INFERENTIAL STATISTICS

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Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

1. What is the probability that a randomly chosen player would suffer an injury?

P(Randomly chosen player suffering an injury) = Total injured players / Total players

P(Randomly chosen player suffering an injury) = 145 / 235 = 0.617

Therefore, the probability that a randomly chosen player will suffer an injury is 61.7%

2. What is the probability that a player is a forward or a winger?

P(Randomly chosen player being a Forward or Winger) = (Total Forwards + Wingers)/ Total players

P(Randomly chosen player being a Forward or Winger) = 123 / 235 = 0.523

Therefore, the probability that a randomly chosen player is a Forward or Winger is 52.3%.

3. What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

P(Randomly chosen player being both a Striker and Injured) = Injured Strikers / Total players

P(Randomly chosen player being both a Striker and Injured) = 45 / 235 = 0.191

Therefore, the probability that a randomly chosen player is both a Striker and Injured is 19.1%.

4. What is the probability that a randomly chosen injured player is a striker?

P(Randomly chosen Striker suffering an injury) = Injured Strikers / Total Strikers

P(Randomly chosen Striker suffering an injury) = 45 / 145 = 0.310

Therefore, the probability that a Striker will be injured is 31.0%.

Problem 2

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

11% of the gunny bags have a breaking strength of less than 3.17 kg per sq cm

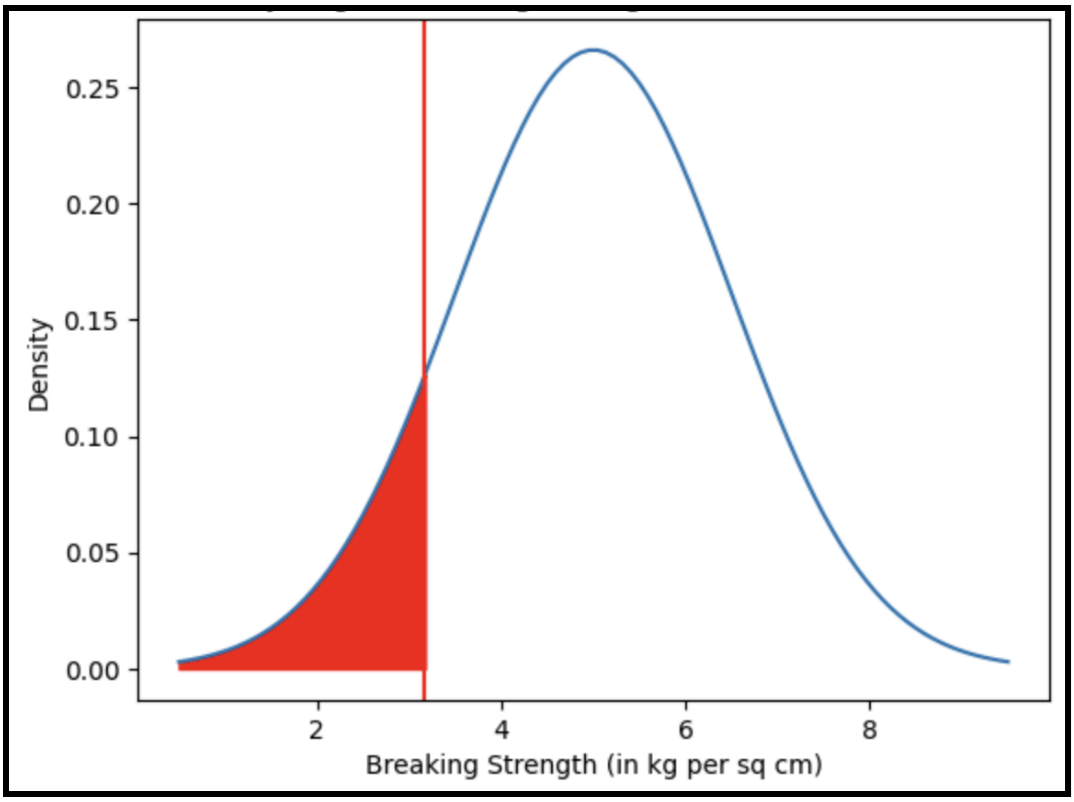

Fig 1: Gunny Bags - Breaking Strength < 3.17 kg per sq cm

2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?

82% of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.

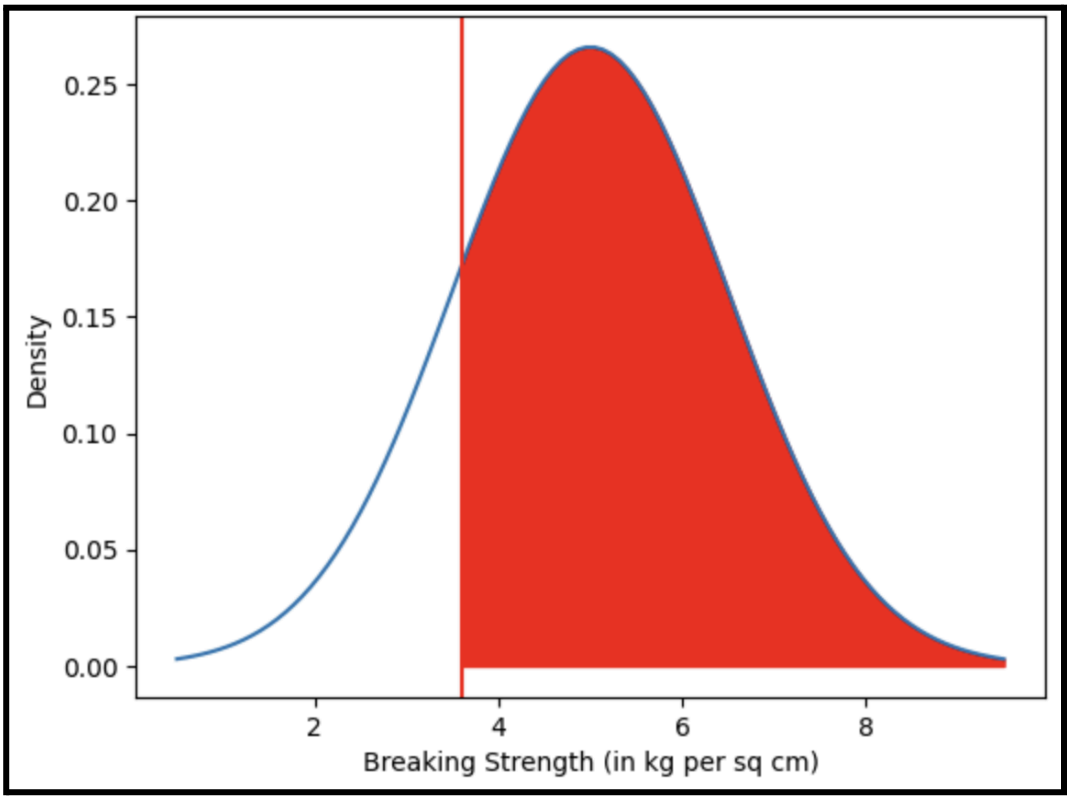

Fig 2: Gunny Bags - Breaking Strength >= 3.6 kg per sq cm

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

Only 13% of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm

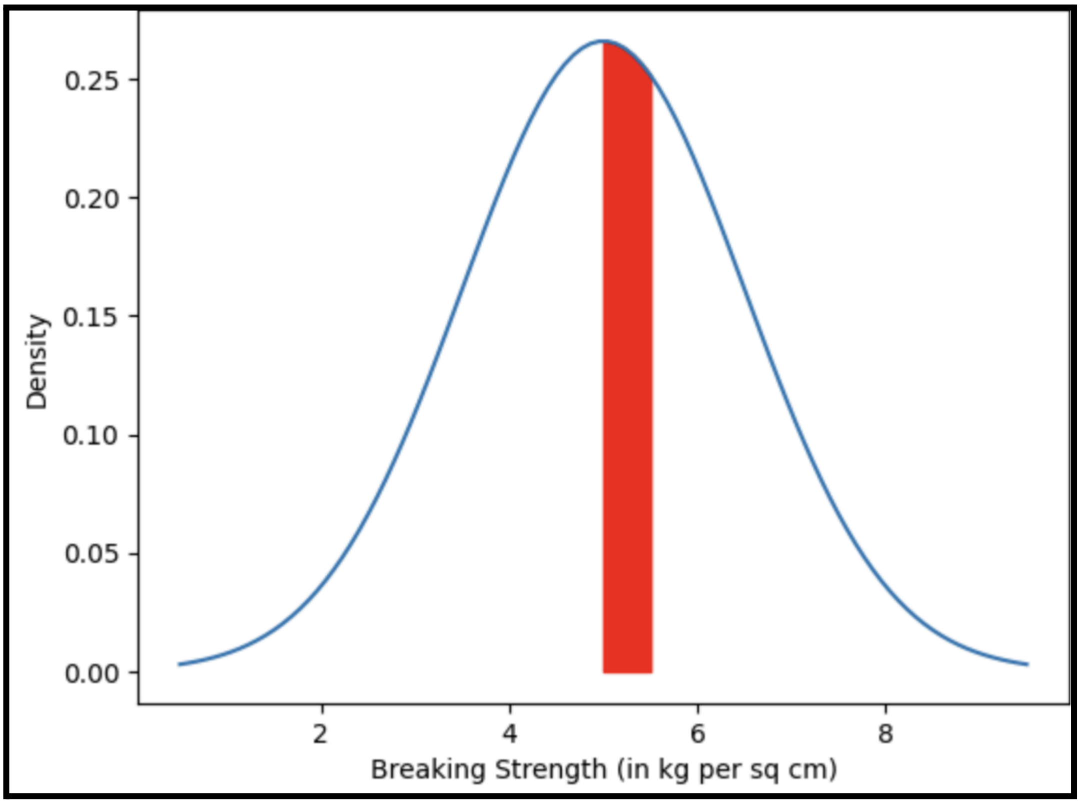

Fig 3: Gunny Bags - Breaking Strength >= 5 and <= 5.5 kg per sq cm

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm. ?

About 14% of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm

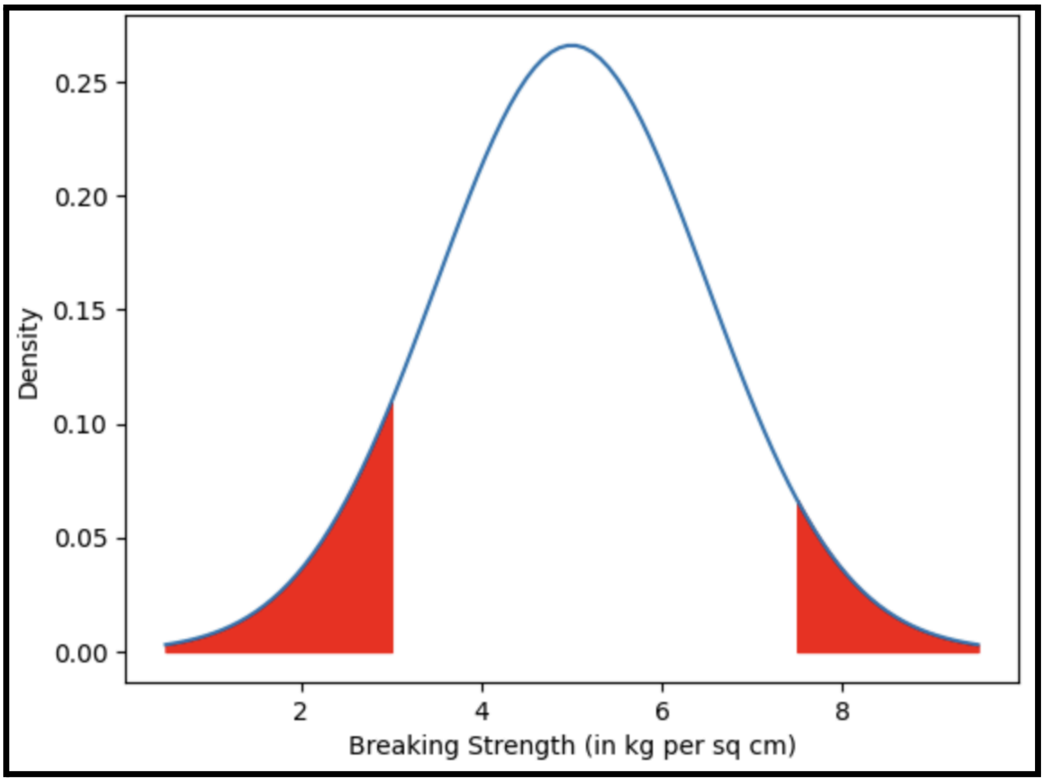

Fig 4: Gunny Bags - Breaking Strength > 3 and <= 7.5 kg per sq cm

Problem 3

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level)

Given : Level of Significance, α = 0.05

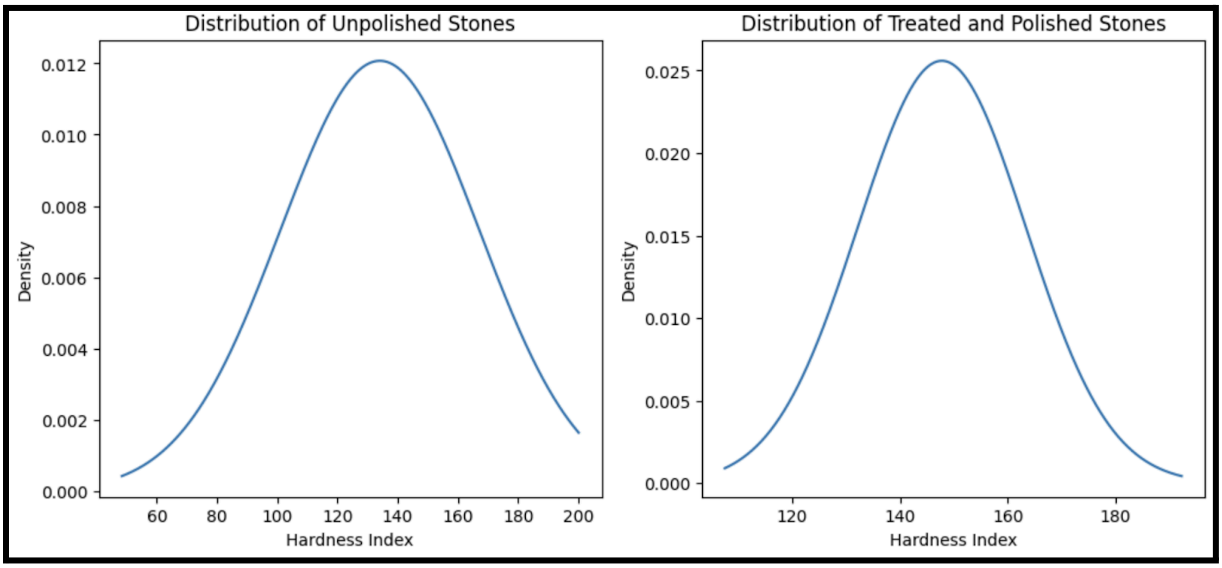

Fig 5: Normal Distribution of Polished and Unpolished Stones

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

<u>Null Hypothesis</u>, H_0: $\mu >= 150$

<u>Alternate Hypothesis</u>, H_a: $\mu < 150$

μ - mean hardness index of unpolished stones

Assumptions of One Sample T-test satisfied :

1. Sample size > 30, the distribution of sample means approaches normal distribution as per the Central Limit Theorem(CLT)
2. Randomly chosen data
3. Unknown Population standard variation
4. Continuous data - hardness index of the unpolished stones

Conclusions from One Sample T-test performed :

1. Since the p-value (0.00004) is far less than the significance level, we reject the null hypothesis.
2. The mean hardness index of unpolished stones is below 150, providing evidence that they are not suitable for printing.

3.2 Is the mean hardness of the polished and unpolished stones the same?

Let $\mu 1, \mu 2$ denote the mean hardness of the polished and unpolished stones respectively

Null Hypothesis, $H_0 : \mu 1 = \mu 2$

Alternate Hypothesis, $H_a : \mu 1 \neq \mu 2$

Observations :

1. The mean hardness index of the polished stones is : 147.79
2. The mean hardness index of the unpolished stones is : 134.11
3. The standard deviation of the polished stones is : 15.59
4. The standard deviation of the unpolished stones is : 33.04

Assumptions of Two Independent Sample T-test for Equality of Means (Unequal Standard Variance)

1. Continuous data - the hardness index of the stones
2. Normal Distribution is assumed as sample size > 30, the distribution of sample means approaches the normal distribution irrespective of the population distribution
3. Unequal Standard Variances
4. Independent populations and Random Sampling

Conclusions :

1. The p-value (0.0016) is much smaller than the significance level, so we reject the null hypothesis.
2. There is sufficient evidence to conclude that the mean hardness index of unpolished and polished stones is unequal.

Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

Assume : Level of Significance, α = 0.05

4.1 How does the hardness of implants vary depending on dentists?

For Alloy 1 :

1. Hypothesis, H_0 : Mean hardness of implants for Alloy 1 is equal across all dentists
2. Alternate Hypothesis, H_a : At least one of the doctors have different mean hardness for Alloy 1
3. Assumptions of One Way ANOVA :
    a. Population is normally distributed
    b. Population variances are equal
4. Conclusion :
    a. The p-value (0.1166) is greater than the significance level, so we fail to reject the null hypothesis.
    b. There is no significant difference in the mean hardness of Alloy 1 implants across the five doctors.

For Alloy 2 :

1. Hypothesis, H_0 : Mean hardness of implants for Alloy 2 is equal across all dentists
2. Alternate Hypothesis, H_a : At least one of the doctors have different mean hardness for Alloy 2

3. Assumptions of One Way ANOVA :

   a. Population is normally distributed
   b. Population variances are equal

4. Conclusion :

   a. The p-value (0.71803) is greater than the significance level, so we fail to reject the null hypothesis.
   b. There is no significant difference in the mean hardness of Alloy 2 implants across the five doctors.

4.2 How does the hardness of implants vary depending on methods?

<u>For Alloy 1 :</u>

1. Hypothesis, H_0 : Mean hardness of implants for Alloy 1 is equal across all Methods
2. Alternate Hypothesis, H_a : At least one of the Methods have different mean hardness for Alloy 1
3. Assumptions of One Way ANOVA :
   a. Population is normally distributed
   b. Population variances are equal
4. Conclusion :
   a. The p-value (0.0041634) is less than the significance level, so we reject the null hypothesis.
   b. At least one of the methods have different mean hardness for Alloy 1
5. Methods for which Mean Hardness differs - Multi Comparison Pairwise Tukey HSD:
   a. Mean hardness of Methods 1 and 2 are similar
   b. However, the mean hardness of Method 3 differs from the hardness of the other methods

For Alloy 2 :

1. Hypothesis, H_0 : Mean hardness of implants for Alloy 2 is equal across all Methods
2. Alternate Hypothesis, H_a : At least one of the Methods have different mean hardness for Alloy 2
3. Assumptions of One Way ANOVA :
   a. Population is normally distributed
   b. Population variances are equal

4. Conclusion :

   a. The p-value (5.415871e-06) is less than the significance level, so we reject the null hypothesis.

   b. At least one of the methods have different mean hardness for Alloy 2

5.Methods for which Mean Hardness differs - Multi Comparison Pairwise TukeyHSD

   a. Mean hardness of Methods 1 and 2 are similar
   b. However, the mean hardness of Method 3 differs from the hardness of the other methods

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

For Alloy 1 : There is some interaction effect between Dentists and Method used for Alloy 1 as the plot shows crossing lines

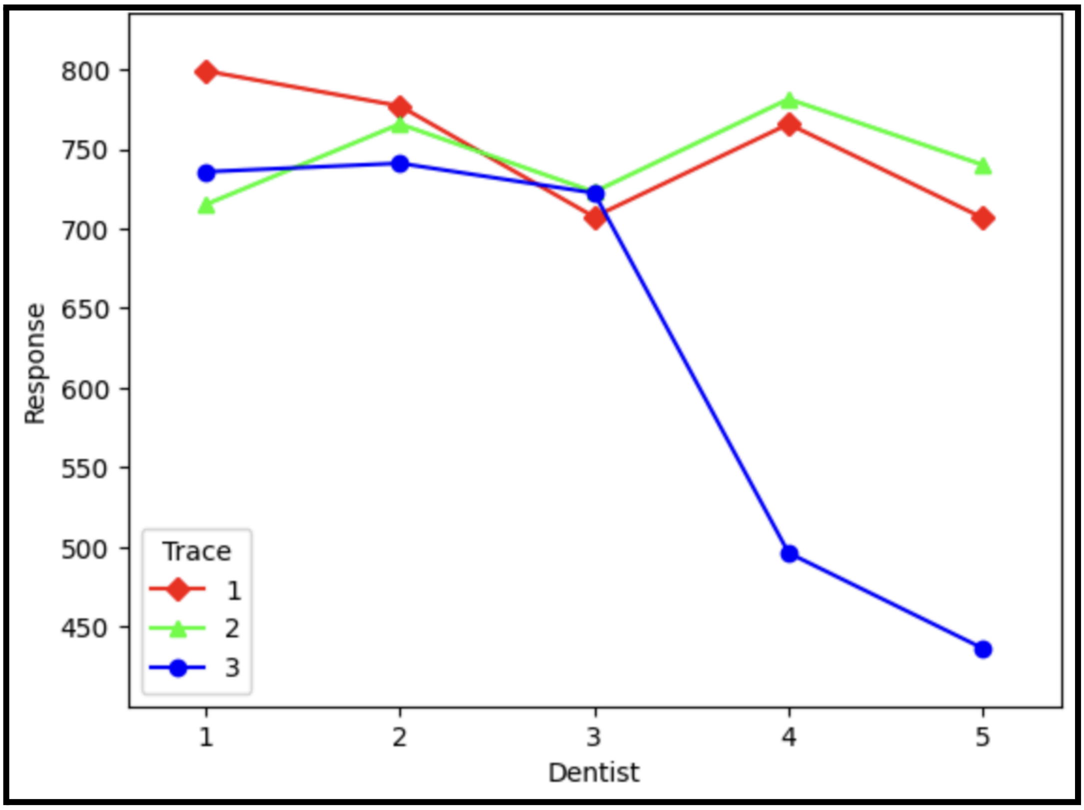

Fig 6 : Interaction Effect of Dentist and Method (Alloy 1)

For Alloy 2 :

a. Method 3 yields lower hardness of implants for all Dentists
b. Methods 1 and 2 show some interaction effect between Dentists and Method

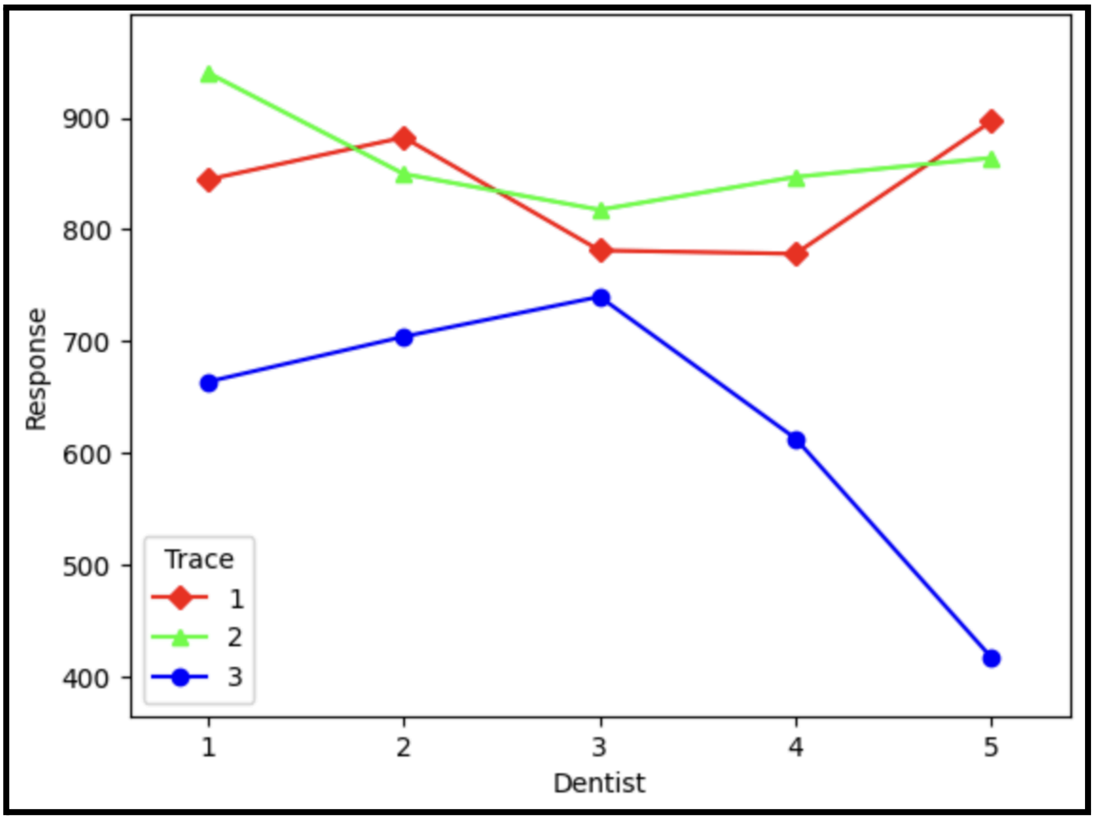

Fig 7 : Interaction Effect of Dentist and Method (Alloy 2)

4.4 How does the hardness of implants vary depending on dentists and methods together?

<u>For Alloy 1 :</u>

1. Null Hypothesis, H_0 : There is no interaction effect between Dentists and Method used on the mean hardness of the implants.ie, the effect of Dentists on the mean hardness of the implants does not depend on the effect of the Method variable for Alloy 1
2. Alternate Hypothesis, H_a : There is an interaction effect between Dentists and Method used on the mean hardness of the implants for Alloy 1
3. Assumptions for the two-way ANOVA Test : As the two-way ANOVA is a type of linear model we need to satisfy pretty much the same assumptions as we did for a one-way ANOVA
    a. Shapiro Wilk's Test For Normality Assumption of ANOVA
    b. Levene's Test for Equality of Variance in ANOVA

4. The formula : ***Response ~ C(Dentist) + C(Method) + C(Dentist):C(Method)*** .
   It builds and trains a linear regression model that explains how Response changes with Dentist, Method, and their interaction.
5. Conclusion :
   a. As the p-value (0.00679274) is less than the level of significance, we reject the null hypothesis.
   b. There is an interaction effect between Dentists and Method used on the mean hardness of the implants for Alloy 1

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

Fig 8: Interactions Output for Alloy 1

<u>For Alloy 2 :</u>

1. Null Hypothesis, H_0 : There is no interaction effect between Dentists and Method used on the mean hardness of the implants.ie, the effect of Dentists on the mean hardness of the implants does not depend on the effect of the Method variable for Alloy 2
2. Alternate Hypothesis, H_a : There is an interaction effect between Dentists and Method used on the mean hardness of the implants for Alloy 2
3. Assumptions for the two-way ANOVA Test : As the two-way ANOVA is a type of linear model we need to satisfy pretty much the same assumptions as we did for a one-way ANOVA
   a. Shapiro Wilk's Test For Normality Assumption of ANOVA
   b. Levene's Test for Equality of Variance in ANOVA
4. The formula : ***Response ~ C(Dentist) + C(Method) + C(Dentist):C(Method)*** .
   It builds and trains a linear regression model that explains how Response changes with Dentist, Method, and their interaction.

5. Conclusion :
    a. As the p-value (0.0932340) is greater than the level of significance, we fail to reject the null hypothesis.
    b. No interaction effect between Dentists and Method used on the mean hardness of the implants for Alloy 2

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

Fig 9: Interactions Output for Alloy 2