16-720B Homework 2 Write-up

Gu, Qiao

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Q1.2

Please see Figure. 1 for the DoG Pyramid of `model_chickenbroth.jpg`.

Figure 1: The DoG Pyramid of `model_chickenbroth.jpg`

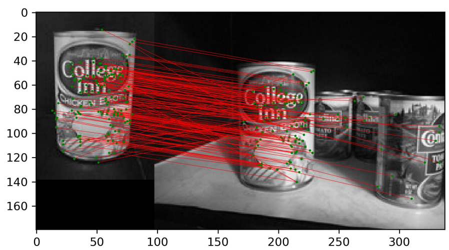
Q1.5

Please see Figure. 2 for the detected keypoints of `model_chickenbroth.jpg`.

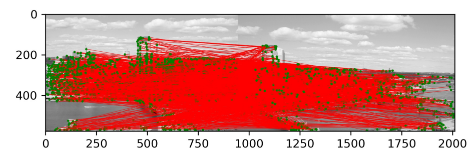
Figure 2: The detected keypoints on image `model_chickenbroth.jpg`

Q2.4

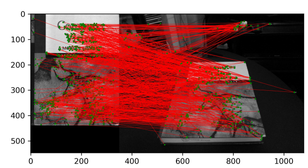
Please see Figure 3 for the matching results. From the results, we can see that if the object is rotated, the matching performance will be much worse than those with little or no rotation (compare Figure. 3 (d)(g) with Figure. 3 (c)(f)). I suspect this is because the BRIEF descriptor does not encode the patch into a rotation-invariant space and thus have poor ability to match rotated patches.



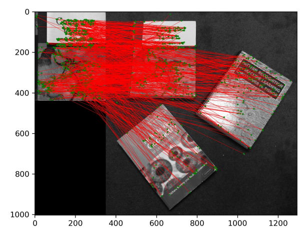
(a) match of two of the `chickenbroth` images.



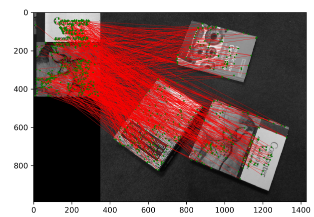
(b) match of the `incline` images.



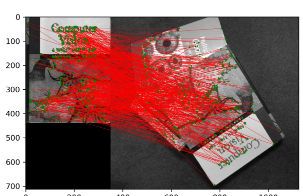
(c) match of `pf_scan_scaled.jpg` against `pf_desk.jpg`.



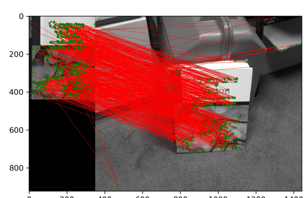
(d) match of `pf_scan_scaled.jpg` against `pf_floor.jpg`.



(e) match of `pf_scan_scaled.jpg` against `pf_floor_rot.jpg`.



(f) match of `pf_scan_scaled.jpg` against `pf_pile.jpg`.



(g) match of `pf_scan_scaled.jpg` against `pf_stand.jpg`.

Figure 3: Results of Feature Match.

Q2.5

Please see Figure 4 for the matching results. As we can see, the number of matches reaches its maximum when there is no rotation between two images, and drops quickly when the rotation angle starts to increase. This is probably because the BRIEF descriptor we Implement only treats each keypoint patch as a rectangle grid, and thus when the rectangle is rotated, the whole grid structure can be messed up. Therefore the number of matches drops quickly as with rotation.

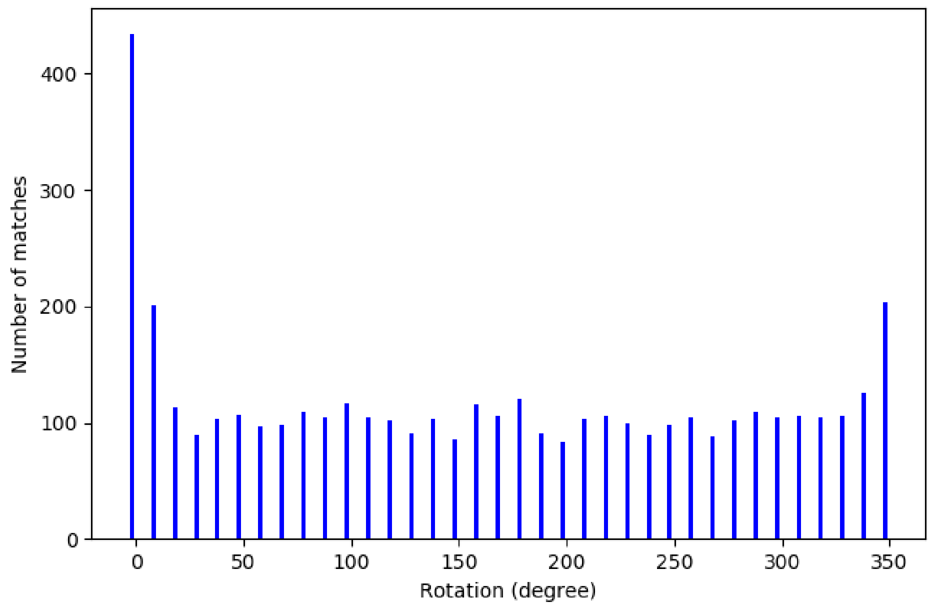


Figure 4: The rotation angle vs the number of correct matches

Q3.1

Q3.1.1

From the given equation $\lambda_n \tilde{\mathbf{x}_n} = \mathbf{H}\tilde{\mathbf{u}_n}$, we can get

$$\lambda_n \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} u_n \\ v_n \\ 1 \end{bmatrix}. \tag{1}$$

And change it to equations:

$$\lambda_n x_n = H_{11}u_n + H_{12}v_n + H_{13} \tag{2}$$
$$\lambda_n y_n = H_{21}u_n + H_{22}v_n + H_{23} \tag{3}$$
$$\lambda_n = H_{31}u_n + H_{32}v_n + H_{33}. \tag{4}$$

Then elinimate λ_n:

$$H_{31}u_n x_n + H_{32}v_n x_n + H_{33}x_n = H_{11}u_n + H_{12}v_n + H_{13} \tag{5}$$
$$H_{31}u_n y_n + H_{32}v_n y_n + H_{33}y_n = H_{21}u_n + H_{22}v_n + H_{23}, \tag{6}$$

which is equivalent to:

$$\begin{bmatrix} u_n & v_n & 1 & 0 & 0 & 0 & -u_n x_n & -v_n x_n & -x_n \\ 0 & 0 & 0 & u_n & v_n & 1 & -u_n y_n & -v_n y_n & -y_n \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = \mathbf{0} \tag{7}$$

Repeat the equation above for $n = 1...N$, then we can get:

$$\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1 x_1 & -v_1 x_1 & -x_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1 y_1 & -v_1 y_1 & -y_1 \\ & & & & & \cdots & & & \\ u_N & v_N & 1 & 0 & 0 & 0 & -u_N x_N & -v_N x_N & -x_N \\ 0 & 0 & 0 & u_N & v_N & 1 & -u_N y_N & -v_N y_N & -y_N \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = \mathbf{0} \tag{8}$$

The $2N \times 2$ matrix on the left hand side is the $\mathbf{A}$ required, and the other one is $\mathbf{h}$

Q3.1.2

There are 9 elements in $\mathbf{h}$,

Q3.1.3

Since we can add a constraint that $\mathbf{h} = 1$. Therefore there are 8 free elements left in $\mathbf{h}$, and thus 8 degress of freedom in $\mathbf{H}$.

We only need 4 correspondence to solve this system, since each point correspondence provide 2 equations that solve 2 degree of freedom.

Q3.1.4

The problem can be formulated as

$$\mathbf{Ah} = \mathbf{0} \quad \text{s.t. } \|\mathbf{h}\|_2 = 1. \tag{9}$$

When we have more than 4 correspondence, a accurate solution cannot be found and thus the problem becomes a least square problem as follows:

$$\arg\min_{\mathbf{h}} \|\mathbf{Ah}\|_2^2 \quad \text{s.t. } \mathbf{h}^T\mathbf{h} = 1 \tag{10}$$

$$\Leftrightarrow \arg\min_{\mathbf{h}} \frac{\mathbf{h}^T\mathbf{A}^T\mathbf{Ah}}{\mathbf{h}^T\mathbf{h}} \quad \text{s.t. } \mathbf{h}^T\mathbf{h} = 1 \tag{11}$$

Since $\mathbf{B} = \mathbf{A}^T\mathbf{A}$ is a symmetric positive semidefinite matrix ($\mathbf{h}^T\mathbf{A}^T\mathbf{Ah} = ||\mathbf{Ah}||_2^2 \geq 0$), it has n mutually orthonormal eigenvectors $\mathbf{x}_1, \ldots, \mathbf{x}_n$ with associating eigenvalues $\lambda_1, \ldots, \lambda_n$ and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$. Assume $\mathbf{h} = \sum_{i=1}^n k_i\mathbf{x}_i$, then $\mathbf{h}^T\mathbf{h} = \sum_{i=1}^n k_i\mathbf{x}_i^T \sum_{i=1}^n k_i\mathbf{x}_i = \sum_{i=1}^n k_i^2 = 1$ and

$$\begin{aligned}
\frac{\mathbf{h}^T\mathbf{A}^T\mathbf{Ah}}{\mathbf{h}^T\mathbf{h}} &= \frac{\sum_{i=1}^n k_i\mathbf{x}_i^T\mathbf{B}\sum_{i=1}^n k_i\mathbf{x}_i}{\sum_{i=1}^n k_i\mathbf{x}_i^T\sum_{i=1}^n k_i\mathbf{x}_i} \\
&= \frac{\sum_{i=1}^n k_i\mathbf{x}_i^T\sum_{i=1}^n \mathbf{B}k_i\mathbf{x}_i}{\sum_{i=1}^n k_i\mathbf{x}_i^T k_i\mathbf{x}_i} \\
&= \frac{\sum_{i=1}^n k_i\mathbf{x}_i^T\sum_{i=1}^n \lambda_i k_i\mathbf{x}_i}{\sum_{i=1}^n k_i^2} \\
&= \frac{\sum_{i=1}^n k_i\mathbf{x}_i^T\lambda_i k_i\mathbf{x}_i}{1} \\
&= \sum_{i=1}^n \lambda_i k_i^2\mathbf{x}_i^T\mathbf{x}_i \\
&= \sum_{i=1}^n \lambda_i k_i^2 \\
&\geq \sum_{i=1}^n \lambda_n k_i^2 = \lambda_n
\end{aligned} \tag{12}$$

The above deduection shows that the Rayleigh quotient $\frac{\mathbf{h}^T\mathbf{A}^T\mathbf{Ah}}{\mathbf{h}^T\mathbf{h}}$ has its minimum λ_{min} of $\mathbf{A}^T\mathbf{A}$, when $\mathbf{h} = \mathbf{x}_n$. Therefore, if we can find the singular value decomposition $\mathbf{A} = \mathbf{U\Sigma V}^T$, the solution $\mathbf{h}$ will be the right singular vector v_9 corresponding to the minimum singular value $\sigma_{min} = \sigma_9$ of $\mathbf{A}$. (Note that $\mathbf{A} \in \mathbb{R}^{2N\times 9}$ and $\mathbf{A}^T\mathbf{A} \in \mathbb{R}^{9\times 9}$)

Q6.1

Please see Figure. 5 for the results. Here we simply extend the width of the image to contain more part of the warped image, which will be enhanced in the next section.

(a) The warped image.

(b) The generated panorama.

Figure 5: Results for **Q6.1**.

Q6.2

Please see Figure. 6 for the result.

Figure 6: The panorama with no clipping on image borders.

Q6.3

Please see Figure. 8 for the result.

Figure 7: The generated panorama.

Q7.2

Please see Figure. 8 for the result of putting the ball at the center of “O”.

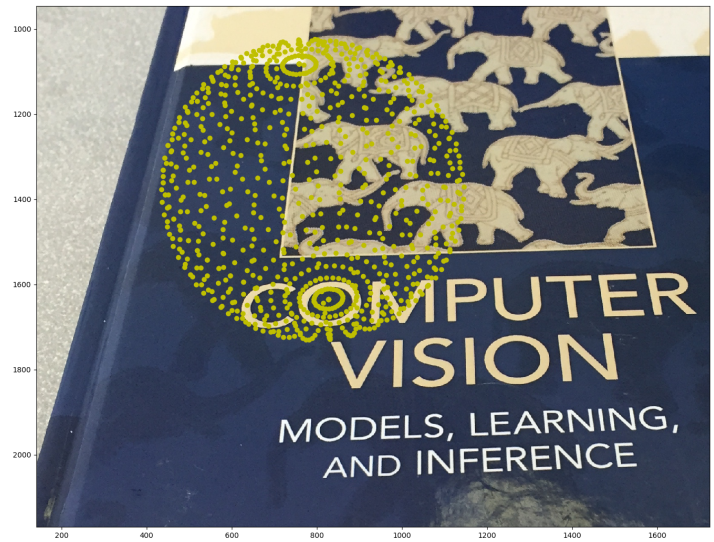


Figure 8: The augmented reality result. The image has been enlarged to see details.