Financial Economics

8 Empirics and the Puzzles

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Overview

Arrow-Debreu
general equilibrium,
welfare theorem,
representative agent

Radner economies
real/nominal assets,
market span,
risk-neutral prob.,
representative good

von Neumann Morgenstern
measures of risk
aversion, HARA class

Finance economy
SDF, CCAPM,
term structure

Data and the Puzzles

Empirical resolutions

Theoretical resolutions

Introduction

- In this chapter you will be told that the theory you have learned so far fails miserably when confronted with the data!
- The empirical failure of the model, whose most famous incarnation is the *equity premium puzzle* , was a great disappointment for the profession
- The puzzles have generated a rich research effort that has produced many new and interesting ideas

Collecting the Right Data

- Risk-free Rate ρ
  - Examples: Treasury bills, inflation-indexed government bonds
  - Real risk-free rate: Nominal rate adjusted for inflation
  - Equation: $r_f = \frac{1+r_{nominal}}{1+\pi_{inflation}} - 1$
  - Issues: Default risk, inflation risk
- Stock indices: capital versus wealth R
  - Capital Index: Weighted average of stock prices
  - Wealth Index: Capital index with dividends reinvested
  - Major indices: S&P 500, FTSE 100, Nikkei, DAX, CAC 40
  - Calculation of returns: $R = \frac{P_t + D_t - P_{t-1}}{P_{t-1}}$
  - Importance of dividends in total return

Endowment or output or consumption? w and g

- Our asset pricing theory explains saving and risk taking as "consumption based"
  - What do we mean by "consumption"?
  - Do things other than consumption enter utility as well?
- Consumption includes non-durable goods, durable-goods, government consumption, etc...
- Consumption is not the only element that enters people's preferences: leisure? a composite of consumption growth and leisure?
- It has become accepted practice, for the purpose of empirical asset pricing, to identify consumption with the expenses for non-durable consumption goods

Some Stylized Facts

- Mehra & Prescott (1985) is a seminal work that tests the CCAPM
  - Risk-free nominal interest rate: yield of 3-month government Treasury bills
  - Mehra & Prescott use the annual averages of the Standard & Poor 500
  - Endowment (consumption) growth is measured as the growth rate of real consumption of non-durables and services divided by total population
- Real per capita consumption grows 1.83% per year on average, with a standard deviation of 3.57%
- The real risk-free return rate is 0.80% on average, with a standard deviation of 5.67%
- The real return rate on the risky asset is 6.98%, with a standard deviation of 16.54%

Equity Premium Puzzle

- The benchmark model of utility is the CRRA specification

$$\ln \rho \approx \gamma E\{g\} - \ln \delta$$

$$E\{R^j\} - \rho \approx \gamma^* cov(g, R^j)$$

with $\gamma^* := \frac{\gamma}{1-\gamma E\{g\}}$

- The equation with respect to the risk-free rate is actually compatible with the real world data
- The problem lies with equation with respect to the risk premium

Equity Premium Puzzle

Table 7.1. *Relative risk aversion implied by equation (5.38).*

country	sample	ρ	$E\{g\}$	γ (a)	γ (b)
Australia	1970.1 to 1998.4	1.02054	2.071%	0.98	0.98
Canada	1970.1 to 1999.1	1.02713	2.170%	1.23	1.14
France	1973.2 to 1998.3	1.02715	1.212%	2.21	1.62
Germany	1978.4 to 1997.3	1.03219	1.673%	1.89	1.52
Italy	1971.2 to 1998.1	1.02371	2.273%	1.03	1.01
Japan	1970.2 to 1998.4	1.01388	3.233%	0.43	0.57
Netherlands	1977.2 to 1998.3	1.03377	1.671%	1.99	1.58
Sweden	1970.1 to 1999.2	1.01995	1.001%	1.97	1.45
Switzerland	1982.2 to 1998.4	1.01393	0.559%	2.47	1.48
U.K.	1970.1 to 1999.1	1.01301	2.235%	0.58	0.71
U.S.A.	1970.1 to 1998.3	1.01494	1.802%	0.82	0.88
U.S.A.	1947.2 to 1998.3	1.00896	1.951%	0.46	0.65

Note: Column (a) reports the maximum coefficient of relative risk aversion that is compatible with (5.38), assuming that $\delta \leqslant 1$. Column (b) reports the same assuming that true inflation is 1.1% smaller than measured, as suggested by the Boskin report. The data for ρ and $E\{g\}$ are taken from table 5 of Campbell (2003).

Equity Premium Puzzle

$$\ln \rho \approx \gamma E\{g\} - \ln \delta$$

$$E\{R^j\} - \rho \approx \gamma^* cov(g, R^j)$$

- The results establish that risk-free rate equation requires coefficients of relative risk aversion of between 0.4 and 2.5
- Given the low covariance between the return rate of equity and aggregate consumption, we need to a huge γ^*
- But again, this suggests a very large risk-free rate
- The bottom line is this: either we assume that γ is moderately small, in which case risk premium equation fails, or we assume that γ is very large, in which case risk-free rate equation fails
- The true puzzle is that there is no γ that satisfies these two equations simultaneously

Equity Premium Puzzle

- Alternatively, we could accept a large coefficient of risk aversion, then choose an appropriate δ to fit the data of risk-free rate

$$\ln \delta = \gamma E\{g\} - \rho$$

- But this requires $\ln \delta > 0$, hence $\delta > 1$
- An agent with$\delta > 1$ is more than infinitely patient: he actually likes waiting!

Mehra and Prescott's Binomial Formulation

- Mehra & Prescott (1985) use the standard general equilibrium model and assume constant relative risk aversion
- They simplify the model by assuming that there are only two states of the world, each with equal probability ($\pi_1 = \pi_2 = 1/2$)
- They calibrate the growth rates of these two states so that the mean and the variance of their binomial model fits the empirical mean and variance of U.S. per capita consumption growth
  - $g_1 \coloneqq +5.4\%, g_2 \coloneqq -1.8\%$
- With only two states, we can easily compute the exact solutions:

$$\rho = \frac{1}{E\{M\}} = \frac{1}{\delta[(1+g_1)^{-\gamma} + (1+g_2)^{-\gamma}]/2}$$

Mehra and Prescott's Binomial Formulation

- Besides the risk-free bond, Mehra and Prescott consider only one additional asset: a very broadly defined "equity" —a Lucas tree— whose cash flow equals state-contingent per capita consumption, $r_s := w^s$

$$q = \delta E\left\{\frac{w^{-\gamma}}{(w^0)^{-\gamma}}w\right\} = \delta E\{(1+g)^{1-\gamma}\}w^0$$

- The expected return rate is

$$E\{R\} = \frac{E\{1+g\}w^0}{\delta E\{(1+g)^{1-\gamma}\}w^0} = \frac{1+(g_1+g_2)/2}{\delta[(1+g_1)^{-\gamma}+(1+g_2)^{-\gamma}]/2}$$

- Then, we can compute the equity premium

$$E\{R\} - \rho = \frac{(g_1+g_2)/2}{\delta[(1+g_1)^{-\gamma}+(1+g_2)^{-\gamma}]/2}$$

Mehra and Prescott's Binomial Formulation

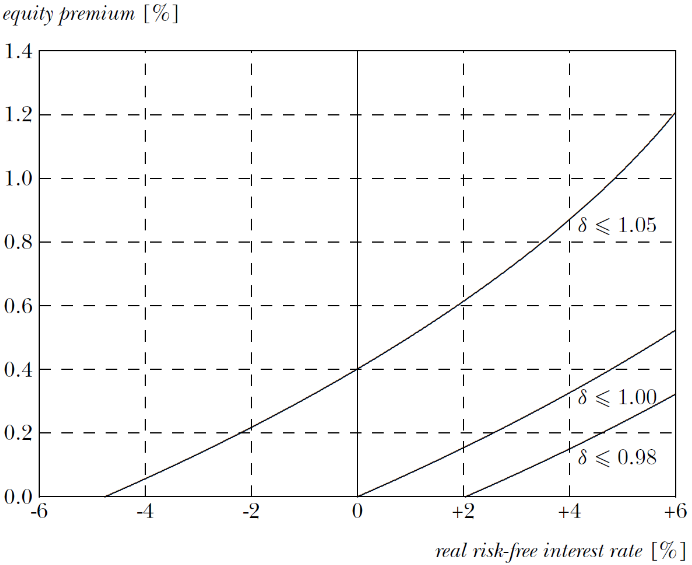

Figure 7.1. *Mehra and Prescott's (1985) plot (based on model without serial correlation).*

Equity Premium Puzzle

- Why is a risk premium of 6.2% per year "too much"?
- Requiring a risk premium of 6.2%, given an annual standard deviation of returns for stocks of 16.5% implies a relative risk aversion coefficient for the representative investor of more than 30
  - A person who has a starting wealth of zero, having the utility function posited by Mehra and Prescott and a coefficient of relative risk aversion of 30, would, when faced with an uncertain gain that involves
    - A payout of \$50,000 with 50% probability
    - A payout of \$100,000 with 50% probability
  - and the choice of receiving \$51,209 for certain, would select the certain sum over the uncertain game!
  - Starting with a wealth level of \$500K, the payout at which the same individual would prefer the certain outcome would be \$61,808. A more reasonable amount, but still seems too low.
  - Few people can be this afraid of risk

Hansen–Jagannathan Bounds

- Hansen & Jaganathan (1991) use the CCAPM to put a bound on the volatility of the SDF

$$cov(-M, R^j) = \sigma_M \cdot \sigma_{R^j} \cdot corr(-M, R^j)$$

- From this and the CCAPM pricing formula, we have

$$\frac{E\{R^j\} - \rho}{\sigma_{R^j}} = corr(-M, R^j) \cdot \frac{\sigma_M}{E\{M\}} \leq \frac{\sigma_M}{E\{M\}} = \rho\sigma_M$$

- The left-hand side is the Sharpe ratio. The large Sharpe ratio we observe in the market implies a large volatility of the SDF
- To a first-order approximation, σ_M equals $\gamma\sigma_g$
- According to the Mehra–Prescott data, $\rho = 1.008$ and $\sigma_g = 3.57\%$, but the Sharpe ratio of the risky asset is $6.18\%/16.54\% = 0.37$. Thus, $\gamma \approx 10$ is required to make Hansen–Jagannathan bounds compatible with the data

Siegel's Extended Sample

- Siegel (1992) has argued that Mehra & Prescott' s sample from 1889 to 1979 covered an exceptional period
- Siegel extends the period back to 1802
  - Over 1802-1870, 1871-1925, and 1926- 1990, real compound equity returns were 5.7, 6.6, and 6.4 percent, respectively
  - However, returns on short-term government bonds have fallen dramatically, the figures for the same three time periods being 5.1, 3.1, and 0.5 percent
  - There was no equity premium in the first two-thirds of the nineteenth century (because bond returns were high), but over the last 120 years, stocks have had a significant edge
- He argues that the risk-free interest rate was substantially lower between 1926 and 1980 than the long-term historical average owing to unanticipated inflation after World War II and in the 1970s

Unobserved Disaster State

- The equity premium puzzle basically says that the price of risk is much higher than what can be justified by our beliefs about the behavior of people facing
- Rietz (1988) advanced the idea that there is a disaster state in which endowment falls dramatically. This state, however, has a very small probability, which is why it was not observed uncertainty
  - But his third "disaster" state is always very extreme
  - The representative agent has a coefficient of relative risk aversion somewhat in excess of 5 (still high!); he is rather impatient ($\delta < 0.9$); and every year, with 0.3% probability, 50% of output is destroyed

Survival Bias

- Significant catastrophes, however, have been observed outside the United States
- In hindsight, investing in American stocks was very profitable
  - Siegel and Thaler (1997) :
    "Suppose your great grandmother had some money lying around at the end of 1925 and, with rational expectations, anticipated your birth and decided to bequeath you $1000. Naturally, since you weren't born yet, she invested the money, and being worried about the speculative boom in stocks going on at the time, she put the money in Treasury bills, where it remained until December 31, 1995. On that date it was worth $12,720.
    Imagine, instead that she had invested the money in a (value-weighted) portfolio of stocks. You would now have $842,000, or 66 times as much money."

Survival Bias

- Siegel (1998) has constructed return series for British, German, and Japanese stocks for the period of 1926–1997
- Comparing this with the performance of American stocks, he concludes that the performance of German stocks fell 60 basis points short of American ones, that of British stocks was 100 basis points, and that of Japanese stocks 380 basis points worse than American stocks. This clearly justifies the hypothesis of a survival bias
- But Siegel also claims that the survival bias cannot explain why the true equity premium should be overestimated by the historic data

Follow-on Studies

- Ilmanen (2011) –Sample includes post-2000 weak period for equities

Table 8.1. Compound average U.S. equity returns and equity premia over 200+ years

	Nominal equity market return GM (and AM) (%)		*Real equity market return GM* (%)	*Equity premium vs. cash (ERPC) GM* (%)	*Equity premium vs. bond (ERPB) GM (and AM)* (%)	
1802–2009	7.90	(9.42)	6.33	NA	2.68	(4.17)
1802–1899	6.00	(7.01)	6.21	NA	0.50	(1.42)
1900–1999	10.75	(12.68)	7.47	6.37	5.93	(7.79)
2000–2009	0.95	(0.28)	−3.38	−3.65	−7.21	(−5.23)
1926–2009	9.94	(12.22)	6.70	5.99	4.54	(6.87)
1960–2009	9.52	(10.70)	5.22	3.90	2.38	(3.51)

Sources: Arnott–Bernstein (2002), Bloomberg.
Note: GM = Geometric mean, AM = Arithmetic mean.

Forward-looking Return Rates

- Our asset pricing relationships concern expected values
  - the risk-free interest rate is a function of expected consumption growth
  - the expected equity premium is a function of the covariance of consumption growth and the stochastic discount factor
  - Yet, we test these relationships work with historical averages
- One reason for the puzzle may therefore be that the historical average equity premium is an imprecise or even distorted proxy for the expected premium
- One obvious way of measuring expectations is by asking professionals
  - Surveys performed between October 1997 and November 1999 by the New York Times, Fortune, and Gallup/Paine-Webber report expected premia of between 16% and 22%!
  - Surveys on academic financial economists around 1997 report the 30-year horizon equity premium was largely between 4% and 11% (the minimum was -2%, the maximum 15%), and the median was 7.1%

Three important points by Mehra (2011) on the Puzzle

- The risk-free asset for a typical investor is not T-bills
  - If one uses longer-term maturity Treasury securities such as TIPS or mortgage-backed securities backed by housing agencies (GNMA, etc.) the premium narrows by roughly 2 percent!
- The marginal investor is likely to be a borrower, not a lender
  - Think of a household with a mortgage balance. Its opportunity cost when judging investing a marginal dollar of savings into stocks is the rate on its mortgage loan, which is on average 2% higher than the lending rate that the household is faced with
  - So points 1 and 2 combined narrow the equity premium by close to 4%!

Three important points by Mehra (2011) on the Puzzle

- The premium should depend on the life cycle stage of the marginal investor
  - Young people looking forward at the start of their lives have uncertain future wage and equity income; furthermore, the correlation of equity income with consumption will not be particularly high as long as stock and wage income are not highly correlated
  - For the middle-aged, wage uncertainty has largely been resolved. Their future retirement wage income is either zero or deterministic, and the innovations (fluctuations) in their consumption occur from fluctuations in equity income. At this stage of the life cycle, equity income is highly correlated with consumption

Empirical Failures and Anomalies

- Researchers have uncovered an impressive collection of even stranger patterns in financial market data
- Anomalies: Patterns in financial data that deviate from standard theory
- Examples:
  - Weekend effect: Negative returns from Friday to Monday
  - Size effect: Small-cap stocks outperform large-cap stocks
  - January effect: Abnormal returns in January
- Such patterns seem difficult or even impossible to reconcile with the mainstream model of rational behavior, because they seem to offer arbitrage opportunities
- Limits to arbitrage: Noise trader risk
- Data mining: long-term reliability of anomalies

Anomalies and Market Efficiency

- Market efficiency hypothesis: Prices reflect all available information
- Challenges from anomalies: Arbitrage opportunities
- Limited arbitrage: Noise trader risk, implementation costs
- Example studies: French (1980) on weekend effect, Banz (1981) on size effect
- Future research directions: Data mining vs. genuine patterns

Remedies?

Arrow-Debreu
general equilibrium,
welfare theorem,
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Radner economies
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Data and the Puzzles

Empirical resolutions

Theoretical resolutions