Fundamentals of Algebra and Calculus (2019-2020)[SEM1]

Question **1**

Not answered

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The sequence defined by $u_n = \dfrac{1}{2\,n+2}$ is:

- Increasing
- Decreasing
- Neither increasing nor decreasing

and is:

- Bounded
- Unbounded

The first few terms are $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$.

We can see that the terms are decreasing, and this is true for all terms in the sequence because the denominators are increasing from one term to the next.

We can see that all of the terms are positive, so the sequence is definitely bounded below. Also, because the sequence is decreasing, all of the terms are definitely smaller than the first one, which is smaller than 1. So the sequence is bounded.

A correct answer is:

- Decreasing

A correct answer is:

- Bounded

Question **2**

Not answered

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The sequence defined by $u_n = n + \dfrac{2}{n}$ is:

- Increasing
- Decreasing
- Neither increasing nor decreasing

and is:

- Bounded
- Unbounded

The sequence $u_n = n + \dfrac{2}{n}$ is increasing because for each n we have $u_n < u_{n+1}$ since $u_n < n+1 < u_{n+1}$.

The sequence is unbounded since there is no upper bound on the size of terms - for instance, if we want a term larger than M then u_M would work.

A correct answer is:

- Increasing

A correct answer is:

- Unbounded

Question **3**

Not answered

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Which of the following sequences are increasing?

- **(a)** $u_n = n!$
- **(b)** $u_n = n^2 + n$
- **(c)** $u_n = e^{-n}$
- **(d)** $u_n = (-1)^n$

To see if the sequence is increasing, check whether $u_{n+1} > u_n$ for every n.

A correct answer is:

- **(a)** $u_n = n!$
- **(b)** $u_n = n^2 + n$

Question 4

Not answered

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Give an example of a sequence which is increasing and bounded above.

(Give your answer as a formula for the nth term.)

$u_n =$

We want the sequence to be increasing, so we could try something like $u_n = n$, where each term is 1 larger than the previous term. However that will not be bounded above.

To get a sequence which is bounded above, the amount we add on each time needs to go down as n increases.

For example, $0, \frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \ldots$ would work since the terms will never go above 1. This is written $u_n = 1 - \frac{1}{2^n}$.

There are many other possible examples.

A correct answer is $1 - \frac{1}{2^n}$, which can be typed in as follows: `1-1/2^n`

G1

Question 5

Not answered

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Give the formula for the nth term of a sequence $u_1, u_2, u_3, \ldots$ which is:

- increasing

  $u_n =$
- increasing and bounded above

  $u_n =$
- increasing, bounded above, and has $u_4 = 16$

  $u_n =$

There are many possible examples in each case.

- One example of an increasing sequence is $10, 11, 12, 13, 14, \ldots$, which can be written $u_n = 9 + n$.
- To get a sequence which is bounded above, the amount we add on each time needs to go down as n increases.

  For example, $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$ would work since the terms will never go above 1. This is written $u_n = 1 - \frac{1}{2^n}$.
- To make sure $u_4 = 16$ we could just modify our previous example.

  That has $u_4 = \frac{15}{16}$ so we could subtract this, then add 16. So $u_n = \frac{257}{16} - \frac{1}{2^n}$.

There are many other possible examples.

A correct answer is $n + 9$, which can be typed in as follows: `n+9`

A correct answer is $1 - \frac{1}{2^n}$, which can be typed in as follows: `1-1/2^n`

A correct answer is $\frac{257}{16} - \frac{1}{2^n}$, which can be typed in as follows: `257/16-1/2^n`

G2

Question 6

Not answered

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Give the formula for the nth term of a sequence $u_1, u_2, u_3, \ldots$ which is:

- decreasing

  $u_n =$
- decreasing and bounded below

  $u_n =$
- decreasing, bounded below, and has $u_4 = 1$

  $u_n =$

There are many possible examples in each case.

- One example of a decreasing sequence is $10, 9, 8, 7, 6, \ldots$, which can be written $u_n = 11 - n$.
- To get a sequence which is bounded below, we could start with a positive number and make each term half as big as the one before.

  For example, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ is decreasing and will never go below 0. This is written $u_n = \frac{1}{2^n}$.
- To make sure $u_4 = 1$ we could just modify our previous example.

  That has $u_4 = \frac{1}{16}$ so we could subtract this, then add 1. So $u_n = \frac{1}{2^n} + \frac{15}{16}$.

There are many other possible examples.

A correct answer is $11 - n$, which can be typed in as follows: `11-n`

A correct answer is $\frac{1}{2^n}$, which can be typed in as follows: `1/2^n`

A correct answer is $\frac{1}{2^n} + \frac{15}{16}$, which can be typed in as follows: `1/2^n+15/16`

G3

Question **7**

Not answered

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For each of the following, find an example of a sequence with the given property.

Enter the **first 8 terms** of your sequence as a list, e.g. `[a,b,c,d,e,f,g,h]`.

- increasing, and different from the examples above:

  What is different about your example?

- not increasing:
- not increasing, and not decreasing:

There are many possible examples in each case.

- The sequence $0.1, 0.2, 0.3, \ldots$ is increasing. It is different from previous examples since we have used decimals for the terms.
- The sequence $-1, -2, -3, -4, \ldots$ ($u_n = -n$) is not increasing -- in fact, it is decreasing.
- The sequence $2, 4, 2, 4, \ldots$ which alternates between 2 and 4 is neither increasing nor decreasing.

There are many other possible examples.

A correct answer is $[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8]$, which can be typed in as follows: `[0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8]`

A correct answer is $\text{The terms are decimals}$, which can be typed in as follows: `The terms are decimals`

A correct answer is $[-1, -2, -3, -4, -5, -6, -7, -8]$, which can be typed in as follows: `[-1,-2,-3,-4,-5,-6,-7,-8]`

A correct answer is $[2, 4, 2, 4, 2, 4, 2, 4]$, which can be typed in as follows: `[2,4,2,4,2,4,2,4]`

G4a

Question **8**

Not answered

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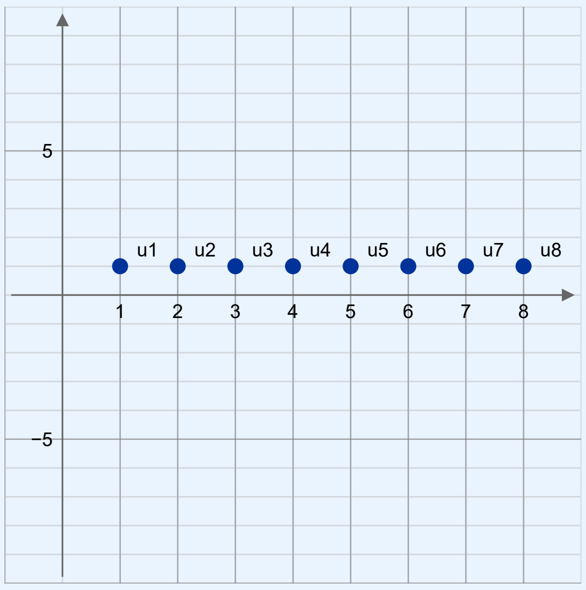
Drag the points $u_1, \ldots, u_8$ so that they show the first 8 terms of an increasing sequence.



A correct answer is $[1, 2, 3, 4, 5, 6, 7, 8]$, which can be typed in as follows: `[1,2,3,4,5,6,7,8]`

G4b

Question **9**

Not answered

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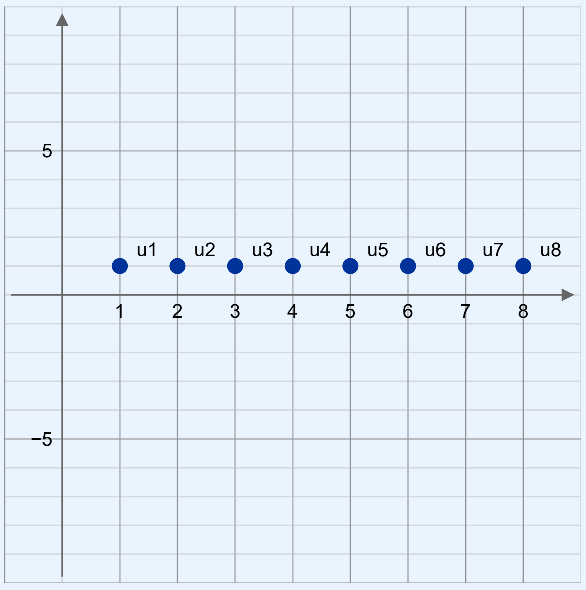
Drag the points $u_1, \ldots, u_8$ so that they show the first 8 terms of a decreasing sequence.



A correct answer is $[5, 4, 3, 2, 1, 0, -1, -2]$, which can be typed in as follows: `[5,4,3,2,1,0,-1,-2]`

G4c

Question **10**

Not answered

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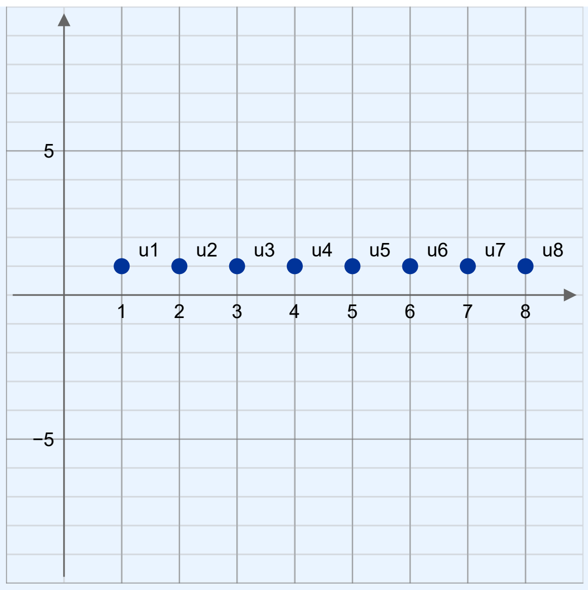
Drag the points $u_1, \ldots, u_8$ so that they show the first 8 terms of a sequence which is neither increasing nor decreasing.



A correct answer is $[2, 4, 2, 4, 2, 4, 2, 4]$, which can be typed in as follows: `[2,4,2,4,2,4,2,4]`

C1-10

Question 11

Not answered

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For each of the following sequences, decide if they are increasing, decreasing, both or neither.

Sequence	Classification
$1, -1, 2, -2, 3, -3, 4, -4, \ldots$	(No answer given)
$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}, \frac{63}{64}, \frac{127}{128}, \frac{255}{256}, \ldots$	(No answer given)
$1, 3, 2, 4, 3, 5, 4, 6, \ldots$	(No answer given)
$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \ldots$	(No answer given)
$-2, -4, -6, -8, -10, -12, -14, -16, \ldots$	(No answer given)
$0, 1, 0, 1, 0, 1, 0, 1, \ldots$	(No answer given)
$6, 6, 7, 7, 8, 8, 9, 9, \ldots$	(No answer given)
$3, 3, 3, 3, 3, 3, 3, 3, \ldots$	(No answer given)
$0, 1, 0, 2, 0, 3, 0, 4, \ldots$	(No answer given)
$1, 4, 9, 16, 25, 36, 49, 64, \ldots$	(No answer given)

The following sequences are increasing, because each term is larger than the previous one:

- $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}, \frac{63}{64}, \frac{127}{128}, \frac{255}{256}, \ldots$
- $1, 4, 9, 16, 25, 36, 49, 64, \ldots$

Similarly, these sequences are decreasing because each term is smaller than the previous one:

- $-2, -4, -6, -8, -10, -12, -14, -16, \ldots$
- $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \ldots$

The following sequences are neither increasing nor decreasing:

- $3, 3, 3, 3, 3, 3, 3, 3, \ldots$
- $6, 6, 7, 7, 8, 8, 9, 9, \ldots$
- $0, 1, 0, 1, 0, 1, 0, 1, \ldots$
- $0, 1, 0, 2, 0, 3, 0, 4, \ldots$
- $1, -1, 2, -2, 3, -3, 4, -4, \ldots$
- $1, 3, 2, 4, 3, 5, 4, 6, \ldots$

The first two have consecutive terms which are equal, so there is neither an increase nor a decrease from one term to the next. The next three sequences all have an alternating behaviour between "high" and "low" values. The final sequence doesn't appear to follow any particular pattern, but we can see it cannot be decreasing (e.g. "1, 3") and also cannot be increasing ("3, 2").

A correct answer is: `"Neither"`

A correct answer is: `"Increasing"`

A correct answer is: `"Neither"`

A correct answer is: `"Decreasing"`

A correct answer is: `"Decreasing"`

A correct answer is: `"Neither"`

A correct answer is: `"Neither"`

A correct answer is: `"Neither"`

A correct answer is: `"Neither"`

A correct answer is: `"Increasing"`

C11-13

Question 12

Not answered

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What are the properties of each of the following sequences?

Sequence	Increasing/decreasing?	Bounded?
$u_n = 5 - \dfrac{1}{2^n}$	(No answer given)	(No answer given)
$u_n = n^2 + 1$	(No answer given)	(No answer given)
$u_n = (-1)^n + 3$	(No answer given)	(No answer given)

- $u_n = (-1)^n + 3$ has terms $2, 4, 2, 4, 2, 4, \ldots$, i.e. it is alternating between 2 and 4.
  Thus it is neither increasing nor decreasing. Also, it is bounded above by 4 and below by 2, hence it is bounded.
- $u_n = n^2 + 1$ has terms $2, 5, 10, 17, 26, 50, \ldots$.
  The terms are increasing, and they are not bounded above (e.g. if you think a natural number M might be an upper bound, note that the term $u_M = M^2 + 1$ is bigger than it - so there can be no upper bound).
- $u_n = 5 - \frac{1}{2^n}$ has terms $4\frac{1}{2}, 4\frac{3}{4}, 4\frac{7}{8}, 4\frac{15}{16}, \ldots$.
  The sequence is increasing, and it is bounded (an upper bound is 5 and a lower bound is 4).

A correct answer is: `"Increasing"`

A correct answer is: `"Bounded"`

A correct answer is: `"Increasing"`

A correct answer is: `"Unbounded"`

A correct answer is: `"Neither"`

A correct answer is: `"Bounded"`

C14-15

Question 13

Not answered

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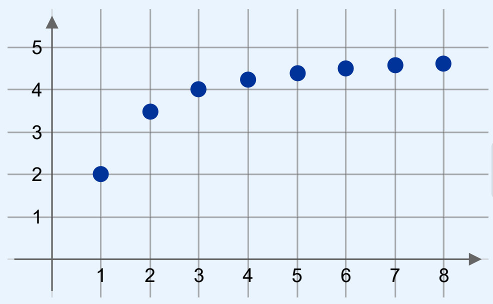
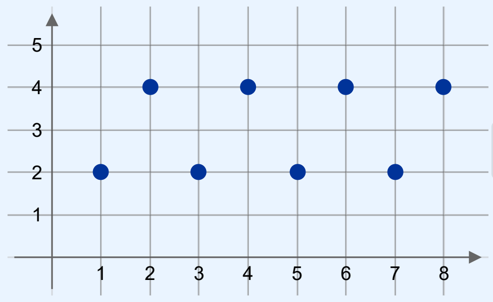
For the sequences shown in the graphs below, decide if they are increasing, decreasing, neither or both.



(No answer given)



(No answer given)

A correct answer is: `"Increasing"`

A correct answer is: `"Neither"`