t_x y_1]$. Since the first elements in both $\mathbf{l}_1$ and $\mathbf{l}_2$ are zero, the epipolar lines are parallel to x axis.

Q1.3

Assume $(\mathbf{R}_i, \mathbf{t}_i)$ and $(\mathbf{R}_i, \mathbf{t}_i)$ are the rotation and translation at time i and time j. Then for a point $\mathbf{w}$ in the 3D world

$$\begin{aligned}
&\lambda_i \tilde{\mathbf{x}}_i = \mathbf{R}_i \mathbf{w} + \mathbf{t}_i, \quad \lambda_j \tilde{\mathbf{x}}_j = \mathbf{R}_j \mathbf{w} + \mathbf{t}_j \\
\Rightarrow &\mathbf{w} = \mathbf{R}_i^T (\lambda_i \tilde{\mathbf{x}}_i - \mathbf{t}_i) \\
\Rightarrow &\lambda_j \tilde{\mathbf{x}}_j = \mathbf{R}_j \mathbf{R}_i^T (\lambda_i \tilde{\mathbf{x}}_i - \mathbf{t}_i) + \mathbf{t}_j \\
\Rightarrow &\lambda_j \tilde{\mathbf{x}}_j = \mathbf{R}_j \mathbf{R}_i^T \lambda_i \tilde{\mathbf{x}}_i - \mathbf{R}_j \mathbf{R}_i^T \mathbf{t}_i + \mathbf{t}_j \\
\Rightarrow &\lambda_j \tilde{\mathbf{x}}_j = \lambda_i \mathbf{R}_{rel} \tilde{\mathbf{x}}_i + \mathbf{t}_{rel} \qquad (5)
\end{aligned}$$

Therefore

$$\mathbf{R}_{rel} = \mathbf{R}_j \mathbf{R}_i^T, \quad \mathbf{t}_{rel} = \mathbf{t}_j - \mathbf{R}_j \mathbf{R}_i^T \mathbf{t}_i \qquad (6)$$

Q1.4