Evaluation of the Parameterized-Response Differential Evolution Trader-Agent

George Herbert

Department of Computer Science
University of Bristol
Bristol, United Kingdom
cj19328@bristol.ac.uk

Abstract—This paper reports results from market experiments containing Paramaterized-Response Differential Evolution (PRDE) trader-agents. Each PRDE trader-agent in a market simultaneously uses differential evolution (DE) to adapt their own trading strategy to maximise profitability. The DE algorithm within each PRDE trader is governed by two parameters: the differential weight coefficient F and the number in population NP. Markets containing a homogeneous population of PRDE traders exhibit different dynamics depending on the values of F and NP The first part of this paper evaluates the effect that these two parameters have on the dynamics of the market; while the latter part of this paper proposes an extension to the PRDE algorithm to further maximise profitability.

Index Terms—Automated Trading, Financial Markets, Adaptive Trader-Agents, Differential Evolution

I. INTRODUCTION

Automated trading accounts for an unprecedented amount of activity in modern financial markets. These algorithms have the ability to execute trades at a frequency simply unachievable for humans beings; thus, the behaviour of many of these markets has significantly shifted. Notably, the 'flash crash' in US financial markets on 6 May 2010 has been partially attributed to high-frequency trading algorithms aggressively reselling short-term positions to one another. Modern automated trading algorithms have the additional quality of being adaptive: they adjust their strategy to extract maximum profit from the market in which they are operating. These contemporary markets—in which competing adaptive algorithms are simultaneously engaged in coninual adjustment to maintain profitability—can be described as *coevolutionary* systems.

Large amounts of research has been conducted to understand the dynamics of these markets. Recently, Cliff introduced the *Parameterized-Response Zero-Intelligence* (PRZI) [1] trader: a nonadaptive generalisation of Gode and Sunder's Zero-Intelligence Constrained (ZIC) [2] trader. The difference lies in the probability mass function (PMF) used to generate quote prices. Each individual ZIC trader samples their quote price from a fixed uniform distribution, whereas each PRZI trader is governed by a strategy parameter $s \in [-1, 1] \in \mathbb{R}$ that determines the PMF the trader samples from; the shape of this PMF determines how 'urgent' or 'relaxed' the trader acts. As $s \to 1$ the distribution is evermore biased towards 'urgent' quote prices—those closest to the least profitable price for the trader, but most likely to attract a willing counterparty—

conversely, as $s \to -1$, the distribution is biased towards 'relaxed' quote prices—those that generate the most profit for the trader, but are considerably less likely to attract a counterparty. When $s = 0$, the PMF is uniform, identical to that of a ZIC trader.

PRZI Stochastic Hillclimber (PRSH) [3] is an adaptive extension to the PRZI also introduced by Cliff. The strategy parameter s is dynamically altered by the algorithm in an attempt to increase profitability. Each PRSH trader maintains a private local population $\mathcal{K}$ of k strategy parameters; each of which it evaluates for a specific period of time via a loop to identify which is most profitable. The most profitable strategy s_0 is 'mutated' $k - 1$ times; these k values comprise the new elements of set $\mathcal{K}$.

PRZI Differential Evolution (PRDE) [4] is further adaptive extension of the PRZI algorithm, and a successor to PRSH; it is the most recent algorithm published by Cliff in the PRZI 'family' of trader-agents. It replaces the simple stochastic hillclimber with a *differential evolution* (DE) optimisation system [5]. Each PRDE trader maintains its own DE system with a population of candidate s-values of size NP ≥ 4, which for trader i can be denoted by $s_{i,1}, s_{i,2}, ..., s_{i,\text{NP}}$; the value NP is referred to as the *number in population*. Once a particular strategy $s_{i,x}$ has been evaluated, three other distinct s-value are chosen at random from the population maintained by trader i: $s_{i,a}$, $s_{i,b}$ and $s_{i,c}$ such that $x \neq a \neq b \neq c$. A new candidate strategy $s_{i,y}$ is constructed as follows:

$$s_{i,y} \leftarrow \max(\min(s_{i,a} + F_i(s_{i,b} - s_{i,c}), 1), -1)$$

where F_i is the trader's *differential weight* coefficient. The fitness of $s_{i,y}$ is evaluated and if it performs better than $s_{i,x}$ then $s_{i,y}$ replaces $s_{i,x}$; otherwise, it is discarded and the next randomly selected strategy is evaluated. This is very similar to the original description of the DE/rand/1 algorithm as described by Storn and Price [5], with a four key differences: max and min functions have been incorporated to constrain the output $s_{i,y}$ to the range $[-1, 1]$; the conventional DE notion of crossover is not relevant because the behaviour of each PRZE trader is governed by only a single scalar value s; the DE algorithm in PRDE is a steady-state evolutionary algorithm as opposed to a generational evolutionary algorithm, which provides a small storage advantage; and Cliff introduced a simple vector-perbutation mechanism to deal with convergence

issues that arised from $s_{i,b} - s_{i,c}$ tending very close to zero. The vector-perbutation mechanism works as follows: if at any time the standard deviation of the candidate s-values in trader i's private population is less than 0.0001, then a randomly selected candidate is provided a value drawn at random from the uniform distribution $\mathcal{U}(-1, 1)$. This can be thought of a 'mega-mutation' to the trader's set of s-values—a countermeasure to convergence.

Cliff's conducted experiments on the *Bristol Stock Exchange* (BSE) (see [6], [7]) to analyse the coevolutionary dynamics of markets populated entirely by PRSH and PRDE traders. BSE is a freely-available, open-source simulation a LOB-based financial exchange. Notably, Cliff identified that markets populated by PRDE traders were approximately 100% more economically efficient than those populated by PRSH traders [4]. However, the PRDE traders implemented in Cliff's experiments did not deviate from a differential weight of $F = 0.8$ and a number in population of NP $= 4$ (i.e. the minimum viable value). He stressed the importance of future work to explore the effects on the market's dynamics of altering these key parameters. Cliff proposed two lines of future research. He proposed an exploration into the effects on the market's dynamics of altering the two key parameters homogeneously—with all traders maintaining the same values of F and NP in a given market session. He also proposed an arguably more intriguing exploration into effects that arise from altering the two key parameters heterogeneously—with different traders being provided different values of F and NP.

II. HOMOGENEOUS EXPLORATION OF F AND NP

A. Combined Effect of F and NP

My initial experiments in this paper focus on the relationship between the number in population NP and the differential weight coefficient F, and the combined effect that these two values have on the dynamic of a coevolutionary market. To do so, I designed a set of experiments with a similar setup as Cliff in [4] in which BSE was used to simulate a financial market for a single abstract tradeable commodity. In each market simulation, I implemented a homogeneous population of $N_T = 30$ PRDE traders with an equal number of buyers N_B and sellers N_S (i.e. $N_B = N_S = 15$). The population in a given experiment was homogeneous with respect to the fact that all trader had the same differential weight F and the same number in population NP. Each simulation had what economists refer to as perfect elasticity of supply and demand: all N_B buyers were provided a limit price of $140 per unit, whilst all N_S sellers were provided a limit price of $60 per unit. This produced a market in which every active trader could, in theory, find a willing counterparty to trade with. In the simulation, after two traders engaged in a trade, they were rendered inactive until their stock was replenished, which occurred approximately every five seconds. I ran each experiment for 100 simulated days on an Apple MacBook Pro with the M1 Pro chip, which took approximately three hours running at 800x real-time.

Figure 1 displays a heatmap showing the combined profit extracted from the market by the population of N_B buyers and N_S sellers. This provides a strong indication of the economic efficiency of the market when populated with homogenous PRDE traders with different values of F and NP. There is a clear relationship between the efficiency of the market, and the values of F and NP. Notably, the market performed worst when F and NP were set to low values, such as $F = 0.0$ and NP $= 4$.
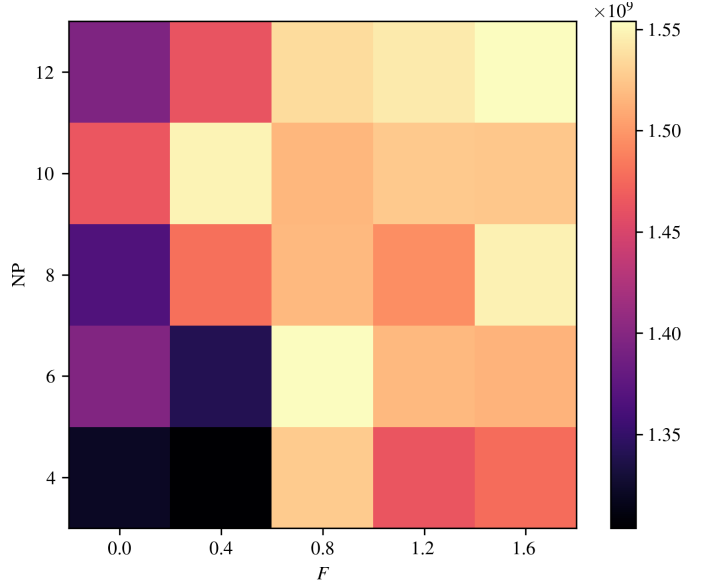

Fig. 1. Plot of profitability data from multiple 100-day experiments in markets populated entirely by homogeneous PRDE traders. Horizontal axis is the differential weight coefficient F of the homogeneous PRDE traders for a given market session; vertical axis is the number in population NP for the homogeneous PRDE traders for a given market session. The intensity of pixel shading represents the total profit extracted from the market during a given 100-day experiment. See text for further discussion.

B. Individual Effect of NP

Figure 2 displays the relationship between the total profit extracted from the market and the number in population NP when $F = 0.8$; there is an evident negative correlation between NP and the total profit extracted. The negative correlation evident from these experiments indicates that markets containing homogeneous PRDE traders with larger private populations NP are less efficient. This can be primarily attributed to an increasingly lengthy adaptive transient at the beginning of the market session. The initial adaptive transient that occurs in homogenous markets of PRDE traders was first reported on by Cliff [4]. There is a pattern in the total profit per second (PPS) extracted from the market π_T by both the buyers and sellers. π_T follows a two-phase pattern: there is an initial adaptive transient whereby π_T increases, followed by a prolonged period whereby π_T remains in approximately the same range. This two-phase pattern is due to the system's initial random conditions being improved upon until the system settles to an ongoing dynamic in which π_T has a zero-sum nature: in general, for the total PPS extracted by the buyers π_B to go

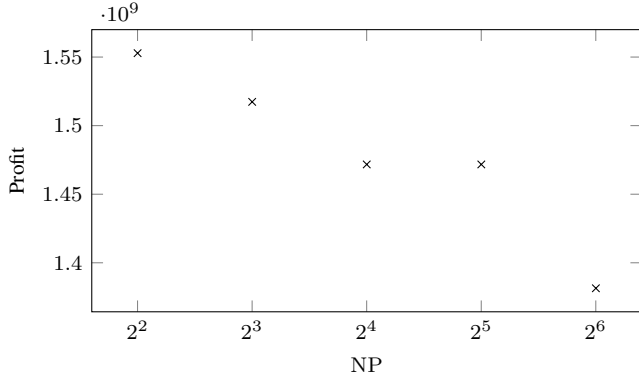up, the total profit extracted by the sellers π_S must go down, and vice versa.

Fig. 2. Plot of profitability data from multiple 100-day experiments in a market populated entirely by PRDE traders with $F = 0.8$. Horizontal axis is the number in population NP of each PRDE trader; vertical axis is the total profit extracted by all 30 PRDE traders. See text for further discussion.

Figure 3 and Figure 4 display the PPS for markets containing homogeneous PRDE traders with NP = 4 and NP = 64, respectively. The PPS was computed every $\Delta t = 3600$ seconds, by dividing the total profit a given trader achieved during the prior Δt seconds by Δt. When NP = 4, the PRDE traders in the market very quickly improved on their initial random s-values to manifestly extract more profit from the market. Conversely, when NP = 64, the PRDE traders take significantly longer.
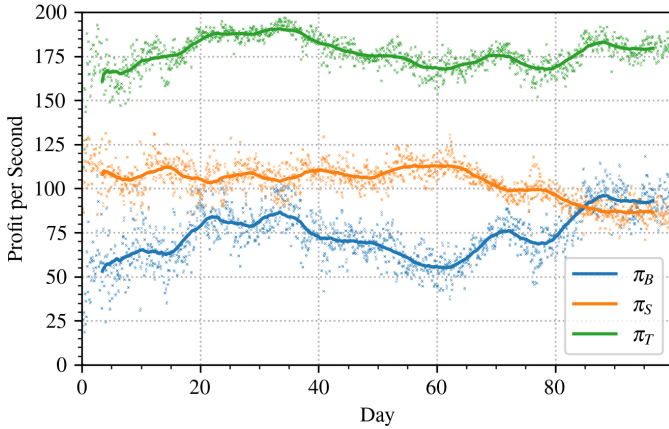

Fig. 3. Plot of profitability data from one 100-day experiment in a market populated entirely by PRDE traders with $F = 0.8$ and NP = 4. Horizontal axis is time, measured in days; vertical axis is PPS. Individual points represent the PPS over the previous Δt seconds; the line is a one-day simple moving average of these points. The line labelled π_S is the total PPS generated by the sub-population of buyers; the line labelled π_S generated by the sub-population of sellers; and the line labelled π_T is the total PPS extracted by all traders, such that $\pi_T(t) = \pi_B(t) + \pi_S(t)$. See text for further discussion.

Intuitively, this makes sense. For a given PRDE trader, the probability than an s-value in the trader's private population is selected to be evaluated next is NP^{-1}. In other words, it is inversely proportional to NP. Therefore, in homogeneous populations of PRDE traders with larger values of NP, it
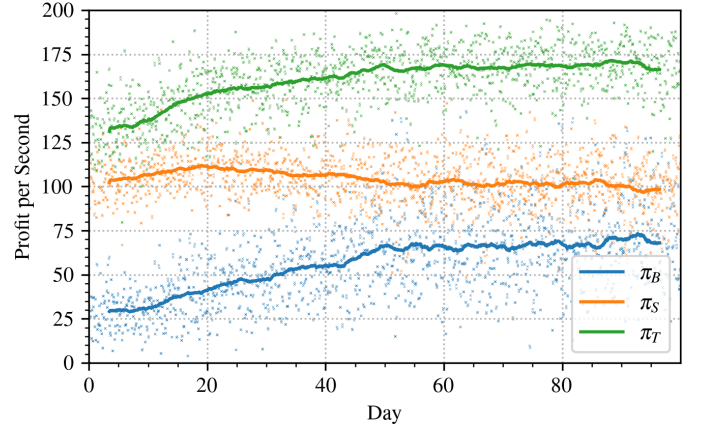

Fig. 4. Plot of profitability data from one 100-day experiment in a market populated entirely by PRDE traders with $F = 0.8$ and NP = 64. Format is the same as for Figure 3. See text for further discussion.

takes significantly longer to iteratively improve on the initial random conditions in the entire local population of s-values. As a result, the total PPS π_T extracted during the overall market session is lower. Markets containing homogeneous PRDE traders with larger private populations are less efficient.

C. Individual Effect of F

I conducted a very similar set of experiments to ascertain the effect the differential weight coefficient F has on the dynamic of a coevolutionary market. For each experiment I similarly used a homogenous population of PRDE traders; I kept the number in population constant at NP = 4 and changed the coefficient F between experiments. Figure 5 displays the relationship between the total profit extracted from the market and the differential weight coefficient F. This relationship is interesting: initially, as F increases from 0 the total profit slowly increases; the trend breaks when F is in the approximate range $[0.75, 0.95]$, whereby there is a sudden spike in profitability; above this range, and the profitability again reduces.
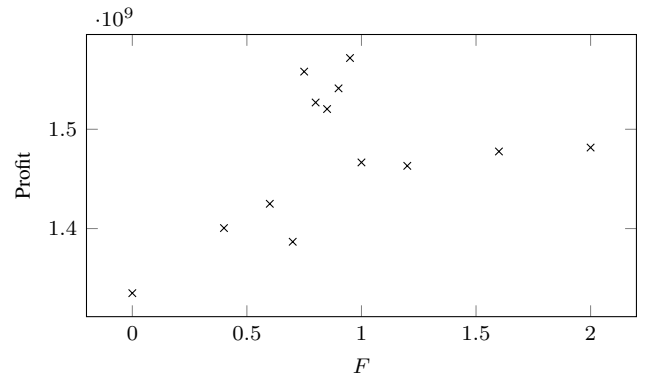

Fig. 5. Plot of profitability data from multiple 100-day experiments in a market populated entirely by PRDE traders with NP = 4. Horizontal axis is the differential weight coefficient F of each PRDE trader; vertical axis is the total profit extracted by all 30 PRDE traders. See text for further discussion.

This relationship can be safely attributed to the strong effect that F has on the distribution of s-values in the entire population of PRDE traders. Figure 6 displays a heatmap of the s-values used by the 15 PRDE buyers in a homogeneous market with $F = 0$. Throughout the 100-day experiment, the distribution of s-values remained approximately uniform in the range $[-1, 1]$. This was because when $F = 0$ the equation to derive a new candidate stratey $s_{i,y}$ to replace $s_{i,x}$ simply becomes $s_{i,y} \leftarrow s_{i,a}$. Therefore, for a given trader i, following the evaluation period of $s_{i,y}$, the value of $s_{i,x}$ can either remain the same, or take on the value of $s_{i,y} = s_{i,a}$, in which case two or more of the s-values in the local population will be identical—the 'genetic diversity' will be reduced. In fact, the only time a new s-value can be introduced into trader i's local population is when the diversity of s-values becomes so constrained a 'mega-mutation' occurs, in which case a new s-value is sampled from $\mathcal{U}(-1, 1)$. As a result, the distribution of s-values in the entire population of PRDE traders remains approximately uniform throughout the 100-day market session. This ultimately produces a market containing a wide range of both 'urgent' and 'relaxed' buyers and sellers, with individual PRDE traders unable to optimise their strategy values. The effect of this is an inefficient market. While the case of $F = 0$ is extreme, I found experimentally that the market increasingly exhibits the inefficient dynamics described here as F tends towards 0.
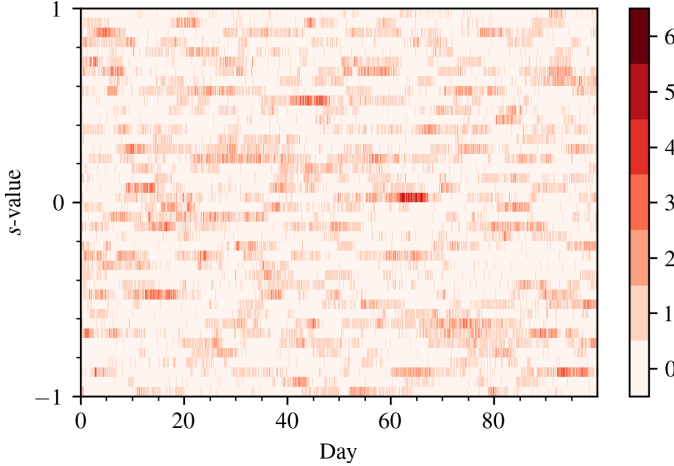

Fig. 6. Heatmap of individual strategy-values for the population of 15 PRDE buyers in a market populated entirely by PRDE traders with $F = 0$ and NP $= 64$. Horizontal axis is time, measured in days; vertial axis is the strategy value pixelated into 40 bins of size 0.05. The intensity of pixel shading increases with the number of PRDE sellers in the population currently trading with a strategy value in a given 0.05 range. See text for further discussion.

On the other extreme of the spectrum when $F = 2$, a very different market dynamic manifests. Figure 7 similarly displays a heatmap of s-values for the 15 PRDE buyers whereby $F = 2$. Unlike the uniform distribution of s-values exhibited when $F = 0$, the s-values were significantly bimodal at the two extremes of $s \approx -1$ and $s \approx 1$; moreso, both the buyers and sellers displayed this behaviour. This creates a market dynamic in which at each stage there is a number of

both extremely 'urgent' and extremely 'relaxed' buyers and sellers. Furthermore, I found experimentally that for larger values of F, this inefficient bimodal behaviour was evermore prominent.
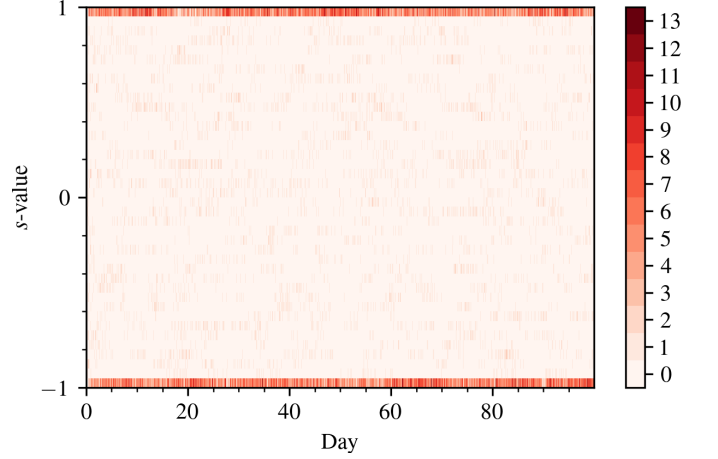

Fig. 7. Heatmap of individual stratey values for the population of 15 PRDE buyers in a market populated entirely by PRDE traders with $F = 2$ and NP $= 64$. Format is the same as for Figure 6. See text for further discussion.

The differential weight coefficient $F = 0.8$ produced a market with the most efficient dynamics. Figure 8 displays a heatmap of individual s-values for the population of 15 PRDE buyers in this market. Arguably, the distribution of the s-values in this market exhibits a degree of balance between the two extremes described previously. There is a clear dominant mode at $s \approx -1$. However, there is also a range of other s-values being trialled by the 15 buyers, most of which are greater than zero. The distribution of s-values for the population of 15 PRDE sellers was the reverse of this. There was a dominant mode at $s \approx 1$, with a range of other s-values, most of which were less than zero. This distribution appears to consistently produce a market with efficient dynamics: the total population of traders were able to extract more profit from the market.

D. Relationship Between F and NP

Thus far, I have evaluated the effects of the differential weight coefficient F and the number in population NP independently. However, there is a level of interplay between these two parameters that I also sought to investigate.

III. HETEROGENEOUS EXPLORATION OF F AND NP

Contemporary real-world markets are not comprised of homogeneous populations of automated trading-agents; real-world markets contain a multitude of different trading algorithms, all using different adaptive algorithms to maximise profitability, as well as a number of human traders.

IV. EXTENDING PRDE

REFERENCES

[1] D. Cliff, "Parameterised-response zero-intelligence traders," Mar. 2021. [Online]. Available: https://ssrn.com/abstract=3823317
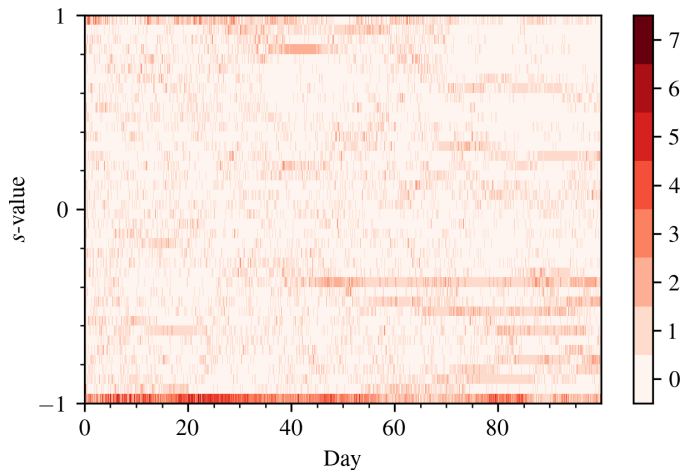
Fig. 8. Heatmap of individual stratey values for the population of 15 PRDE buyers in a market populated entirely by PRDE traders with $F = 0.8$ and NP $= 64$. Format is the same as for Figure 6. See text for further discussion.

[2] D. K. Gode and S. Sunder, "Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality," *Journal of Political Economy*, vol. 101, no. 1, pp. 119–137, 1993. [Online]. Available: http://www.jstor.org/stable/2138676

[3] D. Cliff, "Co-evolutionary dynamics in a simulation of interacting financial-market adaptive automated trading systems," in *European Modeling and Simulation Symposium (EMSS)*, Jul. 2022. [Online]. Available: https://ssrn.com/abstract=4154426

[4] ——, "Metapopulation differential co-evolution of trading strategies in a model financial market," in *European Modeling and Simulation Symposium (EMSS)*, Jun. 2022. [Online]. Available: https://ssrn.com/abstract=4153519

[5] R. Storn and K. Price, "Differential evolution –a simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, vol. 11, no. 4, pp. 341–359, 1997. [Online]. Available: https://doi.org/10.1023/A:1008202821328

[6] D. Cliff. Bristolstockexchange. [Online]. Available: https://github.com/davecliff/BristolStockExchange

[7] ——, "BSE: A minimal simulation of a limit-order-book stock exchange," *CoRR*, vol. abs/1809.06027, 2018. [Online]. Available: http://arxiv.org/abs/1809.06027