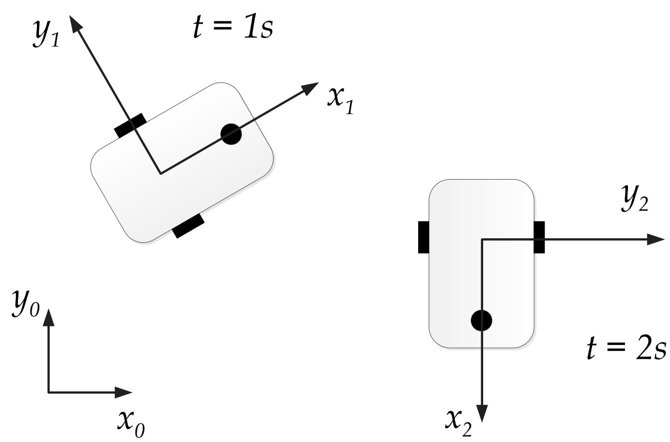
Two configurations for a mobile robot at times $t = 1$ and $t = 2$ seconds are shown below. The robot pose (position and orientation) is given by the vector, $q = [x \quad y \quad \theta]^T$ where x and y are the coordinates of origin of the robot-fixed frame expressed in the global coordinate frame, and θ is the angle between the x-axes of robot-fixed and global coordinate frames measured counterclockwise about the z_0-axis. All units are in seconds, meters and radians.



Given the following information:

$$
{}^0q_1 = \begin{bmatrix} 10 \\ 20 \\ \pi/6 \end{bmatrix}, \quad {}^0q_2 = \begin{bmatrix} 50 \\ 15 \\ -\pi/2 \end{bmatrix}
$$

a) Write the 4×4 homogeneous transformation matrix ${}^0T_1$, relating the robot coordinate frame at time $t = 1$ with respect to the global coordinate frame.

$$
{}^0T_1 = \begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} & 0 & 10 \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 10 \\ 1/2 & \sqrt{3}/2 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

b) Write the 4×4 homogeneous transformation matrix ${}^{0}T_2$, relating the robot coordinate frame at time $t = 2$ with respect to the global coordinate frame.

$$ {}^{0}T_2 = \begin{bmatrix} \cos -\frac{\pi}{2} & -\sin -\frac{\pi}{2} & 0 & 50 \\ \sin -\frac{\pi}{2} & \cos -\frac{\pi}{2} & 0 & 15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 50 \\ -1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

c) The robot's velocity given by the vector, $v = [\dot{x} \quad \dot{y} \quad \dot{\theta}]^T$ where $\dot{x}$ and $\dot{y}$ are the planar, and $\dot{\theta}$ is the angular measured counterclockwise about the z_0-axis. If the robot has a local velocity vector at time $t = 2$, given by ${}^{2}v_{robot} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$. Calculate the robot velocity vector ${}^{1}v_{robot}$ with respect to the robot coordinate frame at time $t = 1$.

For this question we don't need to use the whole homogenous transform since we're not looking for the relative position of the reference frames. The question asks only about the velocity vector, which simply represents magnitude and direction (which way is the robot moving and how quickly?). So we can solve it by only using the rotation matrices, which we can extract from the transform matrices we defined above.

$$ {}^{1}v_{robot} = {}^{1}R_2 \, {}^{2}v_{robot} = {}^{1}R_0 \, {}^{0}R_2 \, {}^{2}v_{robot} = ({}^{0}R_1)^T \, {}^{0}R_2 \, {}^{2}v_{robot} $$

$$ {}^{1}v_{robot} = \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} $$

$$ {}^{1}v_{robot} = \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} $$

$$ {}^{1}v_{robot} = \begin{bmatrix} -1 \\ -\sqrt{3} \\ -1 \end{bmatrix} $$