Structured cospans

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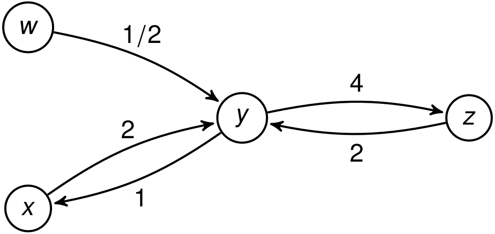
Networks can very often be viewed as sets equipped or 'decorated' with extra structure...

For example,



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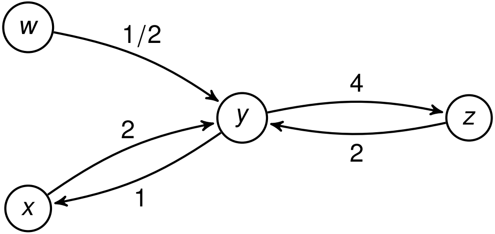

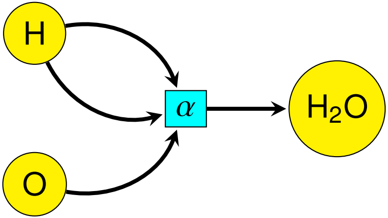

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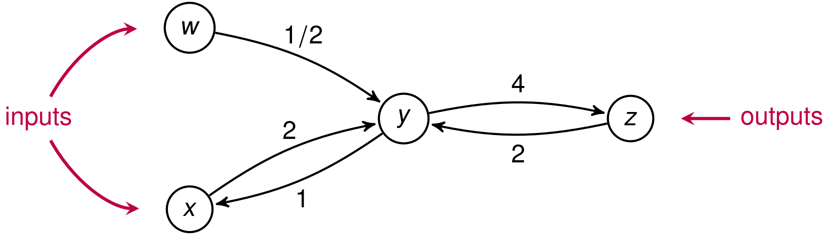

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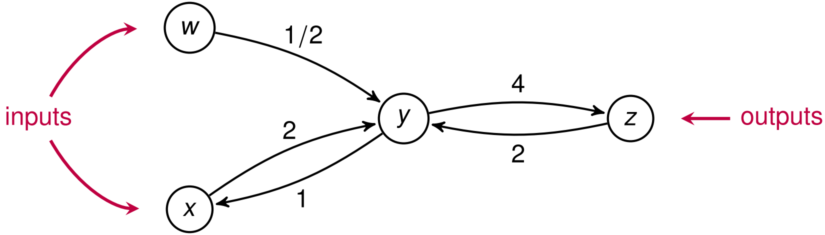
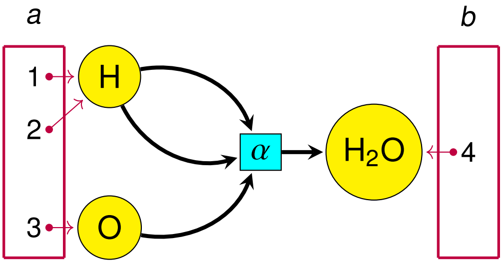
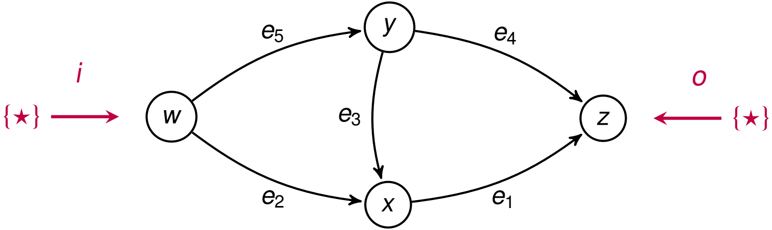




An easy example to have in mind is the example of open graphs:



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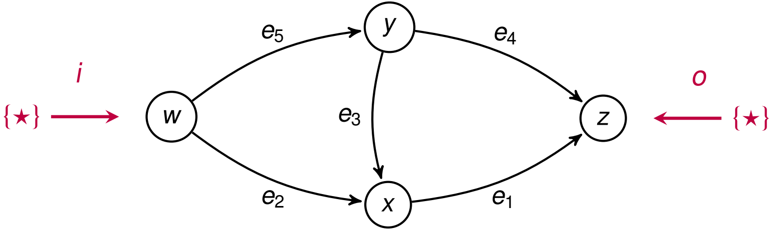

The overall shape of this diagram resembles that of a **cospan**:

$$\begin{tikzcd} & c & \\ a \arrow[ur, "i"] & & b \arrow[ul, "o"'] \end{tikzcd}$$

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Theorem (B. Fong)

Let A *be a category with finite colimits and* $F\colon \mathsf{A} \to \mathrm{Set}$ *a symmetric lax monoidal functor. Then there exists a category* $F\mathrm{Cospan}$ *which has:*

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Let A *be a category with finite colimits and* $F\colon \mathsf{A} \to \mathsf{Set}$ *a symmetric lax monoidal functor. Then there exists a category* $F\mathsf{Cospan}$ *which has:*

- *objects as those of* A *and*
- *morphisms as isomorphism classes of* F*-decorated cospans, where an* F*-decorated cospan is given by a pair:*

$$\begin{tikzcd} & c & \\ a \arrow[ur, "i"] & & b \arrow[ul, "o"'] \end{tikzcd} \qquad d \in F(c)$$

Theorem (B. Fong continued)

Two F-decorated cospans are in the same isomorphism class if the following diagrams commute:

$$
\begin{tikzcd}
 & c \arrow[dd, "f", "\sim"'] & \\
a \arrow[ur, "i"] \arrow[dr, "i'"'] & & b \arrow[ul, "o"'] \arrow[dl, "o'"] \\
 & c' &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & F(c) \arrow[dd, "F(f)"] \\
1 \arrow[ur, "d"] \arrow[dr, "d'"'] & \\
 & F(c')
\end{tikzcd}
$$

Theorem (B. Fong continued)

To compose two morphisms:

$$a_1 \xrightarrow{i} c \xleftarrow{o} a_2 \qquad\qquad a_2 \xrightarrow{i'} c' \xleftarrow{o'} a_3$$

$$d \in F(c) \qquad\qquad\qquad d' \in F(c')$$

we take the pushout in A*:*

$$\begin{tikzcd}
 & & c +_{a_2} c' & & \\
 & & c + c' \arrow[u, "\psi"] & & \\
 & c \arrow[ur, "j"] & & c' \arrow[ul, "j'"'] & \\
a_1 \arrow[ur, "i"] \arrow[uuurr, bend left, "\psi j i"] & & a_2 \arrow[ul, "o"'] \arrow[ur, "i'"] & & a_3 \arrow[ul, "o'"'] \arrow[uuull, bend right, "\psi j' o'"']
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\end{tikzcd}$$

$$d \odot d' \colon 1 \xrightarrow{d_1 \times d_2} F(c_1) \times F(c_2) \xrightarrow{d_1 + d_2} F(c_1 + c_2) \xrightarrow{F(\psi)} F(c_1 +_{a_2} c_2)$$

For example, if we let $F\colon \mathrm{Set} \to \mathrm{Set}$ be the symmetric lax monoidal functor that assigns to a set N the (large) *set* of all graph structures having N as its set of vertices:

$$F(N) = \{ \begin{tikzcd} E \arrow[r, bend left, "s"] \arrow[r, bend right, "t"'] & N \end{tikzcd} \}$$

For an example of this example, if we take $N = \{n_1, n_2, n_3\}$ to be a three element set, then some elements of the (large) set $F(N)$ are given by:

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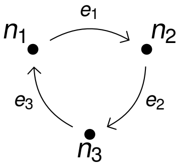
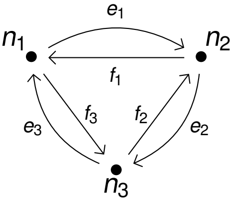
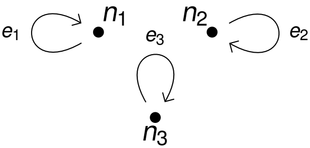


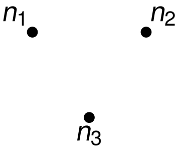
$d_1 \in F(N)$



$d_2 \in F(N)$



$d_3 \in F(N)$



$d_4 \in F(N)$

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The triangle on the right is in Set and commutes on the nose.

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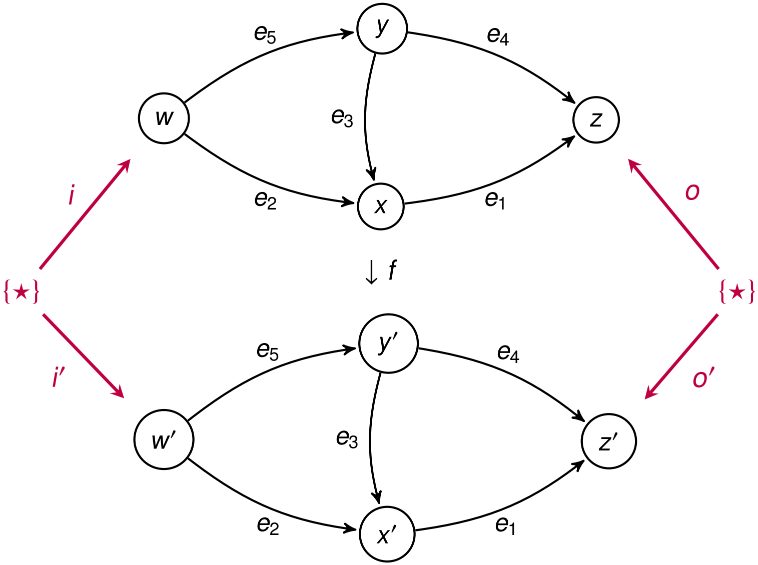
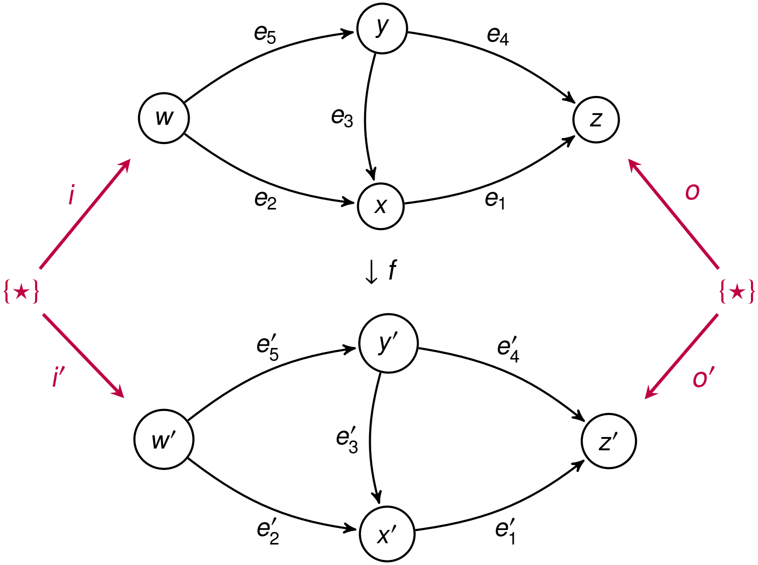
This means that a decoration $d \in F(c)$ together with a bijection $f\colon c \to c'$ determines what the decoration $d' \in F(c')$ must be.

In the context of open graphs, the following two open graphs would be in the same isomorphism class:



But the following two open graphs would *not* be in the same isomorphism class:



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Theorem (Baez, C.)

Let A *be a category with finite coproducts,* X *a category with finite colimits and* $L \colon \mathsf{A} \to \mathsf{X}$ *a finite coproduct preserving functor. Then there exists a category* $_L\mathsf{Csp}(\mathsf{X})$ *which has:*

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- *objects as those of A and*
- *morphisms as isomorphism classes of **structured cospans**, where a structured cospan is given by a cospan in X of the form:*

$$\begin{tikzcd} & x & \\ L(a) \arrow[ur, "i"] & & L(b) \arrow[ul, "o"'] \end{tikzcd}$$

Theorem (Baez, C. continued)

Two structured cospans are in the same isomorphism class if the following diagram commutes:

$$
\begin{tikzcd}
& x \arrow[dd, "\alpha", "\sim"'] & \\
L(a) \arrow[ur, "i"] \arrow[dr, "i'"'] & & L(b) \arrow[ul, "o"'] \arrow[dl, "o'"] \\
& y &
\end{tikzcd}
$$

Theorem (Baez, C. continued)

To compose two morphisms:

$$L(a_1) \xrightarrow{i} x \xleftarrow{o} L(a_2) \qquad L(a_2) \xrightarrow{i'} y \xleftarrow{o'} L(a_3)$$

we take the pushout in X*:*

$$\begin{tikzcd}
 & & x +_{L(a_2)} y & & \\
 & & x + y \arrow[u, "\psi"] & & \\
 & x \arrow[ur, "J"] & & y \arrow[ul, "J'"'] & \\
L(a_1) \arrow[ur, "i"] \arrow[uuurr, bend left, "\psi J i"] & & L(a_2) \arrow[ul, "o"'] \arrow[ur, "i'"] & & L(a_3) \arrow[ul, "o'"'] \arrow[uuull, bend right, "\psi J' o'"']
\end{tikzcd}$$

In the context of open graphs, we take $L\colon \mathrm{Set} \to \mathrm{Graph}$ to be the discrete graph functor which assigns to a set N the edgeless graph with vertex set N.

$$\begin{tikzcd} \emptyset \arrow[r, bend left, "!"] \arrow[r, bend right, "!"'] & N \end{tikzcd}$$

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Both Set and Graph have finite colimits and L is a left adjoint, so we get the following:

Corollary

Let $L : \mathrm{Set} \to \mathrm{Graph}$ *be the discrete graph functor. Then there exists a category* ${}_L\mathsf{Csp}(\mathrm{Graph})$ *which has:*

- *sets as objects and*
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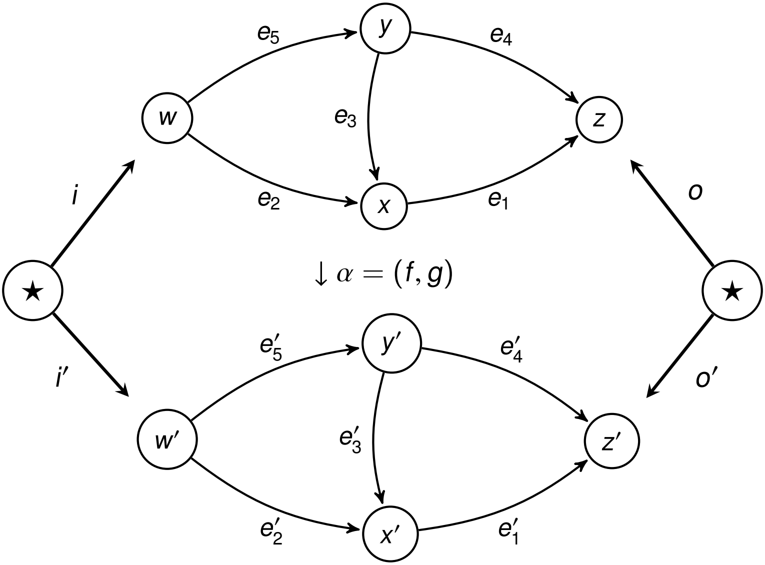
Now, two open graphs are in the same isomorphism class if there exists an isomorphism of graphs $\alpha\colon G_1 \to G_2$ making the following diagram commute:

$$\begin{tikzcd} & G_1 \arrow[dd, "\alpha"', "\sim"] & \\ L(N_1) \arrow[ur, "i"] \arrow[dr, "i'"'] & & L(N_2) \arrow[ul, "o"'] \arrow[dl, "o'"] \\ & G_2 & \end{tikzcd}$$

Here, $\alpha \colon G_1 \to G_2$ is an isomorphism of graphs which is a *pair* of bijections (f, g) making the following squares commute:

$$\begin{tikzcd}
G_1 \arrow[d, "\alpha"'] & = & E \arrow[r, bend left, "s"] \arrow[r, bend right, "t"'] \arrow[d, "g"', "\sim"] & N \arrow[d, "f", "\sim"'] \\
G_2 & = & E' \arrow[r, bend left, "s'"] \arrow[r, bend right, "t'"'] & N'
\end{tikzcd}$$

And now, the following two open graphs are in the same isomorphism class.



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You might be thinking that we should then use a bicategory... and we *could* do this.

But instead, we're going to use a 'double category'!

A double category has figures like this:

$$\begin{tikzcd} A \arrow[r, "M"] \arrow[d, "f"'] \arrow[dr, phantom, "\Downarrow \alpha"] & B \arrow[d, "g"] \\ C \arrow[r, "N"'] & D \end{tikzcd}$$

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$$\begin{tikzcd} A \arrow[r, "M"] \arrow[d, "f"'] & B \arrow[d, "g"] \\ C \arrow[r, "N"'] & D \arrow[from=1-1, to=2-2, phantom, "\Downarrow\alpha"] \end{tikzcd}$$

We have objects, here denoted as A, B, C and D.

Vertical 1-morphisms between objects, here denoted as f and g.

Also, horizontal 1-cells between objects, here denoted as M and N,

and morphisms between horizontal 1-cells, called 2-morphisms, here denoted as α.

These 2-morphisms can be composed both vertically and horizontally.

$$\begin{tikzcd} A \arrow[r, "M"] \arrow[d, "f"'] \arrow[dr, phantom, "\Downarrow\alpha"] & B \arrow[d, "g"] \\ C \arrow[r, "N"] & D \end{tikzcd} \qquad \begin{tikzcd} B \arrow[r, "M'"] \arrow[d, "g"'] \arrow[dr, phantom, "\Downarrow\beta"] & E \arrow[d, "h"] \\ D \arrow[r, "N'"] & F \end{tikzcd}$$

$$\begin{tikzcd} C \arrow[r, "N"] \arrow[d, "f'"'] \arrow[dr, phantom, "\Downarrow\alpha'"] & D \arrow[d, "g'"] \\ G \arrow[r, "O"] & H \end{tikzcd} \qquad \begin{tikzcd} D \arrow[r, "N'"] \arrow[d, "g'"'] \arrow[dr, phantom, "\Downarrow\beta'"] & F \arrow[d, "h'"] \\ I \arrow[r, "P"] & J \end{tikzcd}$$

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$$(\alpha \odot \beta)(\alpha' \odot \beta') = (\alpha\alpha') \odot (\beta\beta')$$

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- *horizontal 1-cells given by* **structured cospans** *which are cospans in* X *of the form:*

$$\begin{tikzcd} & x & \\ L(a) \arrow[ur, "i"] & & L(a') \arrow[ul, "o"'] \end{tikzcd}$$

and

Theorem (Baez, C. continued)

2-morphisms as maps of cospans in X *given by commutative diagrams of the form:*

$$
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The horizontal composite of two 2-morphisms:

$$\begin{tikzcd} L(a) \arrow[r, "i_1"] \arrow[d, "L(f)"'] & x \arrow[d, "\alpha"'] & L(b) \arrow[l, "o_1"'] \arrow[d, "L(g)"] \\ L(a') \arrow[r, "i_1'"] & x' & L(b') \arrow[l, "o_1'"'] \end{tikzcd} \qquad \begin{tikzcd} L(b) \arrow[r, "i_2"] \arrow[d, "L(g)"'] & y \arrow[d, "\beta"'] & L(c) \arrow[l, "i_2"'] \arrow[d, "L(h)"] \\ L(b') \arrow[r, "i_2'"] & y' & L(c') \arrow[l, "o_2'"'] \end{tikzcd}$$

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is given by

$$\begin{tikzcd} L(a) \arrow[r, "J\psi i_1"] \arrow[d, "L(f)"'] & x +_{L(b)} y \arrow[d, "\alpha +_{L(g)} \beta"'] & L(c) \arrow[l, "J\psi o_2"'] \arrow[d, "L(h)"] \\ L(a') \arrow[r, "J\psi i_1'"] & x' +_{L(b')} y' & L(c') \arrow[l, "J\psi o_2'"'] \end{tikzcd}.$$

Theorem (Baez, C. continued)

Monoidal structure:

$$\begin{tikzcd} L(a_1) \arrow[r, "i_1"] \arrow[d, "L(f)"'] & x_1 \arrow[d, "\alpha"'] & L(b_1) \arrow[l, "o_1"'] \arrow[d, "L(g)"] \\ L(a_2) \arrow[r, "i_2"'] & x_2 & L(b_2) \arrow[l, "o_2"] \end{tikzcd} \otimes \begin{tikzcd} L(a_1') \arrow[r, "i_1'"] \arrow[d, "L(f')"'] & x_1' \arrow[d, "\alpha'"'] & L(b_1') \arrow[l, "o_1'"'] \arrow[d, "L(g')"] \\ L(a_2') \arrow[r, "i_2'"'] & x_2' & L(b_2') \arrow[l, "o_2'"] \end{tikzcd}$$

$$= \begin{tikzcd} L(a_1 + a_1') \arrow[r, "(i_1 + i_1')\phi^{-1}"] \arrow[d, "L(f + f')"'] & x_1 + x_1' \arrow[d, "\alpha + \alpha'"'] & L(b_1 + b_1') \arrow[l, "(o_1 + o_1')\phi^{-1}"'] \arrow[d, "L(g + g')"] \\ L(a_2 + a_2') \arrow[r, "(i_2 + i_2')\phi^{-1}"'] & x_2 + x_2' & L(b_2 + b_2') \arrow[l, "(o_2 + o_2')\phi^{-1}"] \end{tikzcd}$$

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- *objects as those of* A*,*
- *vertical 1-morphisms as morphisms of* A*,*
- *horizontal 1-cells as* F*-decorated cospans, which are again pairs:*

$$\begin{tikzcd} & c & \\ a \arrow[ur, "i"] & & b \arrow[ul, "o"'] \end{tikzcd} \qquad d \in F(c)$$

Theorem (Baez, Vasilakopoulou, C. continued)

- *2-morphisms given by maps of cospans in* A*:*

$$
\begin{tikzcd}
a \arrow[r, "i"] \arrow[d, "g"'] & c \arrow[d, "f"] & b \arrow[l, "o"'] \arrow[d, "h"] \\
a' \arrow[r, "i'"'] & c' & b' \arrow[l, "o'"]
\end{tikzcd}
\qquad
\begin{tikzcd}
 & F(c) \arrow[dd, "F(f)"] \\
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1 \arrow[ur, "d"] \arrow[dr, "d'"'] \arrow[r, phantom, "\Swarrow \iota"] & \\
 & F(c')
\end{tikzcd}
$$

together with a 2-morphism ι *which can be viewed as a morphism*

$$\iota \colon F(f)(d) \to d'$$

in $F(c')$*.*

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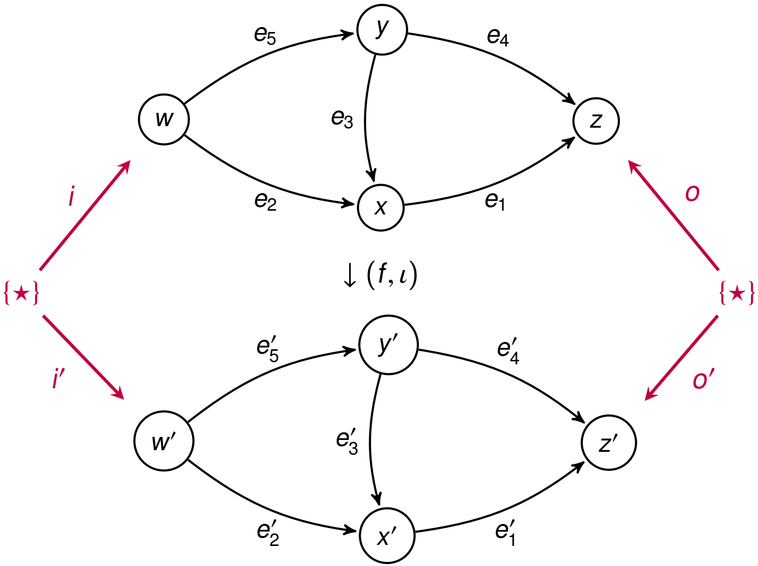

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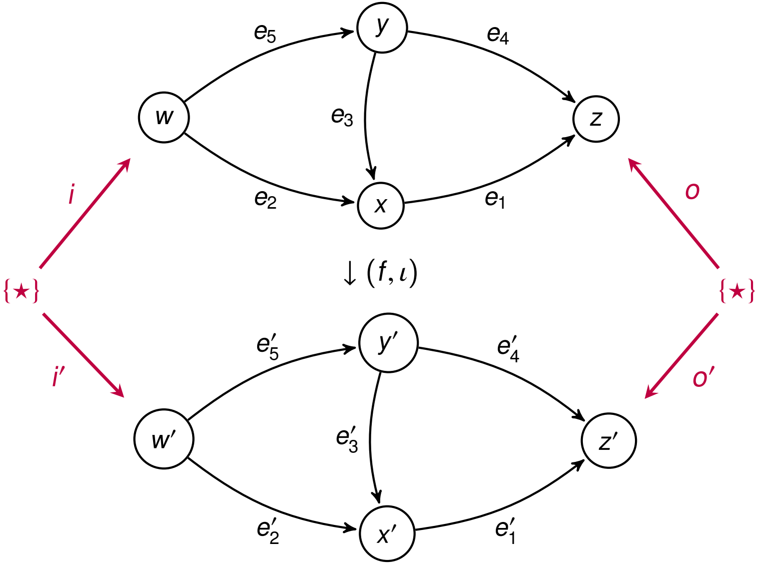

the morphism $\iota \colon F(f)(d) \to d'$ is the map of edges.

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Given a finitely cocomplete category A *and a symmetric lax monoidal pseudofunctor* $F \colon \mathsf{A} \to \mathrm{Cat}$*, if each category* $F(a)$ *is also finitely cocomplete, then there is an equivalence of symmetric monoidal double categories*

$$_{L}\mathbb{C}sp(\textstyle\int F) \simeq F\mathbb{C}sp.$$

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$$ {}_L\mathbb{C}sp(\textstyle\int F) \simeq F\mathbb{C}sp. $$

The functor L used to obtain the structured cospans double category is left adjoint to the Grothendieck construction of the pseudofunctor F:

$$ R\colon \textstyle\int F \to \mathsf{A}. $$

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There exists a left adjoint $L \colon \mathrm{FinSet} \to \mathrm{Circ}$ which we can use to obtain a symmetric monoidal category

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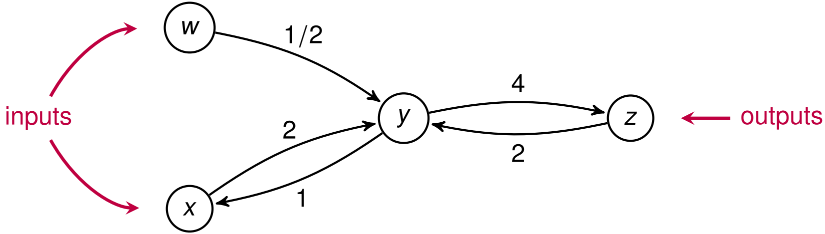

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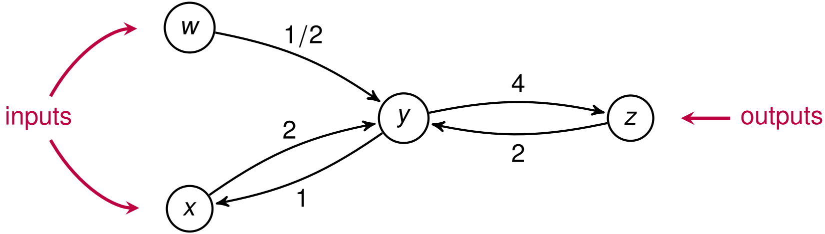
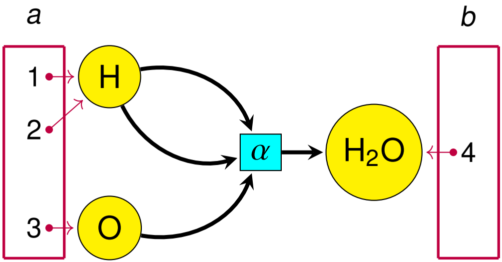


From this, we can obtain a black box functor

$$ \blacksquare\colon {}_L Csp(\mathrm{Circ}) \to \mathrm{Rel}. $$

And likewise for open Petri nets.



$$L\colon \mathrm{Set} \to \mathrm{Petri}$$

$$\blacksquare\colon {}_L\mathsf{Csp}(\mathrm{Petri}) \to \mathrm{Rel}.$$

For more, see my thesis on Dr. Baez's website:

https://tinyurl.com/courser-thesis