Financial Economics

9 Adapting the Theory

LEC, SJTU

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Overview

Arrow-Debreu
general equilibrium,
welfare theorem,
representative agent

Radner economies
real/nominal assets,
market span,
risk-neutral prob.,
representative good

von Neumann Morgenstern
measures of risk aversion, HARA class

Finance economy
SDF, CCAPM,
term structure

Data and the Puzzles

Empirical resolutions

Theoretical resolutions

Mainstream Model Assumptions

1. Markets work without frictions: no transaction costs, no bid-ask spreads, law of one price holds
2. All agents are price takers: prices ensure all markets clear
3. Rational expectations:
    - The data we observe are generated by a Radner equilibrium
    - There is a representative commodity (wealth)
4. Financial markets are complete or quasi-complete
    - There is a representative agent with ordinal utility
5. Time-separable NM expected utility maximization
6. Common beliefs about probability distributions
    - There is a representative NM agent
7. Common discount factor
8. HARA utility with common slope parameter
    - The representative agent, and thus the equilibrium prices, are independent of the inter-personal distribution (Rubinstein aggregation)
9. Moderate coefficient of relative risk aversion
10. CRRA utility function

Mainstream Model Assumptions

- The assumptions that have drawn most attention for being potentially responsible for the empirical failure of the model are 1, 4, and 5, but other assumptions such as 7 or 8 could probably also be held responsible
- Fundamentally, what we need to justify a larger equity premium is more volatility of the SDF. This can be achieved by
  - amending the utility function (somehow people are more risk averse than we think)
  - arguing that individual consumption is more volatile than suggested by aggregate data (people bear some undiversified idiosyncratic risk)
  - breaking the close link between the SDF and asset prices by arguing that the simple firstorder condition captured by the SDF does not hold because of frictions

Non-standard Preferences

- Departure from standard time- and state-separable power utility function
- Non-HARA utility
- Separating cross-state from intertemporal substitution
- Utility with habits
- First-order risk aversion
- Prospect theory

Non-HARA Utility

- Rubinstein's aggregation suggests that the distribution has no effect because the risk tolerance of the representative agent is unaffected by a change of the inter-personal income distribution
  - common cautiousness b
- Inter-personal income distribution can affect asset prices
- Gollier's contributions:
  - Assumes absolute risk tolerance is a concave function of wealth
  - Average risk tolerance is smaller than the risk tolerance of a person with average wealth
  - Greater inequality decreases the representative agent' s risk tolerance, and thus increases the market price of risk
- Gollier estimates that this effect alone may double the theoretical equity premium

Non-HARA Utility

- The effect of the wealth distribution on the risk-free interest rate is more difficult to evaluate
- By assuming that absolute imprudence (the reciprocal of absolute prudence) is a concave function of wealth as well, a mean preserving spread of the inter-personal income distribution reduces the risk-free rate

Separating Cross-State from Intertemporal Substitution

- Elasticity of substitution in production theory
- Application to utility maximization problems
- Epstein-Zin preferences:
  - Separation of risk aversion from intertemporal substitution
  - Implications for equity premium and risk-free rate puzzles

Elasticity of Substitution - Formula

$$\frac{1}{\varepsilon} := -\frac{d\ln(K/L)}{d\ln(\partial_2 f(L,K)/\partial_1 f(L,K))}$$

- Used to measure substitutability of two production factors
- If the relative price of labor and capital changes by 1%, the profit-maximizing capital–labor ratio changes by a factor $\frac{1}{\varepsilon}$
- Can be applied to utility maximization problems

Utility Function Example

- Consider a two-period problem without uncertainty and with additively separable preferences and CRRA function

$$V(y_0, y_1) := \frac{y_0^{1-\gamma}}{1-\gamma} + \delta \frac{y_1^{1-\gamma}}{1-\gamma}$$

- Elasticity of substitution is

$$\frac{1}{\varepsilon} = -\frac{d\ln(y_1/y_0)}{d\ln(\partial_2 V(y_0, y_1)/\partial_1 V(y_0, y_1))}$$

- We have the elasticity of intertemporal substitution $\varepsilon = \gamma$
- Restrictive nature of mainstream specification
  - One can be only moderately averse to cross-state risk (small γ), but at the same time be very averse to intertemporal fluctuations of consumption (largeε), which is consistent with empirical evidence

Epstein-Zin Preferences

- Consider the Epstein-Zin approach. Now R.A. has the following utility function

$$V(y) := u(y_0) + \delta u(v^{-1}(E\{v(y)\}))$$

  - Unlike the standard case, the argument that is used to evaluate future utility is the certainty equivalent of future state-contingent consumption, computed using a different utility function v
  - v captures risk aversion, while u captures intertemporal substitution
- $u(z) := \frac{z^{1-\varepsilon}}{1-\varepsilon}$
- $v(z) := \frac{z^{1-\gamma}}{1-\gamma}$
- γ is coefficient of relative risk aversion
- $1/\varepsilon$ is elasticity of intertemporal substitution

Epstein-Zin Preferences

- These preferences give rise to an SDF of the following form:

$$
\begin{aligned}
M_s &= \delta \overbrace{\frac{u'(v^{-1}(E\{v(y)\}))v'(y_s)}{v'(v^{-1}(E\{v(y)\}))}}^{[\partial V(y)/\partial y_s]/\pi_s} \frac{1}{u'(y_0)} \\
&= \delta[v^{-1}(E\{v(y)\})]^{-\varepsilon+\gamma} y_s^{-\gamma} y_0^{\varepsilon} \\
&= \delta E\{y^{1-\gamma}\}^{\frac{\gamma-\varepsilon}{1-\gamma}} y_s^{-\gamma} y_0^{\varepsilon} \\
&= \delta E\{(1+g)^{1-\gamma}\}^{\frac{\gamma-\varepsilon}{1-\gamma}} y_0^{\gamma-\varepsilon} y_s^{-\gamma} y_0^{\varepsilon} \\
&= \delta E\{(1+g)^{1-\gamma}\}^{\frac{\gamma-\varepsilon}{1-\gamma}} (1+g_s)^{-\gamma}.
\end{aligned}
$$

Epstein-Zin Preferences

- Approximately,

$$-\ln M_s \approx \varepsilon E\{g\} + \gamma(g_s - E\{g\}) - \ln \delta$$

- The risk-free interest rate is

$$\ln \rho \approx -E\{\ln M\} \approx \varepsilon E\{g\} - \ln \delta$$

Epstein-Zin Preferences

- The CCAPM formula becomes

$$\begin{aligned} E\{R^j\} - \rho &= \rho \mathrm{cov}(-M, R^j) \\ &= \rho\delta \underbrace{E\{(1+g)^{1-\gamma}\}^{\frac{\gamma-\varepsilon}{1-\gamma}}}_{\approx e^{(\gamma-\varepsilon)E\{g\}}} \underbrace{\mathrm{cov}(-(1+g)^{-\gamma}, R^j)}_{\approx \gamma \mathrm{cov}(g, R^j)}. \end{aligned}$$

- Apply the approximation $\rho \approx e^{\varepsilon E\{g\}}\delta^{-1}$, we have

$$E\{R^j\} - \rho \approx \gamma^{**} cov(g, R^j)$$

with

$$\gamma^{**} := \gamma e^{\gamma E\{g\}}$$

Habits

- We develop standards and habits about consumptions that we are not easily willing to give up
- Consider the standard habit specification

$$v(y, x) := \frac{z(y, x)^{1-\gamma}}{1-\gamma}$$

  where z is a function, y is consumption, and x is some habit level. $\partial_1 v > 0, \partial_2 v < 0$

- This implies that the SDF is given by

$$M_s = \delta \left(\frac{z(y_s, x_s)}{z(y_0, x_0)} \right)^{-\gamma} \times \frac{\partial_1 z(y_s, x_s) + \partial_2 z(y_s, x_s) \frac{\partial x_s}{\partial y_s}}{\partial_1 z(y_0, x_0) + \partial_2 z(y_0, x_0) \frac{\partial x_0}{\partial y_0} + \delta z(y_{\dot{s}}, x_{\dot{s}})^{-\gamma} \partial_2 z(y_{\dot{s}}, x_{\dot{s}}) \frac{\partial x_s}{\partial y_0}}.$$

- The $\partial x / \partial y$ terms capture the idea that the consumption choice of the agent (y) might affect his reference level (x)

Habits

- Two functional forms for the relation between z and y have been pursued in the literature
  - Additive habits: $z := y - hx$
  - Multiplicative habits: $z := yx^{-h}$
  - $0 \leq h \leq 1$ is a parameter that governs the strength of the habit motive
- For x, also, different specifications have been pursued in the literature
  - One possibility is to set the habit level equal to the consumption of the previous period, $x_t := y_{t-1}$
  - A variation of this idea is to specify the reference consumption level not as past own consumption, but as the average consumption of other people
    - ★ Imagine a situation in which you are earning \$100, 000 a year, and you feel pretty comfortable. Then suppose you learn that all your colleagues at work earn 50% more than you. Suddenly you feel very dissatisfied. Depending on character and temper, you may become bitter or aggressive or depressed. What matters to you is not only absolute wealth, but also wealth relative to your neighbors. This observation is captured by the idea that people have a motivation for *keeping up with Joneses*

Joneses' Preferences

- Joneses' preferences (or external habits, as they are sometimes called) are simpler than the internal habit model discussed earlier
- The individual's consumption decision has no effect on the habit level

$$M_s = \delta \left(\frac{z(y_s, x_s)}{z(y_0, x_0)} \right)^{-\gamma} \frac{\partial_1 z(y_s, x_s)}{\partial_1 z(y_0, x_0)}$$

- Consider a multiplicative Joneses model that contains keeping up and catching up at the same time:

$$z_t(i) := \frac{y_t(i)}{(w_t)^{(1-\lambda)h}(w_{t-1})^{\lambda h}}$$

- Looking at the representative agent ($y = w$), we get

$$M_s = \delta(1+g_s)^{-\gamma-(1-\lambda)h(1-\gamma)}(1+g_0)^{-\lambda h(1-\gamma)}$$

- or, in logs, approximately

$$\ln M_s \approx \ln \delta - [\gamma + (1-\lambda)h(1-\gamma)]g_s - \lambda h(1-\gamma)g_0$$

Joneses' Preferences

- A short lag ($\lambda \to 0$) reduce the equilibrium price of risk, while a long lag ($\lambda \to 1$) does not affect the risk premium

$$\ln M_s \approx \ln \delta - [\gamma + (1-\lambda)h(1-\gamma)]g_s - \lambda h(1-\gamma)g_0$$

- The time preference, however, is affected. For $\lambda = 1$, we have

$$\ln M_s \approx \ln \delta - h(1-\gamma)g_0 - \gamma g_s$$

- If $h > 0$ and $\gamma > 1$ and $g_0 > 0$, then Joneses motive increases patience, thus increases risk-free rate
- If it is not only absolute consumption that matters to you, but also consumption relative to other people, then you will want to consume more tomorrow than today if aggregate consumption grows through time ...
- ... because if you do not you will fall back relative to the mean, and this will hurt if you are trying to catch up with the Joneses.

Joneses' Preferences

- Next we consider the implications of the additive lagged Joneses model used by Campbell & Cochrane (1999)
- Define the consumption surplus ratio, $\psi_s(i) \coloneqq \frac{y_s(i) - hx_s}{y_s(i)} > 0$
- Now, $z = \psi y$, the SDF of the RA is

$$M_s = \delta \left(\frac{\psi_s w_s}{\psi_0 w_0} \right)^{-\gamma}$$

- We assume the habit level is the last period's per capita consumption, $x_t \coloneqq w_{t-1}$

$$\psi = \frac{w_s - hw_0}{w_s} = \frac{1 + g_s - h}{1 + g_s}$$

- When habit is strong ($h \to 1$), ψ_s / ψ_0 is much more volatile than g, thus increase the price of risk

First-order Risk Aversion

- Consider a person with wealth of \$75,000 who faces a 50–50 chance of winning or losing \$25,000, which is a third of his initial wealth and therefore a substantial risk
- If $\gamma = 30$, then the agent is willing to pay \$24,000 to avoid the risk
  - This is clearly absurd, $\gamma = 2$ gives rise to a more sensible decision, because the willingness to pay for an insurance against this risk then drops to \$8,333
  - But now consider a small risk, a lottery in which the agent stands to win or lose \$375, or 0.5% of his wealth. $\gamma = 2$ now induces a willingness to pay for insurance of only 1.88, which seems too small
- This example suggests that most people are much more averse to small gambles than to large gambles

First-order Risk Aversion

- The risk premium for small gambles is proportional to the variance of the lottery

$$w - c \approx A(w)\frac{Var(x)}{2}$$

- Such preferences are called second-order risk averse
- The preferences for which the risk premium is proportional to the standard deviation of the lottery is called first-order risk averse
- Since for small risk standard deviation is much larger in magnitude comparing to variance, it effectively makes people more risk averse, thus helps to increase the model predicted equity premium

Prospect theory and the house money effect

- Kahneman & Tversky (1979) have proposed a far more general theory of risky decision making than the mainstream von Neumann–Morgenstern model: Prospect theory
  - Agents draw utility not from wealth, but from gains and losses defined with respect to some reference level. This is called the endowment effect
  - A loss hurts more than an equally large gain produces joy. This is called loss aversion
  - Agents are risk averse over gains, but risk loving over losses
  - Agents weight low probability states too much, and high probability states too little
- The endowment effect is essentially the same as the idea of habits
- Loss aversion is a particular form of first-order risk aversion

Prospect theory and the house money effect

- Barberis et al. (2001) apply these ideas to equilibrium asset pricing
- Together with the idea of Thaler & Johnson (1990) that a person who has made a gain in the recent past is more likely to take risks than a person who has not. They call this the house money effect

$$v(s, r, z) := \begin{cases} s(R-\rho) & \text{if } R > \rho \\ \lambda(z)s(R-\rho) & \text{if } R < \rho \end{cases}$$

  where s is the amount of wealth invested in the risky asset, R is the return rate of the risky asset, ρ is the return rate of the risk-free bond, z measures prior gains or losses, $\lambda(z) > 1$ measures loss aversion
- Increase in stock prices leads to a gain, making investors less risk averse and holding more stocks
- On the other hand, a loss increases risk aversion and makes investors want to decrease their exposure to the risky asset
- The result is a very volatile stock price. This large volatility then makes a large equity premium necessary in order for stocks to be held in equilibrium