Sub-graph Isomorphism in GPU

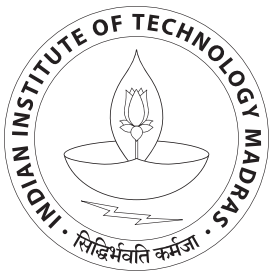
A Project Report

submitted by

GEORGE JOSEPH

in partial fulfilment of the requirements
for the award of the degree of

MASTER OF TECHNOLOGY



DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS.

May 2017

THESIS CERTIFICATE

This is to certify that the thesis entitled **Sub-graph Isomorphism in GPU**, submitted by **George Joseph**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bonafide record of the research work carried out by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Prof. Rupesh Nasre
Research Guide
Professor
Dept. of Computer Science and Engineering
IIT-Madras, 600 036

Place: Chennai

Date:

ACKNOWLEDGEMENTS

I would like to to express my sincere thanks and deep sense of indebtedness to my guide Dr. Rupesh Nasre for his guidance and motivation throughout my work. His inspiring suggestions motivated me to solve problems efficiently. I am also grateful to my guide also for providing access to the Libra servers without which none of my experiments could have been performed.

I would also thank Vinod Raju, Jithin K M, Nikhil Stephen,Hasit Bhatt,Vivek V P and all my colleagues who always helped me whenever needed in my research.

Lastly, I am thankful to god for giving me enough luck to get into IIT Madras and I am thankful to my parents for all the moral support and the amazing oppurtunities they have given me over the years.

ABSTRACT

In Sub-Graph Isomorphism we are trying to find whether a sub-graph of Data Graph is isomorphic to the query graph. Sub-Graph Isomorphism is a NP-Hard problem. Because of the lots of applications of the problem in many Data mining and pattern matching, the problem is well studied and different methods are proposed in the past years. We studied many of the state-of-art techniques used to solve the problem. Then parallelised one of them in GPU. We then tried to solve the dynamic version of the problem. In the dynamic version we remove or add edges on the go and we would like to get the output on the current problem.

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ABBREVIATIONS

NEC	Neighbourhood Equivalence Class
CVS	Candidate vertex Set
NDS	Neighbouring Discriminating Substructure
BFS	Breadth First Search

CHAPTER 1

INTRODUCTION

1.1 Overview

Consider a pattern which you want to know where it is coming in the large data. The pattern can be easily represented as a graph. The actual data can be image video, network of people, etc. The graphs can be used to represent any form of data. The pattern matching has a wide applications. It is not an easy problem. It need a lots of checking at each node. It is similar to placing the pattern at each node in the graph and trying to rotate, flip, etc... to find a similar representation. There are lots of possibilities that can match but there may be only small number of actual matching. We need to find them.

1.2 Motivation

As I mentioned before this problem is having lots of applications since the current era is trying to extract lots of features from images, videos,audios,etc using many pattern matching techniques. Since the problem is NP Hard researchers, have focussed on efficiently solving the problem in practice. The numerous cores of GPUs may help us to solve this problem faster. Each node search is independent so they can be done in parallel. This is the primary motivation on trying to do the sub-graph isomorphism in GPUs.

1.3 Major Contribution

- Implemented an efficient solution in GPU
- Dynamic SubGraph Queries are handled in parallel

1.4 Organization of Thesis

Subgraph Isomorphism chapter 2 discuss the problem and the state-of-art algorithms. A detailed comparison of various algorithms in terms of different pruning techniques is performed. Parallel implementation 3 discuss the parallel version of the TurboIso algorithm. Dynamic Operations 4 discuss the implementation of the dynamic version of the problem. The challenges involved in its implementation are discussed in more depth in each chapter.

CHAPTER 2

Sub-Graph Isomorphism

In this chapter, the problem statement and various known algorithms for Sub-Graph Isomorphism are studied in great depth.

2.1 Problem

Input: A data Graph D, and Query Graph Q. The graphs D and Q are undirected with nodes and edges having labels.See Figure2.1.

Graphs given as adjacency list.

Output: Give all the matching mapping of each node in Q to node in D

In Figure 2.1, 'A' and 'a' represent the node labels and edge labels. A match is present only if the edge labels and node labels are also matched. v1, v2, ...

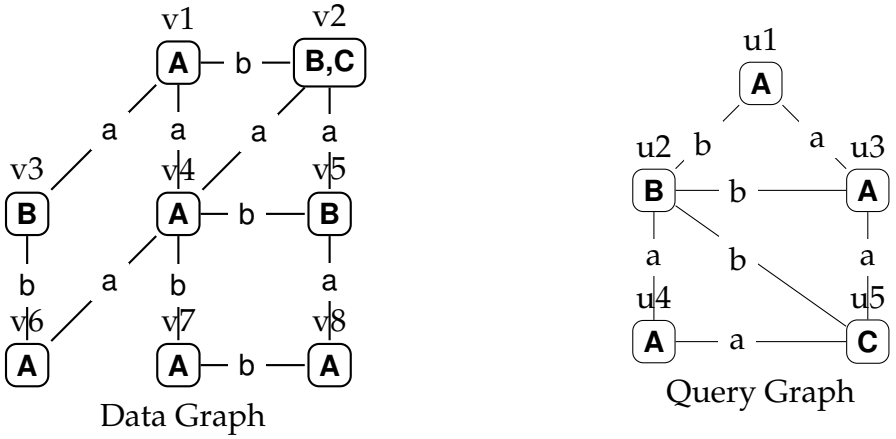

Figure 2.1 Data and Query Graph

and u1, u2, ... are the node ids. They are used to explain the algorithms discussed below.

2.2 Related Work

In this section we are discussing various subgraph isomorphism subgraph isomorphism algorithms. Understanding the differences in these algorithms are very crucial.

2.2.1 Generic Algorithm

The generic algorithm[1] for subgraph isomorphism will help us to study the aspects of state-of-art algorithms in deep. It is presented in Algorithm 1.

Algorithm 1 Subgraph Search

Input: Data Graph D,Query Graph Q.

Output: Mapping of vertices from Q to D.

1. for each vertex v of Q
   (a) C(v)=FindCandidates(v,D)
   (b) If C(v) is empty return
2. SUBGRAPHMATCHING(C,Q,D,ϕ)

Procedure SUBGRAPHMATCHING:

Input: Candidates C,Data Graph D,Query Graph Q Current Map M.

Output: Mapping of vertices from Q to D.

1. if $|M| = |V(q)|$ report M
2. else
   (a) u=NextVertex()
   (b) C_r =RefinedSet(M,u,C(u))
   (c) for each $v \in C_r$
      i. if IsJoinable(M,v)
         A. UpdateState(M,v)
         B. SUBGRAPHMATCHING(q,d,C,M)
         C. RestoreState(M,v)

The procedure FindCandidates finds the vertices in datagraph which can be mapped to query vertex. The procedure NextVertex finds the next vertex in querygraph which should be tried to be mapped.

The RefinedSet prunes out some nodes in the candidate set. The IsJoinable checks whether the map is right. The UpdateState moves to next state (adds new vertex to map).The RestoreState removes the vertex from map and thus restores the state.

If you consider the graphs in Figure 2.1. $C(u1) = \{v1, v6, v7, v8\}$ pruned by node label and degree. Similiarly $C(u4) = \{v1, v6, v7, v8\}$ and $C(u2) = \{v3, v2, v5\}$.

Procedure NextGraph will give the vertices on query graph in some order like $\{u1, u2, u3, u4, u5\}$. It can be even $\{u1, u3, u4, u2, u5\}$. Once $u1$ is mapped to $v1$,the procedure RefinedSet will remove $v1$ from $C(u4)$. If Map has these values $\{(u1, v1)\}$,NextGraph returned $u2$, RefinedSet returned $\{v3, v2, v5\}$ and current v is $v5$ then procedure IsJoinable check whether there is an edge between $v1$ and $v5$ like the one between $u1$ and $u2$.

2.2.2 Ullmann Algorithm

This algorithm[2] is simple.The FindCandidates finds same degree nodes. The NextVertex takes the next node in input.The RefinedSet removes nodes already mapped. The procedure IsJoinable iterates over the neighborhood and checks if corresponding edge exists. The UpdateState and RestoreState adds and removes the vertex from map respectively.

2.2.3 VF2 Algorithm

VF2 algorithm was proposed in [3].The NextVertex takes the next connected vertex. The RefinedSet uses these rules

1. Prune out v if not connected from already mapped vertices.
2. The count of unmatched vertices of neighbors of v in Q must be greater than unmatched vertices of neighbors of u in D
3. The count of neighbors of v who are not neighbors of mapped nodes and not mapped nodes in Q must be greater than neighbors of u who are not neighbors of mapped nodes and not mapped nodes in D

2.2.4 QucikSi Algorithm

QuickSi algorithm was proposed in [4].The $NextVertex$ takes vertices in the most infrequent vertex first order.The $RefinedSet$ uses connectivity to mapped vertices to prune.The $RefinedSet$ only iterates over mapped adjacent vertices.

2.2.5 GADDI Algorithm

GADDI was proposed in [5]. They use the neighboring discriminating substructure(NDS).$\Delta_{NDS}(u, v, P)$ is the number of occurrences of P in induced subgraph $N_k(v) \cap N_k(u)$. $N_k(u)$ is the graph having all the edges in k distance from u. A matrix L is created such that each row corresponds to an induced graph g and each column represent a pattern. See the NDS calculation in Figure 2.2. The dark lines represent the $N_k(v1) \cap N_k(v3)$ with k=2, ie., nodes and edges at a distance of at most 2 from both the vertices $v1$ and $v2$. The $NextVertex$ takes the one next in the DFS Tree from the vertex. The $RefinedSet$ prune based on these conditions. If for each $u' \in N_k(u)$ there is no data vertex $v' \in N_k(v)$ having

1. $L(u') \subseteq L(v')$
2. The shortest distance between v' and v must be greater than or equal to distance between u and u'.

The triangles(P1) present in the $N_k(v1) \cap N_k(v3)$ is 6. The vertices $v1$, $v2$ and $v4$ make the 6 traingles(6 permutation of vertices). The number of lines of length 3(p2) present in the graph is 24. The vertices $v1$, $v2$, $v4$ and $v6$ makes one of the 24 lines. The stars(P3) present are at vertices $v1$ and $v4$. The different combinations gives 12 possibilities.

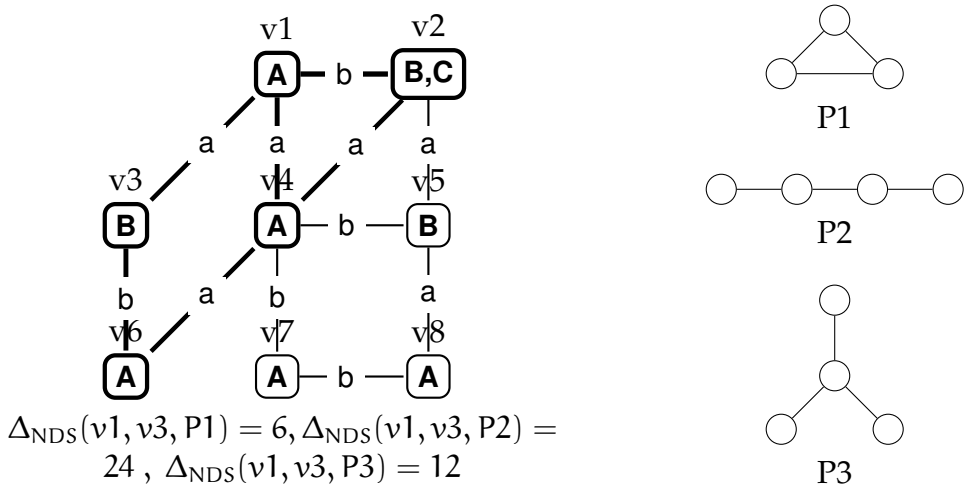

Figure 2.2 GADDI NDS Calculation

2.2.6 GraphQL Algorithm

The GraphWl was proposed in [6]. They use neighborhood signatures. The neighbor of the vertex is encoded as the collection of labels of its neighbors. The $RefinedSet$ pruning is based on this signature. This is a one hop signature. For example the $sig(u1) = \{B, A\}$ in Figure 2.1.

2.2.7 SPath Algorithm

The SPath Algorithm was proposed in [7]. They use signatures till k hop. They store the signature in the form (d, l, c) where d is distance to the neighbor, l the label,c the count.The $RefinedSet$ pruning is based on these signatures.For example the $sig(u1, 2) = \{(1, B, 1), (1, A, 1), (2, A, 1), (2, C, 1)\}$ in Figure 2.1.

2.2.8 STWig Algorithm

The STWig Algorithm was proposed in [8]. Here the query graph is divided into smaller graphs. These smaller graphs are searched in the data graph first.

Their results are combined to get the final result. The graphs are divided such that the root of g_j must be of the children of any of the graphs g_i such that $i < j$. All STWigs are two level trees. See Figure 2.3 for a random STWigs generated for query graph in Figure 2.1. There is no constraint in number of children allowed. So there are many decomposition for a graph.

The candidates can be started from the least frequent pattern and then building up. The splitting of the graphs,matching the small STWigs and then combining can be done in GPU. But the combining of STWig results in the troublesome task. This can lead to the need of large amount of memory too since the candidate set can increase exponentialy.

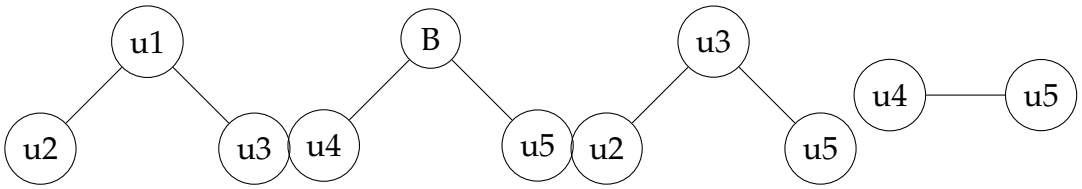

Figure 2.3 STWig Decomposition

2.3 Implemented Algorithm

2.3.1 TurboIso

It was proposed in [9].$\mathtt{Turbo}_{\mathrm{iso}}$ uses neighborhood equivalence class(NEC). Here they make a tree out of the query graph. In this tree they create the NEC. Each node will be part of a unique NEC. Later this tree is searched in the data graph. Then the graph edges are checked.

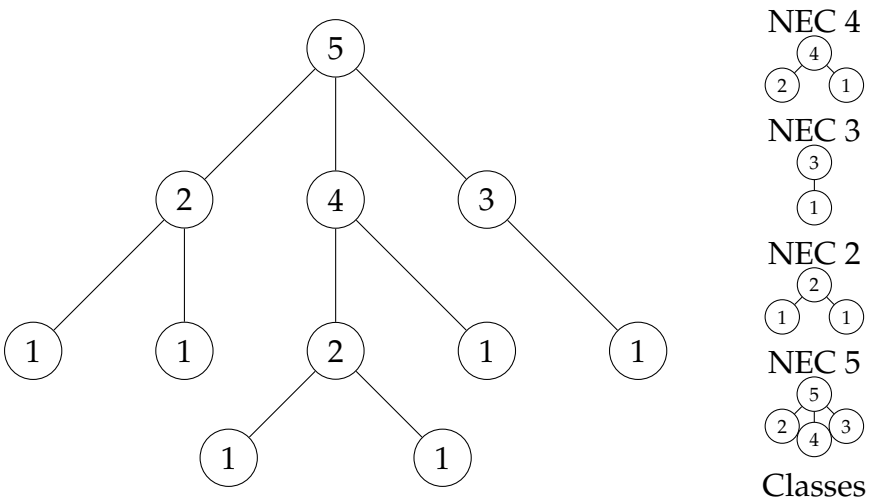

Figure 2.4 NEC Numbering

Algorithm 2 NEC creation

Input: Data Graph D,Query Graph Q.

Output: Mapping of vertices from Q to D.

1. The leaf nodes are given NEC 1
2. for each level upward
   (a) Each new neighborhood will get a new NEC

In figure 2.4, the leaf nodes are given the NEC 1. Then the algorithm 2 moves upward each level. In the succeeding level it finds two more classes NEC 2 and NEC 3. The numbering are given a sequential order. In the next level it find the NEC 5. The corresponding NEC's can be seen on the right of the image. Then CVS for each NEC is found in the data graph. Then for each combination the actual graph is tested for a match.

The CVS is the collection of all matching vertices of data graph for each NEC. This is found out by checking the existence of the NEC children at each data graph node. Each node in data graph is given a NEC 1. Then each higher NECs are checked. After that all possible combinations are checked for existence of the graph. If a match is found, it is printed.

2.3.2 Inferences

All of the above algorithms tried to decrease the total candidate vertices set(CVS) for a vertex in query graph. The permutation and combination of these vertices will result in the final answer. More the CVS, more will be the combinations. Since the answer requires all possible permutations we can't avoid this calculation. So we need to prune out the false candidate as early as possible. This is the reason why intermediate pruning steps are added in $UpdateState$ also. The neighborhood signature is the way seen so far to prune the CVS initially better.

CHAPTER 3

Parallel Implementation

3.1 Introduction

In the parallel implementation the unique NEC numbering, CVS generation in data graph, and checking whether the query graph exists in the sub-graph and the combination generation are done in GPU.

3.2 Algorithm

[H] **Algorithm 3** Parallel $\text{Turbo}_{\text{iso}}$

Procedure: NECGen()

Parallel NEC generation on each node.

Input: Query Graph Q.

Output: NEC.

1. repeat until all nodes got NEC
2. Run parallel on all nodes
   (a) running on node v
   (b) iterate over all neighbors of vertex v
   (c) if not all neighbors have NEC return
   (d) find the hash of neighborhood.set its hash location to 1.
3. assign unique numbering to all 1's in the hash array
4. Run parallel on all nodes
   (a) running on node v
   (b) iterate over all neighbors of vertex v
   (c) if not all neighbors have NEC return

(d) find the hash of neighborhood.Find the unique NEC in hash location
(e) assign it to the vertex

Procedure: CVSGen()

Parallel CVS generation for each NEC.

Input: Data Graph Q,NEC.

Output: CVS.

1. all nodes in data graph is in NEC 1
2. for each NEC from 2 to last
3. Run parallel on all nodes
   (a) running on node v
   (b) iterate over all neighbors of vertex v.
   (c) check the existence of the neighborhood of NEC on the node v.
   (d) if found set the flag 1.

Procedure: PermandComb()

Parallelly check all possible permutations and combinations.

Input: Data Graph D, Query Graph Q, Current vertex index(i),Possibilities(P),Maximum

Possibilities(MP)

Output: Mapping of nodes from query graph to data graph.

1. if i= |V(q)|
2. Report all values in P and return
3. else
   (a) find u,the NEC of the vertex
   (b) Multiply P with CVSGen(u)
   (c) while P >= MP
      i. call CheckMap() on MP elements of P
      ii. call Exclusive-scan on Checkmap output
      iii. Save valid possibilities to newP
      iv. P-=MP (numbers)
   (d) move newP to P.

Procedure: CheckMap()

Parallel check of existence of query graph.

Input: Data Graph D,Map m,Query Graph Q,till vertex v in query graph.

Output: true/false.

1. running on all nodes u if $u <= v$.
2. iterate over all neighbors of vertex u.
3. check the existence of all edges in data graph corresponding to one in query graph.
4. if not all edges present set false.

The procedure NECGen gives unique ids to one tree in the query graph similar to Figure 4. We do a level order traversal on the tree. So at some nodes all its children may not have NEC given we process those nodes in the next iterations. The step 2 finds the neighborhoods that can be processed at the current iteration and finds a hash of the neighborhood. To these unique hashes we assign numbering by SCAN algorithm. Then these numbering are assigned back to each nodes.

The procedure CVSGen finds the candidates for a particular NEC. They check on each node on data graph and checks the neighboring nodes for matching neighbors of the particular NEC in query graph.

The procedure PermandComb finds all possibilities of the query graph. For each vertex of query graph it finds all possibilities(current possibilities x Cvs-Gen(u)). If the possibilities is more than we can store(MP), CheckMap is called on all current possibilities and the wrong ones are removed. This will make the P <MP. At last when i= $|V(q)|$, P has all valid possibilities. The procedure CheckMap checks the existence of non tree edges in the current mapping. If CheckMap returns true it is reported as a correct mapping.

In the figure 3.1 the NEC's in query graph are detected. The first step is the finding of tree in the query graph. The bold lines in the image shows the selected tree in the query graph. In the next step the leaf nodes are given unique NEC's

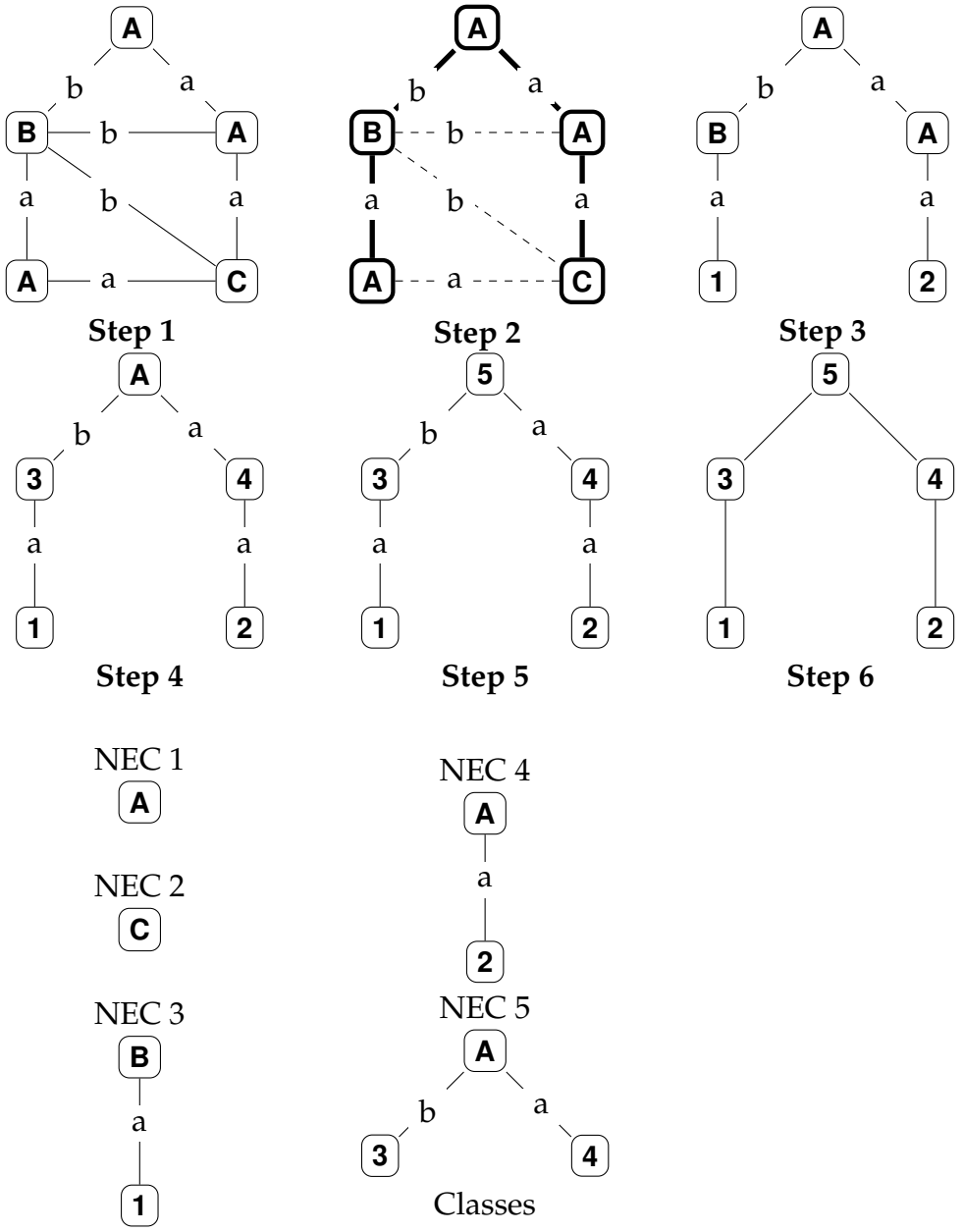

Figure 3.1 Algorithm Query Graph Processing

according to the node label. Then at each step the parent nodes whose all children has NEC defined, is given new NEC's. See the figure 3.1 and the NEC classes are given in the end. The NEC's are taking care of the edge labels so they are no longer required in the graph.

In the data graph, the singleton NEC's are given according to the label of the node. In step 2 and Step 3 the nodes are getting NEC 1 and NEC 2 because the

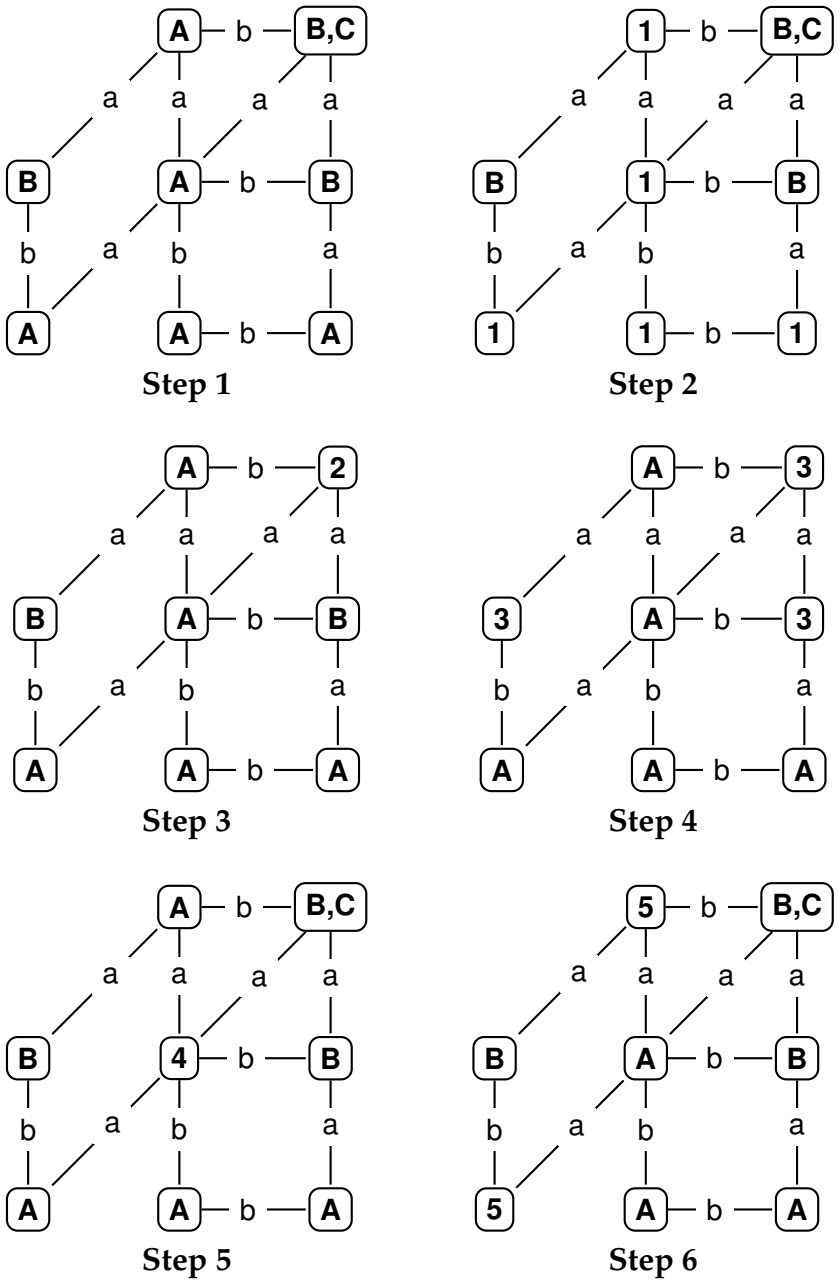

Figure 3.2 Algorithm Data Graph Processing

nodes matches the node label. NEC 1 and NEC 2 has no children to check. In step 4 the nodes are getting NEC 3 if it has a child with NEC 1 by an edge label 'a'. Then in step 5 the nodes are getting NEC 4 if it has a child of NEC 2 by edge label 'a'. In the final step the NEC 5 is given for nodes with two children. Each step describes the candidate set for each of the NEC's. $CVS(NEC1)$ has 5 nodes. $CVS(NEC2)$ has 1 node. $CVS(NEC3)$ has 3 nodes. Then a actual tree matching is done using these

CVS. All the combination of CVS are tested to find all possibilities of matching the tree in data graph. After that the non tree edges are checked.

3.3 Inferences

The parallel version needs to store the mapped NEC for each node in the data graph. This is asking for a space of $O(n * N(q))$ where n is number of nodes in data graph and $N(q)$ is number of NEC in query graph. The time taken for executing complete graphs on various test scenarios are given below. These results are obtained on a NVIDIA CUDA supported GPU with 580 MHz speed. Stage 1 is the NEC finding on Query Graph. Stage 2 is the CVS finding. Stage 3 is the matching.100n1000e means 100 nodes and 1000 edges.

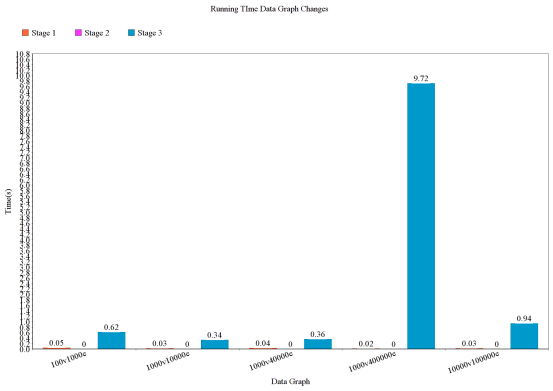

Figure 3.3 Data Graph Size Changes

When the data graph size changes the running times increases rapidly. The running time is depending on the density of the graph. If the size of the data graph increases and the density remains same, the running time doesn't show a spike. Fourth group shows a spike because of the huge density change from the other graphs. When the density changes the number of successful matching also increases causing the Stage 3(matching) to run more.

When the query graph size changes the overall time increases. But the

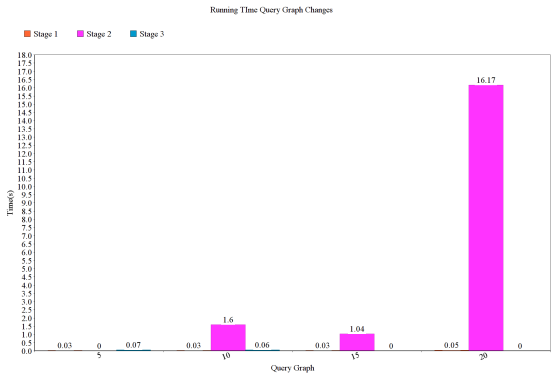

Figure 3.4 Query Graph Size Changes

processing in Stage 2 is increased. The Stage 2 depends on the number of NEC. The NEC count will increase if the number of nodes in query graph increases. The Stage 3 is getting zero because the CVS becomes null set. No candidate would have been found for higher level nodes. This will help to not run Stage 3 making the time zero.

Facebook Data is having 4039 nodes and 88234 edges making it a slightly

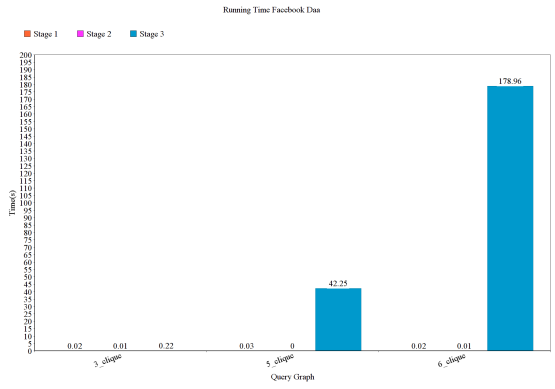

Figure 3.5 Facebook network

dense network. Even if the final number of cliques of size 5 and 6 are in the range of number of triangle. Number of possible checks requires increases. Each node addition will cause to calculate almost 100(from CVS size) more possibilities. The

false candidates are not easy to remove since the graph is fairly dense.

Condense Matter collaboration network Dataset is having 23133 nodes and

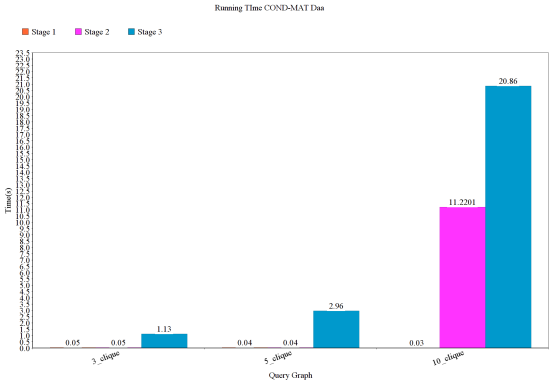

Figure 3.6 Condense Matter collaboration network

93497 edges. The graph is slightly sparse. The number of higher node cliques are less and the possibilities of finding them is also getting reduced due to the sparsity of the graph. The increase in time is not that steep because of the low density.

3.4 Failed Approach

We tried to make the the NEC numbering more informative by using primes and composites. A prime will be assigned to a class if that graph has no other embedding of any previous graphs we came across. If it has the embedding we give product of the prime numbers of the embedding. This method helps to know that if the NEC has a composite number it has some smaller graphs embedded in it. So we won't be needed to search the sub-graphs in this node.
It actually captures all sub-graphs at the root. See the figure 3.7.

$v4$ is getting 2 since it has only one child 1. $v7$ and $v3$ gets 4 since it has two same sub-graphs(sub-graph 2) inside them. $v2$ is getting 10 since it has sub-graph 2 and 5 (see the numbering shown on right). Similarly $v1$ has two 3s($v2$ contributes

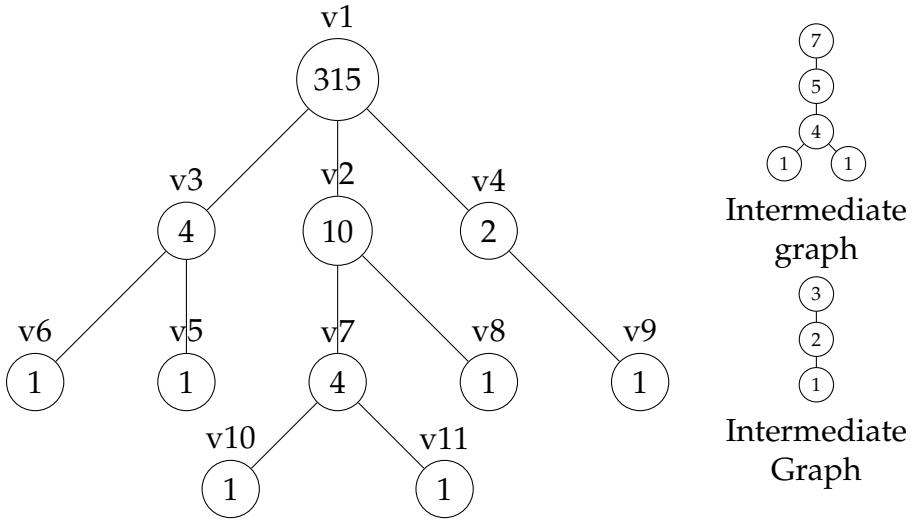

Figure 3.7 NEC based on primes

one 3) ,one 7 and one 5.

But this was not effective.When we consider the data graph we will need to store only one integer the product of all the graphs inside that node. The first problem we faced was the value of composite number can go beyond the long integer limit. So we tried only storing primes. But that also didn't make much difference. By our propagation algorithm in Data Graph, we start by giving id 1 to all nodes in the data graph in the first iteration. In the second iteration every node will get id 2 since every node will have a child of id 1. Then in third every node will get 3 and so on. So every node gets every id present in query graph.

It only helped as in knowing whether there exist a path of length matching the largest length path in query graph. This will lead to all nodes becoming a candidate for the final search we atleast one graph existed in the connected component. So it is not making the CVS tight.

CHAPTER 4

Dynamic Operations

4.1 Introduction

The dynamic operations are add/delete operation on either query or data graph. The dynamic version is also having large applications. The dynamic processing will help to generate the isomorphic mappings without computing the whole answer again. This will give a great improvement in time. The dynamic changes are allowed with one condition that the query or graph will remain connected at any point of time. The dynamic queries are processed non-deterministically meaning the order of execution of the dynamic queries is undefined. The dynamic operations and its difficulties are shown below.

	Query Add Edge	**Query Remove edge**	**Query Unchanged**
Data Add Edge	**Easy**	**Difficult**	**Easy**
Data Remove Edge	**Easy**	**Difficult**	**Easy**
Data Unchanged	**Easy**	**Difficult**	**Static**

Table 4.1 Dynamic Changes Difficulty Level

4.2 Intermediate Answers

The intermediate answers will be saved so that dynamic answers can be processed faster. A, B, C, D are the intermediate answer saved variables. A store

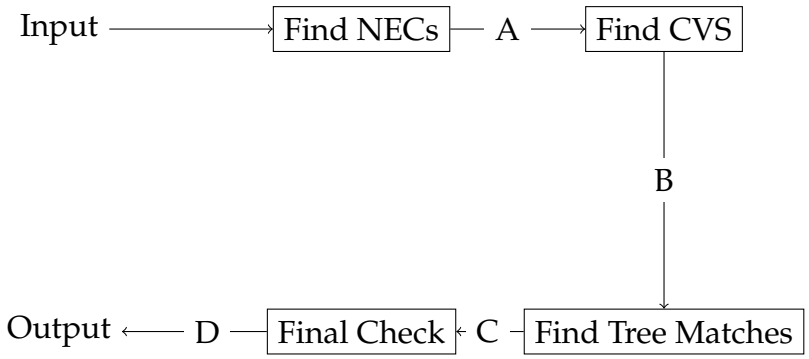

Figure 4.1 Intermediate Answers

the NECs of each vertex in query graph. B store CVS of each query vertex in data graph. C store the possible tree matches. D store the final exact graph maps.

4.3 Adding Edge to Query Graph

Its one of the easiest case because we need to check only the previous cases. The final answer will be a subset of previous answer,ie.,matching will become invalid when the added edge is not present. Only D changes. Multiple queries can be done in parallel by checking the existence of the added edges in all the previous answers. In parallel, on all previous answers check the presence of added edges.

4.4 Deleting Edge from Data Graph

It is also easy because the final answer is subset of previous answer.Multiple Queries can be processed similiar to the previous case.Only D changes.

4.5 Deleting Edge from Query Graph

This is a difficult case since we need to find the mappings that are going to be added. The deleting an edge in query graph can be divided as deleting any of

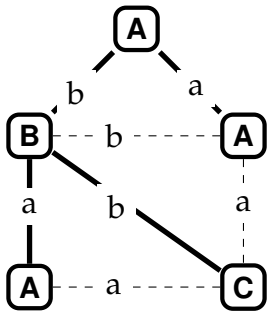

Figure 4.2 Tree in Query Grpah

the dashed edges(non-tree edges) and deleting one of the bold edges(tree edges) in figure 4.2.

4.5.1 Deleting a non-tree edge

The tree inside the query graph remains unchanged. So the tree matches are correct. We need to go through all the tree matching(C) and check for possible additions of maps to final answer. So saving the intermediate answer C helps in finding the solution faster.

4.5.2 Deleting a tree edge

Since the tree is changed here the CVS of vertices may change. So rather than calculating all the CVS there is a more efficient way. If a edge u-v in the tree is deleted and u is parent of v in tree. All the nodes in the path from u to root(parent,grand-parent,.. of u) should recalculate the CVS.

When multiple queries are given the deletion may be from different parts of the tree. But each node should be processed once.

Algorithm 4 Dynamic tree edge deletion of thread t

Input: Data Graph D,Query Graph Q,Delete u-v(u is parent of v).

Output: CVS updates.

1. w=v
2. for each parent of w(till root)
   (a) mark w for t
3. for each parent of w(till root)
   (a) if mark at w is t, acquire lock for w
4. for each parent of w(till root)
   (a) if mark at w is t and able to acquire locks for all childs of w
   (b) Recompute NEC of w
   (c) if not a previously computed NEC then
   (d) C(w)=FindCandidates(w,D) update
   (e) Release all locks

The above algorithm marks all the parent nodes from a particular deleted edge. This marking helps to make sure only one thread process one node. The processing will be done such a way that no parent nodes are processed if any child of the parent is unprocessed. So the processing order will be leaf to root. This will change the values inside A and so as B.

In Figure 4.3 the tree edge deletion process is shown. The step 1 shows the tree in query graph. Thread 1 deletes u2-u4 edge and thread 2 deletes u2-u5 and thread 3 deletes u1-u3(Step 2 in figure4.3). The threads are trying to mark the parents (Step 2 in Algorithm 4). The marking is not atomic so the thread ids got written in the parents are absolutely random. Here Thread 1 and thread 2 got the parents marked. Since thread 3 doesn't have any parent marked (Step 4(a) in Algorithm4),

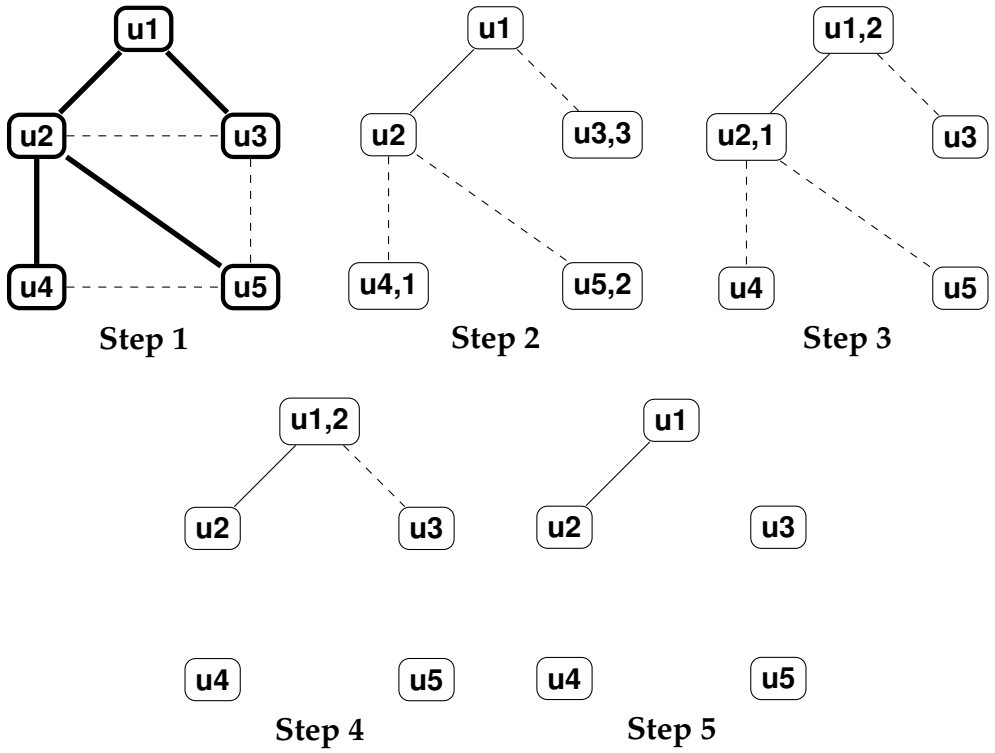

Figure 4.3 Deleting Edge in Query Graph

it will return. Now thread 1 will acquire lock on u2 and thread 2 will acquire lock on u1(Step 3 in figure4.3). Now thread 2 will go to waiting since it can't attain the lock on u1 's children. So only thread 1 is now processing. It will acquire locks on its children(step 4 in figure4.3). Since all of its children are deleted, no locks are required. So thread 1 will recalculate the NEC of u2 and find the candidates in Data graph. After that thread 1 will release the lock and return. Now thread 2 will acquire the locks of all children of u1(step 5 in figure4.3). Then it will recalculate the NEC , CVS and finishes.

4.6 Adding Edge in Data Graph

This will also add entries into B. But A will not be changed since no edges in query graph is changed.

Algorithm 5 Dynamic data edge addition of thread t

Input: Data Graph D,Query Graph Q,Delete u-v(u is parent of v).

Output: CVS updates.

1. w=v (for u also)
2. for each child of w(till |Q| length)
   (a) if acquire locks for all children of w and w
   (b) foreach NEC's as x and w is not in CVS(x)
   (c) CVS(x)=Is_w_candidate(w, x, D) update
   (d) Release all locks

This algorithm moves to |Q| length from both u and v in a BFS fashion. At each Data node it is checked whether it can be added to CVS of any of the NECs. When there is an overlap of regions of different thread locks are used to synchronize them. Since the Data graph is huge and query graph is small and so the possibility of two edge addition happening near(edge length $<$ |Q|) is small, the number of locks waits will be minimal.

Step 1 in figure 4.4 is the data graph. Then 2 edges are added (v6-v7 and v3-v4). Thread 1 starts processing the v3-v4 edge and Thread 2 starts processing v6-v7 edge(Step 3). Since these edges are so close to each other. There will be lots of waits on locks. The acquire lock on w and children of w in step 2(a) of Algorithm 5 becomes very complex. The lower id threads are given priority when trying to acquire locks. So the thread 1 acquires lock of v4 and its neighbors(Step 4). Thread 2 goes to waiting. The locking system can be implemented using an array with atomic operations. Minimum value write operation will help to give lower threads more priority than higher ones. Now thread 1 will recalculate the CVS changes for all NEC for which v4 is not a part of. Now thread 1 will move to other end of the edge (v3). Now thread 2 can acquire locks on v7 and its neighbors(Step 5). Thread 2 will now update the CVS. There is another possibility that thread 1 will ask for

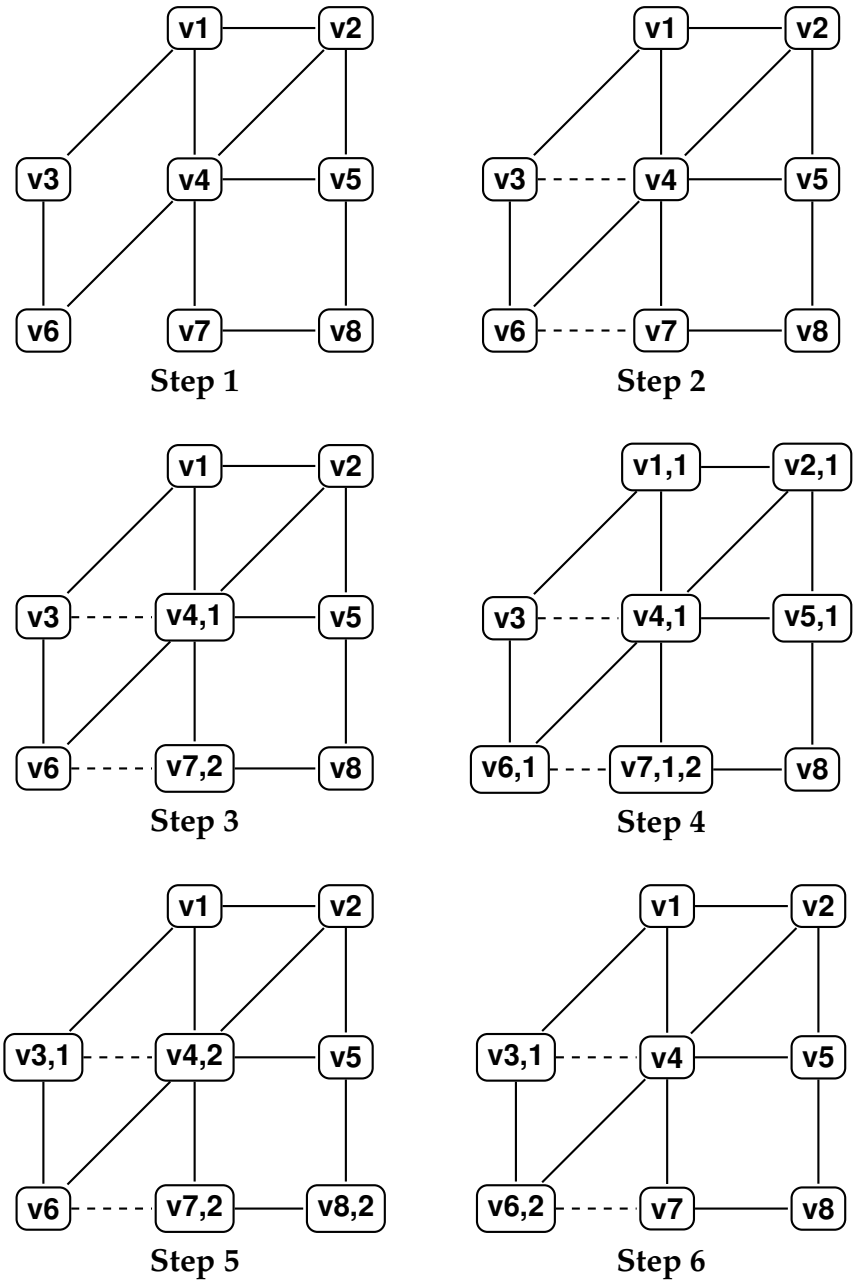

Figure 4.4 Edge Addition Data Graph Processing

lock on v4 because it is a neighbor of v3. Then thread 2 can again go to waiting. These lock waitings are happening because of closeness of both added edges. So now thread 2 will move to v6(Step 6). Then the process continues. Thread 1 will go through v4, v3, v1, v2, v5, v7 and v6. Since it should go through all the nodes at a 2 hopes(length of query tree). Similiarly thread 2 will go through v7, v6, v4,v8 and v3.

4.6.1 Parallel Execution

All the four quires can be processed in parallel since they are operating on different data.Section 4.3 and 4.4 are processing on D while Section 4.5 and 4.6 are processing on A and B. So it is safe to run in parallel. Parallel adding to A and B doesn't make the answer inconsistent. It will add a vertex into the CVS which will be removed when exact matching is performed in the end.

4.6.2 Inferences

So a FindTree match algorithm should be done at each stage of the output to get the new results . But this stage is the costliest of all the four steps. Even if one vertex is added to any two CVS, we need to recompute most of the permutations again. So there is no computational advantage by dynamic processing. Running a sub-graph isomorphism solution after applying all edge updates becomes equally fast as the dynamic version.

CHAPTER 5

CONCLUSION

The state-of-art algorithms for Sub-graph Isomorphism use different pruning techniques to avoid the false candidates as early as possible. The part of the algorithm which checks all possibility is the most time consuming part. Since we can't avoid checking a possibility, the one efficient way of making this part faster is decreasing the number of candidates. The complexity of the pruning technique helps to remove more false candidates thus making the algorithm faster. The possibility checking part is made faster by using GPU by checking different possibilities in different threads. Thus a million possibilities are checked in parallel.

The dynamic version of the problem is trying to answer the problem after many edge deletions and additions. The trivial cases are the adding in query and deletion in data. The other two cases makes the problem hard. Whether there exists a faster method(poly-time) for processing them in parallel still remains as an open problem.

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